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Reliability analyses of a prototype soil nail wall using regression models

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Abstract. Soil nailing technique is being widely used for stabilization of vertical cuts because of its economic, environment friendly and speedy construction. Global stability and lateral displacement are the two important stability criteria for the soil nail walls. The primary objective of the present study is to evaluate soil nail wall stability criteria under the influence of in-situ soil variability. Finite element based numerical experiments are performed in accordance with the methodology of 2^3 factorial design of experiments. Based on the analysis of the observations from numerical experiments, two regression models are developed, and used for reliability analyses of global stability and lateral displacement of the soil nail wall. A 10 m high prototype soil nail wall is considered for better understanding and to highlight the practical implications of the present study. Based on the study, lateral displacements beyond 0.10% of vertical wall height and variability of in-situ soil parameters are found to be critical from the stability criteria considerations of the soil nail wall.

Keywords: soil nailing; factorial design; regression model; soil variability; reliability analysis.

1. Introduction

In India, soil nailing technique has been widely used for the stabilization of road/rail side slopes, basement excavations, support for bridge abutments and side walls of the approach road for subways (e.g., Murthy *et al.* 2002, Sivakumar Babu *et al.* 2002, 2007). Simultaneous advances in the analysis, design and construction aspects of soil nail walls are desirable to address its continuously growing demand in urban infrastructure development and rehabilitation projects. Armour and Cotton (2003) highlighted the role of advances in soil nailing technique towards significant increase in its use as an earth retention system, and reported advances in the following areas: (a) design methods (based on field instrumentation data and pullout tests); (b) construction methodology (e.g., use of architectural reinforced sculpted shotcrete and precast concrete facing panels), and (c) construction materials (e.g., use of hollow injection bars or "self drilling" bars, and fiberglass soil nails). As brought out by Armour and Cotton (2003), a need for the advances in the methods for design, performance assessment and construction is found inevitable. As a contribution to the aforesaid need, Sivakumar Babu and Singh (2009a) demonstrated the use of regression models developed based on the analysis of the observations from the numerical simulations (i.e.,

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experiments) conducted in accordance with the methodology of factorial design of experiments (Montgomery 2001) for the estimation of global stability and the lateral displacement at the top of the soil nail walls.

Since, global stability and lateral displacement are the two main stability criteria for soil nail walls; it is beneficial to perform a site specific analysis of these aspects in the soil nailing design. Moreover, the efficiency of soil nail walls is dependant on the complex nature of interaction between its main components (namely, in-situ soil, reinforcement and facing) as well as on the variability of in-situ soil properties. Complex soil structure interaction in soil nail walls can be accounted by rigorous numerical analysis using computational codes (e.g., Murthy *et al.* 2002, Fan and Luo 2008). Further, the influence of variability in in-situ soil properties on the stability criteria of soil nail walls can be rationally addressed using a reliability based analysis. However, limited studies on the reliability based analysis of soil nail walls (e.g., Yuan *et al.* 2003, Sivakumar Babu and Singh 2009b) are readily available in the existing literature. Therefore, in the present study, it is found desirable to perform reliability based stability criteria analyses of soil nail walls using regression models developed from the analysis of the observations from the numerical experiments (simulations) conducted in accordance with the methodology of factorial design of experiments.

A case of prototype soil nail wall is considered for the illustration and better understanding of regression based methodology for reliability analysis of soil nail walls. 2^3 factorial method (Montgomery 2001) for design of experiments is used for developing two regression models: (a) *FS*-model to compute factor of safety for global stability, and (b) y-model to estimate maximum lateral displacement at the top of soil nail wall. Hasofer-Lind reliability indices (Hasofer and Lind 1974) are determined using performance functions based on both conventional limit equilibrium method as well as developed regression models to study the influence of variability of in-situ soil parameters (namely, cohesion c, angle of internal friction ϕ and unit weight γ) on the stability criteria for soil nail walls.

2. Design details of prototype soil nail wall

The prototype soil nail wall considered for the study was constructed with the primary objective of retaining a vertical excavation of 10 m height so as to enable construction of two basement floors below general ground level for a commercial building. A photograph of the concerned soil nail wall is shown in Fig. 1 and its typical cross-section is shown schematically in Fig. 2. Excerpts from geotechnical investigation report of the construction site reveal that general soil type is fine grained and ground water table is well below the zone of influence. Summary of the various soil nail wall design parameters and in-situ soil properties is given in Table 1. Properties indicated in Table 1 are used for the numerical experimentation as discussed below. In the following section, a detailed discussion on the methodology to conduct numerical experiments based on 2^3 factorial design method and procedure to develop y-model and *FS*-model is presented.

3. Regression models using 2^3 factorial design of experiments

Montgomery (2001) discusses in detail about the factorial design of experiments widely used in the experiments involving several factors; especially, when it is necessary to study the interaction

Fig. 1 Photograph of the prototype soil nail wall

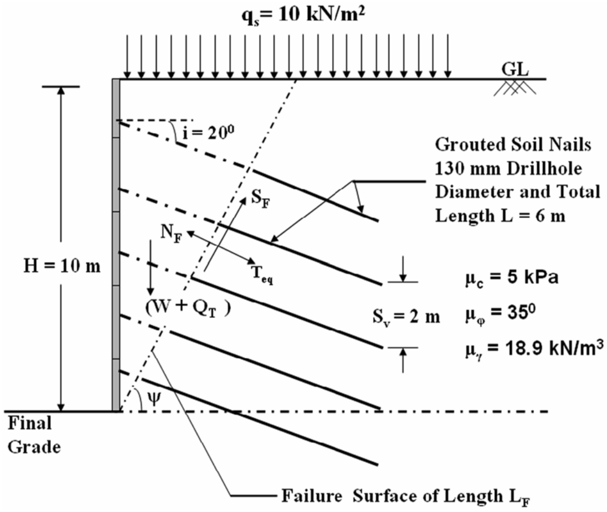

Fig. 2 Schematic typical cross-section of the prototype soil nail wall

Table 1 Excerpts from design document for the prototype soil nail wall

Parameter	Value
Vertical height of wall H (m)	10.0
Face batter wrt vertical α (deg)	0.0
Slope of backfill β_s (deg)	0.0
Surcharge load q_s (kPa)	10.0
In-situ soil cohesion c (kPa)	5.0
In-situ soil friction angle ϕ (deg)	35.0
In-situ soil unit weight γ (kN/m^3)	18.9
Diameter of nail d (mm)	25.0
Drill hole diameter D_{DH} (mm)	130.0
Length of nail L_N (m)	6.00
Nail inclination wrt horizontal i (deg)	20.0
Horizontal nail spacing S_h (m)	1.5
Vertical nail spacing S_v (m)	2.0
Modulus of elasticity of nail E_n (GPa)	200.0
Compressive strength of grout f_{ck} (MPa)	25.0
Allowable bond strength q_a (kPa)	50.0

effect of the factors on a response. A special case of factorial design is that of k factors, each at only two levels. These levels may be quantitative or qualitative. A complete replicate of such a design requires $2 \times 2 \times \ldots \times 2 = 2^k$ observations and is called a 2^k factorial design. The 2^k factorial design provides the smallest number of runs with which k factors can be studied in a complete factorial design. Because there are only two levels for each factor, it is assumed that the response is approximately linear over the range of factor levels chosen. In the present study, soil parameters c,

ϕ and γ each at two levels are considered as the three design factors (i.e., $k = 3$) for the experimental design and therefore, further discussion is restricted to the method of 2^3 factorial design of experiments. Following sub-sections presents the discussion on the steps involved in the development of regression models (y-model and FS-model) using 2^3 factorial design of experiments.

3.1 Fixing levels for design factors

The 2^3 factorial design of experiments need to specify values of each factor at two levels i.e., high and low. Orr (2000) suggest that the characteristic values for geotechnical parameters (in-situ soil cohesion c, friction angle ϕ and unit weight γ) shall be based on the 95% confidence intervals, i.e., the lower limit (or low level x_L) and upper limit (or high level x_h) values are related to mean value μ and standard deviation σ with the relationships $x_L = \mu - 1.65\sigma$ and $x_h = \mu + 1.65\sigma$ respectively. These values of geotechnical parameters are based on the assumption that input soil parameters follow normal distribution and upper and lower limit values have probabilities of 5% and 95% being exceeded. In the present study, the mean values of c, ϕ and γ are 5 kPa, 35° and 18.9 kN/m^3 respectively. Further, according to Phoon and Kulhawy (1999) and Duncan (2000), the possible ranges of coefficient of variation (COV) of c, ϕ and γ are 10-30%, 2-10% and 3-7% respectively. Therefore, to account for the in-situ soil variability, COV values for c, ϕ and γ are rationally adopted as 12% (i.e., 0.12), 6% (i.e., 0.06) and 6% (i.e., 0.06) respectively and used for computing corresponding value of standard deviation (i.e., $\sigma = \mu.\ COV$). Table 2 summarizes three design factors (also called natural variables) considered in the present study.

3.2 Combinations for 2^3 factorial design of experiments

Standard notations are followed to provide clarity with regard to the various terms involved in factorial designs. Therefore, in the present study, three design factors namely in-situ soil cohesion c, angle of internal friction of in-situ soil ϕ and in-situ soil unit weight γ are represented as the A, B and C respectively. Eight design runs for the 2^3 design using the "+ and –" notation (also called the geometric notation) to represent the low and high levels of the factors are shown in Table 3. Factor combinations (or treatment combinations) in the design are usually represented by lower case letters. High level of any factor in the factor combination is denoted by the corresponding lower case letter and that the low level of any factor in the factor combination is denoted by the absence of the corresponding letter. For example, a represents the factor combination of A at high level and B, C at low level, b represents B at high level and A, C at low level, ab represents A, B at high level and C at low level and so on. By convention, (1) is used to denote all factors A, B, and C at

Table 2 Input parameters for the 2^3 factorial design

Natural variable (or design factor)	Design notation	COV	Mean μ	St. dev. σ	Low level value x_L	High level value x_h
Soil cohesion c (kPa)	A	0.12	5.0	0.60	4.01	5.99
Soil friction angle ϕ (degrees)	B	0.06	35.0	2.10	31.54	38.47
Soil unit weight γ (kN/m^3)	C	0.06	18.9	1.13	17.04	20.76

Note: (a) $x_L = \mu - 1.6\sigma$; $x_h = \mu + 1.65\sigma$; $\sigma = \mu\ .\ COV$

(b) For reliability analysis, all natural variables are assumed to follow lognormal distribution

Table 3 Observations from numerical experiments for 2^3 factorial design

Run number	Factor			Run label	Observed displacement y (mm)	Observed factor of safety FS
	A	B	C			
1	−	−	−	(1)	39.67	1.56
2	+	−	−	*a*	26.40	1.61
3	−	+	−	*b*	18.31	2.01
4	+	+	−	*ab*	17.13	2.08
5	−	−	+	*c*	48.31	1.52
6	+	−	+	*ac*	36.18	1.58
7	−	+	+	*bc*	22.60	1.99
8	+	+	+	*abc*	20.54	2.01

Note: Positive sign indicates corresponding factor at high level (i.e., at maximum value) and negative sign indicates corresponding factor at low level (i.e., at minimum value).

the low level. In Table 3 column 'Run label' indicates the standard order of eight experimental run labels for different factor combinations as (1), *a*, *b*, *ab*, *c*, *ac*, *bc*, and *abc*.

3.3 Numerical experimentation

PLAXIS (2006), a finite element based two-dimensional geotechnical engineering computational code, is used for the numerical experimentation (i.e., simulations) of prototype soil nail wall to conduct 2^3 factorial design experiments. Shiu *et al.* (2006) and Fan and Luo (2008) are some of the recent example studies that have successfully used PLAXIS for the study of soil nail walls. Tan *et al.* (2005) studied in detail the effects of 2D modeling of 3D soil nailing problem. Soil nail wall considered in the present study is well within the guideline for 2D analysis range suggested by Tan *et al.* (2005).

Mohr-Coulomb material model is used to represent the behavior of in-situ soil. Elastic material model is used for nails and facing. Long term behaviour of the soil nail wall is simulated by adopting drained analysis condition using effective stress soil parameters. Nails and facings are modeled using plate elements. Fifteen node triangular elements with medium mesh density are used to discretise the soil domain. Sequence of construction is simulated using staged construction option by activating or deactivating the elements/clusters. Global stability factors of safety are determined using strength reduction technique (Matsui and San 1992). Strength reduction technique is advantageous as it identifies the critical failure mechanism automatically, which is normally assumed in the conventional analysis. A detailed procedure for simulating soil nail walls using the computational code adopted in the present study can be found elsewhere (e.g., Sivakumar Babu and Singh 2009a, 2009c, 2010). An illustrative numerically simulated soil nail wall of 10 m height is shown in Fig. 3 (Ex 1, Ex 2, …, Ex 5 in Fig. 3 represents construction stages). Observations corresponding to the global factor of safety and maximum lateral displacement of the soil nail wall are noted for each numerical experiment.

3.4 Experimental runs and analysis of observations

As mentioned earlier in section 3.2, eight experimental runs based on the combinations of design

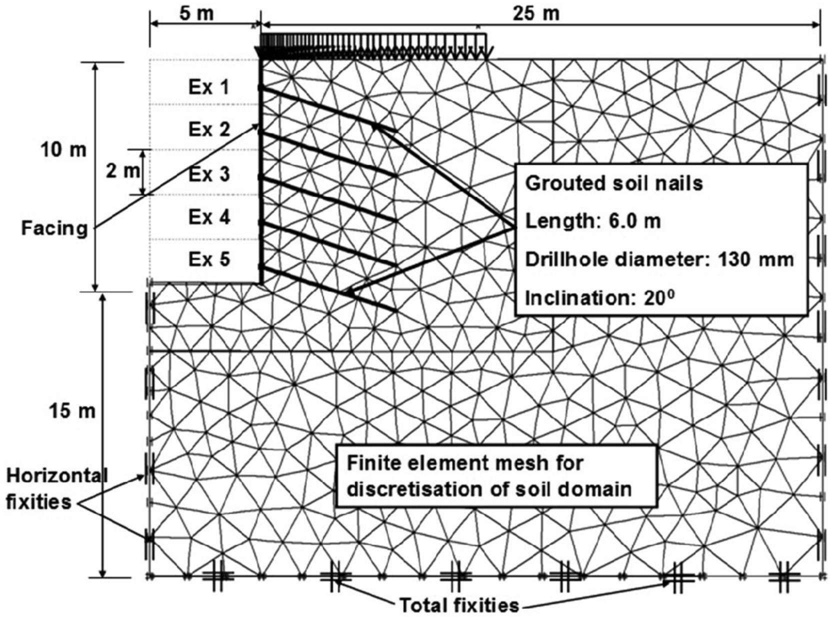

Fig. 3 Illustrative numerically simulated soil nail wall of 10 m height

factors A, B and C (i.e., c, ϕ and γ respectively) at high or low level are given in Table 3. Experiments (i.e., numerical simulations of soil nail wall) are performed for each design combination and the observations of the response quantities, namely global factor of safety (*FS*)and lateral displacement at the top of soil nail wall (y) are tabulated (see Table 3). To develop a regression model for any response quantity using observations from experimental runs based on 2^3 factorial design, contrast constants, effect estimates and percent contribution of main factors (A, B and C) and interaction factors (AB, AC, BC and ABC) are to be determined and most significant main factors/interaction factors affecting the response of the particular quantity are to be identified. Steps involved to develop y-model are discussed below; similar steps are followed to arrive at the *FS*-model.

3.4.1 Steps involved in development of y-model from experimental observations

Step 1: Contrast constants for main factors/interaction factors are determined. For example, contrast constant (also called as the total effect, gives the net influence of particular main factor or interaction factors on the desired response over various design combinations considered for experimentation) for main factor A is equal to algebraic sum of observations for experimental runs for each of the eight factor combinations. Column A of Table 4 shows the algebraic sign convention for the algebraic sum to determine contrast of A. Therefore

$$\text{Contrast } A = [-(1) + a - b + ab - c + ac - bc + abc] \quad (1)$$

Similarly, contrast constants for other main factors/interaction factors can be determined as the algebraic sum of the observations following sign convention given in corresponding column of Table 4.

Step 2: For each main factor/interaction factor, its effect estimate and percent contribution to the

Table 4 Algebraic signs for determination of contrast constants

Run level	*A*	*B*	*AB*	*C*	*AC*	*BC*	*ABC*
(1)	−	−	+	−	+	+	−
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
ab	+	+	+	−	−	−	−
c	−	−	+	+	−	−	+
ac	+	−	−	+	+	−	−
bc	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+

response quantity are calculated using Eqs. (2)-(5). For example, for main factor *A* corresponding parameters are determined as

$$\textit{Effect estimate}, A = \frac{1}{4n}\ [\textit{Contrast } A] \tag{2}$$

$$\textit{Percent contribution } A = \frac{SS_A}{SS_T} \tag{3}$$

$$SS_A = \frac{(\textit{Contrast } A)^2}{8n} \tag{4}$$

where n = number of experiment replicates (= 1 in the present case), SS_A = sum of squares for *A* and SS_T = total sum of squares given by

$$SS_T = (\textit{sum of square of each observation}) - \left[\frac{\textit{square sum of all observations}}{8n}\right] \tag{5}$$

Factor effect estimates and percent contribution values for both regression models are summarised in Table 5.

Table 5 Factor effect estimates and percent contribution

Model term	Desgin factor	Displacement model (*y*-model)			Factor of safety model (*FS*-model)		
		Effect estimate	Sum of squares	Percent contribution	Effect estimate	Sum of squares	Percent contribution
A	c	-7.16	102.53	**11.25**	0.05	0.01	**1.18**
B	ϕ	-18.00	647.64	**71.04**	0.46	0.41	**97.88**
AB	cϕ	5.54	61.38	**6.73**	0.00	0.00	0.01
C	γ	6.53	85.28	**9.35**	-0.04	0.00	**0.76**
AC	cγ	0.06	0.01	0.00	-0.01	0.00	0.05
BC	$\phi\gamma$	-2.68	14.36	1.58	0.00	0.00	0.01
ABC	c$\phi\gamma$	-0.51	0.51	0.06	-0.02	0.00	0.11

Note: Bold numbers indicate most influencing main factor/interaction factor

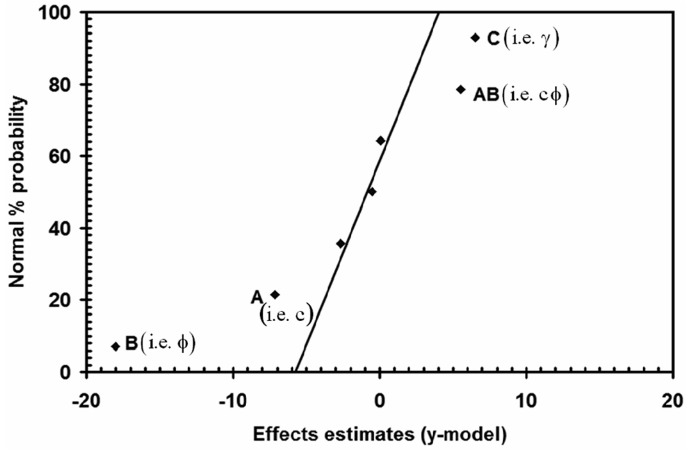

Fig. 4 Normal probability plot for the effect estimates for displacement response

Step 3: Normal probability plot (Montgomery 2001) of the effect estimates is drawn to identify main factors/interaction factors, affecting significantly the response of lateral displacement y at the top of the soil nail wall. The effects that are negligible are normally distributed, with mean zero and variance σ^2 will tend to fall along (or very close to) a straight line, whereas significant effects will have nonzero means and will not lie along (or lie far from) the straight line. Normal probability plot for the effect estimates for displacement observations is shown in Fig. 4. From the normal probability plot and also from the percent contribution values (see bold values in Table 5), the important factors that emerge out for displacement response are A (i.e., in-situ soil cohesion c), B (i.e., angle of internal friction ϕ), C (i.e., unit weight of in-situ γ) and AB (i.e., interaction between in-situ soil cohesion c and angle of internal friction ϕ). Similarly, Fig. 5 shows the normal probability plot for effect estimates for factor of safety FS observations and the important factors that emerge out for factor of safety response are A (i.e., in-situ soil cohesion c), B (i.e., angle of internal friction ϕ) and C (i.e., unit weight of in-situ γ). Table 5 provides useful information for soil

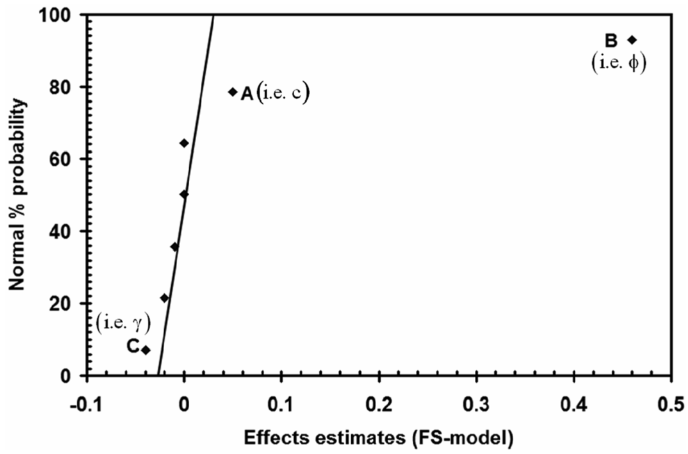

Fig. 5 Normal probability plot for the effect estimates for factor of safety response

nailing practitioners in the form of percent contributions of main factors/interaction factors towards the stability and displacement response of soil nail walls. It indicates that percent contribution of the interaction factors to the response of stability criteria of soil nail wall is marginal in comparison to the main factors.

Step 4: In 2^k factorial design, the results of the experiments can be expressed in terms of a regression model. The regression model for predicting wall displacement (i.e., y-model) is

$$y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 \tag{6a}$$

$$y[mm] = 28.64 - 3.58x_1 - 9x_2 + 3.27x_3 - 2.77x_1x_2 \tag{6b}$$

Similarly, the regression model for global factor of safety (i.e., FS-model) is

$$FS = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \tag{7a}$$

$$FS = 1.80 - 0.025x_1 + 0.23x_2 - 0.02x_3 \tag{7b}$$

where β_o, β_1, β_2, β_3 and β_{12} are the regression coefficients (β_o is the average of all eight observations of the corresponding response quantity given in Table 3 and all other are one-half the effect estimate of the corresponding main factor/interaction factor given in Table 5), and x_1, x_2, and x_3 are the coded factors representing main factors A, B, and C respectively. The term x_1x_2 represent interaction factor AB. The coded factors x_1, x_2, and x_3 can be expressed in terms of design factors as

$$x_1 = (c - 5)/0.99 \quad \text{where} \quad 4.01 \text{ kPa} \le c \le 5.99 \text{ kPa} \tag{8a}$$

$$x_2 = (\phi - 35)/3.47 \quad \text{where} \quad 31.54^0 \le \phi \le 38.47^0 \tag{8b}$$

$$x_3 = (\gamma - 18.9)/1.86 \quad \text{where} \quad 17.04 \text{ kN/m}^3 \le \gamma \le 20.76 \text{ kN/m}^3 \tag{8c}$$

When design factors have only two levels, coded factors given by Eq. (8) produce the familiar ± 1 notation for levels of the coded factors. For example, the relation between design factor in-situ soil cohesion c (i.e., A) and the corresponding coded factor x_1 is given by

$$x_1 = \frac{c - (c_{high} + c_{low})/2}{(c_{high} - c_{low})/2} = \frac{c - (5.99 + 4.01)/2}{(5.99 - 4.01)/2} = \frac{(c - 5)}{0.99} \tag{9}$$

Eq. (9) yields $x_1 = +1$ when c is at high level c_{high} (equal to 5.99 kN/m^2), $x_1 = -1$ when c is at low level c_{low} (equal to 4.01 kN/m^2) and $x_1 = 0$ when c is at mean value (equal to 5.0 kN/m^2). Likewise, coded factors x_2 and x_3 can be interpreted corresponding to the design factors in-situ soil friction ϕ (i.e., B) and unit weight γ (i.e., C) respectively. Coded factors also enable graphical representation of variation of different design factors (between two levels i.e., high and low level) on the same axis.

Step 5: Model adequacy can be checked by means of plots of residuals [residuals are the differences between the observed value form experiment (in present case numerical simulation) and fitted value (i.e., from corresponding regression model) of the response parameter]. Fig. 6 and Fig. 7 show plots of residuals for y-model and FS-model respectively. From the residual plots, it is observed that residuals for predicted displacements are in the range of ±0.02% and for predicted factor of factor safety for global stability are in the range of ±0.02. These observations of residual ranges validate the adequacy of developed regression models to predict corresponding response quantity.

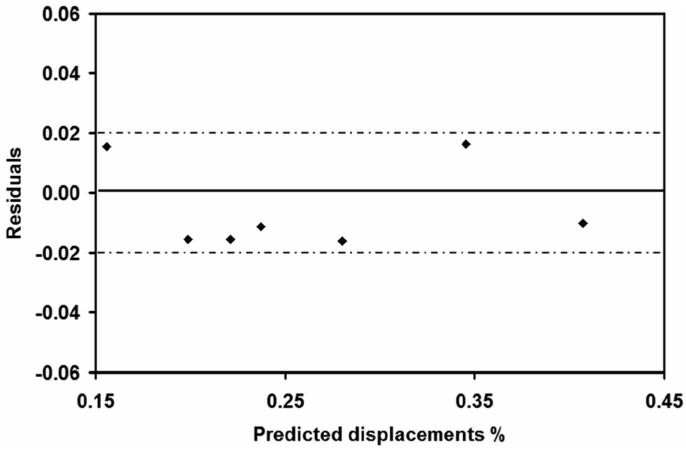

Fig. 6 Plot of residuals for y-model

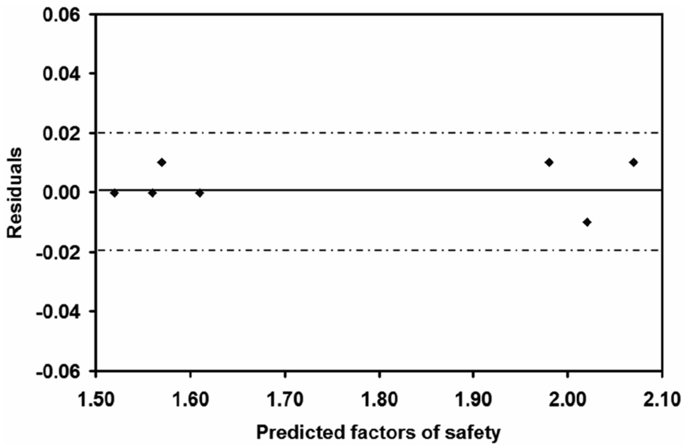

Fig. 7 Plot of residuals FS-model

Regression models given by Eqs. (6) and (7) are used to assess soil nail wall stability criteria for the following two main reasons: (a) they provide reasonably accurate estimates of lateral soil nail wall displacement (y) and global factor of safety (FS) respectively, and (b) they incorporate variability of in-situ soil parameters within the ranges given in Eq. (8). In the following section, an overview of the methods used for the conventional limit equilibrium for determination of global stability and the reliability analyses of the soil nail wall is presented.

4. Methods adopted for conventional and reliability analyses

Single wedge failure mechanism (FHWA 2003, Sheahan and Ho 2003) is considered for limit equilibrium analysis and factor of safety for global stability is determined by considering the equilibrium of various forces (see Fig. 2) along the failure plane. The failure plane is assumed to be inclined at an angle $\psi = 45 + (\phi/2)$ in degrees with respect to horizontal. The factor of safety against global failure FS is expressed as the ratio of the resisting ΣR and driving forces ΣD, which acts tangentially to the potential failure plane

$$FS = \frac{\Sigma R}{\Sigma D} \tag{10}$$

Considering the equilibrium of forces (see Fig. 2) acting along the failure plane and rearranging the terms, we get

$$FS_G = \frac{\Sigma R}{\Sigma D} = \frac{cL_F + T_{eq}\cos(\psi - i) + [(W + Q_T)\cos\psi + T_{eq}\sin(\psi - i)]\tan\phi}{(W + Q_T)\sin\psi} \tag{11}$$

where W = weight of triangular failure wedge, Q_T = surcharge load, T_{eq} = equivalent nail force, L_F = length of failure plane and i = inclination of soil nail with respect to horizontal. In Fig. 2, S_F is the shear force acting along the potential failure plane and N_F is the normal force acting perpendicular to the failure plane. Various terms in Eq. (11) can be determined in accordance with FHWA (2003).

Low (1997a, 2005) illustrated the capability of Microsoft Excel Software and its built-in optimization program SOLVER for conducting reliability analysis. Therefore, Excel's SOLVER program is used to determine Hasofer-Lind reliability indices (Hasofer and Lind 1974). The matrix formulation of the Hasofer-Lind reliability index β is

$$\beta = min_{x \in F}\sqrt{\left[\frac{x_i - \mu_i^N}{\sigma_i^N}\right]^T [\underline{R}]^{-1}\left[\frac{x_i - \mu_i^N}{\sigma_i^N}\right]} \tag{12}$$

where x_i is a vector representing the set of random variables, μ_i^N is the vector of equivalent normal mean values, $\underline{R}$ is the correlation matrix, σ_i^N is the equivalent normal standard deviation, and F is the failure domain. In-situ soil cohesion c, angle of internal friction ϕ and unit weight of soil γ are considered as log-normally distributed correlated random variables (Table 2). Following Chowdhury and Xu (1992) and Sivakumar Babu and Singh (2009b), a negative correlation coefficient of -0.25 is considered between cohesion c and internal friction angle ϕ of in-situ soil whereas, a positive correlation coefficient of +0.25 is considered between shear strength parameters (i.e., c and ϕ) and unit weight γ of in-situ soil. The performance functions (or limit state functions) for global stability of soil nail from conventional method and developed regression models are given in Eqs. (13) and (14) respectively, and for lateral displacement is given by Eq. (15).

(a) Performance function for global stability using conventional limit equilibrium method

$$perfn(1) = R_{conventional} - 1.35L_{conventional} = 0 \tag{13}$$

$$R_{conventional} = cL_F + T_{eq}\cos(\psi - i) + [(W + Q_T)\cos\psi + T_{eq}\sin(\psi - i)]\tan\phi \tag{13a}$$

$$L_{conventional} = (W + Q_T)\sin\psi \tag{13b}$$

where R and L represents the resistance (or capacity) and load (or demand) respectively of the soil nail wall for global stability.

(b) Performance function for global stability using FS-model

$$perfn(2) = (1.80 - 0.025x_1 + 0.23x_2 - 0.02x_3) - 1.35 = 0 \tag{14}$$

Factor 1.35 in the Eqs. (13) and (14) is provided to determine reliability indices so as to incorporate FHWA (2003) criterion for minimum desired factor of safety for global stability (i.e., 1.35) of soil nail walls.

(c) Performance function for lateral wall displacement using y-model

$$perfn(3) = (28.64 - 3.58x_1 - 9x_2 + 3.27x_3 - 2.77x_1x_2) - y_{all} = 0 \tag{15}$$

where y_{all} is the allowable maximum displacement of soil nail wall in millimeters.

5. Stability criteria for soil nail walls – Reliability analysis

FHWA (2003) recommends a minimum factor of safety of 1.35 for global stability of soil nail walls. Further, FHWA (2003) states that for soil nail walls with typical L_N/H ratio between 0.7 and 1.0, negligible surcharge loading, and typical global factors of safety values of 1.50, the maximum long-term horizontal and vertical wall displacements at the top of the wall vary from 0.10 to 0.30 percent of the vertical wall height. Juran (1985) also states that the maximum lateral displacement of soil nailed walls generally does not exceed 0.20% of the vertical height. Therefore, these limits of stability criteria are assessed using reliability analysis for the prototype soil nail wall considered in the study. Influence of variability of in-situ soil parameters (i.e., c, ϕ and γ) on the global stability and lateral displacements of soil nail wall is studied using y-model and FS-model. Further, factor of safety and reliability index for global stability of soil nail are evaluated using the developed regression FS-model, and are compared with those obtained from conventional limit equilibrium method. Finally, y-model is utilised to assess reliability of soil nail wall. Following sub-sections discusses these aspects of the study in detail.

5.1 Influence of construction stage

Soil nailing is essentially different from other earth reinforcement techniques in the sense that it is constructed downward with the soil being reinforced in-situ. Construction proceeds in excavation lifts of 1 m-2 m, depending upon the capacity of in-situ soil to stand unsupported. As the construction of soil nail walls proceeds from first excavation lift to the second, soil nails installed at the first excavation lift starts taking load and contributes to the stability of the retained excavation. Hence, it is desirable that stability criteria for soil nail wall must be satisfied at each construction stage. Therefore, to study the influence of construction stages on the stability criteria, construction stages of soil nail wall are simulated. Observations corresponding to the global factor of safety (FS) and maximum lateral displacement of soil nail wall (y) at the end of each construction stage are noted. A plot showing variation of the global factor of safety (FS) and maximum lateral displacement (evaluated as the percentage of the vertical wall height in the current excavation stage) of soil nail wall (y) with increasing construction stage is shown in Fig. 8. Lateral displacement of soil nail wall is found to reduce significantly from 0.17% to about 0.11% with increase in construction stage from 20% to about 50%. Though, beyond 50% of the construction stage an increase in lateral displacement is observed, for the fully constructed soil nail wall (i.e., 100% construction stage) maximum displacement is about 0.22% of its vertical height. On the other hand, an expected trend in values of factors of safety for global stability (i.e., decrease with increasing construction stage) of each construction stage is observed. It is evident from Fig. 8, that at each

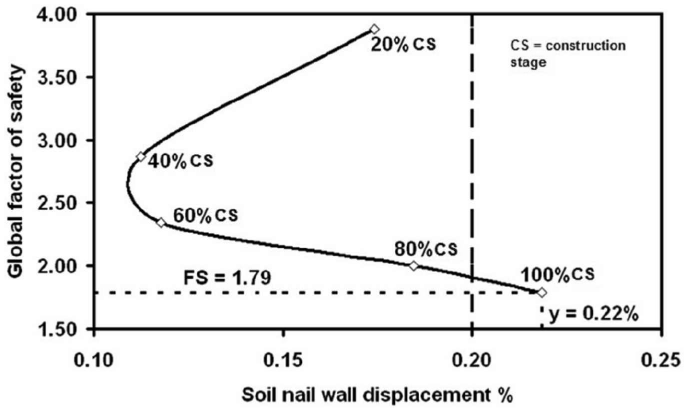

Fig. 8 Influence of construction stage on stability criteria

stage of construction, factor of safety for global stability is above 1.50. From these observations, it may be concluded that the prototype soil nail considered in the present study fulfilled the stability criteria (i.e., factor of safety for global stability above 1.35 and displacement in the range 0.10 to 0.30 percent of vertical wall height) at each construction stage.

5.2 Evaluation of stability criteria using regression models

Soil nailing derives strength to retain an excavation essentially from the interaction of soil nails and the in-situ soil. Consequently, stability criteria of soil nail walls are significantly affected due to the variability of in-situ soil properties. Therefore, stability criteria for the prototype soil nail wall under the influence of variability of in-situ soil parameters (c, ϕ and γ) are studied using the regression models (i.e., y-model and FS-model). Variation of each soil parameters is considered in the range of its low level and high level value as shown in Table 2. Therefore, in-situ soil cohesion c is varied from 4.01 kPa to 5.99 kPa, friction angle ϕ is varied from 31.54^0 to 38.47^0 and unit weight γ is varied from 17.02 kN/m^3 to 20.76 kN/m^3. Only one soil parameter is varied at a time and it is assumed that other two parameters are at their corresponding mean value. Fig. 9 shows the plots of the influence of varying soil parameters on stability criteria of the soil nail wall. From Fig. 9, it is apparent that as the value of in-situ soil friction angle is varied from its high level to low level value, a significant decrease in the factor of safety for global stability and increase in the lateral displacement of the soil nail wall occurs.

For example, at high level value of internal friction angle of in-situ soil (i.e., $\phi = 38.47^0$), global factor of safety and lateral displacement of the soil nail wall are 2.02 and 19.65 mm, respectively. Whereas, at low level value of internal friction angle of in-situ soil (i.e., $\phi = 31.54^0$), corresponding values of global factor of safety and lateral displacement of the soil nail wall are 1.57 and 37.61 mm, respectively. On the other hand, a marginal influence on the global factor of safety (decrease for c and increase for γ) and the lateral displacement (increase for c and decrease for γ) of soil nail wall is observed when in-situ soil cohesion and unit weight are varied from their corresponding high level value to the low level value (see Fig. 9). Thus, based on Fig. 9, it can be concluded that the variation in the value of internal friction angle of the in-situ soil is critical to stability criteria of soil nail walls. At mean value of all soil parameters, FS and y values are 1.80 and 28.64 mm (see

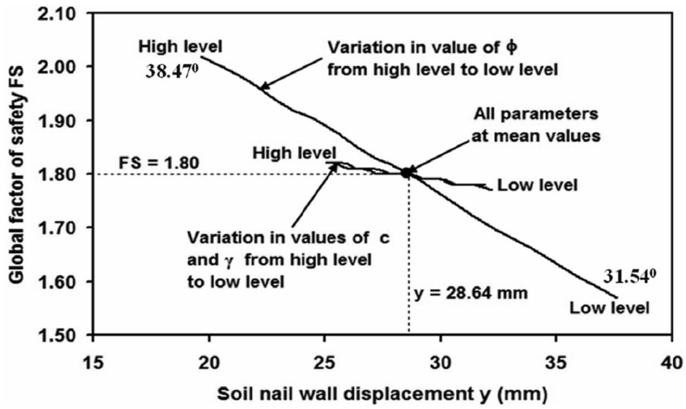

Fig. 9 Influence of high level to low level variation in-situ soil parameters on the stability criteria

Fig. 9), respectively.

5.3 Reliability analysis of soil nail wall global stability

A discussion on the reliability analysis based assessment of the influence of variability of in-situ soil parameters on global stability of soil nail wall is presented in this section. Hasofer-Lind reliability indices are determined using performance functions given by Eqs. (13) and (14). In Figs. 10(a-c), plots of reliability indices accounting for variability of soil cohesion c, friction angle ϕ and unit weight γ are shown respectively. Variability of in-situ soil parameters is accounted by considering one parameter at a time and keeping other two parameters at their mean values. Coefficients of variation of soil parameters c, ϕ and γ are adopted equal to 12%, 6% and 6%, respectively. Reliability indices determined using *FS*-model (as indicated by the dashed lines), and conventional method (as indicated by the continuous lines) show that the global stability of the soil nail wall is found to be significantly influenced with the variation of in-situ soil friction angle and the unit weight in comparison to the in-situ soil cohesion.

5.3.1 Influence of assumed probability density distribution

Random variables (i.e., design factors) considered for the reliability analysis presented above are assumed to obey lognormal distribution. However, to assess the effect of the assumed probability density distribution of variables on the results of the reliability analysis, global stability of the soil nail wall is evaluated considering normal distribution for variables. Fig. 11 shows the corresponding results of the analyses. Abscissa (x-axis) of Fig. 11 represents variation of in-situ soil parameters in term of coded factors given by Eqs. 8(a-c). At a time, mean value for one variable (i.e., design factor) is varied from its low level value to high level value, keeping other two variables at their respective mean values. The dashed and the continuous lines in Fig. 11 indicate the results corresponding to the normal and lognormal probability distribution of variables, respectively. From Fig. 11, it can be noted that for all soil parameters, the dashed lines are very close (slightly less) to the continuous lines. Thus, it can be concluded that a marginal difference in the response (i.e., the influence of varying soil parameters on global stability of the soil nail wall) is observed by the use of two different probability density functions for random variables (i.e., design factors). Further, for

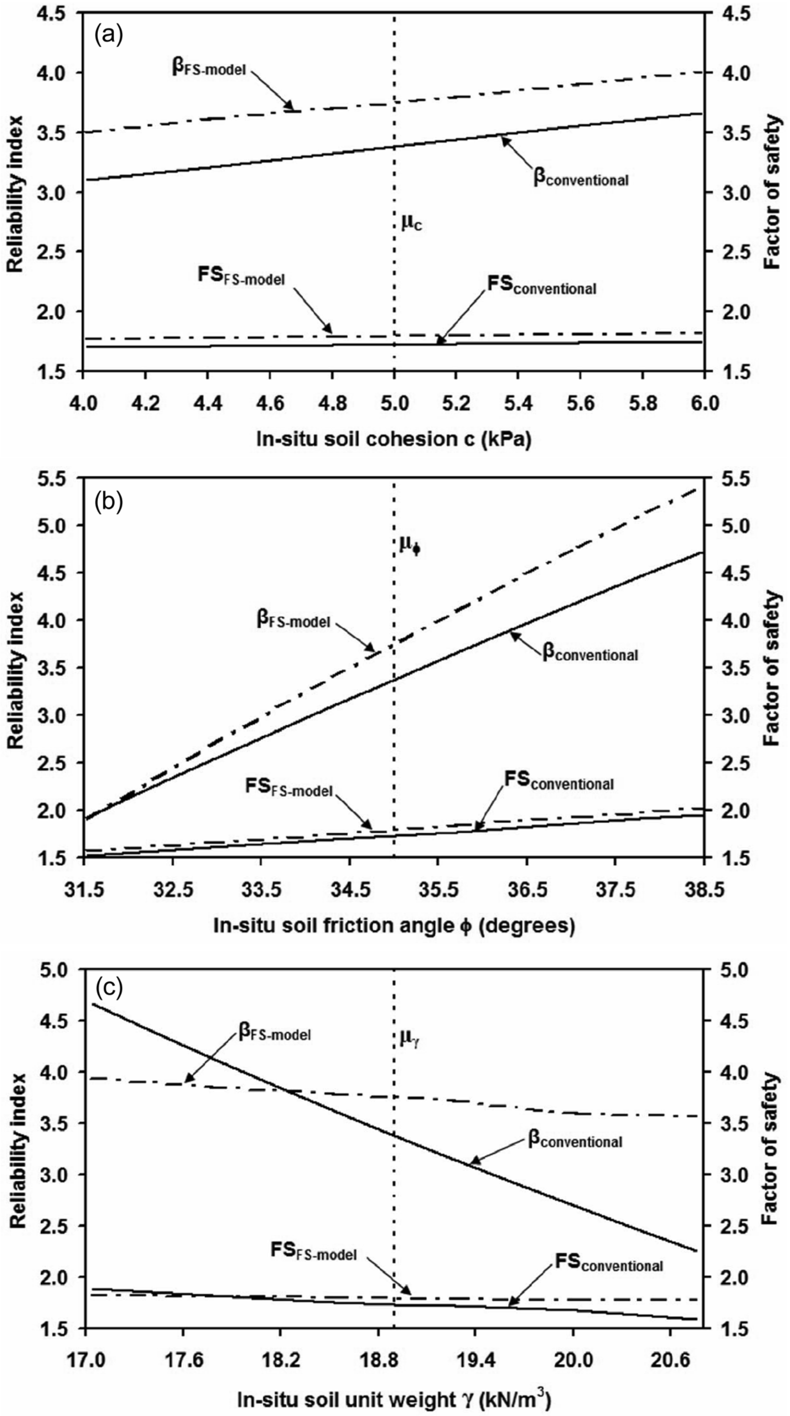

Fig. 10. (a) Influence of variability of in-situ soil cohesion c on global stability, (b) Influence of variability of in-situ soil friction angle ϕ on global stability, (c) Influence of variability of in-situ soil unit weight γ on global stability

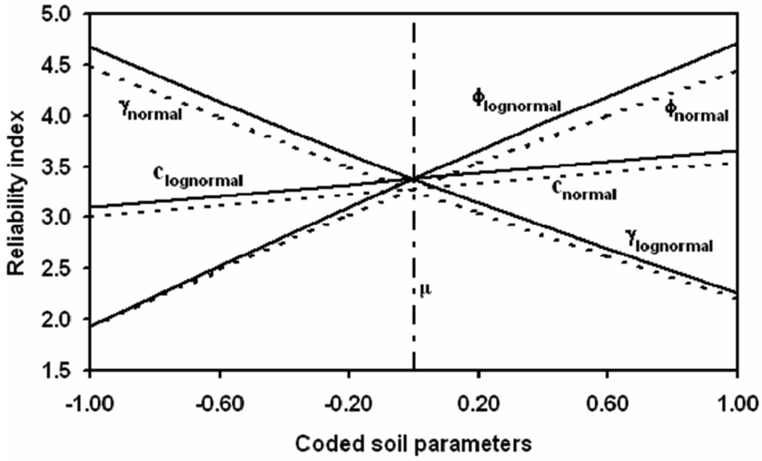

Fig. 11 Assumed probability distribution and global stability response

all variables at their respective mean values, a reliability index equal to 3.36 is obtained indicating that the prototype soil nail wall is safe for the mean values of soil parameters adopted in the conventional design.

5.4 Role of allowable soil nail wall displacements

As mentioned previously, the most commonly assumed serviceability limit for the soil nail walls is that the maximum displacement at the top of the wall should be within 0.10 to 0.30 percent of the vertical wall height (Juran 1985, FHWA 2003). For the prototype soil nail wall considered in the present study, it is already observed that it fulfills both stability criteria. However, it is desirable to ascertain the reliability of the soil nail wall satisfying the above mentioned serviceability limits. To study this aspect, reliability indices are determined using the performance function given by Eq.

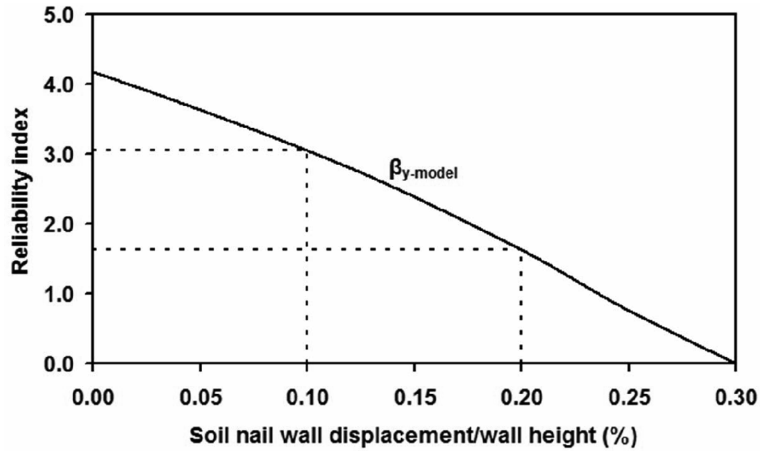

Fig. 12 Reliability assessment of the soil nail wall using y-model

(15) based on *y*-model, and plotted as shown in Fig. 12 with respect to the percent wall displacements determined with respect to the vertical wall height (i.e., 10 m). For the present case, reliability index values are found to decrease significantly with increasing percentage of the lateral displacement (see Fig. 12), and it is observed that the displacement of soil nail wall beyond 0.10% of its vertical height results in reliability indices less than the minimum desirable value of 3.0 for geotechnical engineering structures (Phoon 2004).

6. Conclusions

Two important stability criteria for soil nail walls, namely global stability and maximum lateral displacement are studied with reference to a prototype soil nail wall of 10 m height. This study highlights the use of finite element based numerical simulations of prototype soil nail wall to overcome the limitation of conventional slope stability methods to account for complex soil-structure interaction. Two regression models (namely *y*-model and *FS*-model) using method of 2^3 factorial design of experiments are developed to study influence of variability of in-situ soil parameters on the response of stability criteria for soil nail walls. These models in conjunction with conventional limit equilibrium method for slope stability analysis are used for reliability based assessment of influence of soil variability on the stability criteria for the soil nail walls. Variation in-situ soil properties are found to be critical for both the stability criteria highlighting the need of exhaustive site-specific geotechnical investigation and judicious selection of soil parameters for the soil nail wall design. Though, in general, analyses show that the prototype soil nail wall fulfills both the stability criteria, however, displacement (*y*-model) based reliability analysis suggest that its reliability index is less than the minimum desirable. Since, it is observed that both the regression models provided reasonably accurate estimate of the corresponding response quantity, the methodology developed herein may be used to predict factor of safety for global stability and lateral displacement of other soil nail walls with similar loading and geometric configurations.

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