

# Impact of spatial variability of geotechnical properties on uncertain settlement of frozen soil foundation around an oil pipeline

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**Abstract.** The spatial variability of geotechnical properties can lead to the uncertainty of settlement for frozen soil foundation around the oil pipeline, and it can affect the stability of permafrost foundation. In this paper, the elastic modulus, cohesion, angle of internal friction and poisson ratio are taken as four independent random fields. A stochastic analysis model for the uncertain settlement characteristic of frozen soil foundation around an oil pipeline is presented. The accuracy of the stochastic analysis model is verified by measured data. Considering the different combinations for the coefficient of variation and scale of fluctuation, the influences of spatial variability of geotechnical properties on uncertain settlement are estimated. The results show that the stochastic effects between elastic modulus, cohesion, angle of internal friction and poisson ratio are obviously different. The deformation parameters have a greater influence on stochastic settlement than the strength parameters. The overall variability of settlement reduces with the increase of horizontal scale of fluctuation and vertical scale of fluctuation. These results can improve our understanding of the influences of spatial variability of geotechnical properties on uncertain settlement and provide a theoretical basis for the reliability analysis of pipeline engineering in permafrost regions.

**Keywords:** foundations; spatial variability; geotechnical properties; settlement; permafrost regions

## 1. Introduction

Rocks and soils are formed under the influence of long-term weathering, handling, abrasion and sedimentation. Due to the influence of material composition, sedimentary conditions, geological tectonic movement and internal and external dynamic geological processes, the soil form different spatial structures in different geological periods. It shows local randomness and overall structure (Attia *et al.* 2018, Fatehi *et al.* 2018, Wijerathna and Liyanapathirana 2019, Pan *et al.* 2018a, b, 2019, Pramanik *et al.* 2019, Fei *et al.* 2019). In permafrost regions, the foundation soils are affected by the engineering activities and atmospheric environment, which can accelerate the degradation of permafrost foundation. And the temperature change will lead to a series of mechanical behavior variations of frozen soil (Ming *et al.* 2018, Ren *et al.* 2018, Wu *et al.* 2019, Wang *et al.* 2018, 2019a, b, Zhou *et al.* 2018, Kadivar and Manahiloh 2019). The settlement characteristics of geotechnical materials can affect the safety of geotechnical structures. Hence many studies had focused on the settlement characteristics of geotechnical engineering (Peduto *et al.* 2018, Ghiasi and Moradi 2018, Khanmohammadi and Fakharian 2018, Golpasand *et al.* 2019, Wang *et al.* 2019, Jiang *et al.* 2019, Moeinossadat and Ahangari 2019). For the pipeline engineering, the temperature change will lead to a series of mechanical

behavior changes of frozen soil, which have an adversely impact on the mechanical state of the pipeline, and it can seriously endanger the safety of the buried oil pipeline. Thus far, some scholars have been trying to estimate the deformation for the foundation soils surrounding the warm oil pipeline in permafrost regions, and the settlement analyses of permafrost by layer wise summation method have been developed (Wu *et al.* 2010, Wen *et al.* 2010, Wang *et al.* 2018, Li *et al.* 2019, Cherniavsky 2018, Zhang *et al.* 2019, Hazirbaba 2019). However, all of the researches of deformation characteristic for foundation soils surrounding the warm oil pipeline in permafrost regions are developed under the assumption that the mechanical parameters are deterministic.

In fact, the spatial variability of geotechnical properties is objective existence. The physical and thermodynamic parameters of soil have strong spatial variations. The spatial autocorrelation variations and spatial crosscorrelation variations of soil properties are the specific characteristic, and the spatial variability of soil properties can affect the mechanical properties and reliability of the geotechnical engineering (Bai *et al.* 2018, Bose and Rattan 2018, Zheng *et al.* 2018, Yao *et al.* 2019, Shakir and Talha 2019). Random field theory could be applied to quantify the correlation and uncertainty characteristics of geotechnical properties at different spatial locations. It can scientifically reflect the uncertain spatial variations of geotechnical materials, it is recognized that it can effectively describe the randomness of soil materials. At present, many literatures focus on the spatial variations and correlation structure of the uncertain material properties (Lombardi *et al.* 2017, Ma and Li 2018, Zhang *et al.* 2018, Cheng *et al.* 2018, Chenari

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*et al.* 2019). In permafrost regions, the spatial variability of geotechnical properties can lead to the randomness of the values properties that define the stress-strain relationship (Lai *et al.* 2008, 2012, Kemp *et al.* 2019). The different coefficient of variation and scale of fluctuation have a different effect on the uncertain settlement for the frozen soil foundation around an oil pipeline. Furthermore, some studies reported the stochastic thermal-mechanical characteristics of foundation soil in permafrost regions (Liu *et al.*, 2014; Wang *et al.*, 2018a, 2018b, 2019a, 2019b), and the stochastic thermal regime of frozen soil around an oil pipeline are obtained (Wang *et al.*, 2016). For the frozen soil, the randomness of soil temperature can lead to the randomness of mechanical parameters because they are closely related. Therefore, it is extremely significant to consider the spatial variability of geotechnical properties when the settlement analysis of frozen soil foundation around an oil pipeline is conducted. As a matter of fact, the frozen soil foundation around an oil pipeline is stratified just like rock, which is also caused by crustal movement and external weathering. The vertical scale of fluctuation and the horizontal scale of fluctuation are variable. The coefficient of variation and scale of fluctuation are the key parameters for the spatial variability of geotechnical properties and it can directly affect correlation structure of random field (Zhu *et al.* 2017, Alhasan *et al.* 2018, Titi *et al.* 2018). Therefore, it is necessary to study the influence of coefficient of variation and scale of fluctuation on the stochastic settlement for frozen soil foundation.

In permafrost regions, the temperature of frozen soil is very important to determine the mechanical properties. This paper focuses on the impact of spatial variability of geotechnical properties on settlement for frozen soil foundation around an oil pipeline. Based on the previous study of the random temperature field for the frozen soil foundation around an oil pipeline (Wang *et al.* 2016), considering the effect of stochastic temperature on the stochastic mechanical properties, the elastic modulus, cohesion, angle of internal friction and poisson ratio are taken as four independent random fields. A stochastic analysis of the uncertain settlement characteristic for frozen soil foundation around an oil pipeline is presented. Considering the different combinations for the coefficient of variation and scale of fluctuation, the influences of spatial variability of geotechnical properties on uncertain settlement are estimated. The distributions of mean settlement and variability are obtained, and the influences of spatial variability of geotechnical properties are analyzed. These results can provide an important reference for the safety of pipeline engineering.

## 2. Mathematical model and equations

### 2.1 Finite element equation

It is the principle of virtual work that the total work done by all forces on a system in static equilibrium is zero for a set of infinitesimally small displacements (Davis and Selvadurai 2002). The stress vector for the finite element node are given by

$$\int_V [B]^T \{\sigma\}_t dV = \int_V [N]^T \{f\}_t dV + \int_S [N]^T \{\bar{f}\}_t dS \quad (1)$$

where  $dV$  and  $dS$  are the volume increment and surface increment, respectively;  $[N]$  and  $[B]$  are the interpolation function matrix and geometric function matrix, respectively;  $\{f\}_t$  and  $\{\bar{f}\}_t$  are the body force vector and surface force vector, respectively;  $\delta\{\varepsilon\}$  and  $\delta\{u\}$  are the virtual strain vector and virtual displacement vector, respectively;  $t$  is the load step;  $\{\sigma\}_t$  is the stress vector.

According to Eq. (1), the incremental computational formulae can be expressed as

$$\int_V [B]^T \{\Delta\sigma\} dV = \int_V [N]^T \{\Delta f\} dV + \int_S [N]^T \{\Delta\bar{f}\} dS \quad (2)$$

In the actual numerical calculation, the load effect of the physical column vector is taken into account in the initial condition; therefore, Eq. (2) can be simplified as

$$\int_V [B]^T \{\Delta\sigma\} dV = \int_S [N]^T \{\Delta\bar{f}\} dS \quad (3)$$

### 2.2 Stress-strain relationship

The cohesion and angle of internal friction is the important mechanical parameters for the frozen soil. They can lead to the shear failure. In this paper, the Mohr-Coulomb yielding criteria are adopted, and the yield function can be expressed as

$$f = p \sin \varphi + \frac{q}{6} \left[ (\cos \theta - \sqrt{3} \sin \theta) \sin \varphi - (3 \cos \theta + \sqrt{3} \sin \theta) \right] + c \cos \varphi \quad (4)$$

where  $p$  and  $q$  are the spherical stress and deviatoric stress, respectively;  $\varphi$  and  $c$  are the angle of internal friction and cohesion, respectively;  $f$  and  $\theta$  are the yield surface and Lode angle, respectively.

According to the principles of geotechnical plastic mechanics (Zheng *et al.* 2002), the incremental stress-strain relationship is written as follows

$$d\sigma_{ij} = C_{ijkl}^{ep} d\varepsilon_{kl} \quad (5)$$

$$C_{ijkl}^{ep} = C_{ijkl}^e - \frac{C_{ijmm}^e \frac{\partial g}{\partial \sigma_{mm}} \frac{\partial f}{\partial \sigma_{st}} C_{stkl}^e}{A + \frac{\partial f}{\partial \sigma_{ij}} C_{ijkl}^e \frac{\partial g}{\partial \sigma_{kl}}} \quad (6)$$

where  $C_{ijkl}^{ep}$  is the elastic-plastic stiffness tensor;  $A$  is the hardening function;  $C_{ijkl}^e$  is the elastic stiffness tensor;  $g$  is the plastic potential function, and the yield function is equal to the plastic potential function.

In Eq. (6), the hardening function is determined by the consolidation parameters and potential strength. The detailed analysis and calculation process had been developed (Wang 2015). According to the broad Hooke's law, the elastic stiffness tensor can be expressed as

$$C_{ijkl}^e = \left( K - \frac{2}{3} G \right) \delta_{ij} \delta_{kl} + G (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (7)$$

Substituting Eqs. (4), (6) and (7) into (5), the stress-strain relationship can be determined.

### 2.3 Stochastic analysis process

The uncertainty of the geotechnical properties is objective existence because of the complex geological structure can be applied to quantify the correlation between any two observations in a field. Its essence is to use normal random field to simulate geotechnical parameters, and the variance, variance function, correlation function, scale of fluctuation and correlation distance are used to describe spatial variability and correlation of soil properties (Vanmarcke 2010, Cheng *et al.* 2019). In this study, the elastic modulus, cohesion, angle of internal friction and poisson ratio are taken as four independent 2D random fields, respectively. The rectangular elements can be used to divide the random field, and the local average element is defined as

$$X_e = \frac{1}{A_e} \int_{\Omega_e} X(x, y) dx dy \quad (8)$$

where  $A_e$  is the area; and  $\Omega_e$  is the domain of integration.

According to Eq.(8), the covariance of two elements can be expressed as

$$\text{Cov}(X_i, X_j) = \frac{\sigma^2}{4A_i A_j} \sum_{k=0}^3 \sum_{l=0}^3 (-1)^k (-1)^l (L_{1k} L_{2l})^2 \Gamma^2(L_{1k}, L_{2l}) \quad (9)$$

where  $\sigma^2$  is the variance of random field;  $L_{1k}$  and  $L_{2l}$  are the distances of the relative location for the two elements.  $\Gamma^2(L_{1k}, L_{2l})$  is the variance function of random field. The expression formula of  $\Gamma^2(L_{1k}, L_{2l})$  is

$$\Gamma^2(L_{xi}, L_{yi}) = \frac{4}{L_{xi} L_{yi}} \int_0^{L_{xi}} \int_0^{L_{yi}} \left(1 - \frac{\xi}{L_{xi}}\right) \left(1 - \frac{\eta}{L_{yi}}\right) \rho(\xi, \eta) d\xi d\eta \quad (10)$$

where  $\rho(\xi, \eta)$  is the correlation function of the random field.

Based on random field theory, the anisotropic spatial variations of soils and rocks had been developed (Zhu and Zhang 2013). The correlation structure of random field can be expressed as exponential correlation functions. The expression formula is

$$\rho(\xi, \eta) = \exp\left(-2\sqrt{\frac{\xi^2}{\theta_x^2} + \frac{\eta^2}{\theta_y^2}}\right) \quad (11)$$

where  $\theta_x$  is the horizontal scale of fluctuation;  $\theta_y$  is the vertical scale of fluctuation.

The covariance matrices can be calculated by Eq.(9), and then the stochastic settlement characteristic of frozen soil foundation can be calculated by NSFEM (Wang *et al.* 2018b). After obtaining the settlement of foundation soil for every stochastic simulation, the statistical properties (such as average value, standard deviation and coefficient of variation) can be analyzed by mathematical statistics method. In this study, according to above finite element equation, stress-strain relationship and stochastic analysis process, a calculation program of stochastic settlement characteristic was compiled in MATLAB 7.0. The average value, standard deviation and coefficient of variation can be directly outputted with the compiled program.

### 3. Description of the parameter and boundary conditions

Fig. 1 shows the computational model for a frozen soil

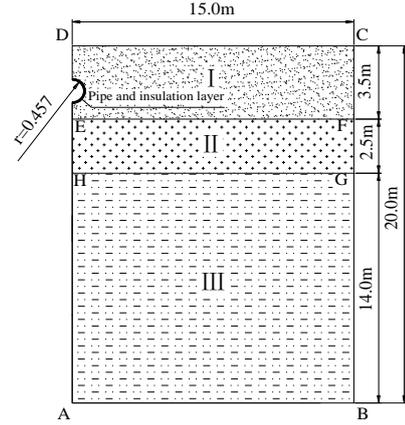


Fig. 1 The numerical computational model. Part I is silty clay; Part II is gravel soil and part III is bed rock

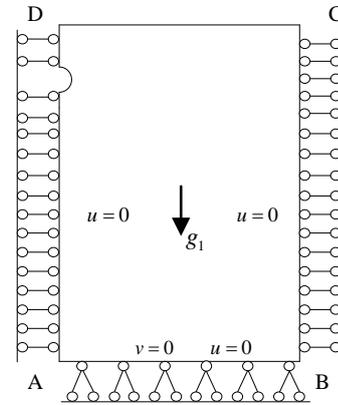


Fig. 2 Load and boundary conditions

foundation around an oil pipeline. It is the numerical analysis model of the Mo-Da oil pipeline in Northeast China. From the stress-strain relationship, the elastic modulus, cohesion, angle of internal friction and poisson ratio need to be determined. As is known to all, these mechanical parameters are closely related to the temperature. Different temperatures have different values for the elastic modulus  $E_T$ , cohesion  $c_T$ , angle of internal friction  $\phi_T$  and poisson ratio  $\nu_T$ . According to the previous studies (Wu *et al.* 1988, Li *et al.* 2009, Liu *et al.* 2019), the mathematic relation between the mechanical parameters and thermal properties can be expressed as

$$\begin{cases} E_T = a_1 + b_1 |T|^m \\ c_T = a_2 + b_2 |T| \\ \phi_T = a_3 + b_3 |T| \\ \nu_T = a_4 + b_4 |T| \end{cases} \quad (12)$$

In Eq.(12),  $a_i$  and  $b_i$  are the test parameters, and the detailed values are shown in Table 1.  $m$  is a nonlinear exponent and it usually equals 0.6. Based on previous studies (Wang *et al.* 2016), the stochastic temperature of the frozen soil foundation around an oil pipeline have been obtained. Therefore, the elastic modulus, cohesion, angle of internal friction and poisson ratio can be calculated by Eq.(12). Fig. 2 shows the calculation model for the load

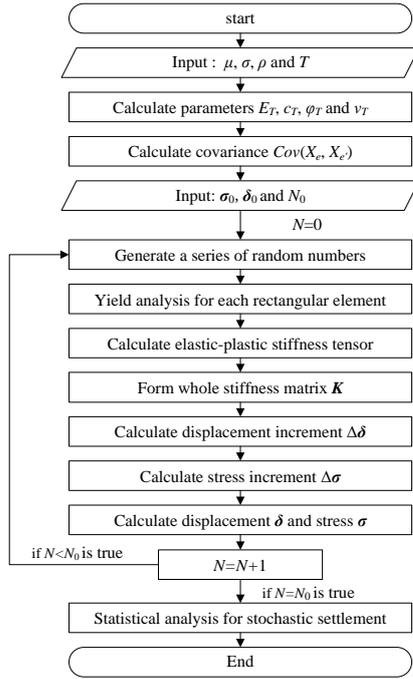


Fig. 3 Calculation flow chart of a load step

Table 1 Basic mechanical parameters of foundation soil

Physical parameters	$\gamma$ (kN·m <sup>-3</sup> )	$a_1$ (MPa)	$b_1$	$a_2$ (MPa)	$b_2$	$a_3$	$b_3$	$a_4$	$b_4$
Silty clay	18.3	58	51	0.03	0.092	23	9.5	0.35	0.007
Gravel soil	19.4	28	26	0.16	0.088	21	8	0.40	0.008
Bed rock	21.6	80	76	0.12	0.241	27	11	0.25	0.004

Table 2 Different groups of coefficients of variation

No.	1	2	3	4	5	6	7	8	9	10	11	12
$E_T$	0.1	0.2	0.3	COV								
$c_T$	COV	COV	COV	0.1	0.2	0.3	COV	COV	COV	COV	COV	COV
$\phi_T$	COV	COV	COV	COV	COV	COV	0.1	0.2	0.3	COV	COV	COV
$v_T$	COV	0.1	0.2	0.3								

Table 3 Different groups of horizontal scale of fluctuation

No.	1	2	3	4	5	6	7	8
$\theta_x$	2	5	10	20	50	100	200	500
$\theta_y$	HD	HD	HD	HD	HD	HD	HD	HD

Table 4 Different groups of vertical scale of fluctuation

No.	1	2	3	4	5	6	7	8
$\theta_x$	VD							
$\theta_y$	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8

effect and boundary constraint. AB, BC, CD and AD represent the bottom boundary, right lateral boundary, upper native surfaces and left lateral boundary. In detail, the horizontal direction and vertical direction of CD are unconstrained; the horizontal direction of AD and BC are constrained while the vertical direction of AD and BC are

free; the horizontal direction and vertical direction of the AB are constrained. For each load step, a stochastic analysis process for the uncertain settlement characteristic need to be conducted. Fig. 3 is the detailed calculation flow chart of a load step.

To study the impacts of coefficients of variation, horizontal and vertical scale of fluctuation on the uncertain settlement of frozen soil foundation around an oil pipeline, Table 2, Table 3 and Table 4 are the different groups in detail.

## 4. Worked examples

### 4.1 Validation and comparison with the computed and measured settlements

To validate the stochastic analysis model of the uncertain settlement characteristic of frozen soil foundation around an oil pipeline, a validation and comparison with the computed and measured settlements is given in Fig.4. The measured settlements of the frozen soil ground for the 1.0 m and 3.0 m away from the oil pipe centerline are obtained by in-situ monitoring method (Chen 2007, Zheng 2011). It can be seen from Fig. 4(a) and 4(b) that the computed mean settlements well agree with the measured mean settlements. Therefore, the stochastic analysis model used in this study can calculate the uncertain settlement characteristic of frozen soil foundation around an oil pipeline. From Fig. 4(a), the maximum difference between the computed and measured settlements for 1.0 m away from the oil pipe centerline is only 0.79 cm. From Fig. 4(b), the maximum difference between the computed and measured settlements for 3.0 m away from the oil pipe centerline is only 0.91 cm. Therefore, the stochastic analysis model has sufficient accuracy.

### 4.2 Distribution of stochastic settlement characteristic

In general, the air temperature is the highest in July and the heat conduction between the permafrost surface and atmosphere is very intense. The air temperature is the higher in October and the heat conduction in foundation soil is very obvious. There is a lot of melting frozen soil in those times and the settlements of foundation soils surrounding

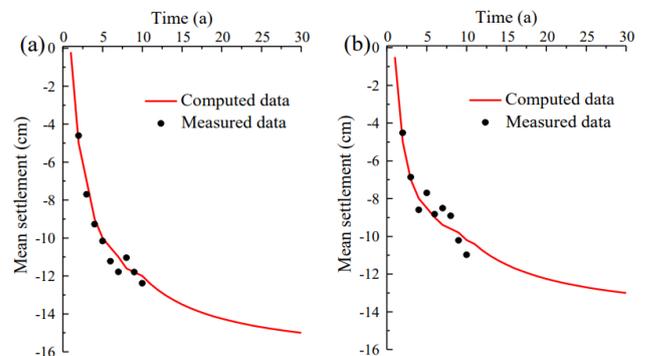


Fig. 4 Comparison between the computed and measured settlements for the frozen soil ground: (a) 1.0 m away from the oil pipe centerline and (b) 3.0 m away from the oil pipe centerline

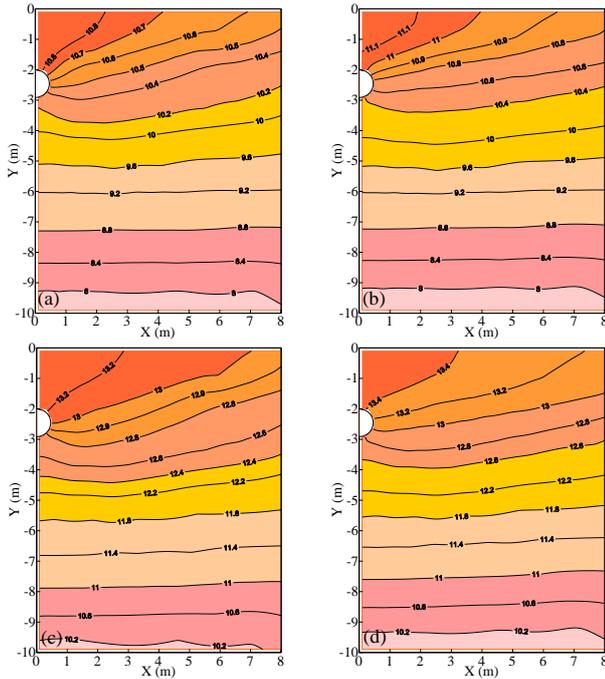


Fig. 5 Distributions of mean settlement around the oil pipe after construction: (a) on July 15 of the 10th year, (b) on October 15 of the 10th year, (c) on July 15 of the 30th year and (d) on October 15 of the 30th year (Unit: cm)

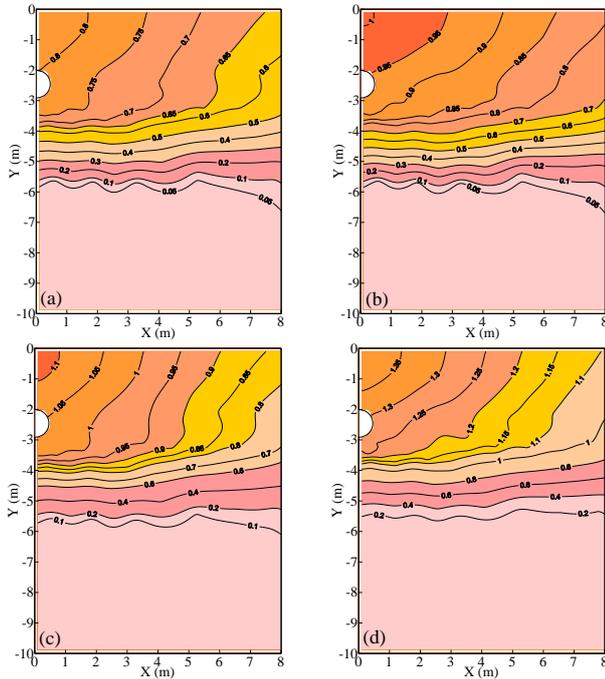


Fig. 6 Distributions of standard deviation around the oil pipe after construction: (a) on July 15 of the 10th year, (b) on October 15 of the 10th year, (c) on July 15 of the 30th year and (d) on October 15 of the 30th year. (Unit: cm)

From the distribution of the mean settlement, the maximum mean settlement is 10.8 cm and 11.1 cm on July 15 and October 15 after 10 years, respectively. The mean settlements are larger above the pipe because of the high temperature of the crude oil pipeline. From Figs. 5(c) and 5(d), the maximum mean settlement is 13.2 cm and 13.4 cm on July 15 and October 15 after 30 years, respectively. As for a further comment, from the 10th year to the 30th year after construction, the maximum mean settlement increases by 2.4 cm on July 15 while it increases by 2.3 cm on October 15 because of the climatic warming. When both the horizontal scale of fluctuation and vertical scale of fluctuation are 2.0 m, Fig. 6 shows the standard deviation for the settlement of the frozen soil foundation. The coefficient of variation is assumed to be 0.1 in the calculation process.

From the distribution of the standard deviation after 10 years of operation, the maximum standard deviation is 0.8cm and 1.0cm on July 15 and October 15, respectively. It can be seen from Fig. 6(c) and 6(b) that the maximum standard deviation is 1.1cm and 1.35cm on July 15 and October 15, respectively. From the 10th year to the 30th year after construction, the maximum standard deviation increases with time. According to the distribution of stochastic settlement characteristic, it can be conclude that the spatial variability of geotechnical properties can lead to the uncertainty of settlement for frozen soil foundation around the oil pipeline, and it can affect the stability of permafrost foundation.

#### 4.3 Impacts of coefficient of variation on standard deviation

In order to elucidate the impact of coefficient of variation of uncertain mechanical parameters on stochastic settlement of frozen soil foundation, different coefficients of variation (0.1, 0.2 and 0.3) for the elastic modulus, cohesion, angle of internal friction and poisson ratio are taken into account, respectively. Table 2 is the different groups. The variations of standard deviation for the settlement are shown in Fig. 7. The variability analysis of the settlement for the surface of frozen soil foundation is very important, and it can directly affect the safety of pipeline foundation. Excessive deformation can cause the pipe to be exposed to the frozen soil surface, so the standard deviation in Fig. 7 represents the settlement for the surface of frozen soil foundation.

When  $COV = 0$ , the separated impact of each coefficient of variation is obtained in Fig. 7(a). It can be seen that the impacts between elastic modulus, cohesion, angle of internal friction and poisson ratio are obviously different. The elastic modulus has a more obvious impact than the cohesion, angle of internal friction and poisson ratio. In detail, the maximum standard deviation of settlement is 1.39cm when the coefficient of variation of elastic modulus is assumed to be 0.3. The angle of internal friction has the minimum effect on the variability of settlement. The minimum standard deviation of settlement is 0.81cm when the coefficient of variation of angle of internal friction is assumed to be 0.1. When  $COV = 0.1, 0.2$  and  $0.3$ , the combined impact of different coefficients of variation are

the oil pipe are larger. Therefore, the mean settlements of foundation soils surrounding the oil pipe on July 15 and October 15 are shown in Fig. 5.

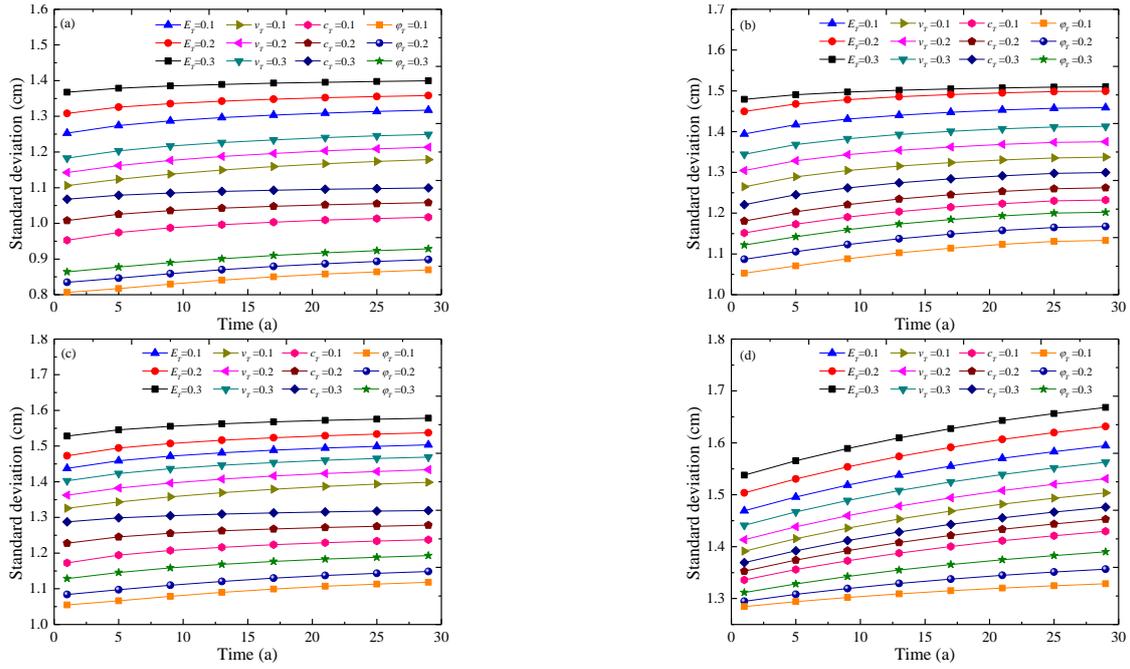


Fig. 7 Variability of settlement with different coefficient of variation: (a)  $COV = 0$ , (b)  $COV = 0.1$ , (c)  $COV = 0.2$  and (d)  $COV = 0.3$

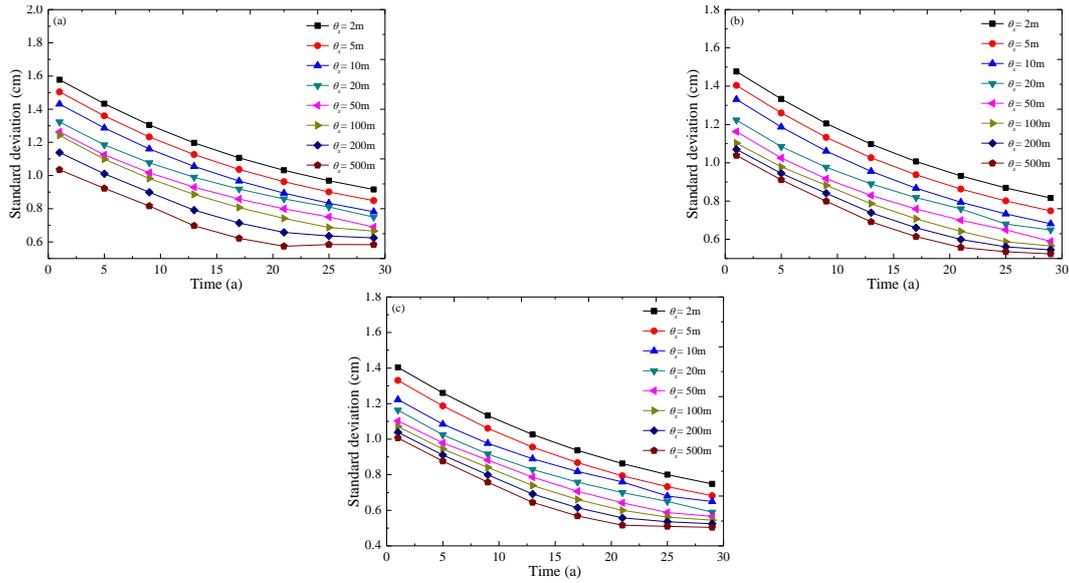


Fig. 8 Variability of settlement with different horizontal scale of fluctuation: (a)  $HD = 0.8$  m, (b)  $HD = 1.0$  m and (c)  $HD = 1.2$  m

obtained in Fig. 7(b)-7(d). It is obvious that the stochastic influences of elastic modulus, cohesion, angle of internal friction and poisson ratio are different. On balance, the elastic modulus has a greatest effect on the stochastic settlement while the angle of internal friction has a least influence on the stochastic settlement. As shown in Fig.7 (b)-7(c), the maximum standard deviation of settlement is 1.51 cm, 1.58 cm and 1.67 cm when the coefficient of variation of elastic modulus is 0.3 and  $COV = 0.1, 0.2$  and  $0.3$ , respectively. the minimum standard deviation of settlement is 1.04 cm, 1.06 cm and 1.28 cm when the coefficient of variation of angle of internal friction is 0.1 and  $COV = 0.1, 0.2$  and  $0.3$ , respectively. From Fig. 7(a)-

7(d), it can be concluded that the deformation parameter (elastic modulus and poisson ratio) have a greater influence than the strength parameters (cohesion and angle of internal friction). Therefore, the traditional settlement analysis can not consider the coefficients of variation and the stochastic settlement analysis is necessary.

#### 4.4 Impacts of horizontal scale of fluctuation on standard deviation

In order to elucidate the impact of coefficient of variation of uncertain mechanical parameters on stochastic settlement of frozen soil foundation, different coefficients

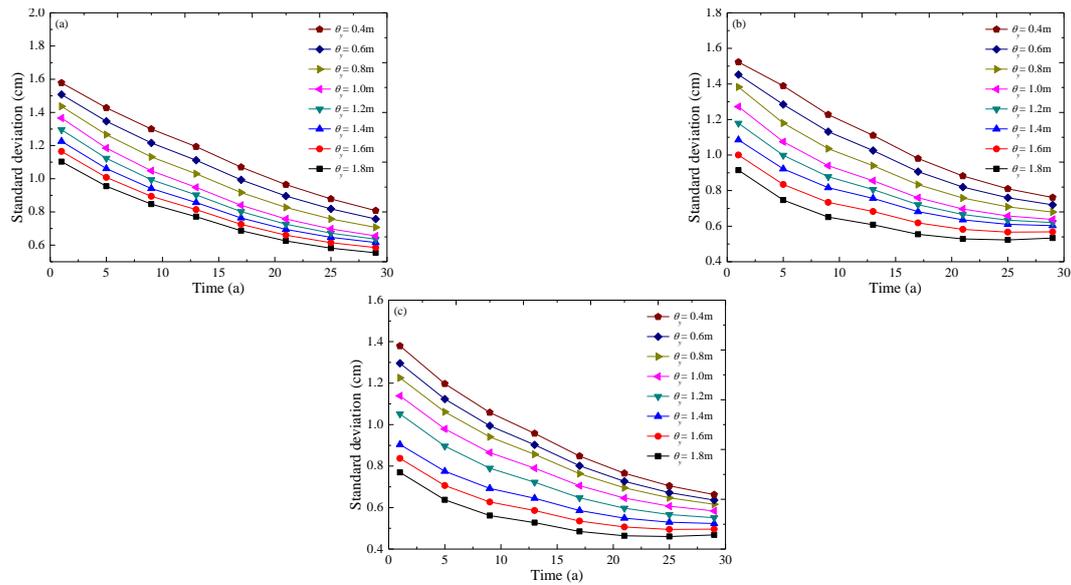


Fig. 9 Variability of settlement with different vertical scale of fluctuation: (a)  $VD = 250$  m, (b)  $VD = 500$  m and (c)  $VD = \infty$

of variation (0.1, 0.2 and 0.3) for the elastic modulus, cohesion, angle of internal friction and poisson ratio are taken into account, respectively. Table 2 is the different groups. The variations of standard deviation for the settlement are shown in Fig.7. The variability analysis of the settlement for the surface of frozen soil foundation is very important, and it can directly affect the safety of pipeline foundation. Excessive deformation can cause the pipe to be exposed to the frozen soil surface, so the standard deviation in Fig.7 represents the settlement for the surface of frozen soil foundation.

The variability analysis of the settlement for the surface is very important, and it can directly affect the safety of pipeline foundation. Excessive deformation can cause the pipe to be exposed to the frozen soil surface and result in the erosion of the pipe, so the standard deviation in Fig.8 represents the stochastic settlement for the surface. From Fig. 8(a), it is clear from the line graph that the variability of settlement reduces with the horizontal scale of fluctuation and time. In detail, when the horizontal scale of fluctuation is 2m and vertical scale of fluctuation is 0.8m, the variability of settlement drops from 1.58 cm to 0.92 cm; when the horizontal scale of fluctuation is 500m and vertical scale of fluctuation is 0.8 m, the variability of settlement falls from 1.04 cm to 0.57 cm. As shown in Fig. 8(b)-8(c), the variability of settlement still reduces with the horizontal scale of fluctuation and time. For example, the variability of settlement drops from 1.41cm to 0.75cm when the horizontal scale of fluctuation is 2m and vertical scale of fluctuation is 1.2 m. From Fig. 8(a)-8(c), it can be concluded that the overall variability of settlement reduces with the increase of vertical scale of fluctuation. As we all know, the bigger the scales of fluctuation, the smaller the variability of mechanical parameter. Therefore, the variability of settlement with different scale of fluctuation is reasonable.

#### 4.5 Impacts of vertical scale of fluctuation on standard deviation

As a matter of fact, the foundation soil is stratified just

like rock, which is also caused by crustal movement and external weathering. The vertical scale of fluctuation is much smaller than the horizontal scale of fluctuation because of this layer. In order to elucidate the impact of vertical scale of fluctuation of mechanical parameter on stochastic settlement of frozen soil foundation, eight values of vertical scale of fluctuation (0.4 m, 0.6 m, 0.8 m, 1.0 m, 1.2 m, 1.4 m, 1.6 m and 1.8 m) for elastic modulus, cohesion, angle of internal friction and poisson ratio are taken into account, respectively. Table 4 is the different groups of vertical scale of fluctuation. The variability of settlement with different vertical scale of fluctuation is obtained in Fig. 9.

From Fig. 9(a), it can be seen from the line graph that the variability of settlement reduces with the vertical scale of fluctuation and construction time. In particular, when the vertical scale of fluctuation is 0.4 m and horizontal scale of fluctuation is 250 m, the variability of settlement falls from 1.61 cm to 0.81 cm; when the vertical scale of fluctuation is 1.8 m and horizontal scale of fluctuation is 250 m, the variability of settlement drops from 1.11 cm to 0.55 cm. As shown in Fig. 9(b), the variability of settlement still reduces with the vertical scale of fluctuation and construction time. For instance, the variability of settlement drops from 1.52 cm to 0.76 cm when the vertical scale of fluctuation is 0.4m and horizontal scale of fluctuation is 500m. In this study, a special case is considered, which the horizontal scale of fluctuation is infinite. It means that only vertical spatial variability is considered. The variability results for the limiting conditions are shown in Fig .9(c). From Fig. 9(a)-9(c), it can be concluded that the overall variability of settlement reduces with the horizontal scale of fluctuation.

## 5. Discussion

The temperature of frozen soil is very important to determine the mechanical properties in permafrost regions. This study focuses on the impact of spatial variability of

geotechnical properties on settlement for frozen soil foundation around an oil pipeline. It is closely related to the previous study (Wang *et al.* 2016). According to Eq.(12), the mechanical parameters of frozen soil are closely related to temperature. In fact, the impact of temperature on mechanical parameters is considered while the impact of mechanical parameters on temperature is not considered i.e., it is one-way stochastic effect. Therefore, this paper is a continuation of the previous study. The deterministic one-way coupling analyses have been presented and the deterministic settlement evaluations have been developed (Wu *et al.* 2010, Wen *et al.* 2010). The stochastic settlement evaluation of this study has a big improvement over previous research. The research results of multi-field coupling can provide a reference for future coupling analysis (Yang *et al.* 2019, Liu *et al.* 2019). Second, the coefficient of variation of the uncertain mechanical parameters needs enough statistical parameter. The Mo-Da oil pipeline located in Greater Khingan Mountains, and the environment is very terrible (very cold, dry, strong ultraviolet and low pressure). Obtaining enough statistical parameter needs a lot of material and financial resources. Based on the distribution laws of mechanical parameter (Li 2008), the coefficients of variation for the elastic modulus, cohesion, angle of internal friction and poisson ratio are made some assumptions at present. Third, the 2D random field can accurately reflect the 2D spatial variability, and a 3D analysis can consider the stochastic interaction of axial direction for the frozen soil foundation around an oil pipeline. Developing 3D random field theory and local average method is very important to estimate the 3D spatial variability of geotechnical properties on settlement for frozen soil foundation around an oil pipeline. However, this study can clarify the influences of spatial variability of geotechnical properties on uncertain settlement of frozen soil foundation.

## 6. Conclusions

In this study, the structural resistance deterioration, such as residual bond strength and load bearing capacity, caused by reinforcement corrosion is investigated. A stochastic deterioration model is then employed to evaluate the failure probability of the corroded RC beam during the service life. The results for the flexural strength deterioration due to reinforcement corrosion are then examined by the experimental and field data available from various sources.

On the basis of the results from the worked examples involving a case study of Ullasund Bridge and RC beam subject to reinforcement corrosion, the following conclusions are drawn: 1) The proposed approach is capable of evaluating the lifecycle performance deterioration of concrete structures subjected to reinforcement corrosion; 2) Flexural strength decreases significantly after critical corrosion level due to significant reduction in bond strength loss. Further progress of corrosion causes significant reduction in rebar size which in turn widens the crack in concrete cover, and consequently reduces both residual bond and flexural strength; 3) The proposed stochastic

deterioration model based on the gamma process can effectively assess the structural reliability and the failure probability of corrosion affected RC structures, depending on many factors such as predefined allowable limit of deterioration, concrete cover depth and confinement of the concrete. The reliability of the corroded structure decreases with the progress of corrosion induced cracking in concrete.

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## References

- Alhasan, A., Ali, A., Offenbacher, D., Smadi, O. and Lewis-Beck, C. (2018), "Incorporating spatial variability of pavement foundation layers stiffness in reliability-based mechanistic-empirical pavement performance prediction", *Transport. Geotech.*, **17**(PartA), 1-13. <https://doi.org/10.1016/j.trgeo.2018.08.001>.
- Attia, M.A., Eltahir, M.A., Soliman, A., Abdelrahman, A.A. and Alshorbagy, A.E. (2018), "Thermoelastic crack analysis in functionally graded pipelines conveying natural gas by an FEM", *Int. J. Appl. Mech.*, **10**(04), 1850036. <https://doi.org/10.1142/S1758825118500369>.
- Bai, T., Hu, X. and Gu, F. (2018), "Practice of searching a noncircular critical slip surface in a slope with soil variability", *Int. J. Geomech.*, **19**(3), 04018199. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001350](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001350).
- Bose, T. and Rattan, M. (2018), "Modeling creep analysis of thermally graded anisotropic rotating composite disc", *Int. J. Appl. Mech.*, **10**(06), 1850063. <https://doi.org/10.1142/S1758825118500631>.
- Chen, Q.L. (2007), "Engineering geological research on the permafrost in high latitude area and its impact on pipeline construction", Ph.D. Thesis, Chinese Academy of Geological Sciences, Beijing, China (in Chinese).
- Chenari, R.J., Fatahi, B., Ghoreishi, M. and Taleb, A.(2019), "Physical and numerical modelling of the inherent variability of shear strength in soil mechanics", *Geomech. Eng.*, **17**(1), 31-45. <http://doi.org/10.12989/gae.2019.17.1.031>.
- Cheng, H., Chen, J. and Li, J. (2019), "Probabilistic analysis of ground movements caused by tunneling in a spatially variable soil", *Int. J. Geomech.*, **19**(12), 04019125. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001526](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001526).
- Cheng, H., Chen, J., Chen, R. and Chen, G. (2018), "Comparison of modeling soil parameters using random variables and random fields in reliability analysis of tunnel face", *Int. J. Geomech.*, **19**(1), 04018184. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001330](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001330).
- Cherniavsky, A. (2018), "Ratcheting analysis of "pipe-freezing soil" interaction", *Cold Reg. Sci. Technol.*, **153**, 97-100. <https://doi.org/10.1016/j.coldregions.2018.05.005>.
- Davis, R.O. and Selvadurai, A.P.S. (2002), *Plasticity and Geotechnics*, Cambridge University Press.
- Fatehi, M.R., Ghanbarzadeh, A., Moradi, S. and Hajnayeb, A. (2018), "Determination of random matrices dispersion parameters for nonparametric modeling of stochastic dynamic systems with experimental verification", *Int. J. Appl. Mech.*,

- 10(09), 1850101. <https://doi.org/10.1142/S1758825118501016>.
- Fei, S., Tan, X., Wang, X., Du, L. and Sun, Z. (2019), "Evaluation of soil spatial variability by micro-structure simulation", *Geomech. Eng.*, **17**(6), 565-572. <https://doi.org/10.12989/gae.2019.17.6.565>.
- Ghiyasi, V., and Moradi, M. (2018), "Assessment the effect of pile intervals on settlement and bending moment raft analysis of piled raft foundations", *Geomech. Eng.*, **16**(2), 187-194. <https://doi.org/10.12989/gae.2018.16.2.187>.
- Golpasand, M.R.B., Do, N.A. and Dias, D. (2019), "Impact of pre-existent Qanats on ground settlements due to mechanized tunneling", *Transport., Geotech.*, **21**, 100262. <https://doi.org/10.1016/j.trgeo.2019.100262>.
- Hazirbaba, K. (2019), "Effects of freeze-thaw on settlement of fine grained soil subjected to cyclic loading", *Cold Reg. Sci. Technol.*, **160**, 222-229. <https://doi.org/10.1016/j.coldregions.2019.02.008>.
- Jiang, H., Li, X., Xin, G., Yao, Z., Zhang, J. and Liang, M. (2019), "Geometry mapping and additional stresses of ballastless track structure caused by subgrade differential settlement under self-weight loads in high-speed railways", *Transport., Geotech.*, **18**, 103-110. <https://doi.org/10.1016/j.trgeo.2018.10.007>.
- Kadivar, M., and Manahiloh, K.N. (2019), "Revisiting parameters that dictate the mechanical behavior of frozen soils", *Cold Reg. Sci. Technol.*, **163**, 34-43 <https://doi.org/10.1016/j.coldregions.2019.04.005>.
- Kemp, J.E., Davies, E.G. and Loewen, M.R. (2019), "Spatial variability of ice thickness on stormwater retention ponds", *Cold Reg. Sci. Technol.*, **159**, 106-122. <https://doi.org/10.1016/j.coldregions.2018.12.010>.
- Khanmohammadi, M. and Fakharian, K. (2018), "Evaluation of performance of piled-raft foundations on soft clay: A case study", *Geomech. Eng.*, **14**(1), 43-50. <https://doi.org/10.12989/gae.2018.14.1.043>.
- Lai, Y.M., Li, J.B. and Li, Q.Z. (2012), "Study on damage statistical constitutive model and stochastic simulation for warm ice-rich frozen silt", *Cold Reg. Sci. Technol.*, **71**(2), 102-110. <https://doi.org/10.1016/j.coldregions.2011.11.001>.
- Lai, Y.M., Li, S.Y., Qi, J.L., Gao, Z.H. and Chang, X.X. (2008), "Strength distributions of warm frozen clay and its stochastic damage constitutive model", *Cold Reg. Sci. Technol.*, **53**(2), 200-215. <https://doi.org/10.1016/j.coldregions.2007.11.001>.
- Li, H., Lai, Y., Wang, L., Yang, X., Jiang, N., Li, L. and Yang, B. (2019), "Review of the state of the art: interactions between a buried pipeline and frozen soil", *Cold Reg. Sci. Technol.*, **157**, 171-186. <https://doi.org/10.1016/j.coldregions.2018.10.014>.
- Li, S.Y. (2008), "Numerical Study on the Thermal-mechanical Stability of Railway Subgrade in Permafrost Regions", Ph.D. Thesis, Graduate University of the Chinese Academy of Sciences, Lanzhou, China (in Chinese).
- Li, S.Y., Lai, Y.M., Zhang, M.Y. and Dong, Y.H. (2009), "Study on long-term stability of Qinghai-Tibet Railway embankment", *Cold Reg. Sci. Technol.*, **57**(2-3), 139-147. <https://doi.org/10.1016/j.coldregions.2009.02.003>.
- Liu, H., Maghoul, P., Shalaby, A. and Bahari, A. (2019), "Thermo-hydro-mechanical modeling of frost heave using the theory of poroelasticity for frost-susceptible soils in double-barrel culvert sites", *Transport. Geotech.*, **20**, 100251. <https://doi.org/10.1016/j.trgeo.2019.100251>.
- Liu, X.Q., Liu, J.K., Tian, Y.H., Chang, D. and Hu, T.F. (2019), "Influence of the freeze-thaw effect on the Duncan-Chang model parameter for lean clay", *Transport. Geotech.*, **21**, 100273. <https://doi.org/10.1016/j.trgeo.2019.100273>.
- Liu, Z.Q., Yang, W.H. and Wei, J. (2014), "Analysis of random temperature field for freeway with wide subgrade in cold regions", *Cold Reg. Sci. Technol.*, **106-107**, 22-27. <https://doi.org/10.1016/j.coldregions.2014.06.004>.
- Lombardi, M., Cardarilli, M. and Raspa, G. (2017), "Spatial variability analysis of soil strength to slope stability assessment", *Geomech. Eng.*, **12**(3), 483-503. <http://doi.org/10.12989/gae.2017.12.3.483>.
- Ma, X.F. and Li, T.J. (2018), "Dynamic analysis of uncertain structures using an interval-wave approach", *Int. J. Appl. Mech.*, **10**(02), 1850021. <https://doi.org/10.1142/S1758825118500217>.
- Ming, F., Yu, Q.H. and Li, D.Q. (2018), "Investigation of embankment deformation mechanisms in permafrost regions", *Transport. Geotech.*, **16**, 21-28. <https://doi.org/10.1016/j.trgeo.2018.06.003>.
- Moeinossadat, S.R. and Ahangari, K. (2019), "Estimating maximum surface settlement due to EPBM tunneling by Numerical-Intelligent approach—A case study: Tehran subway line 7", *Transport. Geotech.*, **18**, 92-102. <https://doi.org/10.1016/j.trgeo.2018.11.009>.
- Pan, Y.T., Liu, Y., Lee, F.H. and Phoon, K.K. (2019), "Analysis of cement-treated soil slab for deep excavation support-A rational approach", *Geotechnique*, **69**(10), 888-905. <http://doi.org/10.1680/jgeot.18.P.002>.
- Pan, Y.T., Liu, Y., Xiao, H.W., Lee, F.H. and Phoon, K.K. (2018a), "Effect of spatial variability on short-and long-term behaviour of axially-loaded cement-admixed marine clay column", *Comput. Geotech.*, **94**, 150-168. <http://doi.org/10.1016/j.compgeo.2017.09.006>.
- Pan, Y.T., Shi, G.C., Liu, Y. and Lee, F.H. (2018b), "Effect of spatial variability on performance of cement-treated soil slab during deep excavation", *Construct. Build. Mater.*, **188**, 505-519. <http://doi.org/10.1016/j.conbuildmat.2018.08.112>.
- Peduto, D., Elia, F. and Montuori, R. (2018), "Probabilistic analysis of settlement-induced damage to bridges in the city of Amsterdam (The Netherlands)", *Transport. Geotech.*, **14**, 169-182. <https://doi.org/10.1016/j.trgeo.2018.01.002>.
- Pramanik, R., Baidya, D.K. and Dhang, N. (2019), "Implementation of fuzzy reliability analysis for elastic settlement of strip footing on sand considering spatial variability", *Int. J. Geomech.*, **19**(12), 04019126. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001514](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001514).
- Ren, J. and Vanapalli, S.K. (2018), "Empirical model for predicting the resilient modulus of frozen unbound road materials using a hyperbolic function", *Transport. Geotech.*, **17**, 66-74. <https://doi.org/10.1016/j.trgeo.2018.09.011>.
- Shakir, M. and Talha, M. (2019), "On the dynamic response of imperfection sensitive higher order functionally graded plates with random system parameters", *Int. J. Appl. Mech.*, **11**(03), 1950025. <https://doi.org/10.1142/S175882511950025X>.
- Titi, H.H., Tabatabai, H., Faheem, A., Tutumluer, E. and Peters, J.P. (2018), "Spatial variability of compacted aggregate bases", *Transport. Geotech.*, **17**(PartB), 56-65. <https://doi.org/10.1016/j.trgeo.2018.06.007>.
- Vanmarcke, E. (2010), *Random Fields: Analysis And Synthesis*. MIT Press, Cambridge, U.K.
- Wang, C., Zhou, S., Wang, B. and Guo, P. (2018), "Time effect of pile-soil-geogrid-cushion interaction of rigid pile composite foundations under high-speed railway embankments", *Geomech. Eng.*, **16**(6), 589-597. <https://doi.org/10.12989/gae.2018.16.6.589>.
- Wang, F., Li, G., Ma, W., Mu, Y., Zhou, Z. and Mao, Y. (2018), "Permafrost thawing along the China-Russia Crude Oil Pipeline and countermeasures: A case study in Jiagedaqi, Northeast China", *Cold Reg. Sci. Technol.*, **155**, 308-313. <https://doi.org/10.1016/j.coldregions.2018.08.018>.
- Wang, S.H., Wang, Q.Z., An, P., Yang, Y.G., Qi, J.L. and Liu, F.Y. (2019b), "Optimization of hydraulic section of irrigation canals in cold regions based on a practical model for frost heave", *Geomech. Eng.*, **17**(2), 133-143. <http://doi.org/10.12989/gae.2019.17.2.133>.

- Wang, S.H., Wang, Q.Z., Qi, J.L. and Liu, F.Y. (2018), "Experimental study on freezing point of saline soft clay after freeze-thaw cycling", *Geomech. Eng.*, **15**(4), 997-1004. <http://doi.org/10.12989/gae.2018.15.4.997>.
- Wang, S.H., Wang, Q.Z., Xu, J., Ding, J.L., Qi, J.L., Yang, Y.G. and Liu, F.Y. (2019a), "Thaw consolidation behavior of frozen soft clay with calcium chloride", *Geomech. Eng.*, **18**(2), 189-203. <http://doi.org/10.12989/gae.2019.18.2.189>.
- Wang, T. (2015), "Study on the analysis model of stochastic temperature fields and displacement fields in permafrost regions", Ph. D. Dissertation, China University of Mining and Technology, Xuzhou, China.
- Wang, T., Zhou, G., Wang, J. and Yin, L. (2018a), "Stochastic thermal-mechanical characteristics of frozen soil foundation for a transmission line tower in permafrost regions", *Int. J. Geomech.*, **18**(3), 06017025. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001087](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001087).
- Wang, T., Zhou, G., Wang, J., Zhao, X. and Yin, L. (2018b), "Stochastic analysis for uncertain deformation of foundations in permafrost regions", *Geomech. Eng.*, **14**(6), 589-600. <https://doi.org/10.12989/gae.2018.14.6.589>.
- Wang, T., Zhou, G., Wang, J., Zhou, Y. and Chen, T. (2019b), "Stochastic coupling analysis of uncertain hydro-thermal properties for embankment in cold regions", *Transport. Geotech.*, **21**, 100275. <https://doi.org/10.1016/j.trgeo.2019.100275>.
- Wang, T., Zhou, G., Yin, L. and Zhou, L. (2019a), "Estimation on the influence of seepage on stochastic thermal regime of frozen ground surrounding the crude oil pipeline", *Cold Reg. Sci. Technol.*, **157**, 13-20. <https://doi.org/10.1016/j.coldregions.2018.09.007>.
- Wang, T., Zhou, G.Q., Wang, J.Z. and Zhao, X.D. (2016), "Stochastic analysis of uncertain thermal characteristic of foundation soils surrounding the crude oil pipeline in permafrost regions", *Appl. Therm. Eng.*, **99**, 591-598. <https://doi.org/10.1016/j.applthermaleng.2016.01.099>.
- Wen, Z., Sheng, Y., Jin, H., Li, S., Li, G. and Niu, Y. (2010), "Thermal elasto-plastic computation model for a buried oil pipeline in frozen ground", *Cold Reg. Sci. Technol.*, **64**(3), 248-255. <https://doi.org/10.1016/j.coldregions.2010.01.009>.
- Wijerathna, M. and Liyanapathirana, D.S. (2019), "Significance of spatial variability of deep cement mixed columns on reliability of column-supported embankments", *Int. J. Geomech.*, **19**(8), 04019087. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001473](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001473).
- Wu, Y., Sheng, Y., Wang, Y., Jin, H.J. and Chen, W. (2010), "Stresses and deformations in a buried oil pipeline subject to differential frost heave in permafrost regions", *Cold Reg. Sci. Technol.*, **64**(3), 256-261. <https://doi.org/10.1016/j.coldregions.2010.07.004>.
- Wu, Z., Zhang, D., Zhao, T., Ma, J. and Zhao, D. (2019), "An experimental research on damping ratio and dynamic shear modulus ratio of frozen silty clay of the Qinghai-Tibet engineering corridor", *Transport. Geotech.*, **21**, 100269. <https://doi.org/10.1016/j.trgeo.2019.100269>.
- Wu, Z.W., Cheng, G.D., Zhu, L.N. and Liu, Y.Z. (1988), *Roadbed Engineering in Permafrost Region*. Lanzhou University Press, Lanzhou, China (In Chinese).
- Yang, R., Ma, T., Liu, W., Fang, Y. and Xing, L. (2019), "Coupled hydro-mechanical analysis of gas production in fractured shale reservoir by random fracture network modeling", *Int. J. Appl. Mech.*, **11**(03), 1950031. <https://doi.org/10.1142/S1758825119500315>.
- Yao, K., Xiao, H., Chen, D.H. and Liu, Y. (2019), "A direct assessment for the stiffness development of artificially cemented clay", *Geotechnique*, **69**(8), 741-747. <https://doi.org/10.1680/jgeot.18.t.010>.
- Yu, W.B., Liu, W.B., Lai, Y.M., Chen, L. and Yi, X. (2014), "Nonlinear analysis of coupled temperature-seepage problem of warm oil pipe in permafrost regions of northeast China", *Appl. Therm. Eng.*, **70**(1), 988-995. <https://doi.org/10.1016/j.applthermaleng.2014.06.028>.
- Zhang, Y., Cheng, Z. and Lv, H. (2019), "Study on failure and subsidence law of frozen soil layer in coal mine influenced by physical conditions", *Geomech. Eng.*, **18**(1), 97-109. <https://doi.org/10.12989/gae.2019.18.1.097>.
- Zhang, Z., Tian, J., Huang, X. and Hua, H. (2018), "Stochastic response analysis of a built-up vibro-acoustic system with parameter uncertainties", *Int. J. Appl. Mech.*, **10**(08), 1850084. <https://doi.org/10.1142/S1758825118500849>.
- Zheng P. (2011), "Numerical simulation for couplings of water, temperature and stress fields of underground oil pipeline in cold region", Ph.D. Thesis, China University of Petroleum, Qingdao, China (in Chinese).
- Zheng, J.J., Liu, Y., Pan, Y.T. and Hu, J. (2018), "Statistical evaluation of the load-settlement response of a multicolumn composite foundation", *Int. J. Geomech.*, **18**(4), 04018015-1. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001124](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001124).
- Zheng, Y.R., Sheng, Z.J. and Gong, X.N. (2002), *Generalized Plastic Mechanics-The Principles of Geotechnical Plastic Mechanics*, China Architecture and Building, Beijing, China (in Chinese).
- Zhou, Z., Yang, H., Xing, K. and Gao, W.Y. (2018), "Prediction models of the shear modulus of normal or frozen soil-rock mixtures", *Geomech. Eng.*, **15**(2), 775-781. <http://doi.org/10.12989/gae.2018.15.2.783>.
- Zhu, H. and Zhang, L.M. (2013), "Characterizing geotechnical anisotropic spatial variations using random field theory", *Can. Geotech. J.*, **50**(7), 723-734. <https://doi.org/10.1139/cgj-2012-0345>.
- Zhu, H., Zhang, L.M., Xiao, T. and Li, X.Y. (2017), "Generation of multivariate cross-correlated geotechnical random fields", *Comput. Geotech.*, **86**, 95-107. <https://doi.org/10.1016/j.compgeo.2017.01.006>.

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