Geomechanics and Engineering, Vol. 2, No. 1 (2010) 29-44
DOI: http://dx.doi.org/10.12989/gae.2010.2.1.029

A comparative study on damping of finite dry and saturated sand stratum under vertical vibrations

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(Received November 30, 2009, Accepted March 11, 2010)

Abstract. Vertical vibration tests were conducted using model footings of different size and mass resting on the surface of finite sand layer with different height to width ratios which was underlain by either rigid concrete base, under both dry and saturated condition. The effect of saturation on the damping ratio of finite sand stratum underlain by a rigid base has been verified and compared with the results obtained for the case of finite dry sand stratum underlain by the rigid base. Comparison of results of the experimental study showed that the damping in both the cases is less than 10%. The damping ratio obtained for finite saturated sand stratum is marginally lower than that obtained on finite dry sand stratum at H/B ratio of 0.5. The difference between the two cases becomes significant when the H/B ratio increases to 3.0, indicating the significant influence of soil moisture on damping ratio of foundation- soil system with increase in the thickness of the finite sand stratum. Comparison of the predicted damping ratio for a homogeneous sand stratum with the experimental damping ratio obtained corresponding to the height to width ratio of 3.0 of the finite sand stratum underlain by the rigid concrete base indicates a significant reduction in damping ratio of the foundation-soil system for both the cases.

Keywords: damping ratio; displacement amplitude; dynamic response; height to width ratio; rigid concrete base.

1. Introduction

Damping in soils mainly consists of two components-material (internal) damping and radiation (geometrical) damping. Material damping is inherent within the material and is due to viscous and hysteretic effects, while the radiation damping is due to loss of energy affected by radial propagation of elastic waves from the immediate vicinity of the source of vibration.

Many researchers (Reissner 1936, Quinlan 1953, Sung 1953, Lysmer 1965, Lysmer and Richart 1966, Novak 1971, Moore 1971, Gandhi 1986, Hardin and Drenevich 1972, Sheriff *et al.* 1977, Gazetas and Rosset 1979, Gazetas 1980, Dobry and Gazetas 1986, Gazetas and Stokoe 1992, Azam

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et al. 1996) have carried out investigations for the determination of damping ratio in soils. It has been established that, in a foundation-soil system under the vertical mode of vibration, the radiation damping overshadows the material damping. However, the damping in a foundation-soil system is neither purely radiation nor purely material. Generally it is a combination of both, observed during dissipation of waves, which are generated during vibration and hence it can not be obtained individually. Richart *et al.* (1970) have suggested that a value of 5% can be taken as the contribution from material damping that can be added to radiation damping in order to obtain total damping, and is purely empirical, since there are several factors that influences damping, they being the shape and contact area of the foundation, total weight of the foundation, frequency of vibration, excitation level, and properties of the soil medium such as type of the soil as well as its unit weight. By conducting block vibration tests on two and three layered soil system Sridharan *et al.* (1990) suggested a method to estimate the equivalent stiffness and damping of a multi-layered soil system. Baidya and Sridharan (2002) studied the dynamic response of the foundation resting on a stratum underlain by a rigid layer using lumped parameter model and suggested a method to estimate the equivalent stiffness and equivalent damping. Baidya and Muralikrishna (2001) studied the influence of layering and presence of rigid boundary within the soil on the resonant frequency and peak displacement amplitude by conducting model block vibration tests in vertical mode and found that, the damping in the entire test results were within 6.5%, indicating there by the absence of radiation damping. Mandal and Baidya (2004) conducted model block vibration tests on rigid surface footings in order to study the effect of presence of rigid layer at any depth within the soil mass on the dynamic response of foundation under vertical mode of vibration. It was found that the damping in the foundation-soil system was significantly low and remained almost constant for any thickness of the layer. Baidya and Rathi (2004) showed that the inclusion of rigid layer caused a 9.8% reduction in radiation damping with respect to homogeneous condition even for sand layer of thickness equal to four times the width of the footing.

Few researchers have investigated the dynamic response of foundation on non-homogeneous soil system to include the effect of increase in moisture caused by saturation. Further, data pertaining to studies made on the effect of size and mass of the foundation on the damping properties of soil resting on non-homogeneous saturated soil system are very scanty. The present investigation therefore aims to determine experimentally, the effect of saturation on the damping ratio of finite sand stratum underlain by a rigid base and compare the results with that obtained for the case of finite dry sand stratum underlain by a rigid base. Vertical vibration tests were conducted using model footings of different size and mass resting on the surface of finite sand layer, considering different height to footing width ratios (henceforth, called as *height to width ratio* and denoted by H/B) for the two individual cases of finite dry sand and saturated sand underlain by rigid concrete base.

2. Materials and methods

The sand used in the present investigation was locally available river sand, whose properties were determined as per Indian Standards: 1498-1970 and are presented in Table 1. A rigid concrete base of M20 grade and 300 mm thickness was cast and cured at the testing site to simulate the presence of a rock stratum occurring in nature. The location of this is represented by different height to width ratio of the sand stratum. Tests were performed on the same model footings under similar loading

Table 1 Properties of sand used

Sl. No	Particulars		
1	Specific gravity		2.64
2	Grain size analysis	Coarse sand (%)	0.03
		Medium sand (%)	60.8
		Fine sand (%)	37.8
		Silt and clay (%)	1.31
3	Co-efficient of uniformity (Cu)		2.6
4	Coefficient of curvature (Cc)		1.4
5	Maximum dry density, $(\gamma_d)_{max}$ kN/m^3		18.3
	Minimum dry density $(\gamma_d)_{min}$ kN/m^3		14.6

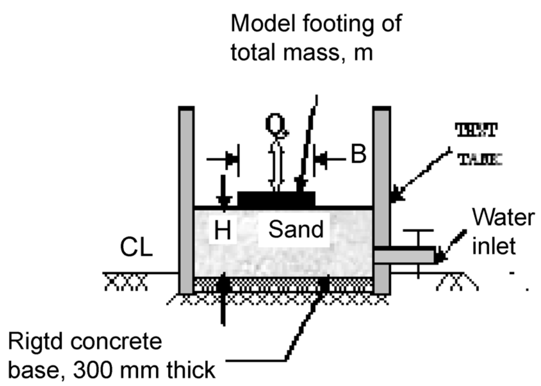


Fig. 1 Schematic experimental set up

conditions considering the same height to width ratios of the sand stratum but with footings resting on the finite saturated sand stratum underlain by the rigid concrete base. The results were compared for both the cases.

A test tank of plan size having internal dimensions 1.6 m × 1.6 m and height 1.2 m with vertical walls was constructed at the testing site situated near the Geotechnical Engineering Laboratory of the Bangalore University Campus, using solid concrete blocks after removing the top loose soil to a depth of 300 mm in the natural ground formation and compacting the soil at the site so as to minimize the relative displacements below the rigid concrete base. Based on the maximum size of the model footings used, the optimized size of the test tank was adopted so as to minimize the boundary reflections. A small opening of size 500 mm × 600 mm was made on one side of the test tank to facilitate easy fixing of the flexible shaft connecting the oscillator (source of vibration mounted on the model footing) with the D.C. motor.

For tests on saturated sand, a network of perforated PVC pipes of 12 mm diameter were made abutting the top of the rigid base and connected to a water inlet, through which water was allowed to flow in the upward direction, maintaining uniform and laminar flow conditions. Saturation was achieved by flooding the sand overnight for 12 hours. Excess water collected at the surface was removed using sponge and the water level in the tank was kept in flush with the top surface of

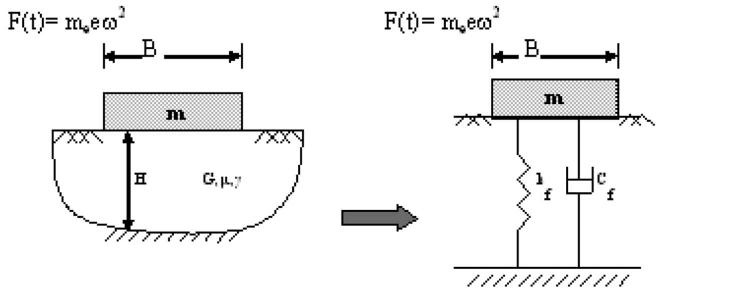
Table 2 Specifications of model footings used

Sl.No.	Series index	Size [L × B × H] (mm)	Total mass including oscillator assembly (kN)	Static weight (kN)	Average static pressure (kN/m^2)
1	SI(A)	200×200×100	1.276		106.9
2	SI(B)	200×200×200	1.408	3.0	110.0
3	SI(C)	200×200×300	1.693		110.8
4	SII(A)	300×300×100	1.431		49.23
5	SII(B)	300×300×200	1.739	3.0	52.66
6	SII(C)	300×300×300	1.861		54.66

sand. Soil samples at different locations (both from the surface as well below the surface of sand) and at different time intervals during the experimental program were removed, to determine the water content by oven drying method. The degree of saturation, S_r was calculated using the water content, voids ratio and specific gravity values. Results of such measurements showed that the degree of saturation achieved was well above 96%. Fig. 1 show the schematic set up used for the present experimental investigation.

Two series of reinforced model footings of square shape and of different thickness using M20 grade cement concrete were cast and cured. Table 2 shows the specification of the model footings used in the present experimental investigation. All the vertical vibration tests on the model footings were carried out using a constant static weight of 3.0 kN, provided by steel plates that were fixed on top of the model footing, along with the oscillator assembly. The mass of footings were so fabricated that, each series of model footings along with the constant mass of the oscillator and the static weight of 3.0 kN generated almost equal intensity of static contact pressure at its base when placed over the finite sand stratum, so as to avoid any consequential effects of variation in contact pressure during vibration.

Calculated weight of sand was poured into the test tank in layers of 100 mm thickness so as to maintain a uniform condition throughout the test program. Each layer was compacted to maintain uniform density of 17 kN/m^3 so that the desired relative density of 70% was achieved. For compaction, a square steel plate of 400 mm width and 10 mm thickness was placed on sand and was tampered by uniformly distributed 16 blows of a 4.5 kg compaction rammer falling through a height of 457.2 mm. Several trials were made initially to know the unit weight γ_d of the sand and the corresponding relative density (D_r) achieved, before starting the test. After filling sand in the test tank, the precast concrete model footing was placed concentrically and the Lazan oscillator assembly along with the static weight (in the form of steel plates) were placed on top of the model footing and secured to it so that the whole set up acted as a single unit, with the center of gravity of whole system and that of the footing to lie in the same vertical line. A piezoelectric type vibration pickup was placed on top of the footing to measure the displacement amplitude and was connected to 'Data Acquisition System (DAS)' using a low noise cable provided for the purpose. The oscillator was then run slowly by the motor using a speed control unit after setting the desired force rating in the oscillator. The model footings were subjected to vertical mode of vibration at four selected force ratings of 0.016 N-sec^2, 0.024 N-sec^2, 0.032 N-sec^2 and 0.039 N-sec^2. The thickness of sand over the rigid concrete base was varied to have height to width ratios (H/B) of 0.5, 1.0, 2.0 and 3.0, where H is the height of sand stratum over the finite base and B is the width of the model



(a) Actual system (b) Idealized mass-dashpot system

Fig. 2 Foundation resting on finite sand stratum

footing under consideration.

Using the data acquisition system, the frequency in Hertz and the corresponding amplitude of vibration were recorded at regular intervals. Sufficient time gap was allowed between two successive measurements to get a stabilized reading. Finally the frequency versus amplitude curves was plotted for each of the different tests. The four different force ratings were used in order to simulate different intensities of dynamic excitation. In all the tests performed, the frequency corresponding to maximum displacement amplitude was taken as the resonant frequency.

3. Determination of damping ratio

The vibrating foundation-soil system can be idealized and modelled using mass-spring-dashpot system as shown in Fig. 2. Fig. 2(a) shows the actual system resting on finite sand stratum of thickness H and Fig. 2(b) shows the idealized mass-spring-dashpot system with the finite sand stratum replaced by the static stiffness (k_f) and the damping coefficient (c_f).

The governing equation of motion is (Richart *et al.* 1970)

$$m\ddot{z} + c_f\dot{z} + k_f z = F(t) = Q_o\sin(\omega t) \tag{1}$$

$$F(t) = m_e\omega^2\sin(\omega t) \tag{2}$$

where, m = total mass of the vibrating block on the soil

z = displacement = A_z sinωt

A_z = displacement amplitude

ω = circular frequency of excitation

t = time variable

Q_o = amplitude of the exciting force

$F(t)$ = the vertical dynamic force induced by the oscillator on the footing at any instant of time, t, and

m_e = eccentric mass of the oscillator

The solution to the Eq. (1) yields the displacement amplitude, given by

$$\frac{(A_z m)}{(m_e e)} = M_r = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2D_f\frac{\omega}{\omega_n}\right)^2\right]^{0.5}} \tag{3}$$

where, M_r = magnification factor

D_f = damping ratio for the foundation-soil system = c_f/c_c, in which $c_c = 2(k_f m)^{1/2}$

ω_n = circular natural frequency = $2\pi f_n = (k_f/m)^{1/2}$, and

f_n = natural frequency of the foundation-soil system

the maximum displacement amplitude $Az_{\max}$ can be obtained from Eq. (3) as

$$\frac{(Az_{\max} m)}{(m_e e)} = \frac{1}{2D_f(1-D_f^2)^{0.5}} \tag{4}$$

which occurs at a frequency ratio

$$\frac{(f_{nr})}{(f_n)} = \frac{1}{(1-2D_f^2)^{0.5}} \tag{5}$$

It can be noted that when damping in the system is low (less than 15%), f_{nr} will be approximately equal to f_n.

4. Results and discussions

The vertical vibration tests were carried out to measure the resonant frequency and the corresponding displacement amplitude. Tables 3 and 4 show the experimental results obtained for model footings of series SI and SII corresponding to finite dry sand stratum and finite saturated sand stratum respectively. As can be seen from these Tables, higher the contact area of the footing, higher is the measured values of the resonant frequency and for a constant area of the model footing, the larger the mass of the footing, lower is the resonant frequency corresponding to a given H/B ratio. Further, the maximum displacement amplitude increases with increase in the contact area of the footing, for a constant H/B ratio and at a constant force rating.

Figs. 3 and 4 show the comparative response curve obtained for model footings resting on finite saturated sand and finite dry sand stratum corresponding to force ratings of 0.016N-sec^2 and 0.024N-sec^2 for model footings of series SI (A), at H/B ratios of 0.5 and 3.0 respectively. As can be seen from Fig. 3 which corresponds to H/B ratio of 0.5, the displacement amplitude is marginally higher and resonant frequency is marginally lower for model footings resting on finite saturated sand stratum than that obtained on finite dry sand stratum. Further, it can be observed from Fig. 3 which corresponds to H/B ratio of 3.0, the maximum displacement amplitude is significantly larger for the case of finite saturated sand stratum as there is large variation in the measured values of displacement amplitude between finite dry sand stratum and finite saturated sand stratum corresponding to resonant frequency.

The damping ratio (D_f) of finite sand stratum has been obtained using Eq. (3) and Eq. (4), for the

Table 3 Resonant frequency and maximum displacement amplitude for series SI

Force rating (N-sec^2)	Height to width ratio (H/B)	Size of the footing: L×B×H in mm					
		200×200×100 SI(A)		200×200×200 SI(B)		200×200×300 SI(C)	
		f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)	f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)	f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)
0.016	0.5	13.5 0.82	13.0 0.86	13.1 0.83	12.6 0.87	12.4 0.75	12.1 0.82
0.024		13.2 0.85	12.9 0.88	12.9 0.85	12.2 0.89	12.2 0.77	11.8 0.86
0.032		12.8 0.93	12.6 0.96	12.7 0.87	12.1 0.91	12.1 0.84	11.6 0.89
0.039		12.4 0.96	12.2 0.98	12.5 0.88	11.8 0.93	11.4 0.87	11.0 0.91
0.016	1.0	10.8 0.65	10.2 0.78	10.4 0.62	10.0 0.72	10.2 0.61	9.7 0.70
0.024		10.6 0.68	10.0 0.82	10.1 0.65	9.6 0.78	9.9 0.64	9.2 0.74
0.032		10.0 0.69	9.7 0.86	9.8 0.66	9.4 0.82	9.2 0.65	9.0 0.80
0.039		9.2 0.71	8.8 0.88	9.6 0.68	9.0 0.86	8.6 0.68	8.6 0.84
0.016	2.0	6.8 0.50	6.2 0.68	6.2 0.48	6.0 0.62	6.0 0.47	5.8 0.61
0.024		6.4 0.51	6.0 0.72	5.9 0.50	5.6 0.66	5.7 0.49	5.6 0.64
0.032		6.2 0.52	5.8 0.74	5.8 0.51	5.4 0.70	5.5 0.51	5.3 0.68
0.039		5.7 0.55	5.2 0.80	5.2 0.53	5.2 0.75	5.1 0.53	5.0 0.71
0.016	3.0	5.9 0.51	5.6 0.71	5.4 0.50	5.0 0.78	5.0 0.47	4.6 0.62
0.024		5.4 0.54	5.2 0.74	4.9 0.51	4.8 0.70	4.8 0.49	4.4 0.66
0.032		4.9 0.55	4.7 0.76	4.5 0.53	4.4 0.72	4.6 0.51	4.2 0.68
0.039		4.5 0.57	4.2 0.78	4.2 0.55	4.2 0.74	4.3 0.53	4.0 0.71

f_{nr} = Resonant frequency in Hertz
Az_{max} = Maximum displacement amplitude in mm

Table 4 Resonant frequency and maximum displacement amplitude for series SII

Force rating (N-sec^2)	Height to width ratio (H/B)	Size of the footing: L×B×H in mm					
		300×300×100 SI(A)		300×300×200 SI(B)		300×300×300 SI(C)	
		f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)	f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)	f_{nr} Az_{max} (Dry sand)	f_{nr} Az_{max} (Saturated sand)
0.016	0.5	19.0 0.84	18.2 0.88	18.6 0.74	18.0 0.82	14.4 0.70	13.6 0.81
0.024		18.0 0.86	17.6 0.92	17.3 0.75	16.9 0.85	14.0 0.71	13.4 0.83
0.032		16.1 0.87	14.5 0.97	14.5 0.76	14.0 0.90	13.4 0.73	13.0 0.86
0.039		15.2 0.89	14.2 0.98	13.5 0.77	13.0 0.94	12.7 0.75	12.0 0.90
0.016	1.0	13.6 0.79	13.2 0.82	13.4 0.70	13.0 0.81	13.3 0.69	12.8 0.78
0.024		13.2 0.81	13.0 0.84	13.1 0.72	12.7 0.82	13.0 0.74	12.4 0.81
0.032		13.0 0.85	12.6 0.86	12.9 0.74	12.4 0.84	12.8 0.77	12.2 0.84
0.039		12.5 0.88	12.1 0.90	12.2 0.77	11.8 0.88	12.4 0.77	11.6 0.86
0.016	2.0	10.4 0.58	9.9 0.69	9.6 0.56	9.4 0.62	8.0 0.54	7.8 0.60
0.024		9.8 0.60	9.4 0.71	9.5 0.58	9.2 0.68	7.8 0.56	7.2 0.66
0.032		9.4 0.64	9.1 0.76	9.4 0.61	9.0 0.71	7.6 0.58	6.8 0.70
0..039		9.4 0.66	9.0 0.80	9.2 0.64	8.6 0.74	7.4 0.60	6.4 0.72
0.016	3.0	8.6 0.52	8.4 0.64	7.6 0.51	7.2 0.58	6.8 0.50	6.6 0.56
0.024		8.4 0.53	8.2 0.68	7.3 0.55	7.0 0.64	6.4 0.52	6.1 0.58
0.032		8.2 0.55	7.9 0.71	7.1 0.56	6.7 0.68	6.00 0.54	5.8 0.62
0.039		7.8 0.57	7.4 0.74	6.9 0.58	6.4 0.71	5.7 0.55	5.4 0.66

f_{nr} = Resonant frequency in Hertz
Az_{max} = Maximum displacement amplitude in mm

entire series of model footings tested. The influence of saturation of the sand on the damping property of finite sand stratum has been analyzed by comparing them with the results obtained for the case of finite dry sand stratum underlain by the same rigid concrete base.

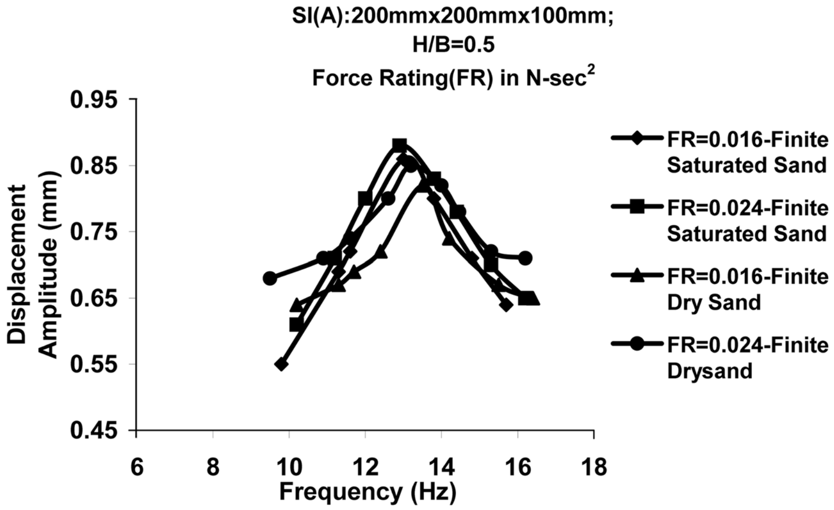


Fig. 3 Frequency response curve for finite saturated sand and dry sand of series SI (A) at H/B=0.5

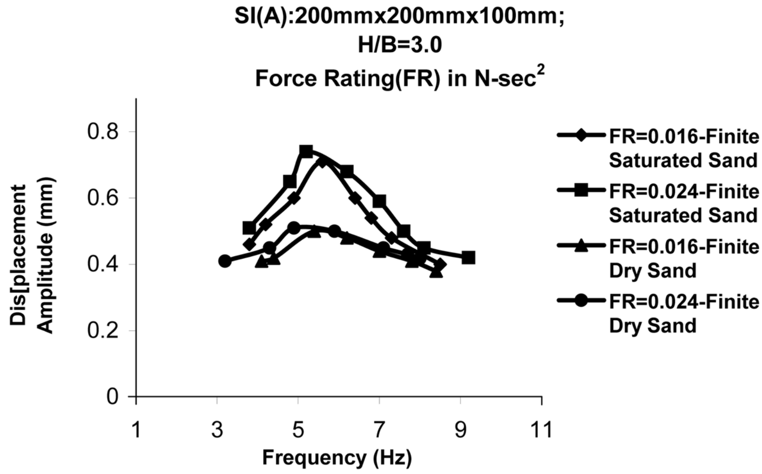


Fig. 4 Frequency response curves for finite saturated sand and dry sand of series SI (A) at H/B=3.0

4.1 Effect of saturation

Laboratory investigations reported by Barkan (1962) have shown that the Poisson's ratio of soil does not depend on moisture content. Thus, in the present investigation the saturated density of soil has been considered with a value of Poisson's ratio equal to 0.33 for saturated sand which same as that for dry sand in all the calculations made for determining the damping of finite saturated sand underlain by rigid base.

Table 5 and Table 6 show the value of damping ratio obtained for model footings of series SI and SII resting on finite dry sand and finite saturated sand stratum underlain by rigid concrete base. It can be seen that the damping ratio obtained is less than 10% for both the cases, but the damping ratio corresponding to the finite dry sand stratum is marginally higher than those obtained for finite

Table 5 Comparison of damping ratio for model footings of series SI

Force ratings (N-sec^2)	Height to width ratio (H/B)	Size of the footing: L×B×H					
		200×200×100 SI(A)		200×200×200 SI(B)		200×200×300 SI(C)	
		D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)	D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)	D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)
0.016	0.5	2.24	2.14	2.15	2.05	2.23	2.04
0.024		3.24	3.13	3.14	3.00	3.26	2.92
0.032		3.94	3.82	4.09	3.91	3.98	3.75
0.039		4.76	4.66	5.04	4.77	4.79	4.58
0.016	1.0	2.83	2.36	2.88	2.48	2.75	2.39
0.024		4.05	3.36	4.11	3.43	3.92	3.39
0.032		5.32	4.26	5.39	4.34	5.14	4.18
0.039		6.45	5.19	6.53	5.16	6.13	4.96
0.016	2.0	3.68	2.71	3.72	2.87	3.57	2.75
0.024		5.41	3.83	5.35	4.05	5.13	3.92
0.032		7.06	4.96	6.99	5.08	6.56	4.92
0.039		8.33	5.72	8.39	5.92	7.86	5.87
0.016	3.0	3.61	2.59	3.57	2.62	3.57	2.70
0.024		5.11	3.72	5.25	3.82	5.13	3.81
0.032		6.67	4.83	6.72	4.94	6.56	4.92
0.039		8.04	5.86	8.08	5.99	7.88	5.87

saturated sand stratum.

4.2 Effect of location of rigid concrete base

Fig. 5 shows the typical variation of damping ratio with H/B ratio for both the series of model footings SI (A) resting on finite saturated sand stratum, at force ratings of 0.016 N-sec^2 and 0.024 N-sec^2. The damping ratio for both the case is significantly low at H/B=0.5 with only a marginal difference between the obtained values for both the cases but increases with increase in H/B ratio. It can be observed that the damping ratio obtained corresponding to finite saturated sand stratum is significantly lower than the one obtained on finite dry sand stratum corresponding to H/B ratio of 3.0, indicating the significant influence of soil moisture on damping ratio of foundation- soil system with increase in the thickness of the finite sand stratum above the rigid concrete base. Further, it can be observed that the damping ratio increases with increase in force rating for both the cases. Same trends have been observed for all the series of model footings tested.

4.3 Predicted and observed damping ratio

According to Lysmer's analogy (Richart *et al.* 1970), the radiation damping for footings under

Table 6 Comparison of damping ratio for model footings of series SII

Force ratings (N-sec^2)	Height to width ratio (H/B)	Size of the footing: L×B×H					
		300×300×100 SII(A)		300×300×200 SII(B)		300×300×300 SII(C)	
		D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)	D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)	D_f^* (%) (Finite dry sand)	D_f^{**} (%) (Finite saturated sand)
0.016	0.5	2.11	2.02	2.24	2.02	2.31	1.99
0.024		3.09	2.89	3.32	2.92	3.41	2.92
0.032		4.07	3.65	4.35	3.67	4.42	3.75
0.039		4.96	4.50	5.36	4.39	5.36	4.46
0.016	1.0	2.25	2.16	2.37	2.05	2.34	2.00
0.024		3.28	3.17	3.45	3.03	3.28	2.99
0.032		4.16	4.11	4.47	3.94	4.19	3.84
0.039		5.01	4.90	5.36	4.69	5.22	4.68
0.016	2.0	3.06	2.57	2.96	2.68	2.99	2.69
0.024		4.43	3.75	4.29	3.65	4.33	3.67
0.032		5.26	4.66	5.43	4.66	5.57	4.60
0.039		6.69	5.52	6.45	5.57	6.67	5.58
0.016	3.0	3.41	2.77	3.25	2.86	3.24	2.89
0.024		5.02	3.91	4.52	3.89	4.66	4.18
0.032		6.44	4.98	5.91	4.87	5.98	5.21
0.039		7.55	5.97	7.12	5.81	7.32	6.09

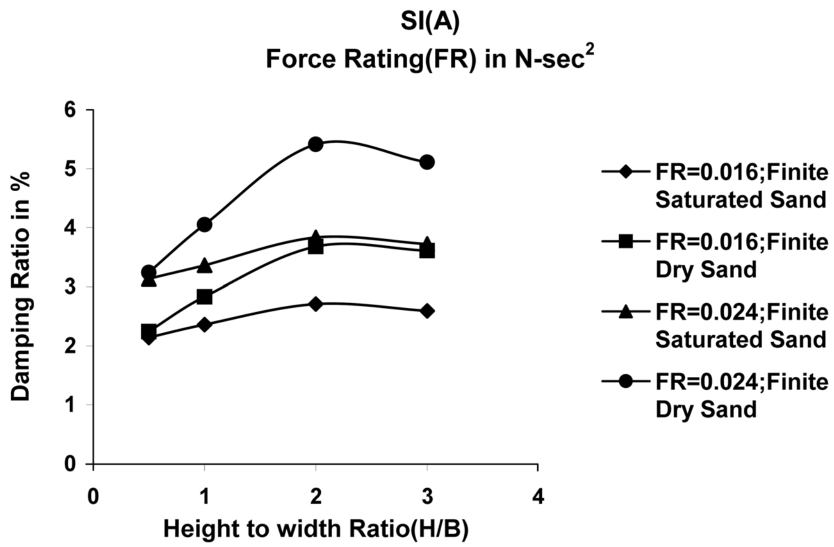


Fig. 5 Comparison of damping ratio with H/B ratio of series SI (A)

Table 7 Predicted damping ratio and observed damping ratio

Series index	Size of the model footing in mm	Modified mass ratio (B_z)	Predicted damping ratio for homogeneous sand** (%)	Observed damping ratio obtained for H/B=3.0 (%)	Difference
SI(A)	200×200×100	24.21	5+8.60=13.60	5.86	4.53
SI(B)	200×200×200	25.13	5+8.47=13.47	5.99	4.36
SI(C)	200×200×300	26.76	5+8.22=13.22	5.87	4.49
SII(A)	300×300×100	7.56	5+15.46=20.46	5.97	11.07
SII(B)	300×300×200	8.08	5+14.95=19.95	5.81	11.02
SII(C)	300×300×300	8.29	5+14.76=19.76	6.09	10.90

**A constant value of 5% for material damping has been added to radiation damping obtained using eq. (4.1) to obtain total damping ratio for homogeneous sand stratum.

vertical mode of vibration can be obtained using the following set of equations

$$D_{\mathrm{rad}} = \frac{0.425}{(B_z)^{0.5}} \tag{6a}$$

$$B_z = \frac{(1-\mu)}{4} b \tag{6b}$$

$$b = \frac{m}{\rho r_o^3} \tag{6c}$$

$$r_o = \left(\frac{A}{\pi}\right)^{0.5} \tag{6d}$$

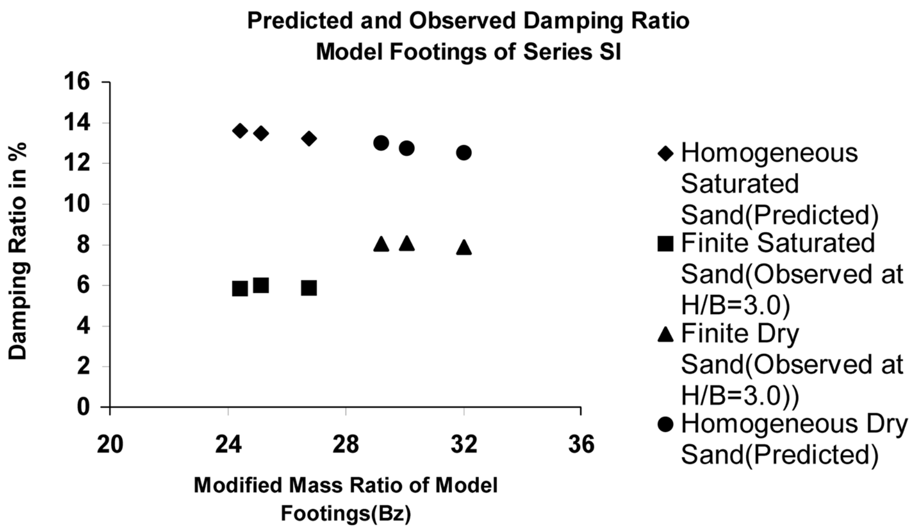


Fig. 6 Comparison of predicted and observed damping ratio for model footings of series SI

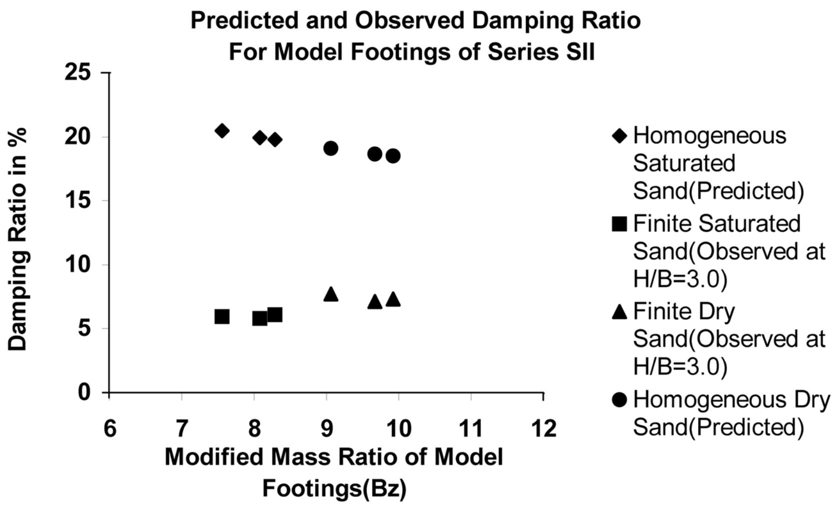


Fig. 7 Comparison of predicted and observed damping ratio for model footings of series SII

where, B_z = modified mass ratio of the footing

b = mass ratio of the footing

m = total mass of the vibrating footing

ρ = mass density of the vibrating in-phase soil, and

r_o = equivalent radius of the footing

A = contact area of the footing

The total damping ratio of a homogeneous sand of infinite depth was estimated for each series of footing tested making use of Eqs. (6a), (6b), (6c) and (6d) for radiation damping component and a suggested value of 5% (Richart *et al*. 1970) for material damping component, which is henceforth called as the *predicted damping ratio*. Since the influence of rigid concrete base decreases with increase in H/B ratio, the damping values obtained using the results of resonant frequency corresponding to H/B ratio of 3.0 is considered to correspond to homogeneous sand stratum, henceforth called as *observed damping ratio*. The predicted damping ratio has been compared with the observed damping ratio, for entire series of model footings used. Table 7 shows the comparison of predicted damping ratio for homogeneous saturated sand of infinite depth and the observed damping ratio obtained corresponding to H/B=3.0, for the saturated sand stratum. It can be observed that, the difference between predicted damping ratio for homogeneous sand and the observed damping ratio ranges from 4.53% for model footing with smaller contact area and lower mass ratio to 11.07% for model footing with larger contact area and higher mass ratio. The reduction in damping indicates the absence of radiation damping in both the case of finite dry sand and finite saturated sand stratum.

Figs. 6 and 7 show the comparison of predicted damping ratio obtained for homogeneous saturated sand and homogeneous dry sand as well as observed damping ratio at H/B=3.0 for both the series of model footings. The predicted damping ratio for both the series of model footings are larger than 10% whereas the observed damping ratio at H/B ratio of 3.0 is less than 10% for both the series of model footings. The observed damping ratio in the case of finite saturated sand is marginally lower than the corresponding values obtained for finite dry sand stratum. The predicted damping ratio for homogeneous saturated sand is marginally higher than the corresponding predicted damping ratio obtained for the case of homogeneous finite dry sand.

5. Conclusions

The results of the experimental study and comparison of results between model footings resting on finite dry sand stratum and finite saturated sand stratum underlain by the rigid concrete base showed that:

1. The rigid base has significant influence on the damping ratio of finite dry sand stratum as well as finite saturated sand stratum.
2. The damping ratio obtained for both the cases is less than 10%. The damping ratio obtained for finite saturated sand stratum is marginally lower than that obtained on finite dry sand stratum at H/B ratio of 0.5. The difference between the two cases becomes significant when the H/B ratio increases to 3.0, indicating the significant influence of soil moisture on damping ratio of foundation-soil system with increase in the thickness of the finite sand stratum.
3. The damping ratio for both the cases is significantly low at H/B=0.5. Thus the presence of rigid base at lower H/B ratio is significant for both the cases, reducing the damping ratio to less than 10% and this is attributed to the absence of radiation damping for both the cases.
4. Comparison of the predicted damping ratio for a homogeneous sand stratum with the observed damping ratio obtained corresponding to the ratio H/B=3.0 of the finite saturated sand stratum underlain by the rigid concrete base indicates a significant reduction in damping ratio of the foundation-soil system with the difference between the two falling in the range of 4 to 11%, irrespective of the contact area and mass ratio of the model footing.

The observed damping ratio in the case of finite saturated sand is lower than the corresponding values obtained for finite dry sand stratum. The predicted damping ratio for homogeneous saturated sand is marginally higher than the corresponding predicted damping ratio obtained for the case of homogeneous finite dry sand.

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Notations

A	: contact area of the footing
b	: mass ratio
B	: width of the model footing
B_z	: modified mass ratio
c_f	: damping coefficient
D_{rad}	: radiation damping
D_f	: damping ratio of foundation-soil system
D_r	: relative density
$F(t)$	: the vertical dynamic force induced by the oscillator at any time, t
f_n	: natural frequency of the foundation-soil system
f_{nr}	: resonant frequency
H	: thickness of the finite sand stratum
H/B	: height to width ratio
M_r	: magnification factor
m	: total mass of the vibrating system
m_e	: eccentric mass provided in the oscillator
$m_e e$	: force rating
Q_o	: amplitude of the exciting force

r_o : equivalent radius of the model footing
ω : circular frequency of excitation
ω_n : circular natural frequency
ρ : mass density of the vibrating in-phase soil