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Field instrumentation and settlement prediction of ground treated with straight-line vacuum preloading

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Abstract. The vacuum preloading method has been used in many countries for ground improvement and land reclamation works. A sand cushion is required as a horizontal drainage channel for conventional vacuum preloading. In terms of the dredged-fill foundation soil, the treatment effect of the conventional vacuum preloading method is poor, particularly in Tianjin, China, where a shortage of sand exists. To solve this problem, straight-line vacuum preloading without sand is widely adopted in engineering practice to improve the foundation soil. Based on the engineering properties of dredged fill in Lingang City, Tianjin, this paper presents field instrumentation in five sections and analyzes the effect of a prefabricated vertical drain (PVD) layout and a vacuum pumping method on the soft soil ground treatment. Through the arrangement of pore water pressure gauges, settlement marks and vane shear tests, the settlement, pore water pressure and subsoil bearing capacity are analyzed to evaluate the effect of the ground treatment. This study demonstrates that straight-line vacuum preloading without sand can be suitable for areas with a high water content. Furthermore, the consolidation settlement and consolidation degree system is developed based on the grey model to predict the consolidation settlement and consolidation degree under vacuum preloading; the validity of the system is also verified.

Keywords: field instrumentation; straight-line vacuum preloading; consolidation settlement prediction; grey model

1. Introduction

With the rapid economic development in the coastal areas of China, the lack of rare land resources has become a problem for the urbanization process. Harbor and waterway dredged-soil reclamation has become a main method to alleviate the shortages of land resources. Dredged soil has a high water content up to 120%, a high clay content of 50%, and certain rheological characteristics (Lei *et al.* 2016). In the absence of suitable ground improvement, an excessive differential settlement and subsequent movement unfavorably affect the stability of the buildings and port infrastructure built on such soft ground (Indraratna *et al.* 2011, Mesri and Khan 2012, Quang and Giao 2014).

To quickly and effectively improve the bearing capacity of dredged-fill foundation soil, a vacuum preloading technique is widely employed to improve the ground soil, which can meet construction requirements (Song and Kim 2004, Lei *et al.* 2017, Cai *et al.* 2018). Recently, China has implemented stringent requirements for environmental protection, and land reclamation is often quickly converted to land for commercial use. The conventional vacuum preloading method has many limitations and challenges in the treatment of dredged-fill ground, including the high sand cost, long construction period, poor improvement effect of new dredged-fill ground and prefabricated vertical drain (PVD) clogging (Zhu and Miao 2002, Khan and Mesri 2014, Wang *et al.* 2016).

To improve the treatment effect, researchers have focused on new types of vacuum preloading methods, such as ground treatment by straight-line vacuum preloading without sand, a vacuum preloading method coupled with air booster vacuum preloading, an underwater vacuum preloading method, a combined method of vacuum preloading and surcharge preloading, the combined method of vacuum preloading and electro-osmosis (Albert *et al.* 2009, Saowapakpiboon *et al.* 2011, Liu *et al.* 2016, Wu *et al.* 2017). Compared with other vacuum preloading technologies, ground treatment by straight-line vacuum preloading without sand has the advantages of a lower engineering cost and better improvement effect.

No sand resources exist in Tianjin, and thus the sand cost is extremely high; sand is purchased from Caofeidian City, Hebei Province, and the environmental protection policy of China is implemented to restrict the overuse of black sand. The ground treatment by straight-line vacuum preloading without sand is needed to widely distribute the sand in the Tianjin Binhai New Area, China. The technique of ground treatment by straight-line vacuum preloading

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without sand has been researched and developed. Scholars have studied the effect of soft soil ground treatment by straight-line vacuum preloading without sand. Relative to conventional vacuum preloading, straight-line vacuum preloading without sand is the key to eliminating the sand cushion. The transfer path of the vacuum pressure is shortened, which improves the utilization efficiency of the vacuum pressure to shorten the consolidation time and enhance the improvement effect. In addition, because the sand cushion is omitted, the construction cost is reduced. Wang *et al.* (2016) demonstrated that straight-line vacuum preloading without sand can shorten the vacuum transfer path and reduce the energy consumption. Liu *et al.* (2017) conducted a series of tests, which showed that straight-line vacuum preloading without sand for soft soil ground treatment can efficiently and economically improve various types of soft soil foundations.

In engineering practice, because there are no corresponding technical regulations for ground treatment by straight-line vacuum preloading without sand, engineers often encounter challenges during design and construction according to the local standard “Technical specification for vacuum preloading technique to improve soft soils” (JTS 147-2-2009) (Ministry of Transport of the People's Republic of China 2009). For example, the PVD spacing and pipe spacing layouts may be unreasonable, often resulting in an excessive pumping time or low subsoil bearing capacity. The scope of application and the implementation details of the soft soil ground treatment by the vacuum preloading without sand are unclear, and blind adoption can result in the failure of the ground treatment and engineering accidents.

In terms of consolidation settlement prediction, the grey model (GM (1, 1)) is often used to predict the settlement in geotechnical engineering (Shahin *et al.* 2005, Xu and Dang 2015). For example, a three-point method and a grey model method were compared and analyzed by Zeng *et al.* (2012), who suggested that predictions of the grey model can be applied to engineering practice to ensure the security of the project. Liu *et al.* (2013) demonstrated that GM (1, 1) was precise and effective for predicting the ground settlement. However, few scholars have used this method to develop a system of consolidation settlement prediction to serve practical engineering.

In summary, due to the shortage of sand resources, the uncertainty of new types of vacuum methods and lack of a national code or industry standards, straight-line vacuum preloading methods without sand must be urgently developed and widely applied in the Tianjin Binhai New Area, China. To date, although many scholars have focused on straight-line vacuum preloading without sand, research remains nonexistent in the exploration phase. In addition, scholars are interested in the consolidation mechanism and numerical simulation, but few have developed a system of consolidation settlement prediction to realize the information construction. Therefore, it is vital to study the field instrumentation and settlement prediction of foundations treated by straight-line vacuum preloading without sand to provide guidelines for large-area implementation and test data accumulation.

This paper systematically studies the pore water pressure dissipation and settlement to explain consolidation behaviors of soft soil foundations via data monitoring. The PVD layout and vacuum pumping method are also discussed to illustrate the optimal spacing and method. Furthermore, the bearing capacity is analyzed to evaluate the effect of the ground treatment. Finally, GM (1, 1) is established to predict the settlement and consolidation degree under vacuum preloading. To control the construction progress and guide the construction arrangement, the consolidation settlement system is developed to guide engineering practice.

2. Engineering properties of dredged fill

A ground treatment field test by straight-line vacuum preloading without sand was conducted in the Lin Gang Area of Tianjin City, China. The treatment area is approximately 121,200 m^2. The ground elevation was surveyed before and after vacuum preloading, with each datum being taken as the mean sea level of the Yellow Sea in China. According to the survey, the initial ground surface was at an elevation of +5.5 m. The field test is divided into 5 sections to evaluate the treatment effect, as shown in Fig. 1.

Samples were obtained from a depth of 20 m below the ground surface at a construction site in each section. Shelby tubes with an internal diameter of 9.8 cm and a tapered end

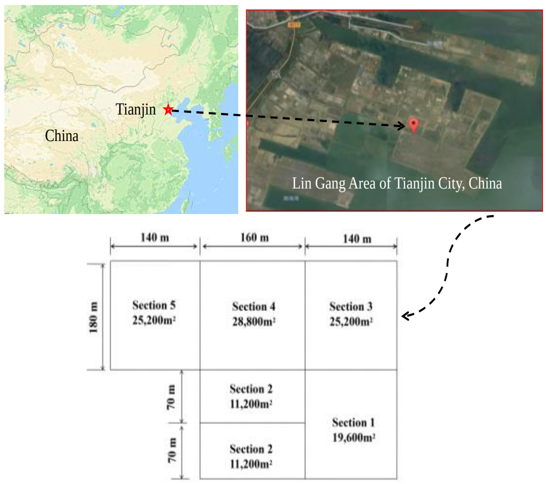


Fig. 1 Field test sections

Table 1 Basic physical properties

Subarea	Water content/%	Specific gravity	Void ratio	Degree of saturation/ %	Liquid limit/%	Plastic limit/%
Section 1	65-72	2.70	1.01-1.24	90-100	31-55	15-30
Section 2-1	68-76	2.72	1.03-1.28	92-99	35-43	12-27
Section 2-2	70-80	2.71	1.11-1.32	95-98	32-40	14-28
Section 3	66-82	2.72	1.07-1.27	92-97	31-52	12-25
Section 4	69-81	2.71	1.08-1.30	92-99	31-44	11-27
Section 5	65-77	2.69	1.06-1.29	91-96	36-52	13-28

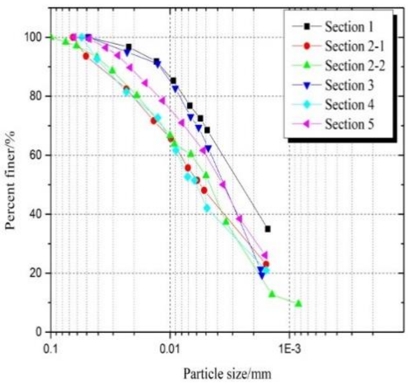


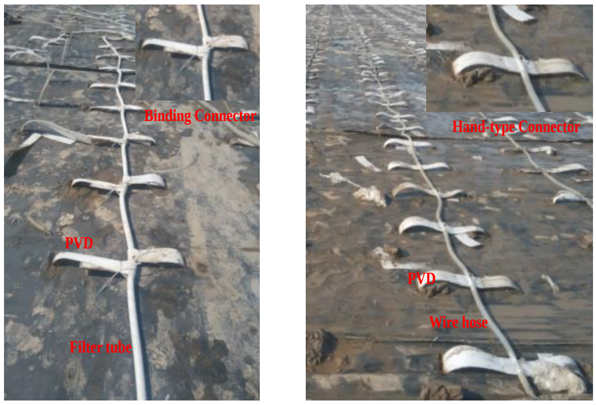
Fig. 2 Particle size distribution curves

of 6 degrees were used to collect soil samples to reduce the sampling disturbance. The length of the tube was 50 cm, and the wall thickness was 0.2 cm. According to the local standard "*Specification of soil test*" (GB/T50123-2019; Ministry of Water Resources of the People's Republic of China 2019), a series of laboratory tests were conducted in an underground engineering laboratory at Tianjin University to obtain the basic physical indices, as shown in Table 1. In general, the water content is approximately 65% to 82%, and the specific gravity is approximately 2.61 to 2.74 g/cm^3 within a depth of 15 m. In the dredged soil, the degree of saturation ranges from 90% to 100%, and the void ratio is high and exceeds 1.0. The liquid limit is from 31% to 55%, and the plastic limit is from 15% to 30%.

In addition, particle size distribution tests were conducted in five sections. The particle size distribution curve of the soil is plotted in Fig. 2, which demonstrates that the particle sizes are distributed over a wide range, the soils are well graded, and the clay content is approximately 43.32% to 86.18%.

3. Ground treatment technique of straight-line vacuum preloading without sand

The straight-line vacuum preloading system consists of a woven geotextile, a vacuum pump, membranes, horizontal pipes and PVDs. Compared with the conventional vacuum preloading technique, the technique of ground treatment by straight-line vacuum preloading without sand adopts a horizontal hose as the horizontal drainage system, which replaces the sand cushion and may increase the cost of the horizontal vacuum tube. At present, there are two connection modes in the construction site. One is the conventional vacuum preloading connection method. The drainage branch pipe is a filter pipe, and PVDs bind with the filter pipe. The other is the straight-line vacuum preloading connection method. The drainage pipe is a wire hose. PVDs and the wire hose are connected by a hand-type connectors, as shown in Fig. 3. However, the second connection method is most used in engineering practice for the ground treatment by straight-line vacuum preloading without sand because the filter tube can easily collapse under the higher vacuum pressure. By contrast, the wire hose can effectively avoid the tube collapse since there are



(a) Conventional vacuum preloading (b) Straight-line vacuum preloading

Fig. 3 Particle size distribution curves

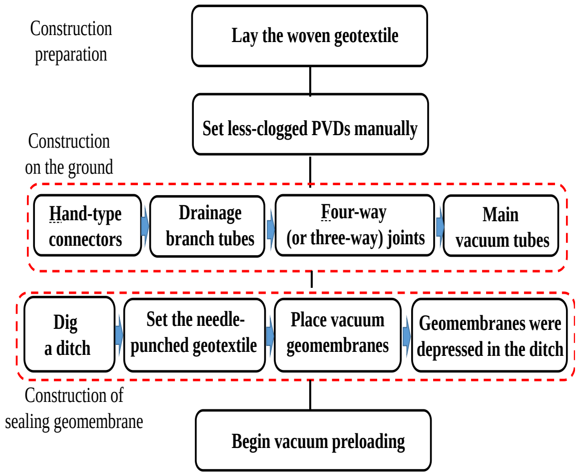


Fig. 4 Construction sequence of ground treatment by straight-line vacuum preloading without sand

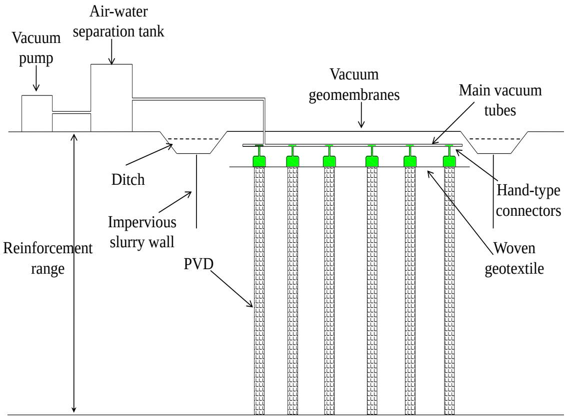
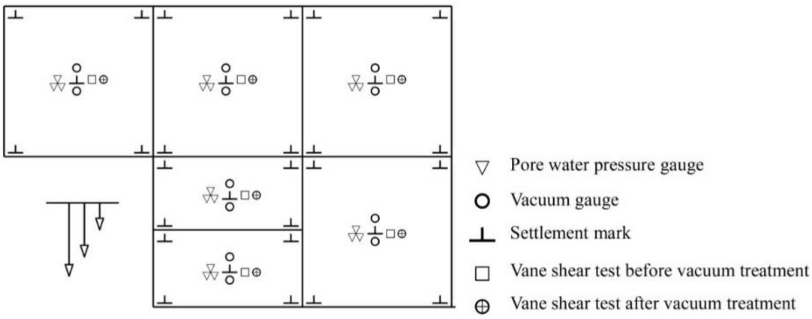


Fig. 5 Cross section of ground treatment by straight-line vacuum preloading without sand

more steel wires inside the hose. According to the construction contract, the cost of wire hose is approximately 1.5 RMB/m higher than that of the filter tube. Therefore, a horizontal drainage system is formed by a horizontal hose instead of a sand cushion in straight-line vacuum preloading

Table 2 Cases tested

Subarea	Treatment area and PVD layout for deep treatment			Vacuum pumping method
	Area/m2	PVD design	Design parameter	
Section 1/Section 3	16,800/19,600	Space/m	0.8 (square configuration)	Air-water separation tank combined with jet vacuum pump
		Depth/m	20	
Section 2-1	9,800	Space /m Depth/m	0.8 (square configuration)	Jet vacuum pump
			20	
Section 2-2	9,800	Space /m	1 (triangular arrangement)	Jet vacuum pump
		Depth/m	20	
Section 4/Section 5	22,400/19,600	Space /m	1 (triangular arrangement)	Air-water separation tank combined with jet vacuum pump
		Depth/m	20	
Predicted preloading time/day			168	



(a) Plan view

(b) Pore water pressure

(c) Settlement marks

(d) Vane shear machine

Fig. 6 Distribution of monitoring devices

without sand. The project cost of the horizontal vacuum tube is slightly higher than that of conventional vacuum preloading.

The construction process of ground treatment by straight-line vacuum preloading without sand includes the construction preparation, construction on the ground and the construction of the sealing geomembrane. The construction sequence of the treatment was as follows: (1) the woven geotextile was laid to separate the dredged soil and to provide a work platform for the construction (2) The less-clogged PVDs were manually rooted. The PVDs should be 0.5 meters long above the ground, according to the test plans. (3) Hand-type connectors were added to the top of the PVDs, which is the key to connecting drainage branch tubes (wire hose). (4)Hand-type connectors were connected with drainage branch tubes, and then four-way (or three-way) joints were connected between the main vacuum tubes and drainage branch tubes. (5) The main vacuum tubes were connected to the air-water separating tank and subsequently connected to the vacuum pump. (6) A ditch is dug to seal the area around each section. (7) A needle-punched geotextile was laid to prevent the breakage of the vacuum geomembranes. (8) Two layers of vacuum geomembranes were placed on the top of the geotextile. (9) The geomembranes were depressed in the ditch. (10) The vacuum preloading began according to the test plans. The construction sequence of the treatment is shown in Fig. 4.

The cross-section of the ground treatment by straight-line vacuum preloading without sand is presented to elucidate the construction sequence of the treatment, as shown in Fig. 5.

4. Test plans

Pore water pressure gauges and settlement marks were

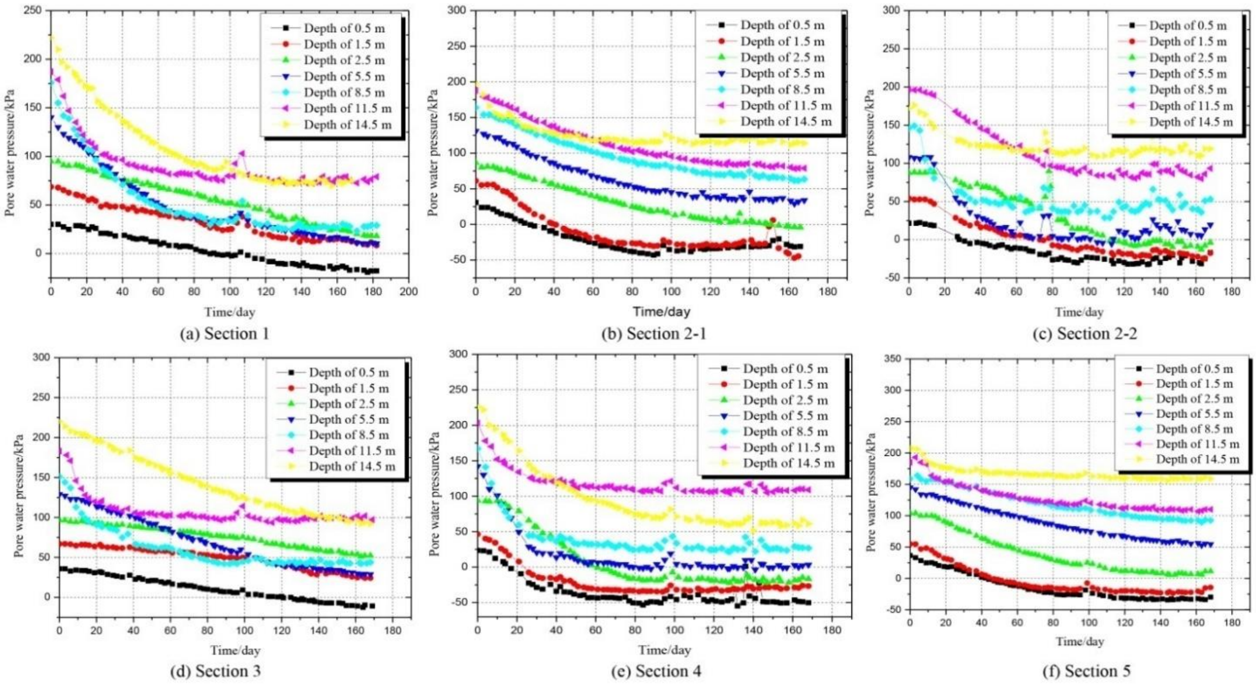


Fig. 7 Pore water pressure vs. time

arranged, and a series of vane shear tests were conducted at the field test site. Pore water pressure gauges adopt a KYJ30 series vibrating string (made in Tianjin, China), which is a tool to measure the fluid pressure. To obtain accurate data and reasonably evaluate this vacuum preloading method, pore water pressure gauges were rooted at depths of 0.5 m, 1.5 m, 2.5 m, 5.5 m, 8.5 m, 11.5 m and 14.5 m. Settlement marks were placed in the four corners at central locations in each section. The vane shear strengths before and after the vacuum treatment were measured by a vane shear test to assess the bearing capacity of the foundation. The monitoring devices were distributed as detailed in Fig. 6.

To evaluate the improvement effect of the ground treatment, the vacuum pumping method and PVD layout are designed as shown in Table 2. The PVDs on each 0.8 m square configuration in Section 1, Section 2-1, and Section 3 are selected, and the PVDs on each 1.0 m triangular arrangement in Section 2-2, Section 4, and Section 5 are chosen to compare the effect of the ground treatment. The PVDs are rooted at a depth of 20 m. With respect to the vacuum pumping method, a jet vacuum pump and an air-water separation tank combined with a jet vacuum pump are also used.

According to the construction contract, the ground treatment by the vacuum method should satisfy three principles such that the degree of consolidation reaches 90%, the surface subsidence rate is not greater than 2.5 mm/d for 5 days and the subsoil bearing capacity exceeds 50 kPa at the ground surface. The predicted preloading time is approximately 168 days.

5. Test results

In this section, the ground treatment field test results by straight-line vacuum preloading without sand are analyzed. Indices of the pore water pressure, settlement and subsoil bearing capacity determined by the vane shear strength are chosen to evaluate the effect of the ground treatment.

5.1 Pore water pressure analysis

The pore water pressure characterizes the consolidation behaviors of the dredged fill, as shown in Fig. 7. The pore pressure dissipates with time, and the pore pressure dissipation decreases with the increasing depth.

A comparison of Section 2-1 and Section 2-2 shows that the increment values of the pore water pressure dissipation are generally close, which implies that the PVD spacing and PVD configuration have little effect on the pore water pressure dissipation. However, the essence of the pore pressure dissipation tends to be related to the influence area of the PVDs, which is idealized as a circle according to a suggestion proposed by Barron (1948), as shown in Fig. 8. The diameter of the circle is determined using Eqs. (1) and (2) as shown below:

Triangular arrangement:

$$d_e = \sqrt{\frac{2\sqrt{3}}{\pi}}l \tag{1}$$

Square configuration:

$$d_e = \sqrt{\frac{4}{\pi}}l \tag{2}$$

where d_e is the circle diameter and l is the PVD spacing. When the configuration is square, l is 0.8 m and d_e is 0.9 m; when arrangement is triangular, l is 1.0 m and d_e is 1.05 m. However, if l is the same for every section, the triangular arrangement

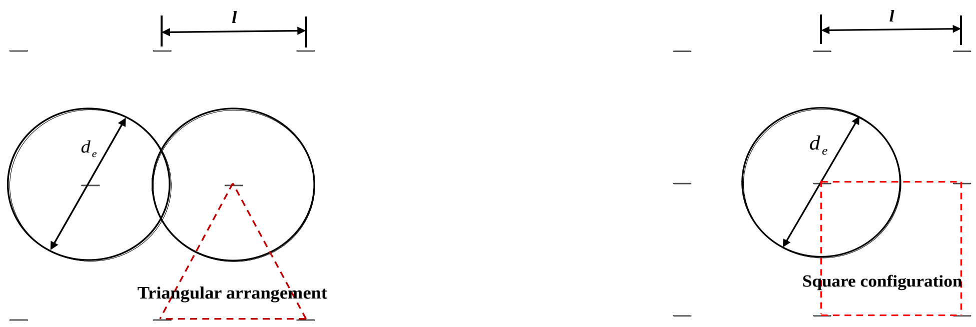


Fig. 8 Cross section of ground treatment by straight-line vacuum preloading without sand

Table 3 Pore water pressure dissipation results

Section	Depth/m	Initial pore water pressure /kPa	Final pore water pressure/ kPa	Time/Day	Increment of pore pressure dissipation /kPa	Pore water pressure dissipation rate/ (kPa/day)
Section 1	0.5	30.1	-17.8	168	47.8	0.28
	1.5	68.6	8.46		60.14	0.36
	2.5	94.9	18.1		76.8	0.46
	5.5	140	10.4		129.6	0.77
	8.5	176	29		147	0.88
	11.5	187	79		108	0.64
	14.5	222	75.4		146.6	0.87
Section 2-1	0.5	30.9	-30.9	168	61.8	0.37
	1.5	61.1	-44.6		105.7	0.63
	2.5	86.6	-4.2		90.8	0.54
	5.5	131	33.8		97.2	0.58
	8.5	164	63.5		100.5	0.60
	11.5	188	78.9		109.1	0.64
	14.5	197	114		83	0.49
Section 2-2	0.5	21.7	-16.9	168	38.6	0.23
	1.5	54	-17.5		71.5	0.43
	2.5	87.7	-3.67		91.37	0.54
	5.5	108	19.4		88.6	0.53
	8.5	146	53.1		92.9	0.55
	11.5	197	93.4		103.6	0.62
	14.5	172	119		53	0.32
Section 3	0.5	35.7	-10.8	168	46.5	0.28
	1.5	66.9	24.3		42.6	0.25
	2.5	96.4	52.2		44.2	0.26
	5.5	129	28.8		100.2	0.60
	8.5	151	43.8		107.2	0.64
	11.5	183	97.3		85.7	0.51
	14.5	220	92.2		127.8	0.76
Section 4	0.5	23.5	-50.7	168	73.7	0.44
	1.5	46.3	-56.7		73	0.43
	2.5	94	-16.7		110.7	0.66
	5.5	142	3.24		138.76	0.83
	8.5	167	26.7		140.3	0.84

Table 3 Continued

Section	Depth/m	Initial pore water pressure /kPa	Final pore water pressure/ kPa	Time/Day	Increment of pore pressure dissipation /kPa	Pore water pressure dissipation rate/ (kPa/day)
Section 4	11.5	203	109	168	94	0.56
	14.5	226	60.9		165.1	0.98
Section 5	0.5	36.5	-29.8	168	66.3	0.39
	1.5	54.9	-14.4		69.3	0.41
	2.5	104	11.4		92.6	0.55
	5.5	145	54.5		90.5	0.54
	8.5	159	92.6		66.4	0.40
	11.5	185	110		75	0.45
	14.5	207	159		48	0.29

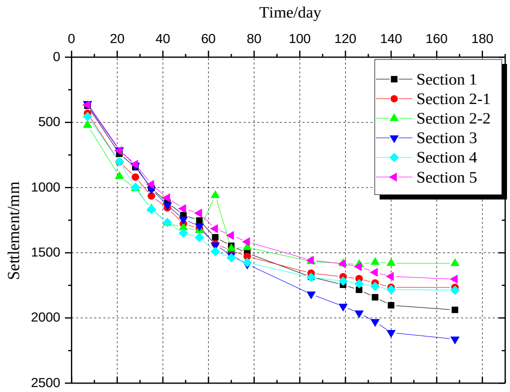


Fig. 9 Settlement vs. time

Table 4 Settlement results in the various sections

Section	Section 1	Section 2-1	Section 2-2	Section 3	Section 4	Section 5
Cumulative consolidation settlement/mm	1938	1765.9	1581.6	2162.8	1787.7	1702.1

might play an important role in the consolidation of the soil, which can improve and increase d_e. For example, when l is 1.0 m, d_e is 0.74 m for the square configuration and d_e is 1.05 m for the triangular arrangement; d_e is increased by 42%. Therefore, to save construction costs and reduce the consolidation time, a triangular arrangement is recommended in engineering practice.

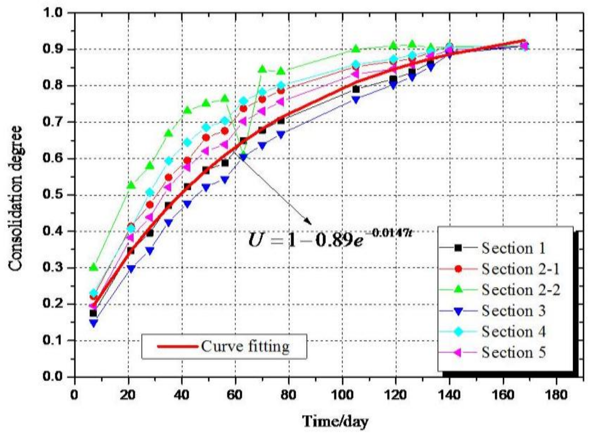


Fig. 10 Consolidation degree vs. time

However, the vacuum pumping method of the air-water separation tank combined with the jet vacuum pump may influence the consolidation effect. To elucidate the influence of the vacuum pumping method on the consolidation, the pore pressure dissipation results are listed in Table 3, and the vacuum pumping method is found to play a significant role. For example, jet vacuum pumps are used to strengthen the ground in Section 2-1 and Section 2-2, and the pore water pressure dissipation rate first increases and then decreases with the increasing depth. However, a vacuum pump coupled with an air-water separation tank is adopted in Section 1, Section 3, and Section 4, and the pore water pressure dissipation rate continues to increase.

These results imply that the vacuum preloading technique using an air-water separation tank combined with a jet vacuum pump can improve the ground treatment effect. However, the pore water pressure dissipation rate is relatively small in Section 5 because silt accumulated in the PVDs. Thus, the vacuum was not efficiently transmitted to the deep foundation. Actually, the largest advantages of the vacuum pump coupled with an air-water separation tank are the power and money savings because the jet vacuum pump is always operating during construction (Wang 2015).

5.2 Settlement analysis

Fig. 9 shows that the settlement of the underground surfaces increases over time. The maximum cumulative deformation approached 2.2 m in Section 3; however, the minimum cumulative consolidation settlement was 1.58 m when the consolidation time was 168 days in Section 2-2. The settlement results in different sections are listed in Table 4.

The consolidation degree is important to evaluate the unloading standard in engineering practice. According the principle of effective stress (Terzaghi 1925), the soil deformation is related only to the effective stress. Thus, the average consolidation degree can be defined as follows:

$$U_t = \frac{\int_0^H \sigma'(z,t)dz}{\int_0^H u_0(z)dz} = \frac{s_t}{s_\infty} \quad (3)$$

where $\sigma'(z,t)$ is the effective stress of the vacuum preloading, $u_0(z)$ is the initial pore water pressure of the vacuum preloading over 1 day, s_t is the consolidation settlement at time t, and s_∞ is the consolidation settlement at time ∞.

To calculate s_∞, the following equation is offered by Wu *et al.* (2009):

$$s_\infty = \sum_{i=1}^{n} \xi h_i (e_{0i} - e_{1i})/(1 + e_{0i}) \quad (4)$$

where e_{0i} is the void ratio before treatment, e_{1i} is the void ratio after treatment, h_i is the thickness of the soil layer in the i deposit or the treatment depth, and ξ is an empirical coefficient. According to engineering practice, ξ is equal to 1.4 (Wu *et al.* 2009), e_{01} is 1.24 and e_{11} is 1.03 for Section 1, and e_{01} is 1.28 and e_{11} is 1.07 for Section 2-1, e_{01} is 1.32 and e_{11} is 1.06 for Section 2-2, e_{01} is 1.27 and e_{11} is 1.01 for Section 3, e_{01} is 1.30 and e_{11} is 1.04 for Section 4, e_{01} is 1.29, and e_{11} is 1.03 for Section 5. In addition, h_1 is 20.0 m.

Fig. 10 shows that the average consolidation degree increases over time and can reach 90% according to Equation (3). The surface subsidence rate is not greater than 2.5 mm/d for 5 days in the different sections. Zeng and Yang (1959) summarized the general relation between the average consolidation degree and time according to the following analytic solution:

$$U = 1 - \alpha e^{-\beta t} \quad (5)$$

where U is the average consolidation degree and α and β are experimental parameters.

This empirical formula can be used to adequately predict the average consolidation degree and settlement. When the consolidation time increases from 0 to 168 (days), Equation (6) is obtained via curve fitting and a regression analysis, where α is 0.89, β is 0.0147 and the correlation degree can reach 88%.

$$U = 1 - 0.89e^{-0.0147t} \quad (0 < t < 168) \quad (6)$$

5.3 Subsoil bearing capacity analysis

The main purpose of the ground treatment is to satisfy the subsoil bearing capacity requirement related to the vane shear strength to some degree. According to the local code "*Technical specification for vacuum preloading technique to improve soft soils*" (JTS_147-2-2009) (Ministry of Transport of the People's Republic of China, 2009), empirical formula (7) is as follows:

$$f_{ak} = M_c C_u \quad (7)$$

where f_{ak} is the characteristic value of the ground bearing capacity; M_c is the coefficient of the bearing capacity, which is recommended to be $(\pi+2)/2$ or determined using the loading plate test; and C_u is the average vane shear strength. Therefore, C_u is a significant index to evaluate the subsoil bearing capacity.

According to the "*Code for in-situ measurement of railway engineering geology*" (TB10041-2003) (Ministry of

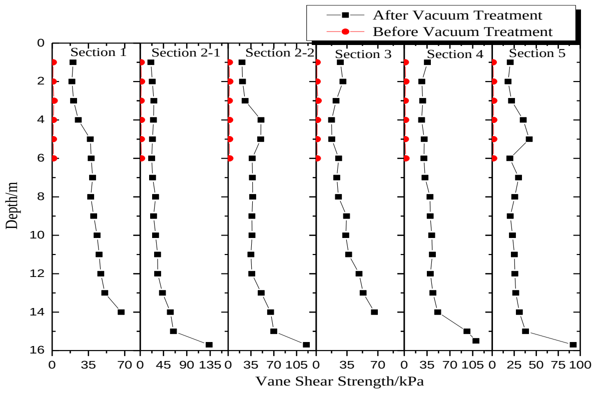


Fig. 11 Vane shear strength vs. depth

Table 5 Characteristic values of the ground bearing capacity at a depth of 1 m

Section	Section 1	Section 2-1	Section 2-2	Section 3	Section 4	Section 5
Characteristic values of the ground bearing capacity/kPa	51.91	53.20	54.74	70.68	91.49	52.69

Transport of the People's Republic of China, 2009), when the plastic index (*PI*) is less than 20, the vane shear strength is corrected by a correction factor of 0.9, and when the *PI* is more than 20 and less than 40, the vane shear strength is corrected by a correction factor of 1.0. Before and after vacuum preloading, the plasticity index of soft soil is approximately 23~25 according to a geological survey report; therefore, the correction coefficient of the vane shear strength should be 1.0.

Relative to the vane shear strength before the vacuum treatment, the vane shear strength of the foundation soil is greatly improved with the ground treatment. The vane shear strength of the foundation soil is generally more than 30 kPa at the surface. The vane shear strengths are 20.2 kPa, 20.7 kPa, 21.3 kPa, 27.5 kPa, 35.6 kPa, and 20.5 kPa at a depth of 1 m in the different sections, as shown in Fig. 11.

Therefore, the characteristic values of the subsoil bearing capacity are calculated at a depth of 1 m according to Equation (7), as shown in Table 5. The effect of the ground treatment in Section 3 is better than that of the other methods, and the maximum characteristic value of the subsoil bearing capacity is approximately 91.49 kPa at a depth of 1 m. The characteristic value of the subsoil bearing capacity is also more than 77.1 kPa at the surface, which can meet the construction contract requirements.

A notable phenomenon appeared before and after the ground treatment. A van weighing less than 1 ton could not be moved on soft ground before the ground treatment. In contrast, a 10-ton crane could be moved on the ground following the treatment. This result demonstrates that the ground treatment by straight-line vacuum preloading without sand can satisfy the engineering requirements and shows the importance of the subsoil bearing capacity evaluations in terms of the vane shear strength.

6. Settlement prediction based on GM (1, 1)

The grey model (GM) is an important calculation method in the discipline of systems engineering. This method mainly uses recursive sequences to increase or decrease the monitoring data to determine the data evolution. To obtain a reasonable result, a differential equation of accumulative and subtraction series is solved. Combined with the results of the differential equation, the prediction results are generated by the accumulative and subtraction series. This method is different from the numerical simulation method. The numerical simulation method needs to obtain material parameters through laboratory tests to predict the soil deformation. The grey model (GM) obtains a mathematical expression of the relationship between the time and settlement according to the changing development of the monitoring data to predict the soil deformation.

GM (1, 1) was proposed by Deng (1994), and the research shows that this model can be applied to predict the settlement of soft soil ground (You 2006). To reasonably evaluate the settlement of soft soil ground treated by straight-line vacuum preloading and provide guidance for engineering decision-making, GM (1,1) is established in this paper. In addition, the establishment of GM (1, 1), $x^{(0)}(k)+az^{(1)}(k)=b$, can be introduced to develop the consolidation settlement and consolidation degree system.

6.1 Basic principle of GM (1,1)

The original settlement data sequence, the 1-AGO data sequence and the contiguous average data sequence are established as follows:

$$X^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n),\ldots,x^{(0)}(n)\right\} \tag{8}$$

$$X^{(1)}=\left\{x^{(1)}(1),x^{(1)}(2),x^{(1)}(3),\ldots,x^{(1)}(k),\ldots,x^{(1)}(n)\right\} \tag{9}$$

$$X^{(1)}=\left\{x^{(1)}(1),x^{(1)}(2),x^{(1)}(3),\ldots,x^{(1)}(k),\ldots,x^{(1)}(n)\right\} \tag{10}$$

where $x^{(0)}(k)\geq 0$, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, and $z^{(1)}=(x^{(1)}(k)+x^{(1)}(k\text{-}1))/2$, (k=1,2,3,…,$n$).

The parameter matrix $\hat{a}=(a,b)^T$ is determined by Eq. (11), which can meet the least squares method.

$$\hat{a}=\left(B^TB\right)^{-1}B^TY \tag{11}$$

where

$Y=\{x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\}^T$, and

$$B=\begin{Bmatrix} -z^{(1)}(2) & -z^{(1)}(3) & \ldots\ldots & -z^{(1)}(n) \\ 1 & 1 & \ldots\ldots & 1 \end{Bmatrix}^T$$

Eq. (12) can be defined as the shadow equation by Deng (1994) as follows:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{12}$$

The solution of Equation (11) can be expressed as follows:

$$x^{(1)}(t)=\left(x^{(1)}(1)-\frac{b}{a}\right)e^{-at}+\frac{b}{a} \tag{13}$$

Therefore, the time sequence solution of the GM (1, 1)

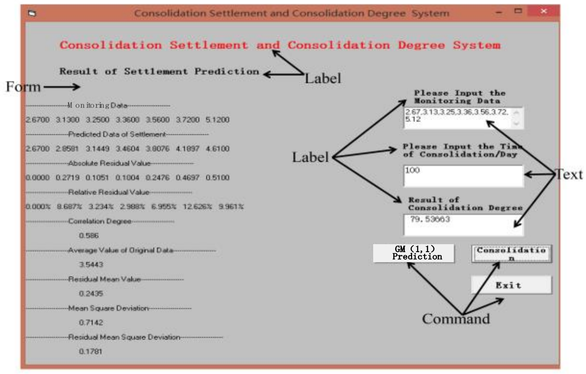


Fig. 12 Operating interface

Table 6 Predicted settlement results

Section	Time/day	Settlement/mm	Predicted settlement/mm	Error value/%	Section	Time/day	Settlement/mm	Predicted settlement/mm	Error value/%
	7	373	373.0000	0.000		7	430.8	430.8000	0.000
	21	740.4	795.5358	7.447		21	804.4	859.1742	6.809
	28	843.8	875.2306	3.725		28	919.8	938.8343	2.069
Section 1	35	1003.9	962.9089	4.083	Section 2-1	35	1064.9	1025.8801	3.664
	42	1114.3	1059.3707	4.929		42	1155.5	1120.9966	2.986
	49	1210.4	1165.49.58	3.710		49	1278.1	1224.9320	4.160
	56	1252.7	1282.2522	2.359		56	1313.5	1338.5040	1.904
	7	522.3	522.3000	0.000		7	356.5	356.5000	0.000
	21	914.1	962.9814	5.347		21	711.6	766.1517	7.666
	28	1007.8	1036.1263	2.811		28	829.8	858.7736	3.492
Section 2-2	35	1162.6	1114.8269	4.109	Section 3	35	1013.9	926.5929	5.060
	42	1272	1199.5054	5.699		42	1134.7	1078.9632	4.912
	49	1306.8	1290.6159	1.238		49	1242.9	1209.4017	2.695
	56	1328.6	1388.6467	4.520		56	1294.9	1355.6093	4.688
	7	454.5	454.5000	0.000		7	366.7	366.7000	0.000
	21	802.3	902.7847	12.525		21	716.9	765.1632	6.732
	28	997.9	993.4010	0.451		28	821.9	844.2075	2.714
Section 4	35	1167.3	1093.1128	6.355	Section 5	35	975.9	931.4174	4.558
	42	1267.5	1202.8331	5.102		42	1078.9	1027.6365	4.751
	49	1348.6	1323.5666	1.856		49	1161.8	1133.7953	2.410
	56	1383.3	1456.4186	5.286		56	1195.9	1250.9208	4.601

model, $x^{(0)}(k)+az^{(1)}(k)=b$, can be expressed as follows:

$$\hat{x}^{(1)}(k+1)=\left(x^{(1)}(1)-\frac{b}{a}\right)e^{-at}+\frac{b}{a} \tag{14}$$

Thus,

$$\begin{aligned}\hat{x}^{(0)}(k+1)&=\alpha^{(1)}\hat{x}^{(1)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)\\&=\left(1-e^{a}\right)\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-at}\end{aligned} \tag{15}$$

As t approaches infinity (∞), the value of the final settlement converges to $\frac{b}{a}$.

6.2 Development of the consolidation settlement and consolidation degree system

To better serve engineering practice, a consolidation settlement and consolidation degree system is established based on GM (1, 1) using Visual Basic (VB) 6.0. This system has two functions; first, it can use monitoring data to predict the consolidation settlement of soft soil foundations treated by straight-line vacuum preloading without sand. Second, the consolidation settlement and consolidation degree system can be used to calculate the consolidation degree that is determined by an empirical formula (5). To achieve the consolidation settlement and consolidation degree system, three command objects are selected, and the caption attributes are "GM (1, 1) Prediction," "Consolidation" and "Exit," where the "GM (1, 1) Prediction" command is used to control the result of the settlement prediction, the "Consolidation" command is employed to obtain the result of the consolidation degree and the "Exit" command implies the end of the program. Three empty texts are listed from top to bottom in the form of the consolidation settlement and consolidation degree system. The first and second texts are used to input the monitoring data of the settlement and consolidation times, respectively, and the third text is to obtain the result of the consolidation degree. Furthermore, five labels are chosen to warn users about operations. An operating interface is designed as shown in Fig. 12. The code program of the consolidation settlement and consolidation degree system is offered in the appendix.

The core program is designed to predict the consolidation settlement determined by GM (1, 1) and the consolidation degree determined by the empirical formula (5) under straight-line vacuum preloading. To verify the correctness of the program, the monitoring data of the consolidation settlement of soft soil ground treatment by straight-line vacuum preloading without sand is provided to analyze the settlement prediction in the 5 sections. The predicted settlement results are listed in Table 6.

Table 6 demonstrates that most error values are lower than 5% in the preloading times of 28 days to 56 days and that several error values of the five sections before the preloading time of 21 days can exceed 5%, which is not acceptable in engineering practice.

The main reason is that the time sequence solution of the GM (1, 1) model is an exponential function and that the change trend of this function is greater in the initial phase to some degree. Nevertheless, the results of the data are conservative; specifically, the consolidation settlement and consolidation degree system can be applied to forecast the consolidation settlement and consolidation degree of soft soil under straight-line vacuum preloading without sand.

7. Conclusions

This paper presents field instrumentation and settlement predictions for a ground treatment by straight-line vacuum preloading without sand in the Binhai New Area of Tianjin, China. By adjusting the PVD layout and the vacuum method, the pore water pressure, settlement and vane shear strength are analyzed in various sections. A consolidation settlement and consolidation degree system is developed, and the validity of the model is verified; the following conclusions are drawn.

(1) With respect to a dredged-fill ground, a triangular PVD arrangement is successful in engineering practice because the pore pressure dissipation is sensitive to the PVD spacing. Compared with a square configuration, the diameter of the influence area can be increased by 42% for the same PVD spacing of the triangular arrangement. Furthermore, the vacuum pumping method of an air-water separation tank combined with a jet vacuum pump may accelerate consolidation. Compared with a jet vacuum pump method in Section 2, the pore water pressure dissipation rate is increased by more than 20% for air-water separation tank combined with a jet vacuum pump method in Sections 1, 3, 4 and 5.

(2) The results of the field instrumentation tests reflect that the pore pressure dissipates over time at a rate that decreases with increasing depth. The range of the pore water pressure dissipation rate is 0.23-0.84 kPa/day, and the underground surface settlement increases with time. When the consolidation time is 168 days, the subsidence range is from 1.58 to 2.2 m. While for 5 days, the average consolidation exceeds 90%, and the surface subsidence rate is not greater than 2.5 mm. The vane shear strength of the ground surface is generally more than 30 kPa, and the characteristic value of the ground bearing capacity is more than 77.1 kPa, which is greater than 50 kPa. All of these indices satisfy the construction contract requirements.

(3) A consolidation settlement and consolidation degree system is developed based on GM (1, 1) to predict the consolidation settlement and consolidation degree under vacuum preloading, and the validity of the system is verified. The majority error values are lower than 5% for a preloading time of 28 days to 56 days, which can meet the requirement of engineering practice.

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Notations

d_e	circle diameter
l	PVD spacing
$\sigma^{'}(z,t)$	effective stress
$u_0(z)$	initial pore water pressure
s_t	consolidation settlement at time t
s_∞	consolidation settlement at ∞
m	empirical coefficient
U	average consolidation degree, α
β	experimental parameters

f_{ak}	characteristic value of the ground bearing capacity
M_c	coefficient of the bearing capacity
C_u	average vane shear strength
$\hat{a}$	parameter matrix, a
b	elements of the parameter matrix
$X^{(0)}$	original settlement data sequence
$X^{(1)}$	1-AGO data sequence
$Z^{(1)}$	contiguous average data sequence

Appendix

Code Program of Consolidation Settlement and Consolidation Degree System

```
Option Explicit
Private Sub Calculate_1_AGO(X_0() As Double,X_1()As Double)'1-AGO sequence is established
    Dim i As Long, TempX As Double, K As Long
    K = UBound(X_0)
    ReDim X_1(K)
    For i = 0 To K
        TempX = TempX + X_0(i)
        X_1(i) = TempX
    Next i
End Sub

Private Sub Calculate_Matrix_B(X_1() As Double, B() As Double) 'Matrix B is set up
    Dim i As Long, K As Long
    K = UBound(X_1) - 1
    ReDim B(K, 1)
    For i = 0 To K
        B(i, 0) = -0.5 * (X_1(i) + X_1(i + 1))
        B(i, 1) = 1
    Next i
End Sub

Private Sub Calculate_Matrix_YN(X_0() As Double, YN() As Double)'Matrix B YN is established
    Dim i As Long, K As Long
    K = UBound(X_0) - 1
    ReDim YN(K, 0)
    For i = 0 To K
        YN(i, 0) = X_0(i + 1)
    Next i
End Sub

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' Function name: Matrix_Transpotation
' Function :  transportation of the matrix
' Parameter:  m    - Integer, the line of the matrix
'             n    - Integer, the columns of the matrix
'             mtxA  - Double,  m x n are two- dimensional array, which deposits the original matrix
'             mtxAT  - Double, n x m are two- dimensional array, return transportation of the matrix
''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
Private Sub Matrix_Transpotation(mtxA() As Double, mtxAT() As Double)
    Dim i As Integer, j As Integer
    Dim M As Integer, N As Integer
    M = UBound(mtxA, 2)
    N = UBound(mtxA, 1)
    ReDim mtxAT(M, N)
    For i = 0 To M
        For j = 0 To N
            mtxAT(i, j) = mtxA(j, i)
        Next j
    Next i
End Sub

''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
' Function name: Matrix_Multiplication
' Function:  multiplication of matrix
' Parameter:  m  - Integer, the row number on the matrix of the multiplicative left
'             n  - Integer, the column number on the matrix of the multiplicative left or the row number on the matrix of the multiplicative right
'             l  -  Integer,  the column number on the matrix of the multiplicative right
'             mtxA  - Double, m x n are two- dimensional array, which deposits the matrix of the multiplicative left
'             mtxB  - Double, n x l are two- dimensional array, which deposits the matrix of the multiplicative right
'             mtxC  - Double,  m x l are two- dimensional array, which returns matrix multiplication
''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
Private Sub Matrix_Multiplication(mtxA() As Double, mtxB() As Double, mtxC() As Double)
    Dim i As Integer, j As Integer, K As Integer
    Dim M As Integer, N As Integer, L As Integer
    M = UBound(mtxA, 1): N = UBound(mtxB, 1): L = UBound(mtxB, 2)
    ReDim mtxC(M, L)
    For i = 0 To M
        For j = 0 To L
            mtxC(i, j) = 0#
            For K = 0 To N
                mtxC(i, j) = mtxC(i, j) + mtxA(i, K) * mtxB(K, j)
            Next K
        Next j
    Next i
End Sub

''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
' Function name: Matrix_Inversion
' Function:  inverse matrix
' Parameter:  n       - Integer, the order of the matrix
'             mtxA    - Double, two- dimensional array, volume is n x n。 '
 Return value：Boolean,  False or True
''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
Private Function Matrix_Inversion(mtxA() As Double) As Boolean
    ' local variable
    Dim N As Integer
    N = UBound(mtxA)
    ReDim nIs(N) As Integer, nJs(N) As Integer
    Dim i As Integer, j As Integer, K As Integer
    Dim d As Double, P As Double
```

```
  For K = 0 To N
      d = 0#
      For i = K To N
          For j = K To N
              P = Abs(mtxA(i, j))
              If (P > d) Then
                  d = P
                  nIs(K) = i
                  nJs(K) = j
              End If
          Next j
      Next i

      ' failure
      If (d + 1# = 1#) Then
          Matrix_Inversion = False
          Exit Function
      End If

      If (nIs(K) <> K) Then
          For j = 0 To N
              P = mtxA(K, j)
              mtxA(K, j) = mtxA(nIs(K), j)
              mtxA(nIs(K), j) = P
          Next j
      End If

      If (nJs(K) <> K) Then
          For i = 0 To N
              P = mtxA(i, K)
              mtxA(i, K) = mtxA(i, nJs(K))
              mtxA(i, nJs(K)) = P
          Next i
      End If

      mtxA(K, K) = 1# / mtxA(K, K)
      For j = 0 To N
          If (j <> K) Then mtxA(K, j) = mtxA(K, j) *
mtxA(K, K)
      Next j
      For i = 0 To N
          If (i <> K) Then
              For j = 0 To N
                  If (j <> K) Then mtxA(i, j) = mtxA(i, j)
- mtxA(i, K) * mtxA(K, j)
              Next j
          End If
      Next i
      For i = 0 To N
          If (i <> K) Then mtxA(i, K) = -mtxA(i, K) *
mtxA(K, K)
      Next i
  Next K

  ' adjust the column sequence
  For K = N To 0 Step -1
      If (nJs(K) <> K) Then
          For j = 0 To N
              P = mtxA(K, j)
              mtxA(K, j) = mtxA(nJs(K), j)
              mtxA(nJs(K), j) = P
          Next j
      End If
      If (nIs(K) <> K) Then
          For i = 0 To N
              P = mtxA(i, K)
              mtxA(i, K) = mtxA(i, nIs(K))
              mtxA(i, nIs(K)) = P
          Next i
      End If
  Next K

  ' success
  Matrix_Inversion = True

End Function

Private Sub Predicted_Value(ByVal X_1_0 As Double,
  ByVal u_value As Double, ByVal a_value As Double, K
  As Long, PV() As Double)
    Dim i As Long
    ReDim PV(K)
    For i = 1 To K + 1
        PV(i - 1) = (X_1_0 - u_value / a_value) * Exp(-
  a_value * (i - 1)) * (1 - Exp(a_value))
    Next i
    PV(0) = X_1_0
End Sub

Private Sub String_to_Array(Data As String, X_0() As
  Double)
    Dim Predict_Data() As String, K As Long, i As Long
    Predict_Data = Split(Data, ",")
    K = UBound(Predict_Data)
    ReDim X_0(K)
    For i = 0 To K
        X_0(i) = Predict_Data(i)
    Next i
End Sub

Private Sub Print_Array(Arrays() As Double, Title As
  String) 'print array
    Dim i As Long
    Form1.Print vbCrLf & String(20, "-") & Title &
  String(20, "-") & vbCrLf
    For i = 0 To UBound(Arrays)
        Form1.Print Format(Arrays(i), "0.0000") & "     ";
    Next i
    Form1.Print
End Sub

Private Sub Print_String(Arrays() As String, Title As
  String) 'Print -Relative_Residual_Error-RRE
    Dim i As Long
    Form1.Print vbCrLf & String(20, "-") & Title &
  String(20, "-") & vbCrLf
    For i = 0 To UBound(Arrays)
        Form1.Print Arrays(i) & "     ";
    Next i
    Form1.Print
```

```
End Sub

Private Sub Print_One_String(S As Double, Title As String)
    Form1.Print vbCrLf & String(20, "-") & Title & String(20, "-") & vbCrLf
    Form1.Print Space(25) & Format(S, "0.#####")
End Sub

Private Sub Absolute_Residual_Error(Array1() As Double, Array2() As Double, ARE() As Double) 'absolute residual
    Dim K As Long, i As Long
    K = UBound(Array1)
    ReDim ARE(K)
    For i = 0 To K
        ARE(i) = Format(Abs(Array2(i) - Array1(i)), "0.0000")
    Next i
End Sub

Private Sub Relative_Residual_Error(Array1() As Double, ARE() As Double, RRE() As String)    ' relative residual
    Dim K As Long, i As Long
    K = UBound(Array1)
    ReDim RRE(K)
    For i = 0 To K
        RRE(i) = Format(ARE(i) / Array1(i), "0.000%")
    Next i
End Sub

Private Sub Relatedness_Test(ARE() As Double, P As Double, R As Double) 'Correlation degree
    Dim i As Long, K As Long, Min As Double, Max As Double, SumR As Double
    Dim Ri() As Double
    K = UBound(ARE)
    ReDim Ri(K)
    Min = ARE(0): Max = ARE(0)
    For i = 0 To K
        If ARE(i) < Min Then Min = ARE(i)
        If ARE(i) > Max Then Max = ARE(i)
    Next i
    For i = 0 To K
        Ri(i) = (Min + P * Max) / (ARE(i) + P * Max)
        SumR = SumR + Ri(i)
    Next i
    R = Format(SumR / (K + 1), "0.0000")

End Sub

Private Function Array_Mean(Array1() As Double) As Double 'average values
    Dim i As Long, K As Long
    K = UBound(Array1)
    For i = 0 To K
        Array_Mean = Array_Mean + Array1(i)
    Next i
    Array_Mean = Array_Mean / (K + 1)
End Function

Private Function Mean_Square_Error(Array1() As Double) As Double 'Mean square deviation
    Dim Average As Double, i As Long, K As Long, Temp As Double
    K = UBound(Array1)
    Average = Array_Mean(Array1())
    For i = 0 To K
        Temp = Temp + (Array1(i) - Average) ^ 2
    Next i
    Mean_Square_Error = Format(Sqr(Temp / (K + 1)), "0.0000")
End Function

Public Sub GM1_1_Predict(Data As String)

    Dim X_0() As Double, X_1() As Double, B() As Double, YN() As Double, PV() As Double
    Dim BT() As Double, BTB() As Double, BTBBT() As Double, A() As Double, ARE() As Double
    Dim RRE() As String, R As Double

String_to_Array Data, X_0
    Print_Array X_0, "Morontoring Data"

    Calculate_1_AGO X_0, X_1
    Calculate_Matrix_YN X_0, YN
    Calculate_Matrix_B X_1, B
    Matrix_Transpotation B, BT
    Matrix_Multiplication BT, B, BTB
    If Not Matrix_Inversion(BTB) Then
        MsgBox "Solution Failure", vbCritical, "Warning"
        Exit Sub
    End If
    Matrix_Multiplication BTB, BT, BTBBT
    Matrix_Multiplication BTBBT, YN, A
    Debug.Print "u=" & A(1, 0) & "," & "a=" & A(0, 0)

    Predicted_Value X_1(0), A(1, 0), A(0, 0), UBound(X_1), PV
    Print_Array PV, "Predicted Data of Settlement"

    Absolute_Residual_Error X_0, PV, ARE
    Print_Array ARE, "Absolute Residual Value"

    Relative_Residual_Error X_0, ARE, RRE
    Print_String RRE, "Relative Residual Value"

Relatedness_Test ARE, 0.5, R
    Print_One_String R, "Correlation Degree"

    Print_One_String Format(Array_Mean(X_0), "0.0000"), "Average Value of Original Data"
    Print_One_String Format(Array_Mean(ARE), "0.0000"), "Residual Mean Value"
    Print_One_String Mean_Square_Error(X_0), "Mean Square Deviation"
    Print_One_String Mean_Square_Error(ARE), "Residual Mean Square Deviation"

End Sub
```

```
Private Sub Test_Print_Array(X() As Double)
    Dim i%, j%
    For i = 0 To UBound(X(), 1)
        For j = 0 To UBound(X(), 2)
            Form1.Print  Format(X(i,  j),  "000.0000")  &  "
  ";
        Next j
        Form1.Print
    Next i
End Sub
Private Sub Command1_Click() 'GM (1,1) model
    Cls
    Dim Data As String
    Dim myspace As String
    Dim i As Integer
    myspace = Space(10)
    For i = 1 To 10
    Form1.Print myspace
     Next i
    Data            =            Text4.Text                             '
  "2.67,3.13,3.25,3.36,3.56,3.72,5.12"
    GM1_1_Predict Data
End Sub
Private Sub Command3_Click()
MsgBox "Are You Sure Close The Window"
End
End Sub
Private Sub Command4_Click()
Dim t As Integer
Dim U As Single
t = Val(Text2.Text)
U = (1 - 0.89 * Exp(-0.0147 * t)) * 100
Text3.Text = Str(U)
End Sub
```