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Estimation of rock tensile and compressive moduli with Brazilian disc test

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Abstract. The elastic modulus is an important parameter to characterize the property of rock. It is common knowledge that the strengths of rocks are significantly different under tension and compression. However, little attention has been paid to the bi-modularity of rock. To validate whether the rock elastic moduli in tension and compression are the same, Brazilian disc, direct tension and compression tests were conducted. A horizontal laser displacement meter and a pair of vertical and transverse strain gauges were applied. Four types of materials were tested, including three types of rock materials and one type of steel material. A comprehensive comparison of the elastic moduli based on different experimental results was presented, and a tension-compression anisotropy model was proposed to explain the experimental results. The results from this study indicate that the rock elastic modulus is different under tension and compression. The ratio of the rock elastic moduli under compression and tension ranges from 2 to 4. The rock tensile moduli from the strain data and displacement data are approximate. The elastic moduli from the Brazilian disc test are consistent with those from the uniaxial tension and compression tests. The Brazilian disc test is a convenient method for estimating the tensile and compressive moduli of rock materials.

Keywords: Brazilian test; uniaxial compression test; uniaxial tensile test; tensile modulus; compressive modulus

1. Introduction

The Brazilian disc test is a simple indirect testing method to measure the tensile strength of rock, concrete and other brittle materials. Since the Brazilian engineer Carneiro (1943) developed it in 1943, the Brazilian disc test is like a work of art drawing the interest of a number of outstanding researchers over the past 70 years (Li and Wong 2013). It has been studied widely using analytical, experimental, and numerical approaches which are presented in detail in Table 1. Most of these studies focused on the stress distribution, location of crack initiation and strength characteristics but ignored the deformation characteristics, particularly the relationship between strain and stress.

Zhao and Li (2000) measured the dynamic tensile modulus of granite using a strain gauge. Ye *et al.* (2009) estimated the rock tensile modulus using the Brazilian disc test by attaching a strain gauge at the horizontal centre of the disc. Compared to the direct tension test, the Brazilian

Table 1 A rough review of the development of Brazilian test on rocks

Authors (year)	Methods	Contributions
Carneiro (1943)	Experimental	Proposing a test method and giving an formula to calculate the tensile strength
Hondros (1959)	Analytical	Determining a tensile strength formula for a homogeneous elastic disc by considering the influences of the bearing strip
Fairhurst (1964)	Analytical	Studying the validity of the Brazilian test and pointing out that the Brazilian tensile strength is lower than the true value
Hudson *et al.* (1972)	Experimental	Observing that failure always initiated directly under the loading points in the Brazilian test if only flat steel platens were used
Wijk (1978)	Analytical	Giving out a three-dimensional correction to the tensile strength of the Brazilian test
Newman and Bennett (1990)	Experimental	Studying the effects of specimen geometry and stress rate on the tensile strength of sandstone by the Brazilian test
Zhu and Tang (2006)	Numerical	Studying the deformation and failure process of Brazilian disk rock using RFPA
Dai and Xia (2010)	Experimental	Investigating the influence of loading rate on the tensile strength of rock using Brazilian test
Cho *et al.* (2012)	Experimental	Studying the influences of anisotropy on the tensile strength of Brazilian disk rock
Yu *et al.* (2014)	Numerical	Using digital image techniques to acquire the inhomogeneous distributions of rocks and applying it to the mechanical analysis of the Brazilian test
Jung *et al.* (2014)	Experimental	Estimating the characteristics of delayed failure and long-term strength of granite by Brazilian test
Roy and Singh (2016)	Experimental	Studying the effects of heat treatment and layer orientation on the Brazilian tensile properties of granitic gneiss

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Table 1 Continued

Authors (year)	Methods	Contributions
Zhou *et al.* (2016)	Experimental	Studying the influence of water content on the tensile strength of Brazilian disk rock
Yang *et al.* (2016)	Numerical	Simulating the fracture propagation of Brazilian disk rock with 3D numerical manifold method
Li *et al.* (2017)	Numerical	Simulating the crack initiation and propagation of Brazilian disk rock with 3D polycrystalline discrete element method
Jiang *et al.* (2017)	Numerical	Simulating crack propagation of Brazilian disk rock using the distinct lattice spring model
Wang *et al.* (2018)	Numerical	Studying the influence of anisotropy and directionality on the tensile behaviours of a jointed rock mass using Brazilian test
Chen and Irfan (2018)	Experimental	Studying Kaiser effect of Brazilian disc under cyclic loading
Liu *et al.* (2018)	Experimental	Investigating the tensile fatigue properties of rocks using the cyclic flattened Brazilian disc test

disc test is more convenient in operation. However, strain gauges have a certain length. The strain measured by a strain gauge is the average of a length and less than the true value at the centre point. Thus, Gong *et al.* (2010) derived a quantitative relationship between the tensile modulus and the total horizontal displacement and proposed a new method to determine the tensile modulus in the Brazilian disc test. Patel and Martin (2018) derived new equations for the displacements in the Brazilian test considering the bi-modularity in the constitutive relations. However, none of them compared the results from Brazilian tests against the ones obtained from uniaxial tension tests. And there is no literature making a comparison between these two methods. Thus, the validity of these methods is lack of proof.

Furthermore, in the Brazilian disc test, a disc of rock is subjected to a pair of diametrically opposite and symmetric loadings. It involves the application of compressive stress along the vertical direction and tensile stress along the horizontal direction. Thus, it is possible to measure the compressive modulus along the vertical direction and the tensile modulus along the horizontal direction simultaneously. The current mainstream view is that the tensile and compressive moduli are identical. Few people distinguished them in rock mechanics. The purpose of this paper is to whether the rock elastic moduli under tension and compression are identical by using the Brazilian disc test and validate its test results against the results of direct tension and compression tests.

2. Theory of Brazilian disc tests

According to elasticity theory, the solution for the Brazilian disc is (Timoshenko and Goodier 2013)

$$\sigma_x = \frac{2P}{\pi L}\left(\frac{x^2\left(y+\frac{D}{2}\right)}{\left(x^2+\left(y+\frac{D}{2}\right)^2\right)^2}+\frac{\left(\frac{D}{2}-y\right)x^2}{\left(x^2+\left(y-\frac{D}{2}\right)^2\right)^2}-\frac{1}{D}\right) \tag{1}$$

$$\sigma_y = \frac{2P}{\pi L}\left(\frac{\left(y+\frac{D}{2}\right)^3}{\left(x^2+\left(y+\frac{D}{2}\right)^2\right)^2}+\frac{\left(\frac{D}{2}-y\right)^3}{\left(x^2+\left(y-\frac{D}{2}\right)^2\right)^2}-\frac{1}{D}\right) \tag{2}$$

$$\tau_{xy} = \frac{2P}{\pi L}\left(\frac{x\left(y+\frac{D}{2}\right)^2}{\left(x^2+\left(y+\frac{D}{2}\right)^2\right)^2}-\frac{\left(\frac{D}{2}-y\right)^2 x}{\left(x^2+\left(y-\frac{D}{2}\right)^2\right)^2}\right) \tag{3}$$

where P is the compression load in the thickness direction of the platen, L and D are the thickness and diameter of the disc, respectively, $-0.5D\le x\le 0.5D$, and $-0.5D\le y\le 0.5D$. The stresses at any point (x, 0) along the horizontal centre line are(Ye *et al.* 2009)

$$\sigma_x = \frac{2P}{\pi DL}\left(\frac{16D^2x^2}{\left(4x^2+D^2\right)^2}-1\right) \tag{4}$$

$$\sigma_y = \frac{2P}{\pi DL}\left(\frac{4D^4}{\left(4x^2+D^2\right)^2}-1\right) \tag{5}$$

$$\tau_{xy} = 0 \tag{6}$$

and the stresses at the centre (0, 0) of the Brazilian disc are

$$\sigma_x = -\frac{2P}{\pi DL} \tag{7}$$

$$\sigma_y = \frac{6P}{\pi DL} \tag{8}$$

The formulas that are used to calculate the tensile strength can be derived by substituting P with P_t in Eq. (7) as follows

$$\sigma_t = -\frac{2P_t}{\pi DL} \tag{9}$$

where P_t is the failure load, and σ_t is the tensile strength. According to the generalized Hooke's law, the strain at the centre of the disc (0, 0) satisfies the following equations

$$\varepsilon_x(0,0) = \frac{\sigma_x - \nu\sigma_y}{E_x} = -\frac{2P}{\pi DL}\frac{(1+3\nu)}{E_x} \tag{10}$$

$$\varepsilon_y(0,0) = \frac{\sigma_y - \nu\sigma_x}{E_y} = \frac{2P}{\pi DL}\frac{(3+\nu)}{E_y} \tag{11}$$

These equations can be rewritten for Young's modulus as follows

$$E_x = -\frac{2P}{\pi DL}\frac{(1+3\nu)}{\varepsilon_x(0,0)} \tag{12}$$

$$E_y = \frac{2P}{\pi DL}\frac{(3+\nu)}{\varepsilon_y(0,0)} \tag{13}$$

Similarly, the strain $\varepsilon_x(x, 0)$ along the horizontal centre line satisfies the following equation (Gong *et al.* 2010)

$$\varepsilon_x(x,0) = \frac{\sigma_x - \nu\sigma_y}{E'_x} = \frac{2P}{\pi DLE'_x}\left\{\left[\frac{16D^2x^2}{(4x^2+D^2)^2} - 1\right] - \nu\left[\frac{4D^4}{(4x^2+D^2)^2} - 1\right]\right\} \quad (14)$$

The strain $\varepsilon_x(x, 0)$ is continuous along the interval $0 \le x \le 0.5D$, so it can be integrated as follows

$$\Delta u = \frac{2P}{\pi DLE'_x}\int_0^{\frac{D}{2}}\{[\frac{16D^2x^2}{(4x^2+D^2)^2} - 1] - \nu[\frac{4D^4}{(4x^2+D^2)^2} - 1]\}dx \quad (15)$$

where Δu is the maximum horizontal displacement on the right side of the Brazilian disc. The following equation can be obtained

$$E'_x = \frac{2P}{\pi DL\Delta u}\int_0^{\frac{D}{2}}\{[\frac{16D^2x^2}{(4x^2+D^2)^2} - 1] - \nu[\frac{4D^4}{(4x^2+D^2)^2} - 1]\}dx \quad (16)$$

Generally, the tensile and compressive moduli of the rock are assumed to be equal. If this assumption is appropriate, then

$$E_x = E'_x = E_y \quad (17)$$

$$\frac{\varepsilon_y(0,0)}{\varepsilon_x(0,0)} = \frac{\sigma_y - \nu\sigma_x}{\sigma_x - \nu\sigma_y} = -\frac{3+\nu}{1+3\nu} \quad (18)$$

Otherwise, the assumption is invalid.

3. Results of Brazilian disc tests

A type of yellow sandstone quarried from Neijiang, Sichuan province of China is used in this study. All the tests were loaded at a constant rate of 30 kN/min until failure. The mechanical parameters of the uniaxial compression tests are listed in Table 2. The mean values of the uniaxial compressive strength, elastic modulus and Poisson's ratio are 70.18 MPa, 9.79 GPa and 0.35, respectively. A horizontal laser displacement meter and a pair of vertical and transverse strain gauges were applied (Fig. 1).

The transverse strain ε_x and transverse displacement Δu were used first. The tensile moduli of the Brazilian discs were determined using Eqs. (12) and (16). Fig. 2 shows the results of the Brazilian disc tests, including curves of *t*-*P*, *t*-ε_x-E_x, and *t*-Δu-E'_x. The loading force *P*, transverse strain ε_x and transverse displacement Δu all increased with time. The tensile modulus E_x from the transverse strain decreased over time (Fig. 2(b)), while the tensile modulus E'_x from the transverse displacement increased (Fig. 2(c)). However,

Table 2 Results of the uniaxial compression test of the yellow sandstone

Specimen	*R* (mm)	*H* (mm)	σ_c (MPa)	Elastic modulus (GPa)	Poisson's ratio
UHS1	25.06	100.12	74.35	8.24	0.35
UHS2	25.10	100.10	56.03	9.22	0.34
UHS3	25.08	100.14	80.17	10.81	0.35
Average			70.18	9.42	0.35

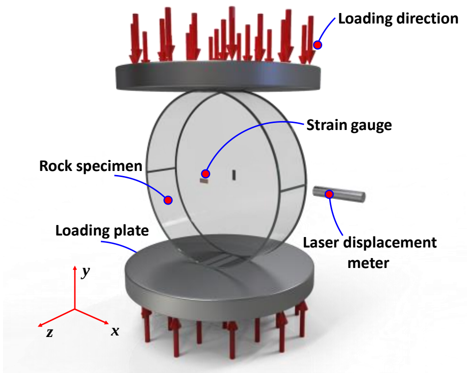
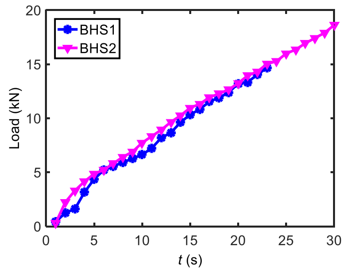
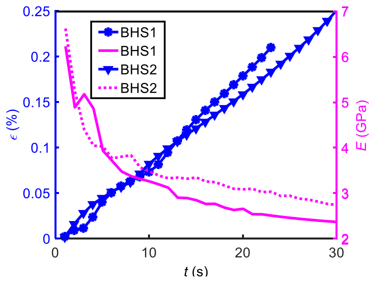


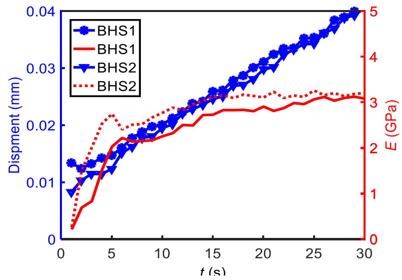
Fig. 1 Sketch map of the experiment setup



(a) *t*-*P*



(b) *t*-ε_x-E_x



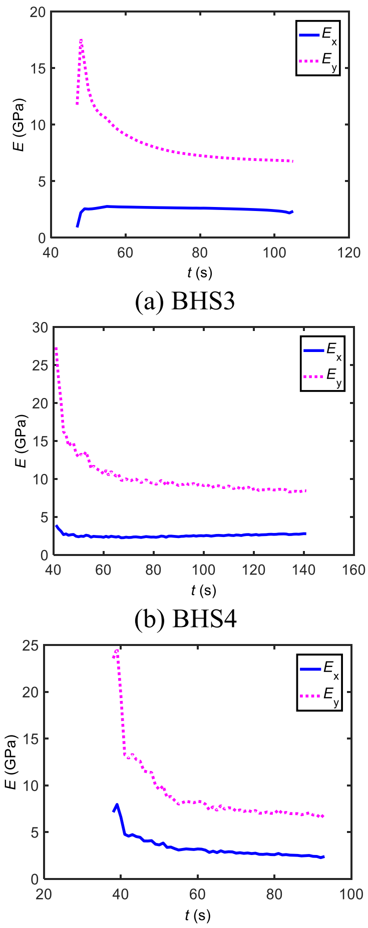
(c) *t*-Δu-E'_x

Fig. 2 The results of the Brazilian disc tests of the yellow sandstone (BHS-Yellow sandstone)

Table 3 Results of tensile moduli from strain and displacement

Specimen	*R* (mm)	Thickness (mm)	σ_t (MPa)	E_x (GPa)	E_x' (GPa)
BHS1	40.10	40.12	2.82	2.51	3.09
BHS2	40.08	40.10	3.67	2.81	3.23

both curves became stable and asymptotically approached a particular value. The data during the last loading stage were used to estimate the rock tensile modulus. The



(a) BHS3

(b) BHS4

(c) BHS5

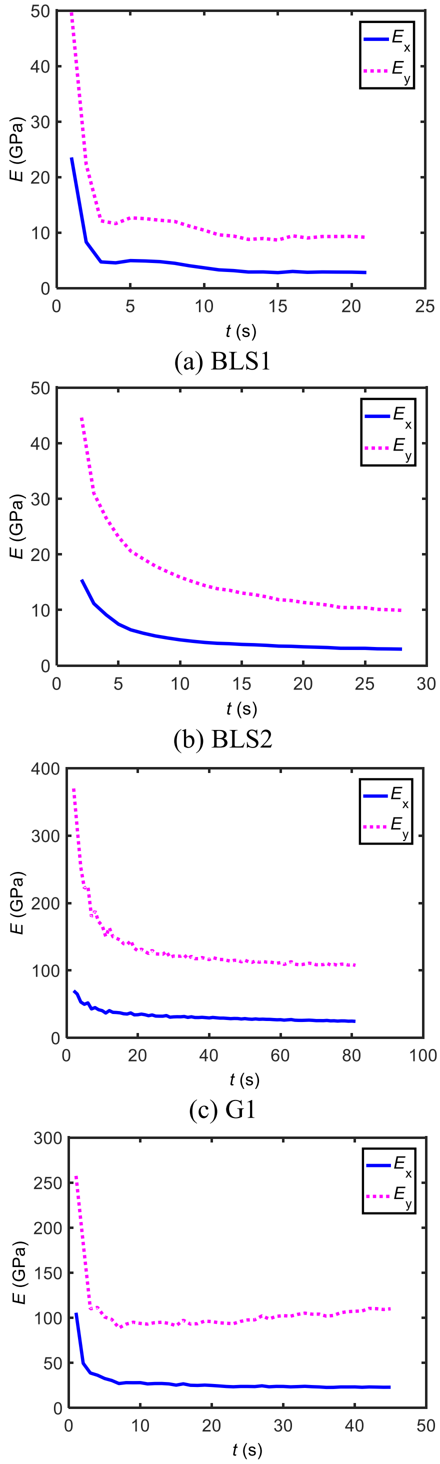
Fig. 3 Curves of the compressive and tensile moduli of the yellow sandstone (BHS-Yellow sandstone)

Table 4 Tensile and compressive moduli of the yellow sandstone

Specimen	R (mm)	Thickness (mm)	σ_t (MPa)	$\varepsilon_y/\varepsilon_x$	E_x (GPa)	E_y (GPa)	E_y/E_x
BHS3	40.08	39.90	3.27	-0.57	2.57	7.22	2.81
BHS4	40.04	39.82	4.96	-0.54	2.43	9.51	3.91
BHS5	40.04	39.90	4.66	-0.96	5.65	8.08	1.43

corresponding values are presented in Table 3. The tensile moduli from the strain and displacement tests were nearly equal; thus, both methods could be used.

The transverse and vertical strains were used next. The tensile and compressive moduli of the Brazilian discs were determined using Eqs. (12) and (13), respectively. Fig. 3 shows the curves of the compressive and tensile moduli for the yellow sandstone during the Brazilian disc tests, and the values are listed in Table 4. The compressive modulus from the Brazilian disc test was approximately equal to that from the uniaxial compression test, but the tensile modulus from the Brazilian disc test was different from the compressive modulus. The ratio of the compressive modulus to the tensile modulus E_y/E_x of the yellow sandstone ranged from 2 to 4 instead of 1, which is notable because the rocks do not contain macro-scale joints, fissures or bedding. Additionally, the ratio of the transverse tensile strain to the vertical compressive strain $\varepsilon_y/\varepsilon_x$ at the centre of the disc was approximately -0.67, but it should have been 1.63 according to Eq. (18).



(a) BLS1

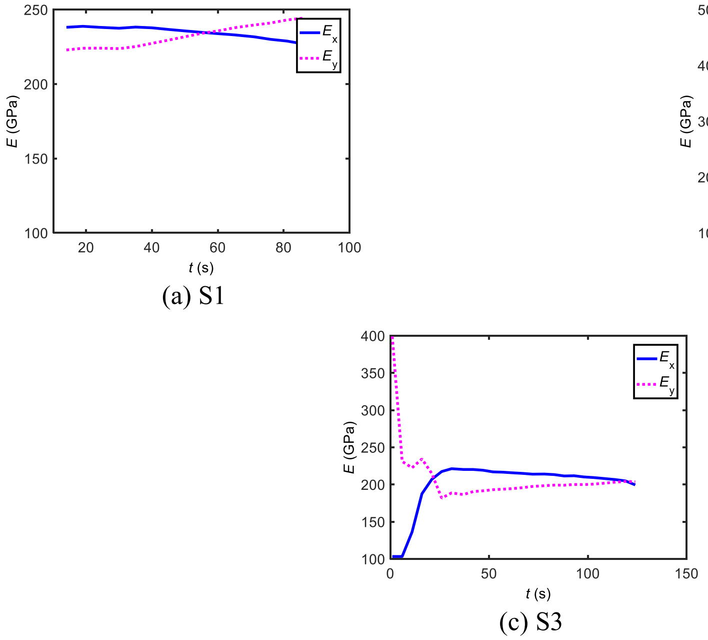
(b) BLS2

(c) G1

(d) G2

Fig. 4 Curves of the compressive and tensile moduli of rocks (BLS-Green sandstone; G-Granite)

Granite and green sandstone specimens quarried from Tangshan, Hebei province and Neijiang, Sichuan province of China, respectively, were used in additional Brazilian disc tests to provide further evidence for this result. Fig. 4 shows curves of the compressive and tensile moduli for the



(a) S1

(b) S2

(c) S3

Fig. 5 Curves of the compressive and tensile moduli of the steel (S-Steel)

granite and green sandstone. The results are consistent with those of the yellow sandstone. The tensile elastic moduli of the 3 types of rock materials are all less than their compressive moduli.

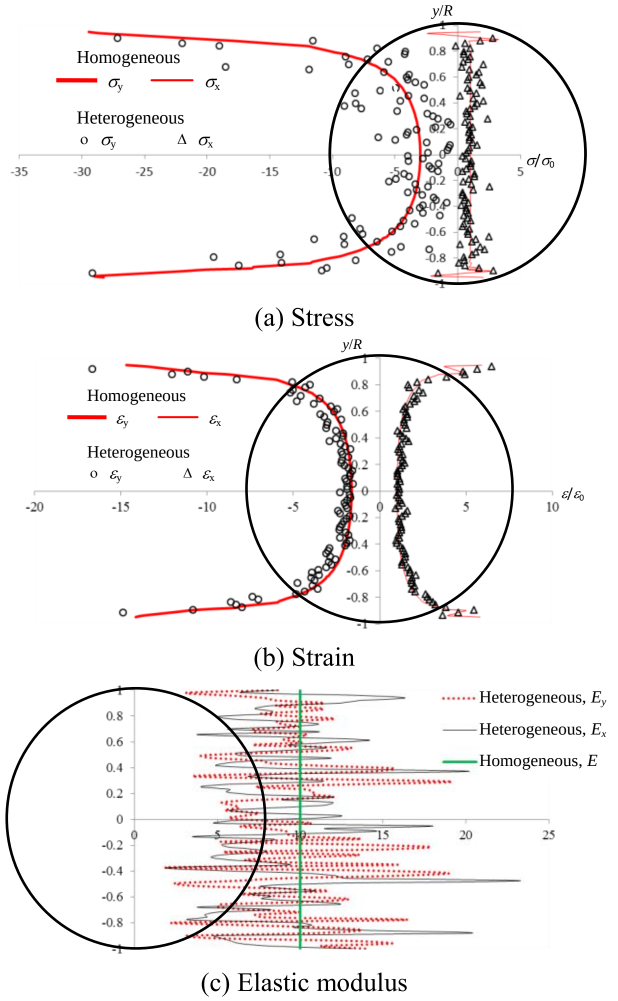
4. Discussions

4.1 Influence of test method

Steel Brazilian discs were used to investigate whether the results are correctly reflecting the rock's characteristics or biased by the test method. In this investigation, the mechanical properties of the disc and the platen were similar. The relative stiffness of the disc and the loading platen influenced the distribution of stress in the disc. However, according to Saint Venant's principle, the relative stiffness has no influence on most part of the disc, particularly on the centre of the disc. Thus, the calculation formulas remain valid. Fig. 5 shows curves of the compressive and tensile moduli versus time for the steel during the Brazilian disc tests. The compressive and tensile moduli of steel are nearly equal, which indicates that the test method is valid and that the results are presenting the characteristics of the rock.

4.2 Influence of heterogeneity

The results presented above indicate that the homogeneous isotropic model is invalid for rock. Rock is a unique material that is different from other solids which is characterized by heterogeneity and anisotropy. Thus, it is difficult to determine which factor influences the experimental results. The influences of heterogeneity and anisotropy on the stresses, strains and elastic moduli of Brazilian discs were studied using numerical simulations. A numerical model was developed in accordance with the



(a) Stress

(b) Strain

(c) Elastic modulus

Fig. 6 Distributions of the stresses, strains and elastic moduli of Brazilian disc across the loading diameter

experimental model. The Young's modulus and Poisson's ratio of the steel platens were 205 GPa and 0.28, respectively. The domain was discretized into 7,448 triangular elements, and the model solved for 30,642 degrees of freedom. The lower boundary was fixed, and a load of 10 MPa was applied to the upper boundary. The other boundaries remained free.

To characterize the heterogeneity of the rock, the material was assumed to be composed of many mesoscopic elements, and the moduli of these elements were assumed to conform to a given Weibull distribution as defined by the following probability density function (Zhu and Tang 2004)

$$f(E)=\frac{m}{E_0}\left(\frac{E}{E_0}\right)^{m-1}\exp\left(-\left(\frac{E}{E_0}\right)^m\right) \tag{19}$$

where E is the elastic modulus of the element, E_0 is related to the average of the element elastic moduli, and the parameter m defines the shape of the distribution function. Using Eq. (19), a heterogeneous rock specimen was produced using a computer simulation, where m = 3, and E_0 = 10 GPa. Poisson's ratio was 0.35, and no heterogeneity was introduced. Fig. 6 shows the distributions of the stresses, strains and elastic moduli in homogeneous and heterogeneous isotropic Brazilian discs across the loading diameter. The stresses, strains and elastic moduli of the heterogeneous rocks all fluctuated near the values of the homogeneous rocks. The values of σ_y/σ_x, $\varepsilon_y/\varepsilon_x$ and E_y/E_x of the heterogeneous rocks were approximately -3, -1.63 and 0.5-2, respectively, which are significantly different from the results presented in Table 4. Thus, the heterogeneous isotropic model cannot explain the experimental results.

4.3 Influence of tension-compression anisotropy

A tension-compression anisotropic model is proposed in this section. During the Brazilian disc test, the tensile stresses are horizontal, and the compressive stresses are vertical. Thus,

$$\begin{aligned} E_y &= E_c \\ E_x &= E_t \\ \nu_{yx} &= \nu_{ct} \\ \nu_{xy} &= \nu_{tc} \end{aligned} \tag{20}$$

where E_t and E_c are the Young's moduli in tension and compression, respectively, ν_{ct} is the Poisson's ratio that characterizes the lateral strain response when a uniaxial compression stress is applied, and ν_{tc} is of inverse. Thus, the tension-compression anisotropic model can be considered to be equivalent to the transverse isotropic model for the Brazilian disc test. The constitutive equations yield (Jaeger *et al.* 2007)

$$\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E_x}-\nu_{yx}\frac{\sigma_y}{E_y} \\ \varepsilon_y &= \frac{\sigma_y}{E_y}-\nu_{yx}\frac{\sigma_x}{E_y} \\ \gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}} \\ \frac{\nu_{yx}}{E_y} &= \frac{\nu_{xy}}{E_x} \end{aligned} \tag{21}$$

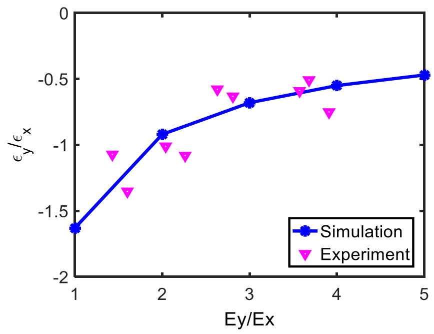


Fig. 7 Curves of $\varepsilon_y/\varepsilon_x$ versus E_y/E_x of the yellow sandstone

Table 5 Results of the uniaxial tension test of the yellow sandstone

Specimen	*R* (mm)	*H* (mm)	σ_t (MPa)	Elastic modulus (GPa)	Poisson's ratio
THS1	25.12	100.10	3.86	2.32	0.10
THS2	25.02	100.08	3.74	2.58	0.11
THS3	25.10	100.04	4.39	2.62	0.12
Average			3.99	2.51	0.11

In the model, E_y=10 GPa, E_y/E_x=1-5, ν_{yx}=0.35, and G_{xy}=3.7 GPa. The values of $\varepsilon_y/\varepsilon_x$ at the centre of the disc are obtained for different values of E_y/E_x using numerical simulations. Fig. 7 shows the curves of $\varepsilon_y/\varepsilon_x$ versus E_y/E_x for the experimental and simulation results. The experimental and simulation results are generally consistent. The values of $\varepsilon_y/\varepsilon_x$ decrease as the values of E_y/E_x increase, which indicates that the tension-compression anisotropic model can properly explain the experimental results and can be used in numerical simulations of Brazilian discs.

4.4 Results of uniaxial tension tests

A series of uniaxial tension tests for the yellow sandstone were conducted subsequently. Table 5 lists the results of the uniaxial tension tests for the yellow sandstone. The mean values of the uniaxial tensile strength, elastic modulus and Poisson's ratio are 3.99 MPa, 2.51 GPa and 0.11, respectively. The results indicate that the elastic moduli of rock during the direct compression and tension tests are different. The tensile modulus from the Brazilian disc test is nearly identical to that from the uniaxial tensile test. Moreover, although there are some errors, the experimental results approximately yield the following

$$\frac{\nu_{ct}}{E_c}=\frac{\nu_{tc}}{E_t} \tag{22}$$

where E_c and ν_{ct} are the Young's moduli and Poisson's ratios shown in Table 1, respectively, and E_t and ν_{tc} are the Young's moduli and Poisson's ratios shown in Table 4, respectively. These results further illustrate the validity of the tension-compression anisotropic model.

5. Conclusions

The tensile and compressive moduli of rock materials were estimated using the Brazilian disc test and compared with the results obtained from the uniaxial tension and compression tests. Moreover, the elastic moduli under tension and compression of Brazilian discs were compared with each other. The following conclusions can be drawn:

• The rock tensile moduli from the strain data and displacement data are similar. The length of the strain gauge has little influence on the tensile moduli.

• The Brazilian disc test is a convenient method for estimating the tensile and compressive moduli of rock materials. The elastic moduli from the Brazilian disc test are consistent with those from the uniaxial tension and compression tests.

• The rock elastic modulus is different under tension and compression. The ratio of the rock elastic moduli under compression and tension ranges from 2 to 4.

• The tension-compression anisotropic model can properly explain the experimental results. Thus, it is better to use the tension-compression anisotropic model in the future analysis and simulation of Brazilian disc tests.

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References

Carneiro, F. (1943), "A new method to determine the tensile strength of concrete", *Proceedings of the 5th meeting of the Brazilian Association for Technical Rules*.

Carneiro, F.L.L.B. (1943), "A new method to determine the tensile strength of concrete", *Proceedings of the 5th meeting of the Brazilian Association for Technical Rules*.

Chen, Y.L. and Irfan, M. (2018), "Experimental study of Kaiser effect under cyclic compression and tension tests", *Geomech. Eng.*, **14**(2), 203-209. https://doi.org/10.12989/gae.2018.14.2.203.

Cho, J.W., Kim, H., Jeon, S. and Min, K.B. (2012), "Deformation and strength anisotropy of Asan gneiss, Boryeong shale, and Yeoncheon schist", *Int. J. Rock Mech. Min. Sci.*, **50**, 158-169. http://dx.doi.org/10.1016%2Fj.ijrmms.2011.12.004.

Dai, F. and Xia, K. (2010), "Loading rate dependence of tensile strength anisotropy of barre granite", *Pure Appl. Geophys.*, **167**(11), 1419-1432. https://doi.org/10.1007/s00024-010-0103-3.

Fairhurst, C. (1964), "On the validity of the 'Brazilian' test for brittle materials", *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, **1**(4), 535-546. https://doi.org/10.1016/0148-9062(64)90060-9.

Gong, F.Q., Li, X.B. and Zhao, J. (2010), "Analytical algorithm to estimate tensile modulus in Brazilian disk splitting tests", *Chin. J. Rock Mech. Eng.*, **29**(5), 881-891.

Hondros, G. (1959), "The evaluation of Poisson's ratio and the modulus of materials of a low tensile resistance by the Brazilian (indirect tensile) test with particular reference to concrete", *Austr. J. Appl. Sci.*, **10**(3), 243-268.

Hudson, J.A., Brown, E.T. and Rummel, F. (1972), "The controlled failure of rock discs and rings loaded in diametral compression", *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, **9**(2), 241-248. https://doi.org/10.1016/0148-9062(72)90025-3.

Jaeger, J.C., Cook, N.G.W. and Zimmerman, R. (2007), *Fundamentals of Rock Mechanics*, 4th Edition, Wiley-Blackwell.

Jiang, C., Zhao, G.F. and Khalili, N. (2017), "On crack propagation in brittle material using the distinct lattice spring model", *Int. J. Solids Struct.*, **118-119**, 41-57. https://doi.org/10.1016/j.ijsolstr.2017.04.024.

Jung, Y.B., Cheon, D.S., Park, E.S., Chan, P., Lee, Y.S., Park, C.W. and Choi, B.H. (2014), "Estimation of the characteristics of delayed failure and long-term strength of granite by brazilian disc test", *Tunn. Undergr. Sp.*, **24**(1), 67-80. https://doi.org/10.7474/TUS.2014.24.1.067.

Li, D.Y. and Wong, L.N.Y. (2013), "The Brazilian disc test for rock mechanics applications: Review and new insights", *Rock Mech. Rock Eng.*, **46**(2), 269-287. https://doi.org/10.1007/s00603-012-0257-7.

Li, X.F., Li, H.B. and Zhao, J. (2017), "3D polycrystalline discrete element method (3PDEM) for simulation of crack initiation and propagation in granular rock", *Comput. Geotech.*, **90**, 96-112. https://doi.org/10.1016/j.compgeo.2017.05.023.

Liu, Y., Dai, F., Xu, N., Zhao, T. and Feng, P. (2018), "Experimental and numerical investigation on the tensile fatigue properties of rocks using the cyclic flattened Brazilian disc method", *Soil Dyn. Earthq. Eng.*, **105**, 68-82. https://doi.org/10.1016/j.soildyn.2017.11.025.

Newman, D.A. and Bennett, D.G. (1990), "The effect of specimen size and stress rate for the Brazilian test—A statistical analysis", *Rock Mech. Rock Eng.*, **23**(2), 123-134. https://doi.org/10.1007/BF01020397.

Patel, S. and Martin, C.D. (2018), "Evaluation of tensile Young's modulus and Poisson's ratio of a bi-modular rock from the displacement measurements in a Brazilian test", *Rock Mech. Rock Eng.*, **51**(2), 361-373. https://doi.org/10.1007/s00603-017-1345-5.

Roy, D.G. and Singh, T.N. (2016), "Effect of heat treatment and layer orientation on the tensile strength of a crystalline rock under Brazilian test condition", *Rock Mech. Rock Eng.*, **49**(5), 1663-1677. https://doi.org/10.1007/s00603-015-0891-y.

Timoshenko, S.P. and Goodier, J.N. (2013), *Theory of Elasticity*, Beijing Higher Education Press, Beijing, China.

Wang, P., Cai, M. and Ren, F. (2018), "Anisotropy and directionality of tensile behaviours of a jointed rock mass subjected to numerical Brazilian tests", *Tunn. Undergr. Sp. Technol.*, **73**, 139-153. https://doi.org/10.1016/j.tust.2017.12.018.

Wijk, G. (1978), "Some new theoretical aspects of indirect measurements of the tensile strength of rocks", *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, **15**(4), 149-160. https://doi.org/10.1016/0148-9062(78)91221-4.

Yang, Y.T., Tang, X.H., Zheng, H., Liu, Q.S. and He, L. (2016), "Three-dimensional fracture propagation with numerical manifold method", *Eng. Anal. Boundary Elem.*, **72**(11), 65-77. https://doi.org/10.1016/j.enganabound.2016.08.008.

Ye, J.H., Wu, F.Q. and Sun, J.Z. (2009), "Estimation of the tensile elastic modulus using Brazilian disc by applying diametrically opposed concentrated loads", *Int. J. Rock Mech. Min. Sci.*, **46**(3), 568-576. https://doi.org/10.1016/j.ijrmms.2008.08.004.

Yu, Q.L., Zhu, W.C., Tang, C.A. and Yang, T.H. (2014), "Impact of rock microstructures on failure processes - Numerical study based on DIP technique", *Geomech. Eng.*, **7**(4), 375-401. https://doi.org/10.12989/gae.2014.7.4.375.

Zhao, J. and Li, H.B. (2000), "Experimental determination of dynamic tensile properties of a granite", *Int. J. Rock Mech. Min.*

Sci., **37**(5), 861-866.

Zhou, Z.L., Cai, X., Cao, W.Z., Li, X.B. and Xiong, C. (2016), "Influence of water content on mechanical properties of rock in both saturation and drying processes", *Rock Mech*. *Rock Eng*., **49**(8), 3009-3025. https://doi.org/10.1007/s00603-016-0987-z.

Zhu, W.C. and Tang, C.A. (2004), "Micromechanical model for simulating the fracture process of rock", *Rock Mech*. *Rock Eng*., **37**(1), 25-56. https://doi.org/10.1007/s00603-003-0014-z.

Zhu, W.C. and Tang, C.A. (2006), "Numerical simulation of Brazilian disk rock failure under static and dynamic loading", *Int*. *J*. *Rock Mech*. *Min*. *Sci*., **43**(2), 236-252. https://doi.org/10.1016/j.ijrmms.2005.06.008.

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