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A novel approach for predicting lateral displacement caused by pile installation

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Abstract. A novel approach for predicting lateral displacement caused by pile installation in anisotropic clay is presented, on the basis of the cylindrical and spherical cavities expansion theory. The K_0-based modified Cam-clay (K_0-MCC) model is adopted for the K_0-consolidated clay and the process of pile installation is taken as the cavity expansion problem in undrained condition. The radial displacement of plastic region is obtained by combining the cavity wall boundary and the elastic-plastic (EP) boundary conditions. The predicted equations of lateral displacement during single pile and multi-pile installation are proposed, and the hydraulic fracture problem in the vicinity of the pile tip is investigated. The comparison between the lateral displacement obtained from the presented approach and the measured data from Chai *et al.* (2005) is carried out and shows a good agreement. It is suggested that the presented approach is a useful tool for the design of soft subsoil improvement resulting from the pile installation.

Keywords: lateral displacement; pile installation; anisotropic clay; cavity expansion; modified Cam-clay model

1. Introduction

The lateral displacement analysis of soil mass around a pile is a fundamental and significant problem in foundation engineering. It is often assumed that lateral displacement analysis of soil mass around a pile is analogous to a cavity expansion problem (Hill, 1950). Since its inception in the 1950s (Hill, 1950), the cavity expansion theory has attracted great interest from geotechnical engineering. Numerous research studies have been conducted and papers were written (e.g., Vesic 1972, Carter and Yeung 1985, Carter *et al.* 1986, Yu 2000, Wang and Yin 2011, Chen and Abousleiman 2012, Silvestri and Abou-Samra 2012, Keawsawasvong and Ukritchon 2016, Kumara *et al.* 2016, Ukritchon *et al.* 2016, Xiao *et al.* 2016, Kim and Choi 2017, Zhou *et al.* 2017, Zou *et al.* 2017, Ahn *et al.* 2018, Fattah *et al.* 2018, Khanmohammadi and Fakharian 2018, Ko *et al.* 2018, Kwon *et al.* 2018, Zou *et al.* 2018, Zou and Wei 2018, Chen *et al.*, 2019a b, Li *et al.* 2019a, b, Li and Zou 2019, Zou *et al.* 2019, Zou and Zhang 2019) that have focused on the mechanism and theoretical models, A few papers have been searched the way to combine cavity expansion theory and pile installation problem. For instance, the process of pile installation has been simulated to be a cylindrical cavity problem in undrained condition (Randolph *et al.* 1979), Randolph (2003) proposed a conceptual and analytical framework to estimate pile capacity. Carter *et al.* (1979) proposed an approach for predicting the disturbance of soil mass around the pile due to pile driving. An approach based on quasi-static cavity expansion was proposed by Yu and Houlsby (1991) in an infinite dilatant elastic-plastic soil mass, and the stress and displacement fields were given during the cavity expansion process. Sagaset and Whittle (2001) proposed an approach for predicting ground movements, when the jacked piles was installed in clay. An approach of lateral displacement was proposed by Chai *et al.* (2005), and this approach was based on the Mohr-Coulomb criterion and isotropic soil mass.

Although these aforementioned cavity expansion approaches have been solved a lot of engineering problem, these approaches also were based on the assumption of isotropic initial in-situ stress. In fact, the initial stress is likely to be anisotropic in natural clay due to the change of sediment and consolidation environments (Zhou *et al.*, 2014), and the initial stress anisotropy and initial stress-induced anisotropy have prominent effects on the soil mass behaviour (Li *et al.*, 2016). More recently, a few published results presented the effect of anisotropy in natural soil mass. For example, Zhou *et al.* (2014) proposed an analytical approach for undrained elasto-plastic cylindrical cavity expansion problem in saturated soil mass under anisotropic initial stress. An anisotropically elasto-plastic approach to the undrained cylindrical cavity expansion in K_0-consolidated clay was proposed by Li *et al.* (2016), and an approximate numerical solution was proposed for practical purposes.

The lateral displacement was calculated by Chai *et al.* (2005), the values of lateral displacement at the bottom of the columns were larger than the measured data. The reason may be the effect of hydraulic fracture at the pile tip (Chai

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et al., 2005). Here, an attempt about the hydraulic fracture problem around the pile is presented for understand the mechanism.

In summary, many published literatures have principally focused on theoretical approach based on the assumption of isotropic initial stress. Only a few published results presented the effect of initial stress anisotropy in natural soil mass and hydraulic fracture of soil mass at pile tip. The main objective of this study is to develop a theoretical approach for predicting lateral displacement caused by pile installation, and the approach is based on cavity expansion theory considering the effect of anisotropic initial stress in undrained soil mass. With the development of strength criterion of soil mass, the MCC model is introduced into the solution of cavity expansion theory in clay (Xiao *et al.* 2011, Chen and Abousleiman 2012, Li *et al.* 2016). The K_0-based modified Cam-clay (K0-MCC) model (Li *et al.* 2016) is applied for the analysis of clay, the pile shaft and pile tip of the expansion process are simulated as the cylindrical and spherical cavity expansion in undrained condition respectively, and the effect of boundary between cylindrical and spherical cavity expansion is neglected. A solution is applied for predicting lateral displacement during the process of single pile and multi-pile installation, and the solution accounts for the effect of hydraulic fracture of anisotropic soil mass at the pile tip. The reliability of the approach is validated by comparing with the data measured by Chai *et al.* (2005), and some parameters of lateral displacement are selected for the parametric analysis. The innovations of this study can be concluded as follows:

(1) A novel approach for predicting lateral displacement caused by single pile installation is applied based on the both spherical and cylindrical cavities expansion, and considers the effect of hydraulic fracture to pile tip of single pile is considered.

(2) The novel approach of lateral displacement in the process of multi-pile installation is applied and considers the effect of anisotropic initial stress.

2. Theory and methodology

2.1 Problem definition and assumptions

The mechanical model of single pile installation is shown in Fig. 1. The anisotropic initial in-situ horizontal and vertical stresses are σ_{h0} and σ_{v0}, respectively. With the increase of the internal pressure σ_a, a plastic region around the cavity occur from the cavity wall to the radius R_p. The initial and final radius of the cavity is R_{u0} and R_u, respectively. The current position and initial position of a point in the plastic region are represented by R_x and R_{x0}. R_{p0} and R_p are the initial and final position of the EP boundary. The radial displacement of the EP boundary is δ_{rp}.

For the K_0-consolidation saturated clay, the initial effective radial and tangential stresses and vertical stress of soil mass around the pile can be expressed,

$$\sigma'_{r0} = \sigma'_{\theta 0} = K_0 \sigma'_{z0} \tag{1}$$

where K_0 is coefficient of earth pressure at rest.

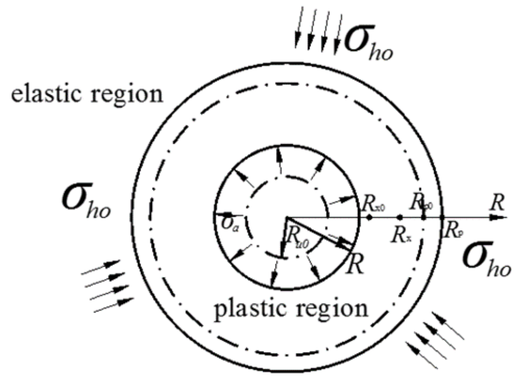

Fig. 1 Mechanical model for pile installation

In both the elastic and plastic regions, the equilibrium equation can be written as,

$$\frac{\partial \sigma'_r}{\partial r} + \varsigma \frac{\sigma'_r - \sigma'_\theta}{r} + \frac{\partial u}{\partial r} = 0 \tag{2}$$

where ζ=1 and ζ=2 are the cylindrical and spherical cavities expansion problem, respectively, u is the pore water pressure.

2.2 Elastic analysis

According to Hooke's law, the elastic stress-strain relationship of soil mass can be expressed,

$$\begin{Bmatrix} d\varepsilon^e_r \\ d\varepsilon^e_\theta \\ d\varepsilon^e_z \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu' & -\nu' \\ -\nu' & 1 & -\nu' \\ -\nu' & -\nu' & 1 \end{bmatrix} \cdot \begin{Bmatrix} d\sigma_r' \\ d\sigma_\theta' \\ d\sigma_z' \end{Bmatrix} \tag{3}$$

where $E = 2G(1+\nu')$ is the elastic modulus, the shear modulus G defined as follows,

$$\begin{cases} G = \left(3(1-2\nu')\upsilon p'\right) / \left(2(1+\nu')\kappa\right) \\ \upsilon = 1+e \end{cases} \tag{4}$$

where κ is a known as the slope of loading-reloading line in the υ-$\ln p'$ plane, υ is the specific volume, e is void ratio of soil mass, ν' is undrained effective Poisson's ratio.

In the elastic region, the stress and displacement of soil mass around the pile can be expressed as,

$$\begin{cases} \sigma'_r = \sigma'_{h0} + \left(\sigma'_{rp} - \sigma'_{h0}\right)\left(\dfrac{R_p}{R}\right)^{\varsigma+1} \\ \sigma'_z = \sigma'_{v0} \\ \sigma'_\theta = \sigma'_{h0} - \dfrac{1}{\varsigma}\left(\sigma'_{rp} - \sigma'_{h0}\right)\left(\dfrac{R_p}{R}\right)^{\varsigma+1} \end{cases} \tag{5}$$

$$\delta_r = \frac{\sigma'_{rp} - \sigma'_{h0}}{2\varsigma G} \bullet \frac{R_p^{\ \varsigma+1}}{R^\varsigma} \tag{6}$$

In undrained condition, the volume of soil mass keep not change, therefore,

$$d\upsilon = 0 \tag{7}$$

According to the Eq. (7), so the excess pore water pressure is zero in elastic region,

$$\Delta u = 0 \tag{8}$$

2.3 Elasto-plastic analysis

In plastic region, the K_0-MCC yield function can be expressed as (Sun *et al.* 2004, Li *et al.* 2016),

$$1+\left(\frac{\eta^*}{M^*}\right)^2 - \frac{p_c'}{p'} = 0 \tag{9}$$

where p'_c is yield stress of K_0-consolidation clay, p' is the mean effective stress, η^* is the relative stress ratio, M^* is the relative stress ratio at critical state, it can be defined as follows,

$$p' = \frac{1}{3}\sigma_{ii}' \tag{10}$$

$$\eta^* = \sqrt{\frac{3}{2}\left(\eta_{ij}-\eta_{ij0}\right)\left(\eta_{ij}-\eta_{ij0}\right)} \tag{11}$$

$$M^* = \sqrt{M^2 - \eta_0^2} \tag{12}$$

$$\eta_0 = \left|\left[3\left(1-K_0\right)\right]/\left(2K_0-1\right)\right| \tag{13}$$

$$\eta_{ij} = \frac{\sigma_{ij}' - p'\delta_{ij}}{p'} \tag{14}$$

$$\eta_{ij0} = \frac{\sigma_{ij0}' - p_0'\delta_{ij}}{p'} \tag{15}$$

$$M = \frac{6\sin\varphi'}{3-\sin\varphi'} \tag{16}$$

where δ_{ij} is Kronecker's delta, σ'_{ij0} and η_0 are the value of σ'_{ij} and $\eta(=q/p')$ at the initial of the anisotropic consolidation, respectively.

Based on the Eqs. (7) and (8), the stress continuity conditions at the EP boundary can be considered,

$$p_p' = p_0' \tag{17}$$

With the combination of Eqs. (17) and (9), the relative stress ratio at the elastic-plastic boundary R_p can be obtained,

$$\eta_p^* = M^*\sqrt{OCR-1} \tag{18}$$

where OCR is the over-consolidation ratio, defined as $OCR = p_{c0}' / p_0'$, and the p_{c0}' is the maximum mean pre-consolidation stress.

Based on the Eq. (11), η^*_P can be obtained,

$$\eta_P^* = \sqrt{\frac{3}{2}\left(\eta_{ijp}-\eta_{ij0}\right)\left(\eta_{ijp}-\eta_{ij0}\right)} \tag{19}$$

where η_{ijp} is the value of the stress ratio η_{ij} at the elastic-plastic boundary.

With the combination of Eqs. (5), (18) and (19), Li *et al.* had solved the cylindrical cavity problem, the effective radial, tangential and vertical stresses at the EP boundary can be expressed based on Li's solution (Li *et al.* 2016),

$$\begin{cases} \sigma_{rp}' = \sigma_{h0}' + \dfrac{\varsigma}{\sqrt{3^{\varsigma}}} p_0'\eta_p^* \\ \sigma_z' = \sigma_{v0}' \\ \sigma_{\theta p}' = \sigma_{h0}' - \dfrac{\varsigma}{\sqrt{3^{\varsigma}}} p_0'\eta_p^* \end{cases} \tag{20}$$

Based on the Eq. (6), the radial displacement δ_{rp} at the EP boundary can be expressed,

$$\delta_{rp} = \frac{\left(\sigma_{rp}' - \sigma_{r0}'\right)R_p}{2\varsigma G_0} \tag{21}$$

According to the cavity expansion of undrained conditions, the volume of soil mass around the pile remains constant, and the process of cavity expansion is shown in Fig. 1. With the increase of the internal pressure σ_a, the current position and initial position of cavity (the cavity's current position r_x and initial position r_{x0}) can be derived,

$$R_u^{\varsigma+1} - R_{u0}^{\varsigma+1} = r_x^{\varsigma+1} - r_{x0}^{\varsigma+1} \tag{22}$$

According to Chai *et al.* (2005),

$$R_u = R_{u0}\left(\frac{E_0}{E}\right)^{1/3} \tag{23}$$

where R_{u0} is initial radius of cavity corresponding to initial elastic modulus E_0, the power of 1/3 was obtained by best fitting the field data cited (Chai *et al.*, 2005).

With the combination of Eqs. (21), (22) and (23), the ratio of the radius of the plastic zone (R_p) to the radius of cavity (R_u) can be given,

$$\begin{aligned} \frac{R_p}{R_u} &= \left(\frac{2\varsigma G_0}{(\varsigma+1)(\sigma_{rp}'-\sigma_{r0}')}\left[1-\left(\frac{R_{u0}}{R_u}\right)^{\varsigma+1}\right]\right)^{1/\varsigma+1} \\ &= \left(\frac{2\varsigma G_0}{(\varsigma+1)(\sigma_{rp}'-\sigma_{r0}')}\left[1-\left(\frac{E_0}{E}\right)^{-(\varsigma+1)/3}\right]\right)^{1/\varsigma+1} \end{aligned} \tag{24}$$

under the condition of the final radius of single pile and the properties of soil mass in the field, the plastic region caused by pile construction can be estimated without the initial radius.

2.4 Development of a modification approach of pile tip

When the limited length of pile installation is simulated, the effect of the free surface and pile tip have to be considered. For simplicity, it is often proposed to take into account the effect of the free surface in an approximate method by using a virtual image approach (Sagaset *et al.* 1997), in this study, the detailed analysis and calculations are not carried out. Because the predicted values at the bottom of columns were larger than the measurement date (Chai *et al.* 2005), this is probably the effect of hydraulic fracture at pile tip. So, the effect of hydraulic fracture to pile tip is considered.

The total stresses (σ_{rx}, $\sigma_{\theta x}$, σ_{zx}) and excess pore pressure Δu_{rx} were proposed based on the approximate anisotropically cylindrical solution (ζ =1) in the plastic region of pile tip (Li *et al.* 2016), the anisotropically cylindrical and spherical solution (ζ =2) can be obtained (Li *et al.* 2016),

$$\sigma_{\theta x}=\sigma_{\mathrm{rp}}+\frac{\sqrt{3^{\varsigma-1}}}{\varsigma+1}\beta p_f' \ln\frac{2\varsigma G_0}{(\varsigma+1)(\sigma_{rp}'-\sigma_{r0}')}\left(\frac{\left(\frac{E_0}{E}\right)^{-(\varsigma+1)/3}-1}{\left(\frac{R_x}{R_{u0}}\right)^{\varsigma+1}}\right)-\beta p_f' \tag{25}$$

$$\Delta u_{\mathrm{rx}}=p_0'\left[\frac{3K_0}{1+2K_0}-\left(\frac{OCR}{2}\right)^{1-\kappa/\lambda}\right]+\frac{\varsigma p_0'\eta_p^*}{\sqrt{3^{\varsigma}}}+ \frac{\sqrt{3^{\varsigma-1}}}{\varsigma+1}\beta p_f' \ln\left(\frac{2\varsigma G_0}{(\varsigma+1)(\sigma_{rp}'-\sigma_{r0}')}\left(\frac{\left(\frac{E_0}{E}\right)^{-(\varsigma+1)/3}-1}{\left(\frac{R_x}{R_{u0}}\right)^{\varsigma+1}}\right)-1\right)\pm\frac{\sqrt{4q_f^2-3\beta^2p_f'^2}}{6} \tag{27}$$

$$\beta=\frac{2\sqrt{3\left[M^2\left(2K_0+1\right)^2-9\left(1-K_0\right)^2\right]}}{\left[3(2K_0+1)\right]} \tag{28}$$

$$p_f'=p_0'\left(\frac{OCR}{2}\right)^{\wedge} \tag{29}$$

$$q_f=Mp_0'\left(\frac{OCR}{2}\right)^{\wedge} \tag{30}$$

where λ is the slopes of compression line, $\sigma_{\theta x}$ is the total radial stress, σ_{zx} is the total radial stress, Δu_{rx} is the excess pore pressure and $p'f$ and q_f are the mean and deviator stresses in the critical state, respectively.

The change of the tangential stress of soil mass can be expressed,

$$\Delta\sigma_{\theta x}=\sigma_{\theta x}-K_0\sigma_{zx} \tag{31}$$

The change of the tangential effective stress of soil mass can be expressed,

$$\Delta\sigma_{\theta x}'=\Delta\sigma_{\theta x}-\Delta u \tag{32}$$

According to Eq. (25), the tangential stress may be negative. Therefore, in the case, the negative tangential stress exceeds tensile strength (usually very small), the first cracks can be created in the soil mass, this can be determined by the critical condition that the effective tangential stress is equal to zero, one can be written as follows,

$$K_0\sigma_{zx}'+\Delta\sigma_{\theta x}'\le 0 \tag{33}$$

When cracks can be created, the $\sigma_{\theta x}$ must to be corrected, the $\sigma_{\theta x}$ is replaced by measured data ($K_0\sigma_{\theta x}$) in the process of pile installation, all measured data are taken from Chai *et al.* (2005) and Chai and Carter (2011), the effective radial stress at the elastic-plastic boundary σ_{rp}' is obtained, and the final position of the EP boundary R_p can be determined again, the effect of correction can be shown in Fig. 4.

3. Lateral displacement caused by multi-pile installation

3.1 The superposition principle to calculate lateral displacement of multi-pile

It is usually necessary to calculate the lateral displacement caused by the installation of multi-piles in foundation engineering, the aforementioned equations is the case of single pile installation. Based on the Chai's paper of the soil-cement column installation, Chai *et al.* (2005) considered that the lateral displacement of multi-pile can be solved according to the superposition principle, the calculation model of single row pile is shown in Fig. 2, it should be noted that multi-pile installation's partial "plane strain" effect on the lateral displacement is neglected and the displacement of chai's method was based on the Vesic's solution (Vesic 1972). In order to verification later, the chai's method needs to be introduced here again.

As shown in Fig. 2, the circle represents the pile cross section, y_i indicates the distance from the i-th pile to the 0 point in the y direction (along the row), δ_{ix} and δ_i, the i-th pile to the point A in the x direction and radial direction expanded displacement, respectively. The value of δ_i can be obtained according to the EP region of the point A, and can be also calculated by the abovementioned presented equations. Thus, the lateral displacement at the point A can be expressed (Chai *et al.* 2005),

$$\delta_{ix}=\delta_i\frac{D}{\sqrt{D^2+y_i^2}} \tag{34}$$

$$\delta_{iA}=\frac{2D}{S}\int_0^L\frac{\delta}{\sqrt{D^2+y^2}}dy \tag{35}$$

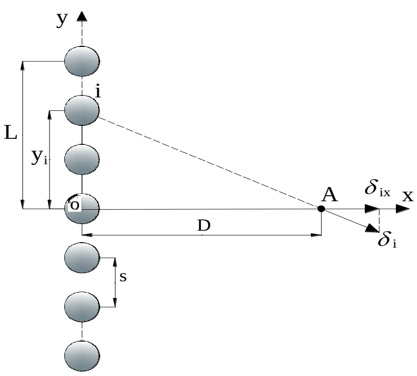

Fig. 2 Calculation model of single row pile (Chai *et al.* 2005)

$$\delta_p = \frac{\sigma_{rp}' - \sigma_{h0}'}{2\varsigma G_0} R_p \tag{36}$$

When $D<R_p$ and $D^2+L^2>Rp^2$, Chai pointed out that the point A is located in plastic region of partial piles surrounding soil mass and elastic region of other piles, the displacement δ_{xA} of the whole row of piles at the point A can be expressed (Chai *et al.* 2007),

$$\begin{aligned}
\delta_{xA} &= \frac{2D}{S}\left(\int_0^{\sqrt{R_p^2-D^2}} \frac{\delta}{\sqrt{D^2+y^2}} dy + \int_{\sqrt{R_p^2-D^2}}^{L} \frac{\delta}{\sqrt{D^2+y^2}} dy \right) \\
&= \frac{2D}{S}\left(\begin{aligned} &\int_0^{\sqrt{R_p^2-D^2}} \frac{1}{\sqrt{D^2+y^2}} \frac{2R_p+\delta_p}{2\sqrt{D^2+y^2}+\delta_p R_p/\sqrt{D^2+y^2}} \delta_p dy \\ &+\int_{\sqrt{R_p^2-D^2}}^{L} \frac{1}{\sqrt{D^2+y^2}} \frac{R_p}{(D^2+y^2)^{1/2}} \delta_p dy \end{aligned} \right) \\
&= \frac{2D}{S}\left(\begin{aligned} &\int_0^{\sqrt{R_p^2-D^2}} \frac{1}{\sqrt{D^2+y^2}} \frac{\left(2+\left(\sigma_{rp}'-\sigma_{h0}'\right)/\left(2\varsigma G_0\right)\right)R_p^{\ 2}}{2\sqrt{D^2+y^2}+\left(\left(\left(\sigma_{rp}'-\sigma_{h0}'\right)/\left(2\varsigma G_0\right)\right)R_p^{\ 2}\right)/\sqrt{D^2+y^2}} \frac{\sigma_{rp}'-\sigma_{h0}'}{2\varsigma G_0} dy \\ &+\int_{\sqrt{R_p^2-D^2}}^{L} \frac{1}{\sqrt{D^2+y^2}} \frac{R_p^{\ 2}}{(D^2+y^2)^{1/2}} \frac{\sigma_{rp}'-\sigma_{h0}'}{2\varsigma G_0} dy \end{aligned} \right)
\end{aligned} \tag{37}$$

When $D<R_p$ and $D^2+L^2 \le Rp^2$, Chai pointed out that the point A is located in plastic region of piles surrounding soil mass, the displacement δ_{xA} of the whole row of piles at the point A can be expressed,

$$\begin{aligned}
\delta_{xA} &= \frac{2D}{S}\int_0^L \frac{1}{\sqrt{D^2+y^2}} \frac{2R_p+\delta_p}{2\sqrt{D^2+y^2}+\delta_p R_p/\sqrt{D^2+y^2}} \delta_p dy \\
&= \frac{2D}{S}\int_0^L \frac{2R_p+\delta_p}{2(D^2+y^2)+\delta_p R_p} \delta_p dy \\
&= \frac{2D}{S}\int_0^L \frac{\left(2+\left(\sigma_{rp}'-\sigma_{h0}'\right)/\left(2\varsigma G_0\right)\right)R_p^{\ 2}}{2(D^2+y^2)+\left(\left(\left(\sigma_{rp}'-\sigma_{h0}'\right)/\left(2\varsigma G_0\right)\right)R_p^{\ 2}\right)} \frac{\sigma_{rp}'-\sigma_{h0}'}{2\varsigma G_0} dy
\end{aligned} \tag{38}$$

When $D \ge R_p$, Chai pointed out that the point A is located in the elastic region of piles surrounding soil mass, the displacement δ_{xA} of the whole row of piles at the point A can be expressed,

$$\begin{aligned}
\delta_{xA} &= \frac{2D}{S}\int_0^L \frac{1}{\sqrt{D^2+y^2}} \frac{R_p}{(D^2+y^2)^{1/2}} \delta_p dy \\
&= \frac{2DR_p\delta_p}{S}\int_0^L \frac{1}{(D^2+y^2)} dy \\
&= \frac{2D\left(\left(\sigma_{rp}'-\sigma_{h0}'\right)/\left(2\varsigma G_0\right)\right)R_p^{\ 2}}{S}\int_0^L \frac{1}{(D^2+y^2)} dy
\end{aligned} \tag{39}$$

3.2 Calculation procedure

The calculation procedure of lateral displacement can be concluded as follows:

1. The physical and mechanical property index of surrounding subsoil are obtained from Chai *et al.* (2005) and Chai and Carter (2011), such as the void ratio of soil mass (e), the undrained effective Poisson's ratio (ν'), the elastic modulus (E), and so forth.

2. The radius of cavity (R_u) is determined by Eq. (23), the initial radius of cavity need to be determined according to the initial elastic modulus E_0 empirically.

3. The radius of plastic region (R_p), the radial displacement (δ_{rp}) are calculated by Eqs. (21) and (24).

4. The radius of plastic region (R_p) can be corrected considering the effect of hydraulic fracture in pile tip, the final position of the EP boundary (R_p) can be determined again by Eqs. (25)-(33).

5. Calculate the lateral displacement of soil mass caused by the multi-pile installation, which are determined by Eqs. (34)-(39).

4. Validation and discussions

4.1 Validation

To confirm the validity and accuracy of the proposed approach, the measured data are taken from Chai *et al.* (2005) and Chai and Carter (2011), and it is shown in Table 1 and Table 2. The comparison results of lateral displacement caused by multi-pile installation (Chai *et al.* 2005) for different way are shown in Fig. 3.

It can be seen from Fig. 3 and Fig. 4 that the results of the presented approach agree well with the measured data (Chai *et al.*, 2005). Furthermore, the difference of the presented approach and the measured data is less than the Chai proposed approach and measured data. For example, the presented approach's the mean of absolute values of difference is reduced by 14.12% compared with the previous approach value in the WJM. The modified presented study predict a smaller value of lateral displacement near the pile tip, which is closer to the measured data (Chai *et al.*, 2005), it may consume some energy due to occur the crack of soil mass at pile tip and resulting in circumferential extrusion to the cracked soil mass, so the radial displacement is relatively smaller.

4.2 Discussions

In addition to validation, some parameters of the lateral displacement of multi-pile are selected for the parametric analysis. The effect of initial radius R_{u0}, over consolidation ratio (OCR) and coefficient of earth pressure at rest K_0 on the lateral displacement of soil mass at the pile tip is analyzed in the dry jet mixing method (DJM). Here, R_{u0} is taken as a variable and the other parameters unchanged. The R_{u0} are taken as 0.15, 0.25, 0.35 and 0.45, and the Fig. 5 is obtained. Similarly, the OCR are taken as 2, 4, 6 and 8, and the Fig. 6 is obtained. The K_0 are taken as 0.25, 0.75, 1.00, 1.25, 2.00 and 4.00, and the Fig.7 is also obtained.

When the different value of R_{u0} can be taken, the lateral displacement is compared as shown in Fig. 5. The effect of R_{u0} on lateral displacement of multi-pile installation significant than the effect of OCR and K_0. With the increase of the initial radius R_{u0}, the lateral displacement of pile surrounding soil is increase significantly. In this study, the initial radius of cavity is related to the injection pressure. The larger the injection pressure is, the larger the initial radius is, and the lateral displacement of soil mass is increase.

When the different over-consolidation ratio (OCR) can be taken, the effect of lateral displacement is compared as shown in Fig. 6. The effect of over-consolidation ratio (OCR) on lateral displacement of multi-pile installation is

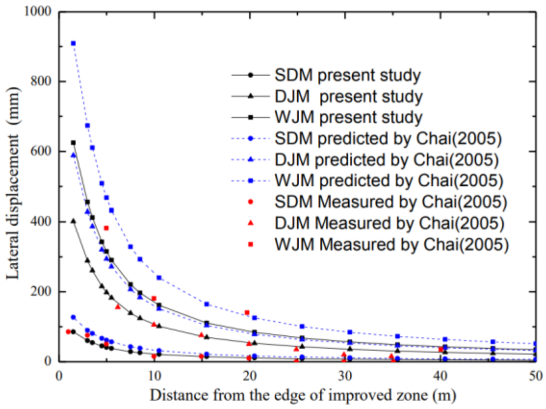

Fig. 3 Comparison of lateral displacement predicted for different installation

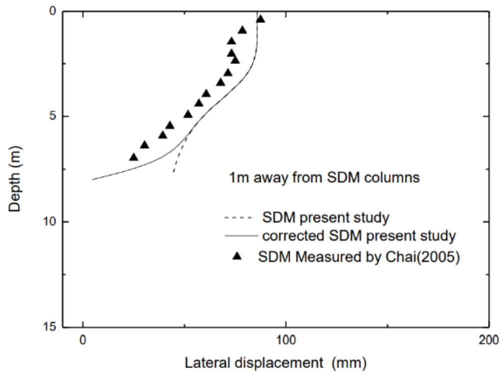

Fig. 4 Comparison of lateral displacement profile predicted for two present study

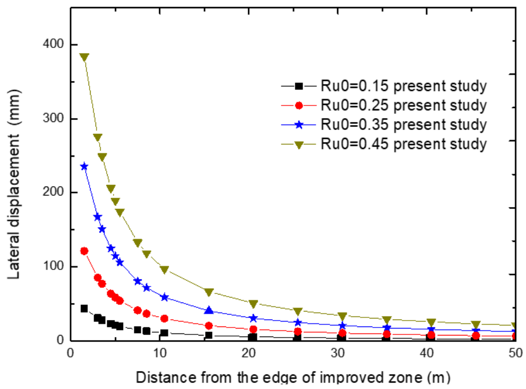

Fig. 5 Comparison effect of lateral displacement for R_{u0} as variable

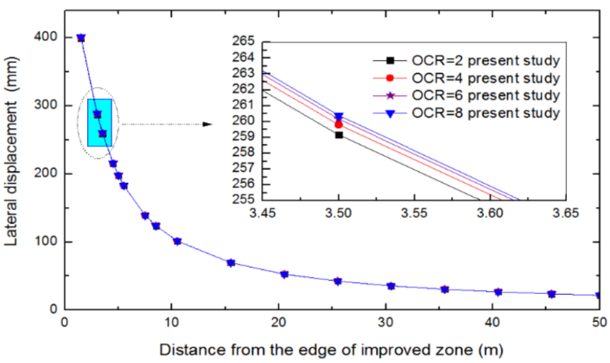

Fig. 6 Comparison effect of lateral displacement for OCR as variable

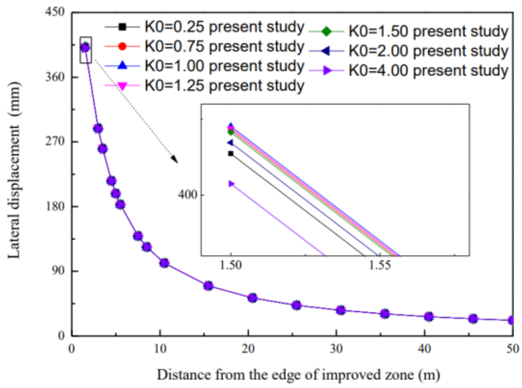

Fig. 7 Comparison effect of lateral displacement for K_0 as variable

not evident. Through the local amplification of the curve can be found: with the increase of over-consolidation ratio, there is a slight increase of lateral displacement. Thus, for the same distance from a pile, the larger the over-consolidation ratio is, the larger the lateral displacement of pile surrounding soil mass is.

When the different value of K_0 can be taken, the effect of lateral displacement is compared as shown the Fig. 7. The effect of K_0 on lateral displacement of multi-pile installation also is not evident. Through the local amplification of the curve can be found: with the increase of K_0, there is a slight increase in the lateral displacement. However, when $K_0 \leq 1$, with the increase of K_0value, the increase of lateral displacement is relatively less in the same increments, when $K_0=1$, the lateral displacement reaches peak value. It is shown that, if the effect of K_0 on lateral displacement is ignored, it will obtain conservative solutions and would lead to an overestimation of lateral displacement value in traditional solution (isotropic condition).

5. Limitations

Although the presented approach considers the effect of anisotropic initial stress and hydraulic fracture to soil mass at the pile tip, and the approach of lateral displacement during the single pile and multi-pile installation process is applied. However, it should be noted that there are still some limitations:

(1) The initial radius of the cavity must be determined empirically.

(2) It have not considered multi-pile installation's partial "plane strain" effect.

(3) It cannot simulate the actual pile incremental installation process.

6. Conclusions

A novel approach is proposed for predicting the lateral displacement caused by pile installation based on cylindrical and spherical cavity expansion theory, the main conclusions are as follows:

(1) A novel approach for predicting lateral displacement caused by single pile installation is applied based on the

both spherical and cylindrical cavities expansion, and considers the effect of hydraulic fracture to pile tip of single pile is considered.

(2) The novel approach of lateral displacement in the process of multi-pile installation is applied and considers the effect of anisotropic initial stress. The reliability of the approach is validated by compare measured data and some parameters of lateral displacement are selected for the parametric analysis.

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References

Ahn, H.Y., Oh, D.W. and Lee, Y.J. (2018), "Behaviour of vertically and horizontally loaded pile and adjacent ground affected by tunneling", *Geomech. Eng.*, **15**(3), 861-868. https://doi.org/10.12989/gae.2018.15.3.861.

Carter, J.P. and Yeung, S.K. (1985), "Analysis of cylindrical cavity expansion in a strain weakening material", *Comput. Geotech.*, **1**(3), 161-180. https://doi.org/10.1016/0266-352X(85)90021-7.

Carter, J.P., Booker, J.R. and Yeung, S.K. (1986), "Cavity expansion in cohesive frictional soils", *Géotechnique*, **36**(3), 345-358. https://doi.org/10.1680/geot.1986.36.3.349.

Carter, J.P., Randolph, M.F. and Wroth, C.P. (1979), "Stress and pore pressure changes in clay during and after the expansion of a cylindrical cavity", *Int. J. Numer. Anal. Meth. Geomech.*, **3**(4), 305-322. https://doi.org/10.1002/nag.1610030402.

Chai, J. C., Miura, N. and Koga, H. (2005), "Lateral displacement of ground caused by soil-cement column installation", *J. Geotech. Geoenviron. Eng.*, **131**(5), 623-632. https://doi.org/10.1061/(ASCE)1090-0241(2005)131:5(623).

Chai, J.C. and Carter, J.P. (2011), *Deformation Analysis in Soft Ground Improvement*, Springer Netherlands.

Chen, G.H., Zou, J.F. and Chen, J.Q. (2019a), "Shallow tunnel face stability considering pore water pressure in non-homogeneous and anisotropic soils", *Comput. Geotech.*, **116**, 103205. https://doi.org/10.1016/j.compgeo.2019.103205.

Chen, G.H., Zou, J.F., and Qian, Z.H. (2019b), "An improved collapse analysis mechanism for the face stability of shield tunnel in layered soils", *Geomech. Eng.*, **17**(1), 97-107. https://doi.org/10.12989/gae.2019.17.1.097.

Chen, S.L. and Abousleiman, Y.N. (2012), "Exact undrained elasto-plastic solution for cylindrical cavity expansion in modified cam clay soil mass", *Géotechnique,* **62**(5), 447-456. http://dx.doi.org/10.1680/geot.11.P.027.

Fattah, M. Y., Salim, N. M. and Al-Gharrawi, A. (2018), "Incremental filling ratio of pipe pile groups in sandy soil", *Geomech. Eng.*, **15**(1), 695-710. https://doi.org/10.12989/gae.2018.15.1.695.

Hill, R. (1950), *The Mathematical Theory of Plasticity*, Clarendon Press.

Keawsawasvong, S. and Ukritchon, B. (2016), "Ultimate lateral capacity of two dimensional plane strain rectangular pile in clay", *Geomech. Eng.*, **11**(2), 235-252. https://doi.org/10.12989/gae.2016.11.2.235.

Khanmohammadi, M. and Fakharian, K. (2018), "Evaluation of performance of piled-raft foundations on soft clay: A case study", *Geomech. Eng.*, **14**(1), 43-50. https://doi.org/10.12989/gae.2018.14.1.043.

Kim, Y.S. and Choi, J.I. (2017), "Nonlinear numerical analyses of a pile-soil system under sinusoidal bedrock loadings verifying centrifuge model test results", *Geomech. Eng.*, **12**(2), 239-255. https://doi.org/10.12989/gae.2017.12.2.239.

Ko, J., Cho, J. and Jeong, S. (2018), "Analysis of load sharing characteristics for a piled raft foundation", *Geomech. Eng.*, **16**(4), 449-461. https://doi.org/10.12989/gae.2018.16.4.449.

Kumara, J.J., Kurashina, T. and Kikuchi, Y. (2016), "Effects of pile geometry on bearing capacity of open-ended piles driven into sands", *Geomech. Eng.*, **11**(3), 385-400. https://doi.org/10.12989/gae.2016.11.3.385.

Kwon, J., Kim, C., Im, J.C. and Yoo, J.W. (2018), "Effect of performance method of sand compaction piles on the mechanical behavior of reinforced soft clay", *Geomech. Eng.*, **14**(2), 175-185. https://doi.org/10.12989/gae.2018.14.2.175.

Li, C. and Zou, J.F. (2019). "Created cavity expansion solution in anisotropic and drained condition based on Cam-Clay model." *Geomech. Eng.*, **19**(2), 141-151. https://doi.org/10.12989/gae.2019.19.2.141.

Li, C., Zou, J.F. and Zhou, H. (2019b), "Cavity expansions in k0 consolidated clay", *Eur. J. Environ. Civ. Eng.*, https://doi.org/10.1080/19648189.2019.1605937.

Li, C., Zou, J.F., and A, S.G. (2019a), "Closed-form solution for undrained cavity expansion in anisotropic soil mass based on the spatially mobilized plane failure criterion", *Int. J. Geomech.*, **19**(7), 04019075. https://doi.org/10.1061/(ASCE)GM.1943-5622.0001458.

Li, L., Li, J. and Sun, D. (2016). "Anisotropically elasto-plastic solution to undrained cylindrical cavity expansion in K0-consolidated clay". *Comput. Geotech.*, **73**, 83-90. https://doi.org/10.1016/j.compgeo.2015.11.022.

Li, L., Li, J. Sun, D. and Yue, Z. (2016), "Pile jacking-in effects considering stress anisotropy of natural clay", *Chin. J. Rock Mech. Eng.*, **35**(5), 1055-1064 (in Chinese).

Randolph, M.F. (2003), "Science and empiricism in pile foundation design", *Géotechnique*, **53**(10), 847-876. https://doi.org/10.1680/geot.2003.53.10.847.

Randolph, M.F., Carter, J.P. and Wroth, C.P. (1979), "Driven piles in clay-the effects of installation and subsequent consolidation", *Géotechnique*, **29**(4), 361-393. https://doi.org/10.1680/geot.1979.29.4.361.

Sagaseta, C. and Whittle, A.J. (2001), "Prediction of ground movements due to pile driving in clay", *J. Geotech. Geoenviron. Eng.*, **127**(1), 55-66. https://doi.org/10.1061/(ASCE)1090-0241(2001)127:1(55).

Sagaseta, C., Whittle, A.J. and Santagata, M. (1997), "Deformation analysis of shallow penetration in clay", *Int. J. Numer. Anal. Meth. Geomech.*, **21**(10), 687-719. https://doi.org/10.1002/(SICI)1096-9853(199710)21:10<687::AID-NAG897>3.0.CO;2-3.

Silvestri, V. and Abou-Samra, G. (2012), "Analytical solution for undrained plane strain expansion of a cylindrical cavity in modified Cam clay", *Geomech. Eng.*, **4**(1), 19-37. https://doi.org/10.12989/gae.2012.4.1.019.

Sun, D.A., Matsuoka, H., and Yao, Y.P. (2004), "An anisotropic hardening elastoplastic model for clays and sands and its application to FE analysis", *Comput. Geotech.*, **31**(1), 37-46. https://doi.org/10.1016/j.compgeo.2003.11.003.

Ukritchon, B., Faustino, J.C. and Keawsawasvong, S. (2016), "Numerical investigations of pile load distribution in pile group foundation subjected to vertical load and large moment", *Geomech. Eng.*, **10**(5), 577-598.

https://doi.org/10.12989/gae.2016.10.5.577.
Vesic, A.S. (1972), "Expansion of cavities in infinite soil mass", *J. Soil Mech. Found. Div.*, **98**(3), 265-290. https://trid.trb.org/view/125818.
Wang, S., and Yin, S. (2011), "A closed-form solution for a spherical cavity in the elastic-brittle-plastic medium", *Tunn. Undergr. Sp. Technol.*, **26**(1), 236-241. https://doi.org/10.1016/j.tust.2010.06.005.
Xiao, Y., Sun, Y., Yin, F., Liu, H. and Xiang, J. (2016), "Constitutive modeling for transparent granular soils", *Int. J. Geomech.,* 04016150. https://doi.org/10.1061/(ASCE)GM.1943-5622.0000857.
Yu, H.S. (2000), *Cavity Expansion Methods in Geomechanics*, Kluwer Academic Publishers.
Yu, H.S. and Houlsby, G.T. (1991), "Finite cavity expansion in dilatant soils: loading analysis", *Géotechnique*, **42**(4), 649-654. https://doi.org/10.1680/geot.1991.41.2.173.
Zhou, H., Liu, H., Kong, G. and Huang, X. (2014), "Analytical solution of undrained cylindrical cavity expansion in saturated soil under anisotropic initial stress", *Comput. Geotech.*, **55**(2), 232-239. https://doi.org/10.1016/j.compgeo.2013.09.011.
Zhou, H., Liu, H., Randolph, M.F., Kong, G. and Cao, Z. (2017), "Experimental and analytical study of X-section cast-in-place concrete pile installation influence", *Int. J. Phys. Model. Geotech.*, **17**(2), 1-19. https://doi.org/10.1680/jphmg.15.00037.
Zou, J. F., Wei, A. and Yang, T. (2018), "Elasto-plastic solution for shallow tunnel in semi-infinite space", *Appl. Math. Model.*, **64**(12), 669-687. https://doi.org/10.1016/j.apm.2018.07.049.
Zou, J.F. and Wei, X.X. (2018), "An improved radius-incremental-approach of stress and displacement for strain-softening surrounding rock considering hydraulic-mechanical coupling", *Geomech. Eng.*, **16**(1), 59-69. https://doi.org/10.12989/gae.2018.16.1.059.
Zou, J.F. and Zhang, P.H. (2019), "Analytical model of fully grouted bolts in pull-out tests and in situ rock masses", *Int. J. Rock. Mech. Min. Sci.*, **113**(1), 278-294. https://doi.org/10.1016/j.ijrmms.2018.11.015.
Zou, J.F., Chen, G. and Qian, Z. (2019), "Tunnel face stability in cohesion-frictional soils considering the soil arching effect by improved failure models", *Comput. Geotech.*, **106**, 1-17. https://doi.org/10.1016/j.compgeo.2018.10.014.
Zou, J.F., Chen, K.F. and Pan, Q.J. (2017), "Influences of seepage force and out-of-plane stress on cavity contracting and tunnel opening", *Geomech. Eng.*, **13**(6), 907-928. https://doi.org/10.12989/gae.2017.13.6.907.

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