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Investigation of the effect of grain size on liquefaction potential of sands

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Abstract. Due to the permanent damage to structures during earthquakes, soil liquefaction is an important issue in geotechnical earthquake engineering that needs to be investigated. Typical examples of soil liquefaction have been observed in many earthquakes, particularly in Alaska, Niigata (1964), San Fernando (1971), Loma Prieta (1989), Kobe (1995) and Izmit (1999) earthquakes. In this study, liquefaction behavior of uniform sands of different grain sizes was investigated by using the energy-based method. For this purpose, a total of 36 deformation-controlled tests were conducted on water-saturated samples in undrained conditions by using the cyclic simple shear test method and considering the relative density, effective stress and mean grain size parameters that affect the cumulative liquefaction energy. The results showed that as the mean grain size decreases, the liquefaction potential of the sand increases. In addition, with increasing effective stress and relative density, the resistance of sand against liquefaction decreases. Multiple regression analysis was performed on the test results and separate correlations were proposed for the samples with mean grain size of 0.11-0.26 mm and for the ones with 0.45-0.85 mm. The recommended relationships were compared to the ones existing in the literature and compatible results were obtained.

Keywords: liquefaction; liquefaction energy; cyclic simple shear test; sand; mean grain size; effective stress; relative density

1. Introduction

Liquefaction occurs in loose and water-saturated sandy and silty soils exposed to strong ground motions caused by events, including earthquakes, ocean waves, explosions and so on. Liquefaction can cause serious damage in various structures, such as the buildings, bridges and retaining walls. In recent years, extensive research has been carried out and is still being conducted to understand the mechanisms of this phenomenon and to determine liquefaction-sensitive soil conditions.

In water-saturated soils, the water in the pores between soil particles exerts a pressure to grains that directly affects the effective stress between the grains. Although this water pressure is relatively low in the absence of repeated loads such as earthquakes, it increases in the presence of seismic vibrations from earthquakes, forcing the grains to relocate and approach. Due to the increase in water pressure between the grains, the effective stress decreases and soil behaves as a liquid by losing its strength. This phenomenon, which is called liquefaction, causes the load capacity of the soil to decrease, and hence, toppling of the superstructure, bending and slippage of retaining structures and collapse dams. The first known earthquakes causing liquefaction are the Alaska (1964), Niigata (1964), San Fernando (1971), Loma Prieta (1989), Kobe (1995) and Kocaeli (1999) earthquakes. The effects of surrounding pressure, relative density, saturation degree of the sampling method and sample size on soil liquefaction are well known from previous studies.

However, the effects of other variables, including soil structure, grading properties, size, shape, distribution and packaging of particles have not been studied adequately in the literature and definitely need further investigation. Although there are studies on mean grain size in the literature, there are no studies based on the energy approach on this topic. Therefore, this study focused on the effect of mean grain size (D_{50}) of sand grains on the liquefaction energy by using the energy-based approach. Some studies in the literature are summarized below.

Chang *et al.* (1982) reported that the dynamic liquefaction resistance of clean sand is strongly influenced by the mean grain size (D_{50}) and the uniformity coefficient (C_u) provided that the average grain size is less than 0.23 mm. In addition Vaid *et al.* (1991) investigated the effect of the coefficient of uniformity (C_u) on the undrained dynamic shear strength of soil samples formed with three different sands of the same average grain size (D_{50}). They reported that the dynamic liquefaction resistance increases with increasing coefficient of uniformity (C_u) at low relative density and they observed a reversed trend at high relative density.

Monkul and Yamamuro (2011) investigated the effect of fines content on the liquefaction potential of the sand mixed with three different non-plastic silts by deformation controlled monotonic undrained triaxial compression tests. They reported that if the ratio of the mean grain size of sand to the mean grain size of silt (D_{50}-sand / d_{50}-silt) is

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sufficiently small, the liquefaction potential of the sand increases steadily with increasing fines content for the studied range (0-20%). In addition, as the D_{50} sand / d_{50}-silt ratio increases, the liquefaction potential of silty sand may be lower than the liquefaction potential of clean sand.

Choobbasti *et al.* (2013) determined that the cyclic shear resistance of the soil can be expressed in terms of the grain size, i.e., D_{10}, D_{30} or D_{60}, instead of the uniformity coefficient (C_u) or the curvature coefficient (C_c) of the soil. In addition, Belkhatir *et al.* (2011) showed that the peak undrained shear resistance and residual shear resistance can be correlated to C_u and D_{50}. In other words, undrained peak shear resistance and undrained residual shear resistance decreased linearly as the uniformity coefficient increased and the mean grain size decreased. They concluded that liquefaction resistance can be expressed in terms of grading properties (D_{10}, D_{50} and C_u) instead of curvature coefficient (C_c).

Belkhatir *et al.* (2014) showed that the granulometric properties (D_{10}, D_{50}, C_u) of sand-silt mixtures have a significant effect on the excess pore pressure development. Moreover, they found that the excess pore pressure may be associated with grading properties as well as the effective size ratio (ESR), mean grain size ratio (MGSR), and the uniformity ratio coefficient (CUR)].

Taiba *et al.* (2015) conducted a series of undrained monotonic triaxial tests on silty sand mixtures to investigate the effect of soil gradation on peak shear strength. From the results obtained, they showed that the grading and particle shape have a significant effect on the undrained shear resistance (liquefaction resistance) of different silty sand mixtures. In addition, they showed that there are direct correlations between liquefaction resistance and different classification properties (D_{10}, D_{50} D_{60}, C_u) of the soils.

Monkul *et al.* (2016) reported that the grading of the base sand used in their study had a significant effect on the static liquefaction potential of clean and silty sands. They also observed that clean sand is more liable to liquefaction as the mean sand grain size becomes smaller and/or the sample becomes more uniform. However, they reported that the liquefaction behavior is reversed when the same base sand is mixed with silt. Accordingly, silty sands become more liable to liquefaction as the mean grain size of the base sand increases and/or the base sand becomes relatively well graded.

Krima *et al.* (2019) performed a series of undrained triaxial tests on the effect of clay content and grading properties of sand-clay mixtures on the liquefaction resistance. The results showed that the undrained shear strength (liquefaction resistance) of sandy clay mixtures was controlled by the percentage of clay content and grading properties of the mixture. However, the results also depicted that the resistance of sand-clay mixtures to liquefaction decrease up to an increase of 15% in clay content. In addition, the undrained shear strength at the peak, the undrained residual strength and the maximum shear strength were established to decrease with increasing clay content. Finally, these parameters were stated to decrease with an increase in the uniformity coefficient and gradation coefficient and with a decrease in the effective diameter and mean grain size.

The aim of this study is to evaluate the liquefaction potentials of sands with different mean grain size (D_{50}), effective stress (50, 100,150 kPa) and relative density (30, 50, 70%), using energy based approach. Furthermore, the present study aims at developing mathematical models based on the effective stress, relative density, mean grain size and cumulative liquefaction energy, according to the test results. The mathematical models obtained were compared to the relations in the literature and the results were interpreted.

2. Material and method

Studies have shown that cumulative energy causing liquefaction is an excellent index for assessing liquefaction potential (Kokusho 2013) The energy-based approach for assessing the soil liquefaction potential was primarily proposed by Nemat-Nasser and Shokooh (1979) alternative to the stress-based approach and later improved continuously (Figueroa *et al.* 1994, Green 2001, Kokusho 2013). In this approach, the energy dissipated by unit volume during the loading process in the presence or absence of liquefaction is directly related to the development of excess pore pressure. The energy accumulated in unit volume (J /m^3) associated with the permanent rearrangement of soil particles is defined as the area within the hysteresis curve for a cycle.

The energy-based approach in liquefaction analysis has the following advantages over the stress- and strain-based approaches. First, energy is a scalar quantity taking the whole spectrum of the ground motion into account. Secondly, the use of energy includes and makes use of all of the strain, stress and material properties of the soil (Law *et al.* 1990, Liang 1995, Baziar and Jafarian 2007, Baziar and Jafarian 2011).

In the research conducted by Liang (1995) the area within the hysteresis cycle was shown to be equal to the energy required for the resettlement of sand grains, causing volume change in drained and increased pore pressure in undrained tests. In addition, Okada and Nemat-Nasser (1994) showed that the energy per unit volume applied to the soil strongly depends on the development of the pore pressure.

A typical cyclic load test provides data for stress, strain and pore water pressure. The strain energy in each loading cycle is equivalent to the area remaining inside the hysteresis curve in Fig. 1 (Ostadan *et al.* 1996, Green 2001). The energy in each cycle and the sum of these energy values up to the initiation of liquefaction is defined as the measure for the capacity of soil sample against liquefaction (Alavi and Gandomi 2012). The shear stress-shear strain hysteresis loop is obtained as a function of time as shown in Fig. 1. In the literature, Equation 1 is frequently used when calculating the area within the hysteresis loop to determine the liquefaction energy:

$$W = \frac{1}{2}\sum_{i=1}^{n}(\tau_n + \tau_{n+1})(\gamma_{n+1} - \gamma_n) \tag{1}$$

W=Cumulative liquefaction energy (J/m^3), τ= Shear stress, γ= Shear strain, n= Number of cycles recorded until liquefaction

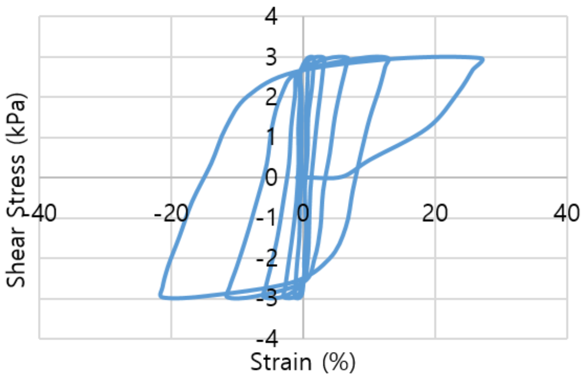


Fig. 1 Typical hysteresis loop

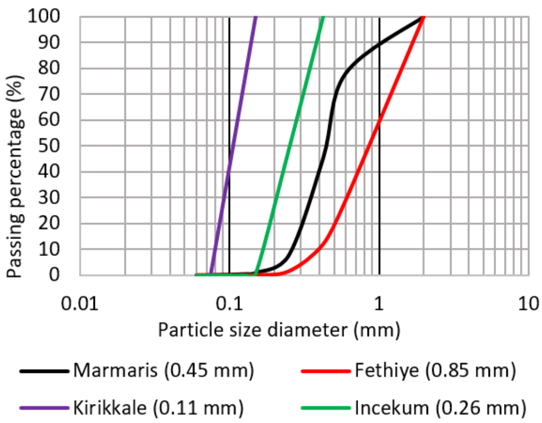


Fig. 2 Gradation curves of sand samples of the study

Table 1 Physical properties of the sands used in the study

Feature	Fethiye	Marmaris	Incekum	Kirikkale
D_{50} (mm)	0.85	0.45	0.26	0.11
G_s	2.65	2.67	2.65	2.68
e_{max}	0.67	0.79	0.78	1.03
e_{min}	0.43	0.54	0.52	0.76
C_u	2.58	1.89	1.64	1.48
C_c	1.57	2.61	4.17	1.27

D_{50}: mean grain size, G_s: spesific gravities, e_{max}: maximum void ratio, e_{min}: minimum void ratio, C_u: coefficient of uniformities, C_c: Coefficient of curvature

In the scope of the study, four clean sand samples with different mean grain size (D_{50}) were used. The specific gravity of each sand sample was determined according to ASTM D854, minimum void ratio according to ASTMD4253 and maximum void ratio according to ASTM D4254. Fig. 2 shows the gradation curves of the sands. Laboratory tests and field studies show that the mean grain size ranges between 0.01 and 2 mm in the liquefied soils (Ye 2017). Therefore, sand samples of 0.11, 0.26, 0.45, and 0.85 mm mean grain size were selected in this study and their physical properties are given in Table 1.

Experiments can be performed as stress controlled or deformation controlled in the evaluation of soil liquefaction under laboratory conditions. Many researchers performed stress-controlled (Wijewichreme *et al.* 2005) and deformation-controlled tests (Silver and Park 1976, Dobry *et al.* 1982) to examine the liquefaction potential of loose and moderately compact sands. In this study, the tests were carried out using cyclic simple shear test with deformation control under undrained conditions. In deformation-controlled tests, cyclic shear stress is applied to the soil sample at a selected deformation amplitude and excess pore water pressure is measured. The deformation-controlled cyclic simple loading test directly correlates liquefaction with pore pressure (Talaganov 1996). In addition, this test can better simulate earthquake loading in the field (Almani *et al.* 2013). The device used in the study is illustrated in Fig. 3. Movahed *et al.* (2011) showed that the frequency does not cause a significant change in the liquefaction energy in their deformation- and stress-controlled experiments with changing range of frequency. The tests of Kusky (1996) on Reid Bedford sand using the hollow cylinder torsional shear apparatus and for the frequency range of 0.2 to 1.0 Hz showed that the frequency has a negligible effect on the energy needed for the initiation of liquefaction. Therefore, in this study, a uniform sinusoidal horizontal shear stress is applied with a frequency of 0.1 Hz, which is recommended for such test instruments, although it is less than the typical earthquake frequency (GDS 2006).

In this study, the desired relative density values were achieved by using the air pluviation method in the preparation of sand samples. In this method, the amount of sample required for the desired relative density is dropped from a certain height (Walker and Whitaker 1967). The height of the sand sample, which is dropped into the test cell, causes different sample heights and consequently different relative density each time if the drop operation is not done carefully. In other words, if the sample is dropped from a higher height, it is dense; otherwise, it becomes loose if it is dropped from a lower height. This causes the increase or decrease in the height of the sample. In this study, a special attention to filling the entire cell and ensuring that the sample height is always the same height (46 mm) has been given since the test cell is 100 mm in diameter and 46 mm in height. Fig. 3 shows the test cell where the sand sample is placed.

Samples, prepared with the desired density, were then given CO_2 for 20 minutes from bottom to top to ensure that they were fully saturated to water and that no air bubbles remained in the sample. After flushing with CO_2, the water, deaerated under low pressure with the help of the deaeration device, was supplied to the sample from bottom to top to ensure that the sample was fully saturated to water. At least 5 times the sample volume of deaerated water was passed through the sample.

After the saturation process was completed, the vertical stress determined for each experiment was applied to the sample. Then, the sample was allowed to consolidate under this vertical stress. In the liquefaction tests performed in the literature, the onset of liquefaction was characterized as the instant, when the excess pore pressure is equal to the effective stress (Figueroa *et al.* 1994). In this study, the liquefaction energies of the tested samples were calculated by taking this criterion into consideration. The calculated values are given in Table 2. Experiments on four different

Fig. 3 The cyclic simple shear test apparatus and cell used in the experiments

Table 2 Test results of the present study

Test Number	D_{50} (mm)	Effective Stress (kPa)	Target Dr (%)	Actual Dr (%)	W (J/m^3)
1	0.85	50	30	31	695
2	0.85	100	30	29	2685
3	0.85	150	30	27	3381
4	0.85	50	50	50	3730
5	0.85	100	50	47	9034
6	0.85	150	50	55	15356
7	0.85	50	70	70	14481
8	0.85	100	70	67	21653
9	0.85	150	70	64	34769
10	0.45	50	30	36	406
11	0.45	100	30	28	1232
12	0.45	150	30	25	3092
13	0.45	50	50	55	1057
14	0.45	100	50	57	2024

Table 2 Continued

Test Number	D_{50} (mm)	Effective Stress (kPa)	Target Dr (%)	Actual Dr (%)	W (J/m^3)
15	0.45	150	50	49	3555
16	0.45	50	70	75	2565
17	0.45	100	70	73	2872
18	0.45	150	70	72	6503
19	0.26	50	30	30	409
20	0.26	100	30	31	890
21	0.26	150	30	28	1304
22	0.26	50	50	54	682
23	0.26	100	50	55	2058
24	0.26	150	50	62	3068
25	0.26	50	70	76	1437
26	0.26	100	70	71	3391
27	0.26	150	70	71	5220
28	0.11	50	30	32	364
29	0.11	100	30	34	686
30	0.11	150	30	35	978
31	0.11	50	50	56	577
32	0.11	100	50	47	1265
33	0.11	150	50	47	1311
34	0.11	50	70	71	710
35	0.11	100	70	67	2107
36	0.11	150	70	67	2655

sand samples (0.11, 0.26, 0.46 and 0.85 mm) were conducted under three different effective stress values (50, 100, 150 kPa) and for three different relative densities (30, 50, and 70%).

3. Research findings

Studies have shown that the amplitude of shear strain does not affect the cumulative liquefaction energy (Figueora 1994, Liang 1995). Furthermore, the sample is assumed to have liquefied in stress-controlled tests when the effective stress is equal to the excess pore pressure or shear strain double amplitude reaches 6% (DeAlba *et al.* 1976, Ishihara 1985). Therefore, the tests were carried out in one amplitude 3% shear deformation. This actually corresponds to double amplitude 6% shear deformation, as in stress-controlled tests. The variation of shear strain with the number of cycles in a typical test is given in Fig. 4. In deformation-controlled tests, amplitude of shear deformation is kept constant throughout the experiment. Fig. 5 shows the relationship between the excess pore water pressure and the number of cycles of a sample with a grain size of 0.26 mm, at a relative density of 30% and under100 kPa effective stress. Due to the applied shear deformation, the excess pore water pressure in the sample continuously increases until it is equal to the vertical effective stress.

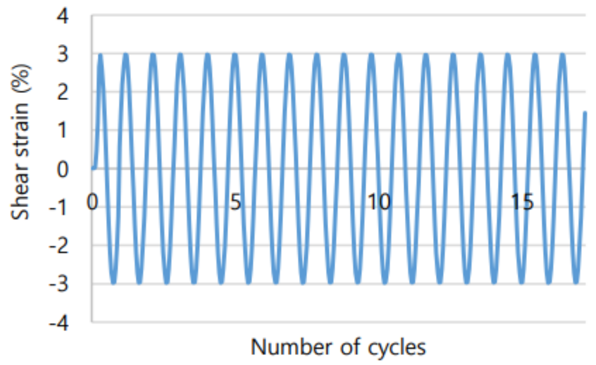


Fig. 4 Shear strain-number of cycles relationship

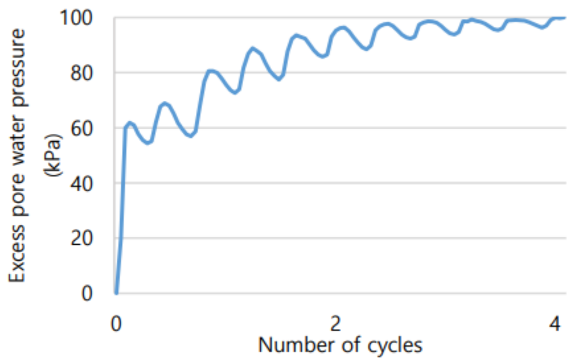


Fig. 5 Relation of excess pore pressure and number of cycles for a sample diameter of 0.26 mm, relative density of 30% and effective stress of 100 kPa

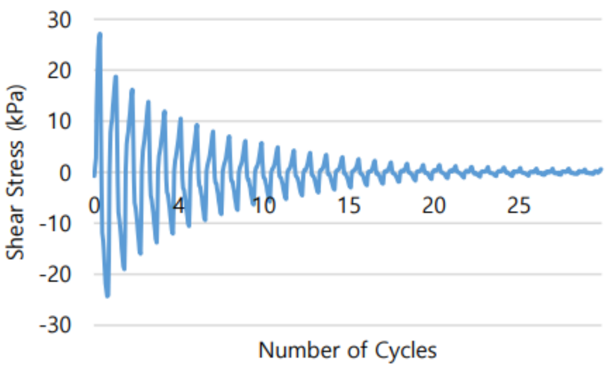


Fig. 6 Shear stress value at liquefaction corresponding to the number of cycles

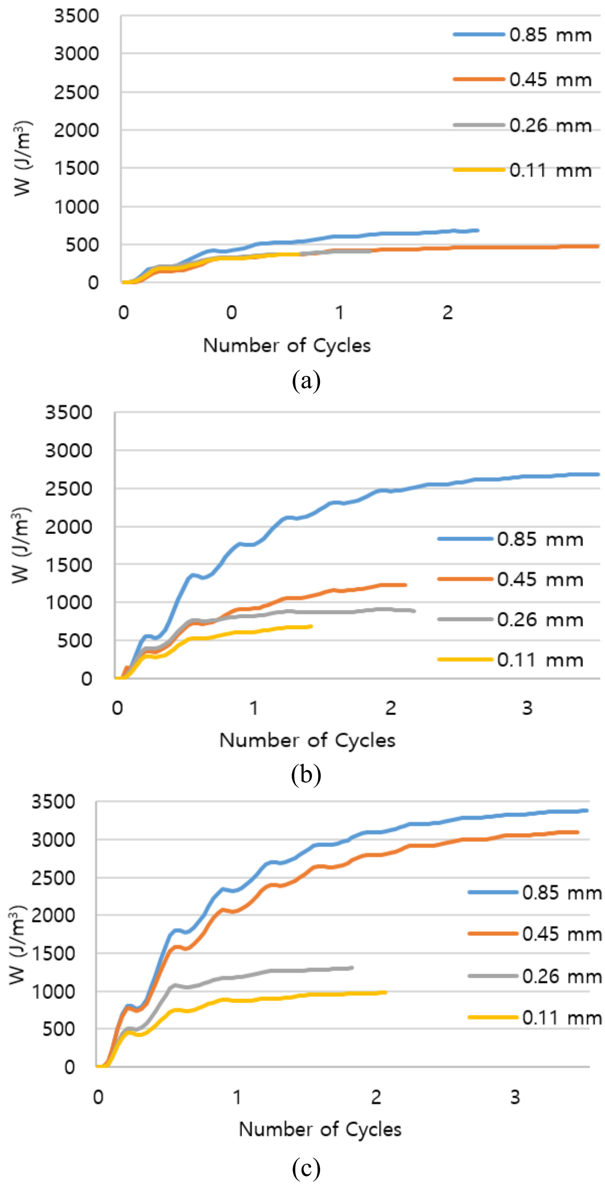


Fig. 7 Variation of the cumulative liquefaction energy with the number of cycles and mean grain size for a fixed relative density of 30% and under an effective stress of (a) 50, (b) 100 and (c) 150 kPa

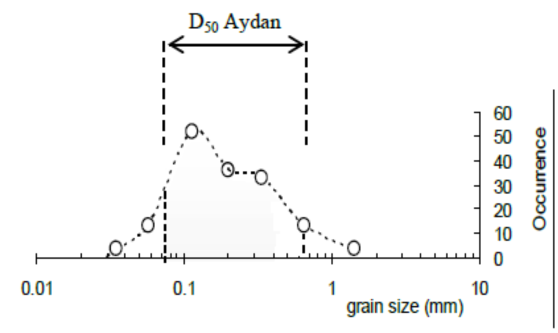
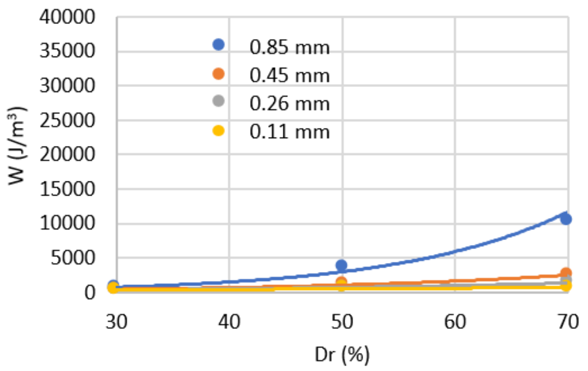


Fig. 8 Relationship between the soil mean grain size and the liquefaction phenomena (Hakam 2016)

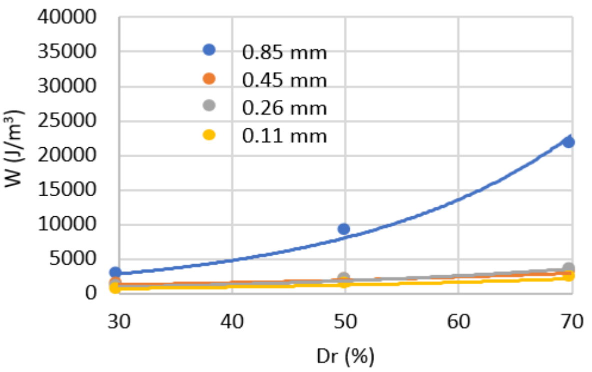
Fig. 6 shows the variation of shear stress with the number of cycles. As the liquefaction approaches with shear strain, applied in a controlled manner, shear stress decreases with decreasing soil resistance and eventually reaches a constant value. Theoretically, it is not exactly zero as in a liquid, which stems from the friction in the test system.

3.1 Mean grain size (D_{50}) - cumulative liquefaction energy (W) relationship

Previous studies have shown that an increase in the mean grain size leads to increased liquefaction resistance (Liang 1995; Dief *et al.* 2001; Baziar and Jafarian 2007; Hakam 2016). Hazout *et al.* (2017) conducted a series of undrained triaxial tests on eighteen natural loose (Dr = 25%) sandy specimens with a fine content of 2% (Ip = 5%) by considering different extreme grain sizes (1.6 mm <=



(a)



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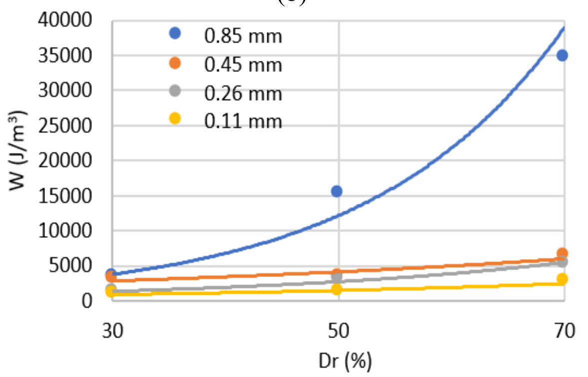


(c)

Fig. 9 Variation of the cumulative liquefaction energy (W) with relative density (Dr) for various mean grain size values and an effective stress value of (a) 50, (b) 100 and (c) 150 kPa

D_{max} <= 4 mm and 0.001 mm <= D_{min}<= 0.63 mm) and two mean grain size ranges (0.25 mm <= D_{50} <= 1.0 mm) and (1.0 mm <= D_{50} <= 2.5 mm). The test results obtained showed that mean grain size (D_{50}) and extreme grain sizes (D_{max} and D_{min}) had a significant effect on the undrained shear strength (known as liquefaction resistance). They also stated that the undrained shear strength and excess pore water pressure could be correlated with the extreme grain sizes (D_{max} and D_{min}) and average grain size (D_{50}) of the tested samples. The mean grain size (D_{50}) and the void ratio (e) of the sand are inversely proportional, i.e. the smaller the mean grain size, the higher the void ratio. With an increase in the void ratio, the amount of water per unit volume increases, the friction resistance decreases, which reduces the cumulative liquefaction energy. The relationship between the cumulative liquefaction energy and the number of cycles in the test under an effective stress of 50, 100 or 150 kPa and at 30% relative density is given in Figs. 7(a), 7(b) and 7(c).

Cumulative liquefaction energy (W) increases with increasing D_{50} under all effective stress values. In Fig. 7(a), the cumulative liquefaction energy of a sample with a relative density of 30% and under an effective stress of 50 kPa can be seen to have a value of 364 j/m^3 for a mean grain size of 0.11 mm; 409 J/m^3 for a mean grain size of 0.26 mm; 474 J/m^3 for a mean grain size of 0.45 mm; and 695 J/m^3 for a mean grain dimeter of 0.85 mm.

When the rates of increase in the cumulative liquefaction energies of the samples are compared, the increase in the energy value from a mean grain size of 0.11 mm to 0.26 mm is 12%, the increase from 0.26 mm to 0.45 mm is 15%, and the respective increase from a mean grain size of 0.45 mm to a size of 0.85 mm is 46%. As can be seen, the liquefaction energy increases logarithmically with increasing grain size. This shows that the increase in D_{50} has a significant effect on the liquefaction energy. The void ratio decreases with increasing D_{50}. As a result of this decrease, the number of grains per unit volume increases and the intergranular friction resistance against grain rearrangement increases during repeated loading. The increase in the resistance also increases the energy required for liquefaction of the sample.

The effect of increasing mean grain size for different relative densities is shown in Figs. 9(a), 9(b) and 9(c). In Fig. 9(b), the cumulative energy value of a sample with a mean grain size of 0.11 mm at a relative density of 30% and under an effective stress of 100 kPa is 686 j/m^3; while the respective value for a mean grain size of 0.26 mm is 890 j/m^3. Similarly, the cumulative energy of a sample with a grain diameter of 0.45 mm is 1232 j/m^3 and the respective value for a mean grain size of 0.85 mm is 2685 j/m3 under the same effective stress and relative density. When the cumulative liquefaction energies of the samples are compared, the energy increases by 29% from a mean grain size of 0.11 mm to a diameter of 0.26 mm; by 38% from 0.26 mm to 0.45 mm; and by 117% from 0.45 mm to 0.85 mm. Liquefaction energy of the samples increases with increasing D_{50}, similar to the influence of the increasing effective stress. However, the sample with a mean grain size of 0.85 mm is distinctly different from the other samples and gives very high liquefaction energy values. The high liquefaction energy of the sample indicates the difficulty of liquefaction of the sample. This situation is shown in the study conducted by Aydan *et al.* (2008), as shown in Fig. 8. Aydan *et al.* (2008) established the interval for D_{50} in which 80% of the liquefaction cases occur. The data obtained in this study also confirms this finding. Soil sample with a mean grain size of 0.85 mm is outside this range and liquefaction is more difficult than samples with other grain diameters.

3.2 Variation of the cumulative liquefaction energy with effective stress

The previous studies in the literature showed that the liquefaction energy increases with increasing effective

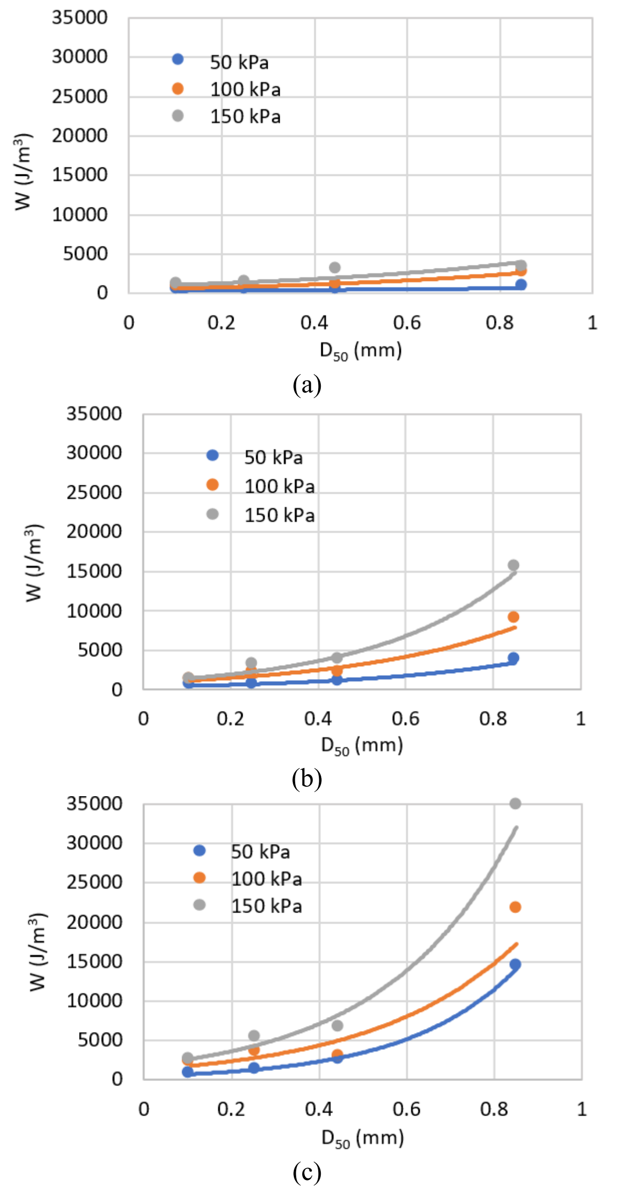


Fig. 10 Relationship between the mean grain size (D_{50}) and cumulative liquefaction energy (W) for different effective stress values and a relative density (Dr) of (a) 30%, (b) 50% and (c) 70%

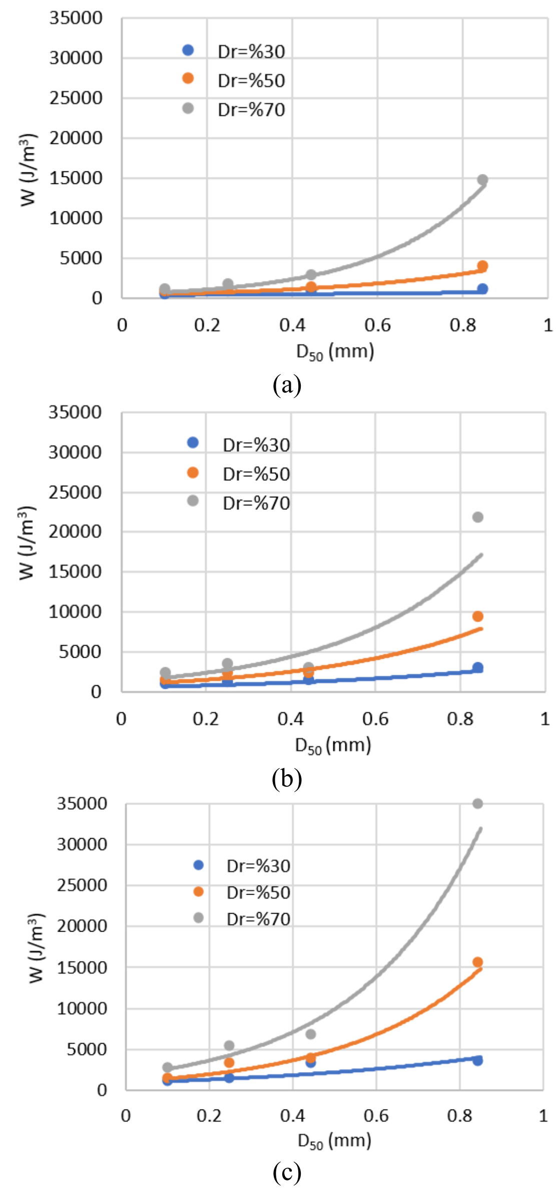


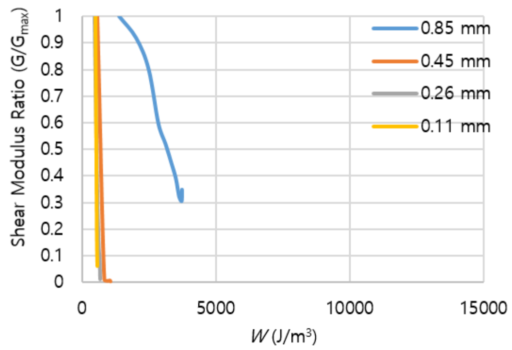
Fig. 11 Cumulative liquefaction energy-Mean grain size relationships for various relative density values and effective stress values of (a) 50 kPa, (b)100 kPa and (c)150 kPa

stress (Figueroa *et al.* 1994, Baziar and Jafarian 2007). With the increase in the effective stress, the load on the soil grains and the resistance of the grains against shear strain increase, which, in turn, increases the cumulative liquefaction energy. The variation of the cumulative liquefaction energy with mean grain size is given in Figs. 10(a), 10(b) and 10(c) for different effective stress values. According to the figures, the liquefaction energy increases with increasing effective stress and grain diameter for all relative density values. The results obtained show that the cumulative liquefaction energy is strongly dependent on effective stress.

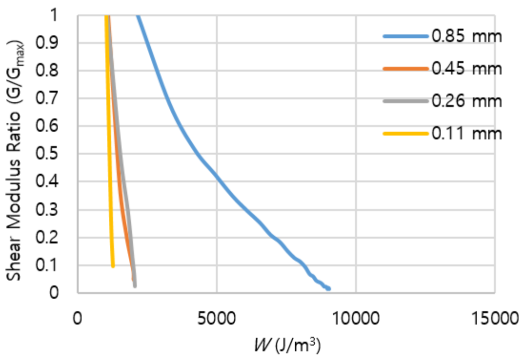
3.3 Variation of the cumulative liquefaction energy with relative density

Previous studies reported that the relative density is a proper parameter for comparison in soil liquefaction analyses (Hazirbaba and Rathje 2009; Carrao *et al.* 2009). As the relative density increases, the cumulative energy required to reach the liquefaction also increases as expected, since the soil grains settle in a more rigid order.

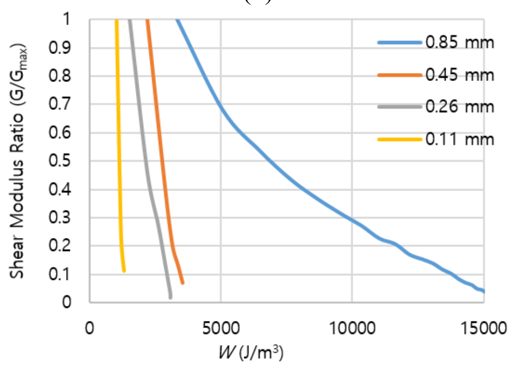
Variation of the cumulative liquefaction energy with the mean grain size is given in Figs. 11(a), 11(b) and 11(c) for different relative density values. Liquefaction energy increases with increasing relative density and grain diameter for all effective stress values. Fig. 11(b) shows that the liquefaction energy of the sample with 0.45 mm grain diameter and 100 kPa effective stress is 1232 j/m^3 for 30% relative density, 2024 j/m^3 for 50% relative density and 2872 j/ m^3 for 70% relative density. As can be seen from the figure, the liquefaction energy increases with increasing relative density. This shows that the cumulative liquefaction



(a)



(b)



(c)

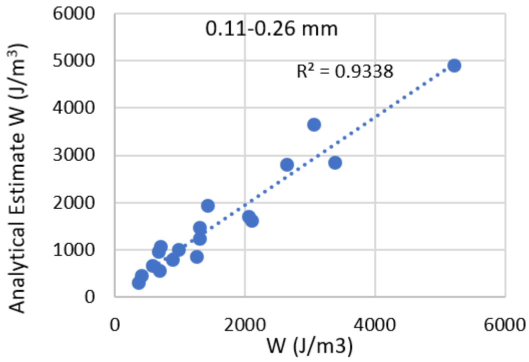
Fig. 12 Variation of the cumulative liquefaction energy with the shear modulus ratio for varying grain dimaters and%50 relative density (a) 50 kPa, (b) 100 kPa and (c) 150 kPa

energy is strongly dependent on the relative density.

3.4 Variation of the cumulative liquefaction energy with shear modulus

The shear modulus (G) is defined as ratio of the shear stress to the shearing strain. When examining the stress-strain relationship of soil, the behavior of soil samples, particularly with no permanent deformations and subjected to symmetrical repetitive loading conditions, are generally determined from the shear modulus and damping ratio properties.

The shear modulus ratio (G/G_{max}) is defined as the ratio of the shear modulus (G) in the cycle at which liquefaction occurs to the maximum shear modulus (G_{max}) (Hardin and Drnevich 1972). Figs. 12(a), 12(b) and 12(c) show the variation of the cumulative liquefaction energy with the



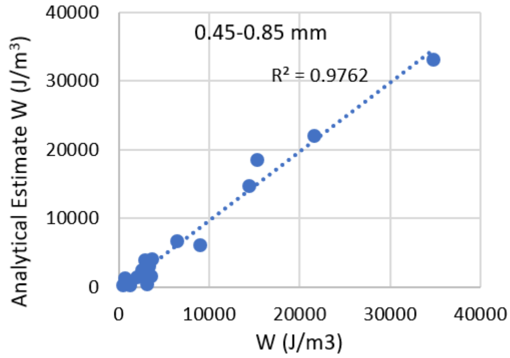


Fig. 13 Relationship between the test results and th analytical estimates from Eqs. (2) and (3)

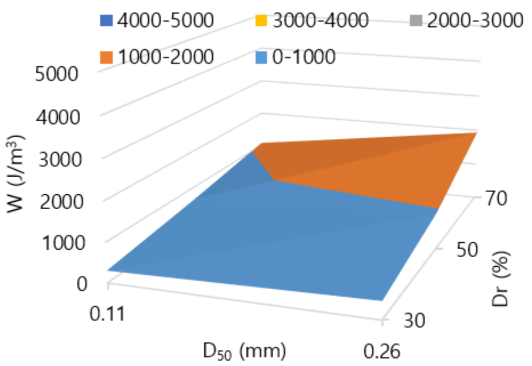
shear modulus ratio for samples of different grain diameters (0.85, 0.45, 0.26 and 0.11 mm) and under a constant effective stress of 150 kPa. As the grain diameter of the sample increases, the shear modulus reduction becomes slower. This is thought to originate from the increased resistance to shear stress as a result of the increase in the grain-to-grain interaction. Increased resistance, in turn, increases the amount of energy required for liquefaction of samples.

3.5 Regression analysis

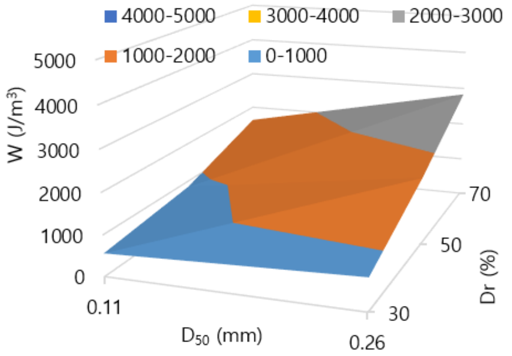
Based on the tests carried out in this study, the effects of mean grain size, effective stress and relative density on liquefaction potential were shown in the previous sections. In this section, on the other hand, a generalized correlation between the mentioned parameters and the liquefaction energy per unit volume is obtained. For this purpose, multiple regression analysis was performed between D_{50} (mean grain size), σ' (effective stress), D_r (relative density) parameters and the W (liquefaction energy per unit volume). In the analysis, the dependent variable is the liquefaction energy per unit volume and the independent variables are the mean grain size, effective stress and relative density. However, due to the differences between the grain diameters and therefore the cumulative liquefaction energy values in the analyses, equations with low correlation coefficient (R^2) were developed. Therefore, separate analyses were performed for 0.11 and 0.26mm grain diameters; and 0.45 and 0.85mm grain diameters.

Equation obtained from regression analysis on samples with mean grain sizes of 0.11-0.26 mm is given in Eq. (2):

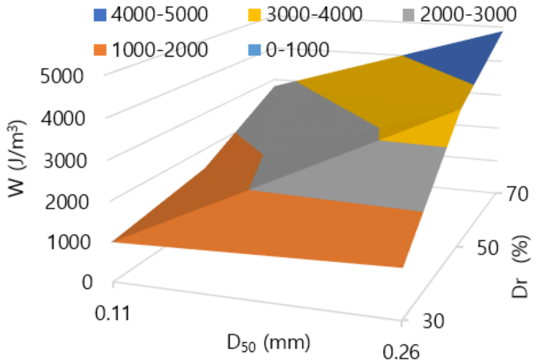
$$W = \exp(0.011\sigma' + 0.032D_r + 2.898D_{50} + 3.834) \quad (2)$$



(a)

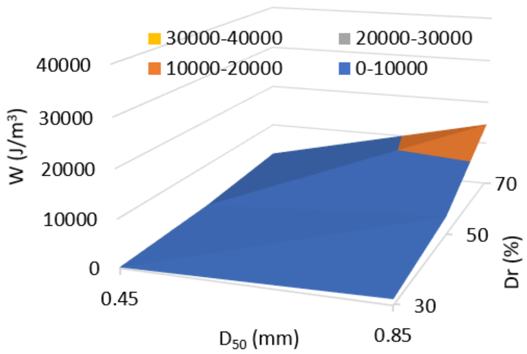


(b)

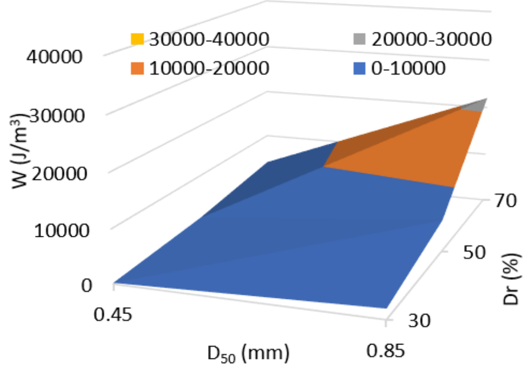


(c)

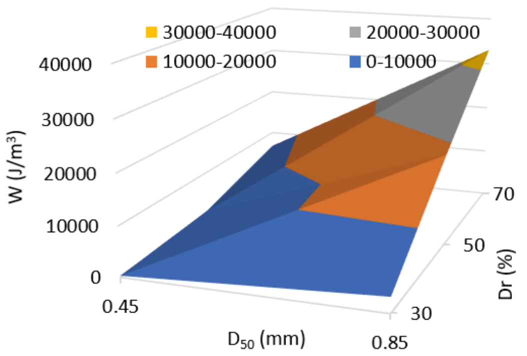
Fig. 14 Variation of the liquefaction energy with grain size and relative density for varying effective stress values of (a) 50 kPa, (b) 100 kPa and (c) 150 kPa



(a)



(b)



(c)

Fig. 15 Variation of the energy with relative density and grain size under an effective stress of (a) 50, (b) 100 and (c) 150 kPa

Equation obtained from regression analysis on samples with mean grain sizes of 0.45-0.85 mm is given in Eq. (3):

$$W = \exp(0.012\sigma' + 0.064D_r + 5.275D_{50} + 0.004) \quad (3)$$

The relationship between the cumulative liquefaction energy values and laboratory test results is given in Fig. 13 for the proposed equations. Correlation coefficients from the analyses are given $R^2 = 0.93$ for the grain diameter of 0.11-0.26 mm and $R^2 = 0.98$ for the grain size of 0.45-0.85 mm. These high correlation coefficients imply a major correlation between the dependent variable, i.e. the cumulative liquefaction energy (W) and the independent variables, i.e., relative density (D_r), mean grain size (D_{50}), and effective stress (σ').

Variation of the cumulative liquefaction energy with mean grain size and relative density is shown in Figs. 14(a), 14(b) and 14(c). The energy values in these plots were calculated from Eq. (2), which was originally developed for the mean grain size range of 0.11-0.26 mm, and using effective stress values of 50, 100 and 150 kPa. Similarly, Figs. 15(a), 15(b) and 15(c) were obtained by using Equation 3, originally developed for mean grain sizes of 0.45-0.85 mm.

The relations for the cumulative liquefaction energy, proposed in the literature, are given in Table 3. In these studies, Figueroa *et al.* (1994) conducted 27 strain-controlled tests with hollow cylinder torsional shear test apparatus using Reid Bedford sand with a mean grain size of 0.26 mm. Similarly, Liang (1995) performed 9 strain-controlled tests on Reid Bedford sand with a mean grain size of 0.26 mm using the hollow cylinder torsional shear test apparatus. Dief and Figuera (2001) conducted 30

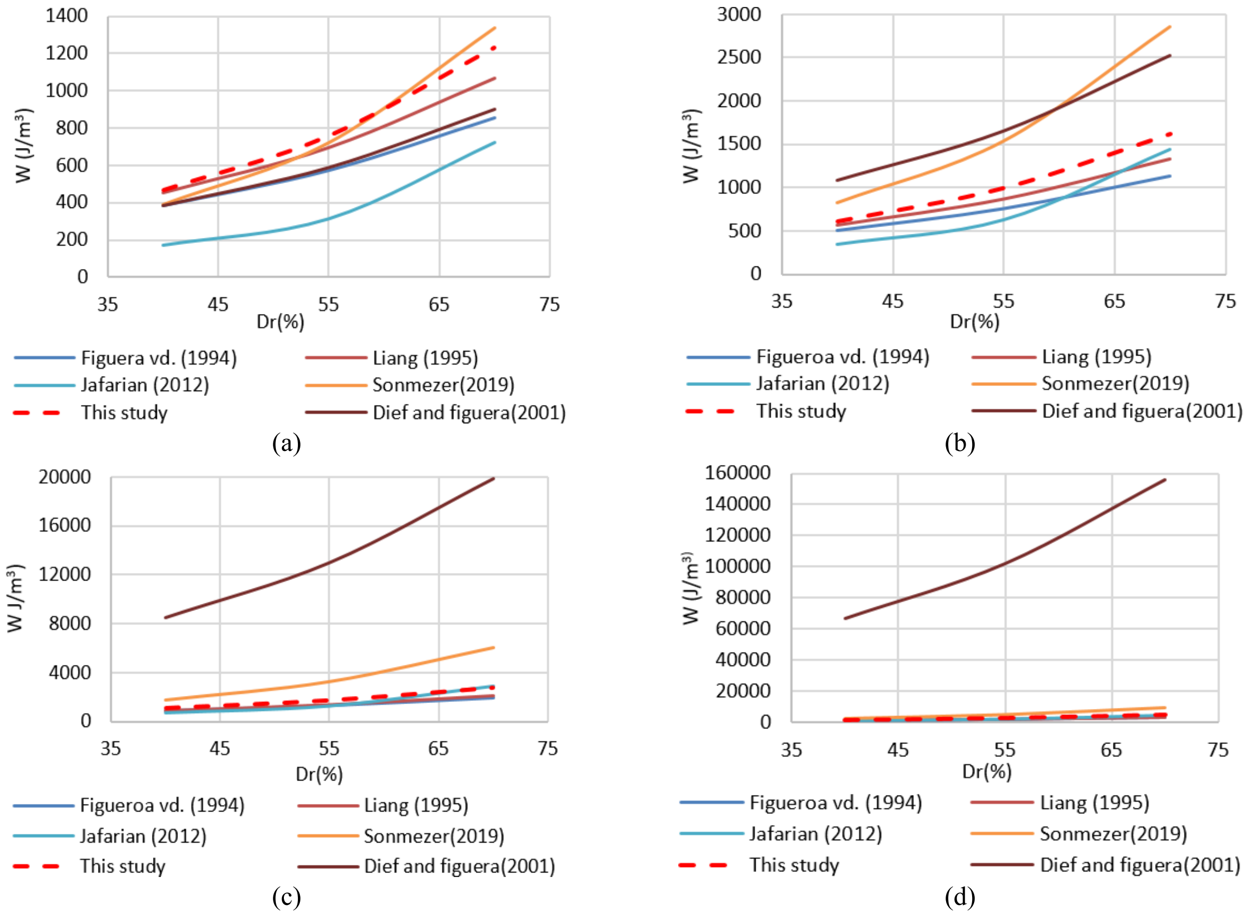


Fig. 16 Variation of the cumulative liquefaction energy with relative density for an effective stress of (a) 25, (b) 50, (c) 100 and (d) 150 kPa according to different formulations

Table 3 The equations in the literature for cumulative liquefaction energy

Figueroa *et al.* (1994)	$\log(W) = 2.002 + 0.00477\sigma'_{mean} + 0.0116D_r$	R^2=0.937
Liang (1995)	$\log(W) = 2.062 + 0.0039\sigma'_{mean} + 0.0124D_r$	R^2=0.925
Dief and Figueroa (2001)	$\log(W) = 1.167 + 0.0179\sigma'_{mean} + 0.0123D_r$	R^2=0.833
Jafarian *et al.* (2012)	$\log(W) = 0.1363P'_0(D_r/100)^{4.925} + 5.375 \times 10^{-3} \times P'_0$	R^2=0.80
Sonmezer (2019)	$W = 2.248 \times (\sigma'_v)^{1.094} \times (1.042)^{D_r}$	R^2=0.94
This study (Equation 2)	$W = \exp(0.011\sigma' + 0.032D_r + 2.898D_{50} + 3.834)$	R^2=0.93
This study (Equation 3)	$W = \exp(0.012\sigma' + 0.064D_r + 5.275D_{50} + 0.004)$	R^2=0.98

W = Cumulative liquefaction energy (J/m³); σ'_{mean} and P'_0 = Mean effective surrounding pressure (kPa); σ'_v= Vertical effective pressure (kPa); D_r= Relative density (%); D_{50}=Mean grain size (mm)

centrifugal liquefaction tests on the Nevada, Reid Bedford and LSFD (Lower San Fernando Dam) sands. Jafarian *et al.* (2012) used the results of 37 hollow cylinder torsional shear and cyclic simple shear tests on Toyouro sand with a grain diameter of 0.20 mm. Finally, Sonmezer (2019) performed 36 strain-controlled tests using the cyclic simple shear test method on a sand with a mean grain size of 0.26 mm.

The results from the relations proposed in the previous studies in literature and given in Table 3 were compared to the results from Eq. (2) for a grain diameter of 0.26 mm. The plots for different equations are compared in Figs. 16(a), 16(b), 16(c) and 16(d). In Fig. 16(a), the liquefaction energy values from the present formulation for different relative density values and under an effective stress of 25 kPa are compared to the results from other formulations in the literature. Although the estimates from the present equation agree well with the estimates from remaining formulations for low relative density values, the present formula yields to lower energy estimates compared to other formulae with increasing relative density values. Figs. 16(a), 16(b), 16(c) and 16(d) depict that the liquefaction energy increases with increasing relative density according to all considered analytical equations.

However, the results of the equation proposed by Dief and Figueroa (2001) are significantly different from the results of the equations proposed in this study and remaining studies in the literature, especially when the effective stress value increases. The equation of Dief and Figueroa (2001) effective pressure (kPa); D_r = Relative density (%) gives higher liquefaction energy values compared to the other estimates for high effective stress values. For effective stress values of 50, 100 and 150 kPa, the estimates from the equations proposed by Figueroa *et al.* (1994), Liang (1995) and Jafarian *et al.* (2012) are in good agreement with the results from the equations proposed in the present study.

4. Conclusions

In this study, a total of 36 deformation-controlled tests were conducted using the energy-based method and the cyclic simple shear test apparatus for the effective stress values of 50, 100, and 150 kPa; relative density values of 30, 50, and 70%; and grain diameters of 0.85, 0.45, 0.26, and 0.11 mm. Furthermore, multiparameter regression analyses were conducted for mean grain size range of 0.11 and 0.26 mm and the range of 0.45 and 0.85 mm. The new proposed equations account for the mean grain size, relative density and effective stress simultaneously, different from the equations existing in the literature. The experimental and analytical studies conducted within this research program yielded to the following conclusions:

• The cumulative liquefaction energy of the samples tested within the scope of the study (0.85, 0.45, 0.26, and 0.11 mm) increased with increasing mean grain size. In other words, liquefaction of the samples became more difficult with increasing grain size. The increase is shown to be logarithmic. This may be due to the increase in the interaction between the grains due to the increased mean grain size and reduced void ratio, and therefore, the need for higher energy values for liquefaction.

• Cumulative liquefaction energy increases with increasing effective stress in samples of identical relative density and grain diameter. This increase is thought to originate from the increased resistance of the grains to rearrangement during deformations, which in turn increases the liquefaction energy.

• Cumulative liquefaction energy increases with increasing relative density in the presence of constant effective stress and grain diameter. This increase might be related to the more dense packing of the grains with increasing relative density and increase in the resistance to liquefaction with increasing intergranular contact.

• Present test results showed that the energy required per unit volume is associated with the increase in the excess pore pressure and the occurrence of liquefaction.

• The shear modulus ratio decreases more gradually with increasing grain diameter. This conclusion is valid for all effective stress values. Similar results were also obtained for the relative density.

• When the results of the correlations derived from this study are compared with the results of the correlations obtained in the literature, Figueroa *et al.* (1994), Liang (1995) and Jafarian (2012) give values that are quite compatible with the results of the proposed correlations.

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