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Fractional order generalized thermoelastic study in orthotropic medium of type GN-III

Parveen Lata[a] and Himanshi Zakhmi*

Department of Basic and Applied Sciences, Punjabi University Patiala, Punjab, India

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Abstract. The present paper is concerned with the investigation of disturbances in orthotropic thermoelastic medium by using fractional order heat conduction equation with three phase lags due to thermomechanical sources. Laplace and Fourier transform techniques are used to solve the problem. The expressions for displacement components, stress components and temperature change are derived in transformed domain and further in physical domain using numerical inversion techniques. The effect of fractional parameter based on its conductivity i.e., ($0<\alpha<1$ for weak, $\alpha=1$ for normal, $1<\alpha\leq 2$ for strong conductivity) is depicted graphically on various components.

Keywords: orthotropic medium; fractional calculus; Laplace transform; Fourier transform; concentrated load; linearly and uniformly distributed loads

1. Introduction

Thermoelasticity deals with the study of the deformation in the body due to both thermal and mechanical sources. It deals with the dynamical system whose interactions with surroundings are limited to mechanical work, external forces and heat exchange. It also comprises the heat conduction, stress and strain that arise due to flow of heat. Nowadays applications of thermoelasticity are widely spread in the areas like in nuclear field, aircrafts, missiles, large steam turbines, shipbuilding, jet and rocket engines. During the last few decades much interest has been given to the thermoelasticity theories which give finite speed of propagation of thermal waves. Catteno (1958) and Vernotte (1958) proposed a thermal wave with a single phase lag in which the temperature gradient after a certain elapsed time was given by $q+\tau_q \ \frac{\partial q}{\partial t} = -K\nabla T$, where τ_q denotes the relaxation time required for thermal physics to take account of a hyperbolic effect within the medium. Here when $\tau_q >0$, the thermal wave propagates through the medium with a finite speed of $\sqrt{\frac{\alpha}{\tau_q}}$, where α is thermal diffusivity. When τ_q approaches zero, the thermal wave propagates with infinite speed and the single phase lag model reduces to the traditional Fourier model. Further Tzou (1995) and Chandrasekharaiah (1998) proposed a dual phase lag model of generalized thermoelasticity. According to this model, the classical Fourier law $q = -K\nabla T$ is replaced by $q(P, t+\tau_q) = -K\nabla T(P, t+\tau_q)$, where the temperature gradient ∇T at point P of the material at time $t+\tau_t$ is equal to the heat flux vector q at the same point P at time $t+\tau_q$, where K is thermal conductivity. The delay time τ_t is due to microstructural interaction which is called phase lag of temperature gradient. The other one is due to fast transient effect of inertia which is called phase lag of heat flux. For τ_t= 0 and $\tau_q= \tau$ this model is referred as single phase lag model. Further Roychoudhuri (2007) developed a new model of generalized thermoelasticity called three phase lag model. This modified Fourier law contains phase lag of heat flux (τ_q), phase lag of temperature gradient (τ_t) and phase lag of thermal displacement (τ_v) respectively.

The generalization of the ordinary differential and integration to a non-integer order give rise to the subject called fractional theory. Fractional order differential equation have been focus of many studies due to their use in different applications in fluid mechanics ,biology, physics, mechanics of solids, control theory etc. The definition of fractional derivatives has been given by Caputo (1967) of order ‘α’ where $0<\alpha\leq1$. Fractional order theory came into existence during the second half of the 19th century and has been used to model polymer materials. In this study, the heat conduction equation with fractional derivative of order ‘α’ was derived. The most important advantage of using differential equation of fractional order is due to their non-localization property. It was Abel, who first introduced the fractional derivatives in the formulation of tautochrone problem. Oldham and Spainer (1974) gave some alternate definitions of fractional thermoelasticity.

Lata and Kaur (2019) had studied the thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperature and without energy dissipation. Adolfsson *et al.* (2005) proposed a model on viscoelasticity theory by using fractional calculus. Ezzat (2010) proposed a model by using Taylor’s series expansion in fractional heat conduction equation with fractional order. Lata and Kaur

*Corresponding author, Ph.D. Scholar
E-mail: heena.zakhmi@gmail.com
[a]Associate Professor
E-mail: parveenlata@pbi.ac.in

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(2019) had studied the effect of rotation and inclined load on transversely isotropic magneto thermoelastic solid. Marin *et al.* (2019) had extended the domain of influence of initially stressed bodies in thermoelasticity. In many generalized thermoelastic problems, the method of finite element was used by (Tian *et al.* (2006), Abbas (2007), Abbas and Abdalla (2008), Youssef and Abbas (2007)).

Rafiq *et al.* (2019) had analyzed the transmission behaviour of plane harmonic waves in an isotropic medium in the context of generalized dual phase lag model of thermoelasticity under the effect of magnetic field. Lata and Kaur (2019) had investigated two dimensional transversely isotropic magneto thermoelastic solid without energy dissipation with two temperatures due to time harmonic sources. Raslan (2015) had investigated the distribution of stresses and temperature when a sudden heat changes occur on the external boundary of a spherical shell in case of one dimensional homogeneous isotropic problem in a thermoelastic medium by using fractional order theory. Abbas and Youssef (2015) had studied the two dimensional problem with the help of fractional order theory with one relaxation time for porous materials. Marin and Craciun (2017) had proved the uniqueness results for a boundary value problem in dipolar thermoelasticity to model composite materials. Kumar *et al.* (2016) had investigated the disturbances in a homogeneous transversely isotropic thermoelastic medium with two temperature with combined effect of magnetic field and hall current due to thermomechanical sources.

Abouelregal (2018) had investigated the effect of temperature dependent physical properties and fractional thermoelasticity on non-local nano beams. Yadav *et al.* (2015) had studied the effect of fractional order strain in a one dimensional viscoelastic solid in the presence of moving heat source and mechanical load. Bachhar *et al.* (2016) had solved the homogeneous isotropic thermoelastic problem by using fractional calculus under the effect of magnetic field with one relaxation time. Roychoudhuri (2007) had studied thermoelastic wave propagation in one dimensional elastic half-space using dual-phase-lag model. Abbas (2018) had also studied the effect of fractional parameter in a two dimensional problem in the context of thermal shock with three types of conductivity weak, normal and strong conductivity. Marin (1998) had studied the uniqueness in thermoelastodynamics on bodies with voids. Alimirzaei *et al.* (2019) had investigated nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions. Medani *et al.* (2019) had analyzed the Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate. Zarga *et al.* (2019) had studied the thermomechanical bending for functionally graded sandwich plates using a simple quasi-3D shear deformation theory. Chaabane *et al.* (2019) had studied the bending and free vibration responses of functionally graded beams resting on elastic foundation. Karami *et al.* (2019) had analyzed the wave propagation of functionally graded anisotropic nanoplates resting on winkler-pasternak foundation. Boulefrakh *et al.* (2019) had studied the effect of parameters of visco-pasternak foundation on the bending and vibration properties of a thick FG plate. Karami *et al.* (2019) had studied the Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates for different boundary conditions. Boukhlif *et al.* (2019) had investigated a simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation. Boutaleb *et al.* (2019) had studied the dynamic analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT. Bourada *et al.* (2019) had investigated the porous functionally graded beam using a sinusoidal shear deformation theory.

Zenkour (2018) had studied generalized thermoelastic two dimensional problem of a thermomechanical loaded beam by using dual phase lag model of heat conduction. Kumar *et al.* (2016) had studied the effect of hall current and two temperatures in a transversely isotropic magnetothermoelastic with and without energy dissipation due to ramp type heat. Othman and Jahangir (2015) had investigated the plane wave propagation in a generalized thermoelastic half space rotating with specific angular frequency. Marin (1999) had proved the existance and uniqueness of solutions for mixed initial-boundary value problems in thermoelasticity for dipolar bodies. Lata and Kaur (2019) had studied the deformation in a transversely isotropic thick circular plate due to thermomechanical sources. Othman and Marin (2017) had studied the effect of thermal loading due to laser pulse on thermoelastic porous medium by using G-N theory. Hassan *et al.* (2018) had studied the exploration of conductive heat transfer and flow characteristics synthesis by Cu–Ag/water hybrid-nanofluids.

In addition to above work we see that not much work has been done using fractional order thermoelasticity in orthotropic medium. In this problem we investigate the thermomechanical interactions in an orthotropic thermoelastic solid by using fractional order heat equation with three phase lags. In three-phase lag model the heat conduction equation consists of three phase lags namely τ_t ,τ_v *and* τ_q i.e. (phase lag of temperature gradient, phase lag of thermal displacement and phase lag of heat flux vector. The components of stress, displacement and temperature change subjected to uniformly distributed load, concentrated load and linearly distributed load are obtained with the help of Laplace and Fourier transform techniques. The effect of fractional parameter on various components has been depicted through graphs.

2. Basic equations

Following Chawla and Kumar (2014) the constitutive relations and basic governing equations of anisotropic three phase lag thermoelastic model in the absence of body forces and heat sources are the following.

$$\sigma_{ij} = c_{ijkm}\, e_{km} - \beta_{ij}\, T, \tag{1}$$

$$\sigma_{ij,j} = \rho\, \ddot{u}_i\ , \tag{2}$$

$$K_{ij}\left(1+\frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial}{\partial t^{\alpha}}\right)\dot{T}_{,ji} + K_{ij}^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right) T_{,ji} = \left[\left(1+\frac{\tau_q^{\alpha}}{\alpha!}\right)+\frac{\tau_q^{2\alpha!}}{2\alpha!}\right]\left[\rho\, C_E \ddot{T} + \beta_{ij}\, T_0\, \ddot{e}_{ij}\right]. \tag{3}$$

In equation (1)-(3) $c_{ijkm}(=c_{kmij}=c_{jikm}=c_{ijmk})$ is the tensor of elastic constant, ρ is the density, T_0 is the reference temperature such that $|\frac{T}{T_0}| \ll 1$, u_i are the components of displacement vector u, C_E is the specific heat at constant strain, u_i are the components of displacement vector u, $\sigma_{ij} = (\sigma_{ji})$ and $e_{ij} = \frac{1}{2}$ ($u_{i,j} + u_{j,i}$) are the components of stress and strain tensors respectively. T (x, y, z, t) is the temperature distribution from the reference temperature T_0. Also τ_q , τ_t and τ_v are respectively, the phase lag of the heat flux, the phase lag of the temperature gradient and the phase lag of the thermal displacement, β_{ij} are tensor of thermal moduli, K_{ij} (= K_{ji}) and K_{ij}^* (= K_{ji}^*) are the components of thermal conductivity and material characteristic constant respectively.

The basis of these symmetries of C_{ijkm} is due to

i. The stress tensor is symmetric, which is only possible if $(C_{ijkm} = C_{jikm})$

ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkm} = C_{kmij}$

iii. From stress tensor and elastic stiffness tensor symmetries infer $(C_{ijkm} = C_{ijmk})$ and $C_{ijkm} = C_{kmij} = C_{jikm} = C_{ijmk}$

In all above equations dot (.) represents the partial derivative w.r.t time and (,) denote the partial derivative w.r.t spatial coordinate.

The Eq. (1) for orthotropic media in Cartesian coordinate system (x, y, z) in component form can be written as

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix} e_{xx}\\ e_{yy}\\ e_{zz}\\ 2e_{yz}\\ 2e_{xz}\\ 2e_{xy}\end{bmatrix} \begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ 0\\ 0\\ 0\end{bmatrix} \mathrm{T}, \quad (4)$$

$$\sigma_{xx} = C_{11}\ e_{xx} + C_{12}\ e_{yy} + C_{13}\ e_{zz} - \beta_1\ T,$$

$$\sigma_{yy} = C_{12}\ e_{xx} + C_{22}\ e_{yy} + C_{23}\ e_{zz} - \beta_2\ T,$$

$$\sigma_{zz} = C_{13}\ e_{xx} + C_{23}\ e_{yy} + C_{33}\ e_{zz} - \beta_3\ T,$$

$$\sigma_{yz} = 2\ C_{44}\ e_{yz},$$

$$\sigma_{xz} = 2\ C_{55}\ e_{xz},$$

$$\sigma_{xy} = 2\ C_{66}\ e_{xy},$$

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + C_{55}\frac{\partial^2 u}{\partial z^2} + (C_{12} + C_{66})\ \frac{\partial^2 v}{\partial x \partial y} + (C_{13} + C_{55})\ \frac{\partial^2 w}{\partial x \partial z} - \beta_1\ \frac{\partial T}{\partial x} = \rho\ \frac{\partial^2 u}{\partial t^2}, \quad (5)$$

$$(C_{12} + C_{66})\ \frac{\partial^2 u}{\partial x \partial y} + C_{66}\ \frac{\partial^2 v}{\partial x^2} + C_{22}\frac{\partial^2 v}{\partial y^2} + C_{44}\frac{\partial^2 v}{\partial z^2} + (C_{23} + C_{44})\ \frac{\partial^2 w}{\partial y \partial z} - \beta_2\ \frac{\partial T}{\partial y} = \rho\ \frac{\partial^2 v}{\partial t^2}, \quad (6)$$

$$(C_{13} + C_{55})\ \frac{\partial^2 u}{\partial x \partial z} + (C_{23} + C_{44})\ \frac{\partial^2 v}{\partial y \partial z} + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\ \frac{\partial T}{\partial z} = \rho\ \frac{\partial^2 w}{\partial t^2}, \quad (7)$$

$$(K_1\ (1+ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\ \dot{T}_{,11} + K_2\ (1+ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\ \dot{T}_{,22} + K_3\ (1+ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\ \dot{T}_{33} + K_1^*(1+ \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\ T_{,11} + K_2^*(1 + \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\ T_{,22} + K_3^*(1+ \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\ T_{,33} = [1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\ ]\ [\rho\ C_E\ddot{T} + T_0\ (\beta_1\ \ddot{u}_{1,1} + \beta_2\ \ddot{u}_{2,2} + \beta_3\ \ddot{u}_{3,3}\ )]. \quad (8)$$

3. Formulation of the problem

We consider a two dimensional homogeneous thermoelastic orthotropic body initially at temperature T_0 with and without energy dissipation in generalized thermoelasticity using three phase lag model. We take a rectangular coordinate axis $(x, y, z)$with z-axis as a axis of symmetry. The components of displacement vector $\vec{u}$, $\vec{v}$ and $\vec{w}$ and temperature change T for the two dimensional problem have the form

$$\vec{u} = u\ (x, z, t),\ \vec{v} = 0,\ \vec{w} = w\ (x, z, t), \text{and}\ T = T(x, z, t), \quad (9)$$

With the help of (9) Eqs (5)-(8) reduce to the form

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{55}\frac{\partial^2 u}{\partial z^2} + (C_{13} + C_{55})\ \frac{\partial^2 w}{\partial x \partial z} - \beta_1\ \frac{\partial T}{\partial x} = \rho\ \frac{\partial^2 u}{\partial t^2}, \quad (10)$$

$$(C_{13} + C_{55})\ \frac{\partial^2 u}{\partial x \partial z} + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\ \frac{\partial T}{\partial z} = \rho\ \frac{\partial^2 w}{\partial t^2}, \quad (11)$$

$$K_1\ (1+ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\ \dot{T}_{,11} + K_3\ (1+ \frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})\ \dot{T}_{33} + K_1^*(1+ \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\ T_{,11} + K_3^*(1+ \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\ T_{,33} = [1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\ ]\ [\rho\ C_E\ddot{T} + T_0\ (\beta_1\ \ddot{u}_{1,1} + \beta_3\ \ddot{u}_{3,3}\ )]. \quad (12)$$

Also

$$\sigma_{11} = C_{11}\ e_{11} + C_{13}\ e_{33} - \beta_1 \mathrm{T}, \quad (13)$$

$$\sigma_{33} = C_{13}\ e_{11} + C_{33}\ e_{33} - \beta_3 \mathrm{T}, \quad (14)$$

$$\sigma_{13} = 2\ C_{55}\ e_{13}\ , \quad (15)$$

where $e_{11} = \frac{\partial u}{\partial x}$, $e_{33} = \frac{\partial w}{\partial z}$, $e_{13} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$, $\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i\ \delta_{ij}$, $K_{ij}^* = K_i^* \delta_{ij}$, i is not summed where i=1, 2, 3 and δ_{ij} is kronecker delta.

To facilitate the solution the following dimensionless quantities are used

$$x' = \frac{x}{L}\ ,\ z' = \frac{z}{L}\ ,\ u' = \frac{\rho c_1^2}{L\ T_0 \beta_1}\ u,\ w' = \frac{\rho c_1^2}{L\ T_0 \beta_1}\ w,\ t' = \frac{c_1}{L}\ t,\ \sigma_{33}' = \frac{\sigma_{33}}{T_0 \beta_1}\ ,\ \sigma_{31}' = \frac{\sigma_{31}}{T_0 \beta_1},\ T' = \frac{T}{T_0}. \quad (16)$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

Using dimensionless quantities given by (16) in Eqs. (10)-(12) and suppressing the primes for convenience yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial^2 u}{\partial z^2} + \delta_2\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \quad (17)$$

$$\delta_3\ \frac{\partial^2 w}{\partial z^2} + \delta_1\frac{\partial^2 w}{\partial x^2} + \delta_2\frac{\partial^2 u}{\partial x \partial z} - \frac{\beta_3}{\beta_1}\frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \quad (18)$$

$$\epsilon_1\ \tau_t'\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_2\ \tau_t'\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right) + \epsilon_3\ \tau_t'\left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_4\ \tau_t'\left(\frac{\partial^2 T}{\partial z^2}\right) = \tau_t''\left[\frac{\partial^2 T}{\partial z^2} + \beta_1^2\ \epsilon_5\ \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \frac{\beta_3}{\beta_1}\frac{\partial w}{\partial z}\right)\right], \quad (19)$$

where $\delta_1 = \frac{c_{55}}{c_{11}}, \delta_2 = \frac{c_{13}+c_{15}}{c_{11}}$, $\delta_3 = \frac{c_{33}}{c_{11}}$, $\tau_t' = (1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha})$, $\tau_t'' = (1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})$, $\tau_t''' = (1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}})$, $\epsilon_1 = \frac{K_1}{\rho\ Lc_1^2 C_E}$, $\epsilon_2 = \frac{K_3}{\rho\ Lc_1^2 C_E}$, $\epsilon_3 = \frac{K_1^*}{\rho\ c_1^2 C_E}$, $\epsilon_4 = \frac{K_3^*}{\rho\ c_1^2 C_E}$, $\epsilon_5 = \frac{T_0}{\rho^2\ c_1^2 C_E}$.

Apply Laplace and Fourier transforms defined by

$$\overline{f}(x,z,s) = \int_0^\infty f(x,z,t)e^{-st}\ dt, \tag{20}$$

$$\hat{f}(\xi,z,s) = \int_{-\infty}^{\infty}\overline{f}(x,z,s)e^{i\xi x}\ dx, \tag{21}$$

On Eqs (17)-(19), we obtain a system of three homogeneous equations. These resulting equations have non- trivial solutions if the determinant of the coefficient ($\hat{u}$, $\hat{w}$, $\hat{T}$) vanishes, which yield to the following characteristic equation.

$$(PD^6 + QD^4 + RD^2 + S)\ (\hat{u},\hat{w},\hat{T}) = 0, \tag{22}$$

where

$D = \frac{d}{dz}$, $P = \{\tau_t''\ \delta_3\ \delta_1\ \ \epsilon_4 + \tau_t'\ \epsilon_2 s\ \delta_1\}$,

$Q = \tau_t'\ [\delta_3\ \epsilon_2 s^3 + s\delta_3\ \epsilon_2\ \xi^2 - \epsilon_1 s\ \delta_3\ \ \xi^2\ \delta_1^2 - \epsilon_2 s^3\ \delta_1^2 - \epsilon_2 s\ \xi^2\ \delta_1^3 + \epsilon_2 s\ \xi^2\ \delta_2^2\ ] + \tau_t''\ [-\delta_3\epsilon_3\xi^2\ s^2 + \delta_3\ \epsilon_4\ \xi^2 - \delta_3\ \epsilon_4 s^2 - \epsilon_4 s^2\ \delta_1^2 - \epsilon_4\xi^2\ \delta_1^3 + \epsilon_4\xi^2\ \delta_2^2\ ] + \tau_t'''\ [-\delta_3 s^2\ \delta_1^2 - s^2\ \delta_1^2\beta_3^2\ \epsilon_5]$,

$R = \tau_t'\ [\delta_3\epsilon_1 s^3\xi^2 - \epsilon_2 s^5 - \delta_1\ \epsilon_2\ \xi^2\ s^3 - \delta_3\ \epsilon_1 s\ \xi^4 - \epsilon_2\ \xi^2\ s^3 - \delta_1\epsilon_2 s\ \xi^4 + \delta_1^2\ \epsilon_1\ \xi^2\ s^3 + \delta_1^3\ \epsilon_1 s\ \xi^4 - \delta_2^2\ \epsilon_1 s\ \xi^4\ ] + \tau_t''\ [-\delta_3\ \epsilon_3\ s^2\ \xi^2 - s^2\ \xi^2\ \delta_1\ \epsilon_4 - \epsilon_4\ s^4 - \epsilon_3\xi^4\ \delta_3 - \epsilon_4\ \ s^2\ \xi^2 - \delta_1\ \epsilon_4\xi^4 + s^2\ \xi^2\ \delta_1^2\ \epsilon_3 + \epsilon_3\ \delta_1^3\ \xi^4 - \epsilon_4\xi^4\ \ \delta_2^2] + \tau_t'''\ [-\delta_3\ s^4 + \beta_3^2\ \epsilon_5\ s^4 - \delta_3\ s^2\ \xi^2 - \beta_3^2\ \epsilon_5\ s^2\ \xi^2 + \delta_1^2\ s^4 + \xi^2\ s^2\ \delta_1^3 - \beta_1\ \beta_3\ \delta_2\epsilon_5 s^2\xi^2 - \beta_1\ \beta_3\ \ \delta_2\epsilon_5\xi^2 + \beta_1^3\ \epsilon_5\ s^2\ \delta_3\xi - \xi^2\ s^2\ \delta_2^2\ ]$,

$S = \tau_t'\ [\ \delta_1\ \epsilon_1\ \xi^4\ s^3 + \epsilon_1\ \xi^4\ s^3 + \epsilon_1\ \xi^2\ s^5 + \epsilon_1 s\delta_1\ \xi^6\ ] + \tau_t''\ [\ \xi^2 s^4\ \epsilon_3 + \xi^4\ s^2\ \delta_1\ \epsilon_3 + s^2\ \xi^4\ \epsilon_3 + \delta_1\ \xi^6\ \epsilon_3] + \tau_t'''\ [\ s^6 + \xi^2\ s^4\ \delta_1 + s^4\ \xi^2 + \delta_1\ \xi^4\ s^2 - \xi^2 s^4\ \beta_1^2\ \epsilon_5 - \beta_1^2\ \ \xi^3\ s^2\ \ \delta_1\epsilon_5]$,

where $\tau_t' = (1+\frac{\tau_t^\alpha}{\alpha!}\ s^\alpha)$, $\tau_t'' = (1+\frac{\tau_v^\alpha}{\alpha!}\ s^\alpha)$, $\tau_t''' = \left(1+\frac{\tau_q^\alpha}{\alpha!}\ s^\alpha + \frac{\tau_q^{2\alpha!}}{2\alpha!}\ s^{2\alpha}\right)$.

The roots of the Eq. (22) are $\pm\ \lambda_i$ (i= 1,2,3); the solution of the equation satisfying the radiation conditions can be written as

$$\tilde{u} = A_1\ e^{-\lambda_1\ z} + A_2\ e^{-\lambda_2 z} + A_3\ e^{-\lambda_3\ z}, \tag{23}$$

$$\widetilde{w} = d_1\ A_1 e^{-\lambda_1 z} + d_2 A_2\ e^{-\lambda_2 z} + d_3\ A_3 e^{-\lambda_3 z}, \tag{24}$$

$$\tilde{T} = l_1\ A_1\ e^{-\lambda_1 z} + l_2\ A_2\ e^{-\lambda_2 z} + l_3\ A_3\ e^{-\lambda_3 z}, \tag{25}$$

where

$$d_i = \frac{\lambda_i^4\ \ A^* + \lambda_i^2\ \ B^*\ + C^*}{\lambda_i^4\ A' + \lambda_i^2\ B' + C'};\ \mathrm{i} = 1, 2, 3 \tag{26}$$

$$l_i = \frac{\lambda_i^4\ P' + \lambda_i^2\ Q' + R'}{\lambda_i^4\ A' + \lambda_i^2\ B' + C'}\ ;\ \mathrm{i} = 1, 2, 3 \tag{27}$$

where

$A^* = \tau_t'\ \delta_1\ [-\epsilon_2\ s] - \tau_t''[\ \ \delta_1\ \epsilon_4\ ]$,

$B^* = -\tau_t'\ [\epsilon_2 s^3 + \xi^2\ \epsilon_2\ s - \delta_1\ \epsilon_1 s\ \xi^2] - \tau_t''\ [\ \epsilon_4 s^2 + \epsilon_4\xi^2 - \delta_1\xi^2\ \epsilon_3\ ] + \tau_t'''\ [\ \delta_1\ s^2]$,

$C^* = \tau_t'\ [\epsilon_1\ \xi^2\ s^3 + \xi^4\ \epsilon_1\ s\ ] + \tau_t''\ [\epsilon_3\xi^2\ s^2 + \xi^4\ \epsilon_3] + \tau_t'''\ [s^4 + \xi^2\ s^2 + \beta_1^2\ \epsilon_5\ \ \xi^2\ s^2]$,

$A' = \delta_3\ \ [\ \tau_t'\ \epsilon_2\ s + \tau_t''\epsilon_4]$,

$B' = \tau_t'\ [\epsilon_1\ s\ \delta_3\xi^2 - \epsilon_2 s^3 - \delta_1\ \epsilon_2 s\ \xi^2\ ] + \tau_t''[-\delta_3\ \epsilon_3\xi^2 - \epsilon_4 s^2 - \delta_1\ \epsilon_4\ \xi^2] + \tau_t'''\ [-\delta_3\ s^2 - \beta_3^2\ \epsilon_5 s^2]$,

$C' = \tau_t'\ [\ \epsilon_1\ s^3\ \xi^2 - \delta_1\ \epsilon_1 s\ \xi^4\ ] + \tau_t''[\ \epsilon_3 s^2\xi^2 + \delta_1\ \epsilon_3\ \xi^4\ ] + \tau_t'''\ [\ s^4 + \delta_1\xi^2\ s^2]$,

$P' = \{-\ \delta_1\delta_3\}$,

$Q' = [-\delta_3\ s^2 - \xi^2\ \delta_3 + s^2\ \delta_1 + \xi^2{\delta_1}^2 + \xi^2{\delta_2}^2]$,

$R' = [s^4 + \delta_1\ \xi^2\ s^2 + \xi^2\ s^2 + \delta_1\ \xi^4]$.

4. Boundary conditions

We apply a normal force and thermal source on the boundary. The boundary conditions are given by

$$\sigma_{33} = -\ F_1\ \psi_1\ (x)\ \delta\ (t), \tag{28}$$

$$\sigma_{31} = 0, \tag{29}$$

$$\frac{\partial T}{\partial z} = F_2\ \psi_2(x)\ \delta\ (t)\ \ at\ \ z = 0, \tag{30}$$

where F_1 *is* the magnitude of force applied, F_2 is the constant temperature applied on the boundary, $\psi_1\ (x)$ *and* $\psi_2\ (x)$ is the source distribution function along x -axis.

By applying Laplace and Fourier transform defined by (20)-(21) on the boundary conditions (28)-(30) and with the help of equations (1),(16),(20)-(21), we obtain components of displacement, normal stress, tangential stress and temperature change as

$$\tilde{u} = -\frac{F_1\ \tilde{\psi}_1\ (\xi)}{\Delta}\ (\ \Delta_1\ e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3\ e^{-\lambda_3 z}\ ) + \frac{F_2\ \tilde{\psi}_2\ (\xi)}{\Delta}\ (\ \Delta_1^* e^{-\lambda_1 z} + \Delta_2^*\ e^{-\lambda_2 z} + \Delta_3^*\ e^{-\lambda_3 z}\ ), \tag{31}$$

$$\widetilde{w} = -\frac{F_1\ \tilde{\psi}_1\ (\xi)}{\Delta}\ (\mathrm{d}_1\Delta_1\ e^{-\lambda_1 z} + \mathrm{d}_2\ \Delta_2 e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3\ e^{-\lambda_3 z}\ ) + \frac{F_2\ \tilde{\psi}_2\ (\xi)}{\Delta}\ (\mathrm{d}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{d}_2\Delta_2^*\ e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3^*\ e^{-\lambda_3 z}\ ), \tag{32}$$

$$\tilde{T} = -\frac{F_1\ \tilde{\psi}_1\ (\xi)}{\Delta}\ (\ \mathrm{l}_1\Delta_1\ e^{-\lambda_1 z} + \mathrm{l}_2\ \Delta_2 e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3\ e^{-\lambda_3 z}\ ) + \frac{F_2\ \tilde{\psi}_2\ (\xi)}{\Delta}\ (\ \mathrm{l}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{l}_2\Delta_2^*\ e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3^*\ e^{-\lambda_3 z}\ ), \tag{33}$$

$$\widetilde{\sigma_{33}} = -\frac{F_1\ \tilde{\psi}_1\ (\xi)}{\Delta}\ (\ \Delta_{11}\ \ \Delta_1\ e^{-\lambda_1 z} + \Delta_{12}\ \Delta_2\ e^{-\lambda_2 z} + \Delta_{13}\Delta_3\ e^{-\lambda_3 z}\ ) + \frac{F_2\ \tilde{\psi}_2\ (\xi)}{\Delta}\ (\Delta_{11}\Delta_1^* e^{-\lambda_1 z} + \Delta_{12}\Delta_2^*\ e^{-\lambda_2 z} + \Delta_{13}\Delta_3^*\ e^{-\lambda_3 z}\ ), \tag{34}$$

$$\widetilde{\sigma_{13}} = -\frac{F_1\ \tilde{\psi}_1\ (\xi)}{\Delta}\ (\ \Delta_{21}\ \Delta_1\ e^{-\lambda_1 z} + \Delta_{22}\ \Delta_2 e^{-\lambda_2 z} + \Delta_{23}\Delta_3\ e^{-\lambda_3 z}\ ) + \frac{F_2\ \tilde{\psi}_2\ (\xi)}{\Delta}\ (\Delta_{21}\Delta_1^* e^{-\lambda_1 z} + \Delta_{22}\Delta_2^*\ e^{-\lambda_2 z} + \Delta_{23}\Delta_3^*\ e^{-\lambda_3 z}\ ), \tag{35}$$

where

$$\Delta = \Delta_{11}\,(\Delta_{22}\,\Delta_{33} - \Delta_{32}\,\Delta_{23}) - \Delta_{12}(\Delta_{21}\,\Delta_{33} - \Delta_{23}\,\Delta_{31}) + \Delta_{13}(\Delta_{21}\,\Delta_{32} - \Delta_{22}\,\Delta_{31}),$$

$$\Delta_1^* = (\Delta_{12}\,\Delta_{23} - \Delta_{13}\,\Delta_{22}),\ \Delta_2^* = (\Delta_{13}\,\Delta_{21} - \Delta_{11}\,\Delta_{23}),\ \Delta_3^* = (\Delta_{11}\,\Delta_{22} - \Delta_{12}\,\Delta_{21}),$$

$$\Delta_{1j} = \frac{c_{13}\quad \xi i}{\rho c_1^2} - \frac{c_{33} d_j \lambda_j}{\rho c_1^2} - \frac{\beta_3}{\beta_1}\, l_j\ ; \quad j = 1, 2, 3$$

$$\Delta_{2j} = \frac{c_{55}}{\rho c_1^2}\,[-\lambda_j + {}_{\mathrm{i}}\xi d_j]\ ; \quad j = 1, 2, 3.$$

4.1 Mechanical force on the surface of half–space

Taking F_2 =0 in Eqs. (31)- (35), we obtain the components of tangential stress, normal stress, displacement, temperature change due to mechanical force.

4.2 Thermal source on the surface of half-space

Taking F_1 =0 in Eqs. (31)- (35), we obtain the components of tangential stress, normal stress, displacement and temperature change due to thermal source.

5. Applications

5.1 Concentrated force

The solution due to concentrated normal force is obtained by setting

$$\psi_1(x) = \delta(x),\ \psi_2\,(x) = \delta(x), \tag{36}$$

where δ(x) is the Dirac delta function. By applying Laplace and Fourier transformations defined in equation (19)-(20) on (35), we get

$$\widehat{\psi_1}\,(\xi) = 1,\ \widehat{\psi_2}\,(\xi) = 1. \tag{37}$$

Using (37) in (31)-(35), we obtain the components of tangential stress, normal stress, displacement and thermodyanamical temperature.

5.2 Uniformly distributed force

The solution due to uniformly distributed force is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{Bmatrix} 1 & if & |x| & \le m \\ 0 & if & |x| > m \end{Bmatrix} \tag{38}$$

The Laplace and Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$with respect to the pair (x,ξ) in case of uniformly distributed load of non-dimensional width 2m applied at origin of co-ordinate system $x = z = 0$ is given by

$$\{\ \widehat{\psi_1}\,(\xi),\ \widehat{\psi_2}\,(\xi)\ \} = [2\,sin(\xi m)/\ \xi]\quad, \xi\ \neq 0. \tag{39}$$

Using (39) in (31)-(35), we get the components of tangential stress, normal stress, displacement, thermodynamical temperature.

6. Inversion of transformation

To obtain the solution of the problem in physical domain, we must invert the transformations in Eqs (31)-(35). Here the displacement components, tangential and normal stresses and thermodynamical temperature are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and are of the form f (ξ, z, s). To obtain the function f (x, z, t) in the physical domain, we first invert the Fourier transform using

$$\bar{f}\ (x, z, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\xi x_1}\ \hat{f}\ (\xi, \mathrm{z}, \mathrm{s})\,\mathrm{d}\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x)\,f_e - \ isin(\xi x) f_0|\ \mathrm{d}\xi, \tag{40}$$

where f_0 and f_e are respectively the odd and even parts of $\hat{f}(\xi,$ z, s). Thus the expression (40) gives the Laplace transform $\bar{f}(x, z, s)$ of the function f(x, z, t) Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(x, z, s)$ can be inverted to f (x, z, t).

The last step is to calculate the integral in Eq (40). The method of evaluating this integral is described in Press et. al. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

For numerical computations, we take the following values of the relevant parameter for an orthotropic thermoelastic material (Biswas *et al.* 2017 and Kumar and Chawla 2014)

c_{11} =18.78 $\times 10^{10}$ $\mathrm{Kg}m^{-1}s^{-2}$,c_{13} =8.0 $\times$ $10^{10}\mathrm{Kg}m^{-1}s^{-2}$, c_{33} =10.2 $\times 10^{10}$ $\mathrm{Kg}m^{-1}s^{-2}$,c_{55} = 10.06 $\times$ $10^{10}\mathrm{Kg}m^{-1}s^{-2}$, T_0=0.293 $\times$ 10^3K, $C_E = 4.27\times 10^2 J/KgK$, β_1= 1.96$\times$ $10^{-5}K^{-1}$, β_3= 1.4$\times$ $10^{-5}K^{-1}$, ρc_{55} = 8.836$\times$ $10^3 Kgm^{-3}$, K_1 =.12$\times$ $10^3\mathrm{W}m^{-1}K^{-1}$, K_3=.33$\times$ $10^3\mathrm{W}m^{-1}K^{-1}$, K_1^*=1.313$\times 10^2$W/s, K_3^*=1.54$\times 10^2$W/s, τ_t=1.5$\times 10^{-7}s$,τ_v=1.0$\times 10^{-8}$s, τ_q =2.0$\times 10^{-7}$ s.

Using above values of parameters, the graphical representation of components of tangential stress, normal stress, tangential and normal displacements and temperature change T with distance $'x\ '$ has been made for an orthotropic body by using different values of fractional parameter α=0.5, α=1.0, α=1.8

(1) The black dashed line with centre symbol triangle (Δ) for an orthotropic material corresponds to α=0.5 refers to weak conductivity.

(2) The red dashed line with centre symbol plus (+) for an orthotropic material corresponds to α=1.0 refers to strong conductivity.

(3) The purple dashed line with centre symbol circle (o) for an orthotropic material corresponds to α=1.8 describes normal conductivity.

8. Mechanical forces on the surface of half space

8.1 Concentrated mechanical force

Figs. 1 and 2 shows the variation of tangential and normal displacements with distance x for different values of fractional parameter α = 0.5, 1.0 and 1.8. The black

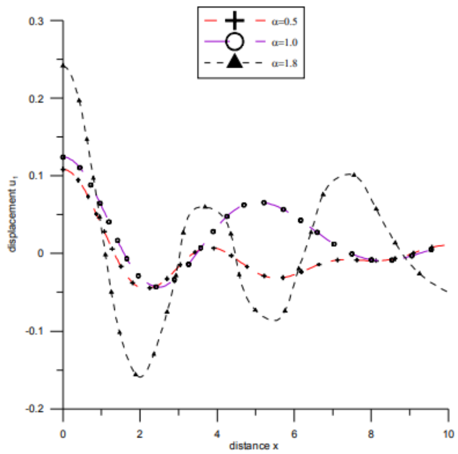

Fig. 1 Variation of tangential displacement u with distance x (concentrated force)

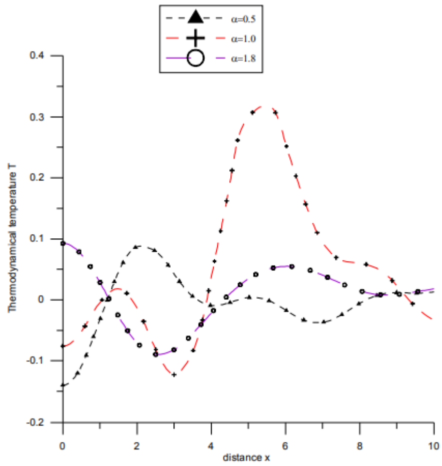

Fig. 4 Variation of temperature change T with distance x (concentrated mechanical force)

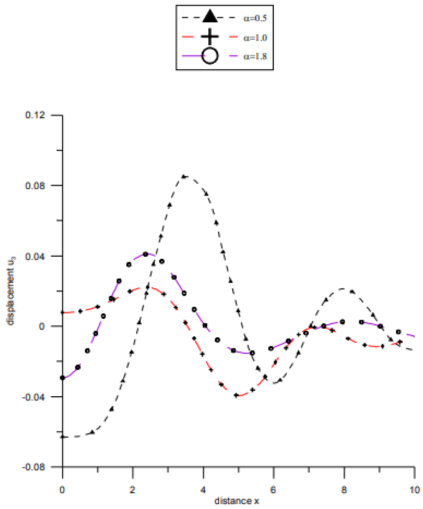

Fig. 2 Variation of normal displacement w with distance x (concentrated force)

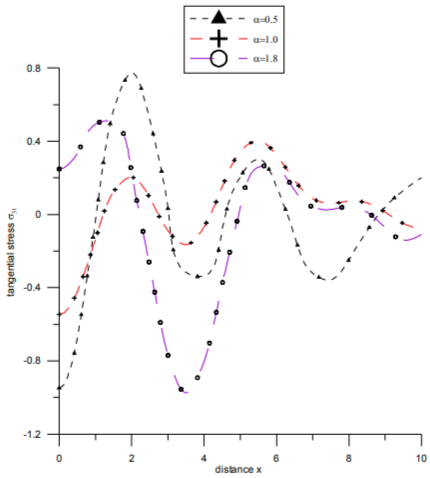

Fig. 5 Variation of tangential stress σ_{31} with distance x (concentrated mechanical force)

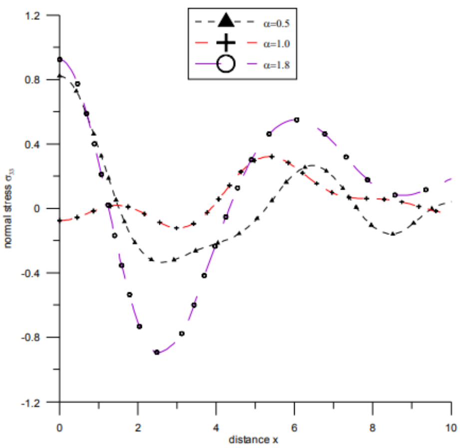

Fig. 3 Variation of normal stress σ_{33} with distance x (concentrated mechanical force)

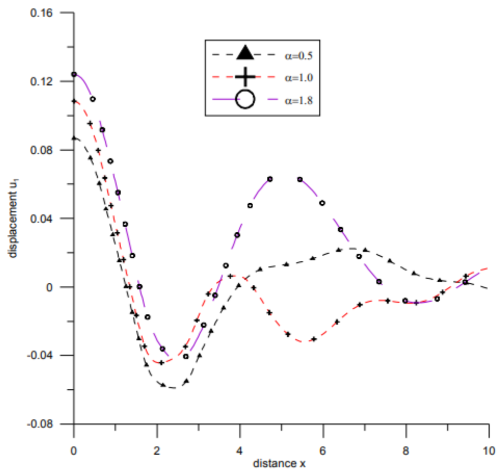

Fig. 6 Variation of tangential displacement u with distance x (uniformly distributed mechanical force)

dashed line refers to weak, red line refers to strong and purple line respectively refers to normal conductivity. It can be seen that at the boundary $x = 0$ the magnitude of tangential stress first decreases then increases corresponding to α=0.5, 1.0 and shows an oscillatory nature. Also, for α= 1.8 graph is oscillatory. It is noticed that in Fig. 2 the behavior of normal displacement is just opposite i.e., first increases and then decreases with

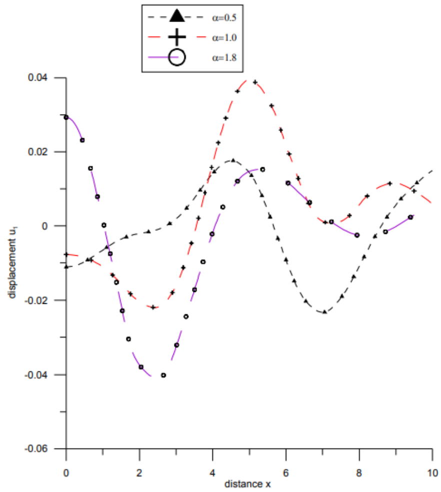

Fig. 7 Variation of normal displacement w with distance x (uniformly distributed mechanical force)

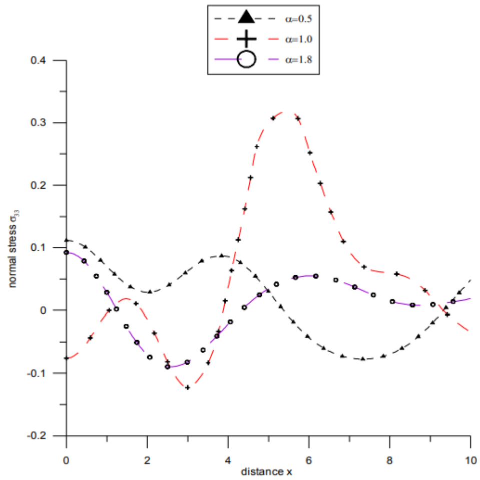

Fig. 8 Variation of normal stress σ_{33} with distance x (uniformly distributed mechanical force)

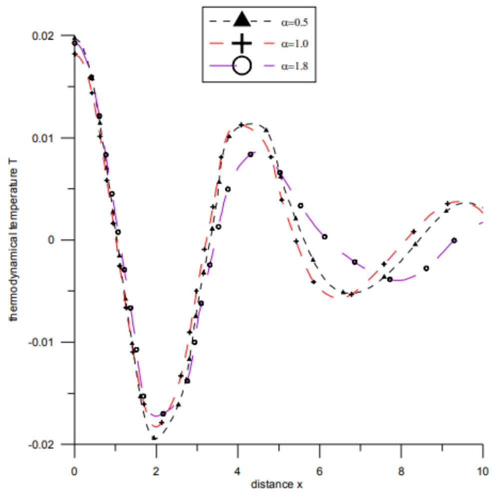

Fig. 9 Variation of temperature change T with distance x (uniformly distributed mechanical force)

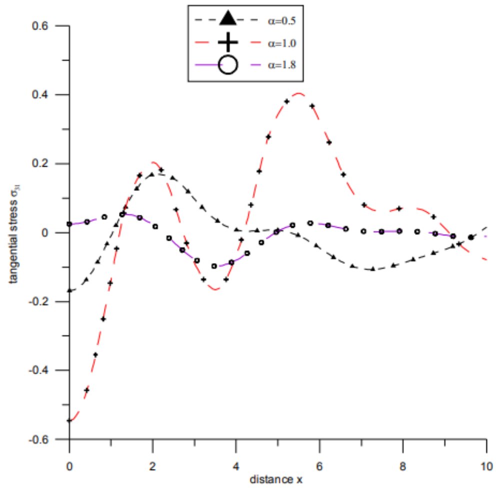

Fig. 10 Variation of tangential stress σ_{31} with distance x (uniformly distributed mechanical force)

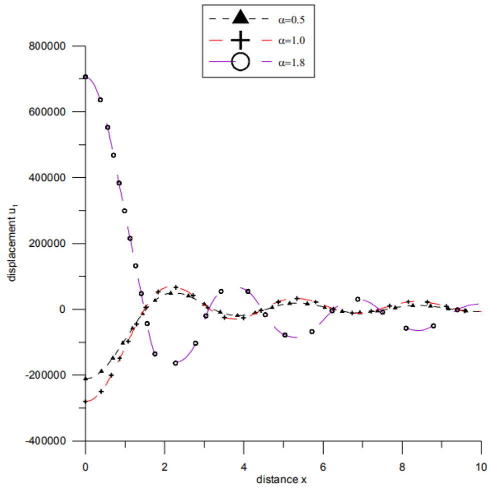

Fig. 11 Variation of tangential displacement u with distance x (concentrated thermal source)

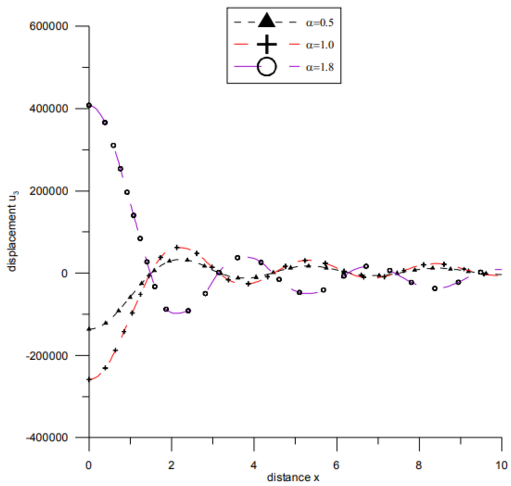

Fig. 12 Variation of normal displacement w with distance x (concentrated thermal source)

increasing value of x and shows an oscillatory behaviour for α= 0.5, 1.0 and 1.8 and there comes a point when all the three curves intersect each other. Fig. 3 shows the variation of normal stress σ_{33} with distance x. It is

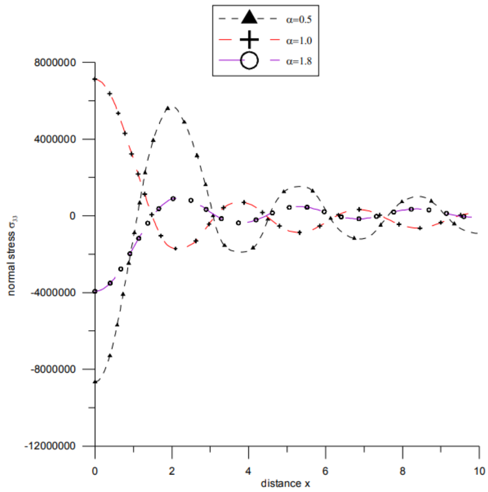

Fig. 13 Variation of normal stress σ_{33} with distance x (concentrated thermal Source)

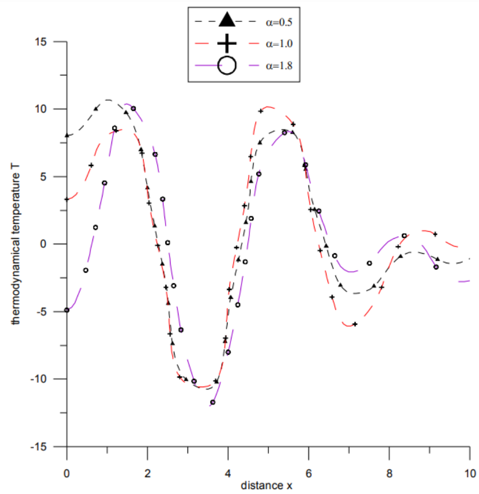

Fig. 14 Variation of temperature change T with distance x (concentrated thermal source)

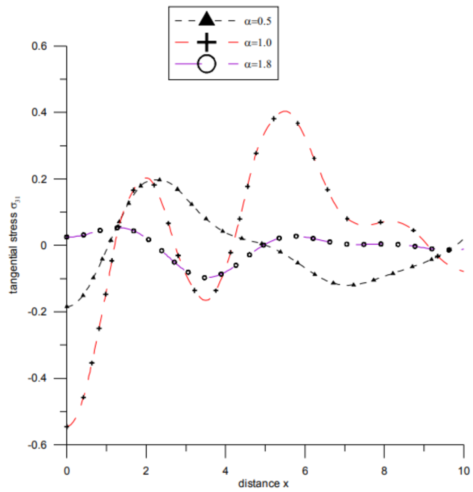

Fig. 15 Variation of tangential stress σ_{31} with distance x (concentrated thermal source)

observe that for α=0.5 first it varies from the maximum value to minimum value then shows an oscillatory behaviour. For α=1.0, 1.8 it is oscillatory. In fig. 4 the behaviour of temperature change has shown and it can be seen that for α=0.5 and α=1.0 shows an oscillatory behavior while for α=1.8 it gradually decreases then increases with increase in the value of distance x and there comes a point where all the three curves meet each other. In fig 5 shows the variation of tangential stress σ_{31} with distance x which shows an oscillatory behavior for α=0.5, 1.0 and α=1.8 respectively.

8.2 Uniformly distributed force

In uniformly distributed mechanical force Figs. 6 to 10 as in case of concentrated force shows the variation of distance x with tangential displacement, normal displacement, normal stress, temperature change and tangential stress respectively. In Fig. 6 variation of tangential displacement first decreases gradually then increases and shows an oscillatory behaviour corresponding to α=0.5, 1.0, 1.8 respectively. In Fig. 7 it can be seen that the magnitude of normal displacement first increases for α=0.5 and then decreases i.e., just opposite to tangential displacement and then shows an oscillatory behaviour. For α=1.0 and 1.8 we see that it varies from maximum to minimum value after that shows an oscillatory behaviour with increasing value of distance x. Fig. 8 describes the variation of normal stress it can be seen that for α = 0.5 and 1.8 at the boundary of surface when x = 0 magnitude of normal stress first decreases and then increases and shows an oscillatory behaviour while for α =1.0 normal stress curve is oscillatory and after attaining a peak value it decreases and there comes a point when all three curves for different fractional parameters value meet each other. Fig. 9 gives the variation of temperature change. It can be seen that all the three curves shows same behaviour i.e., oscillatory with increasing value of distance x. Also in Fig 10 variation of tangential stress σ_{31} with distance x is oscillatory in nature and all the three curves intersect each other.

9. Deformation due to thermal source

9.1 Concentrated thermal source

Fig. 16 shows the variation of tangential displacement u with distance x. Here we noticed that the variation of both the displacements tangential and normal in Fig 16 and Fig 17 with distance x are almost identical when heat is supplied to an orthotropic body. Fig 18 displays the variation of normal stress σ_{33} with distance x and we noticed that the behaviour is oscillatory in nature for all the three cases weak, strong and normal conductivity. Also Fig 19 shows the variation of temperature change with distance x which gradually increases for α=0.5, 1.0, 1.8 respectively and then shows oscillatory behaviour for all. Fig 20 describes the variation of tangential stress with distance x and it can be seen that it is also oscillatory and all the three curves meet each other.

9.2 Uniformly distributed thermal source

As in the above two cases for uniformly distributed thermal source it can be seen that in both Figs. 16 and 17

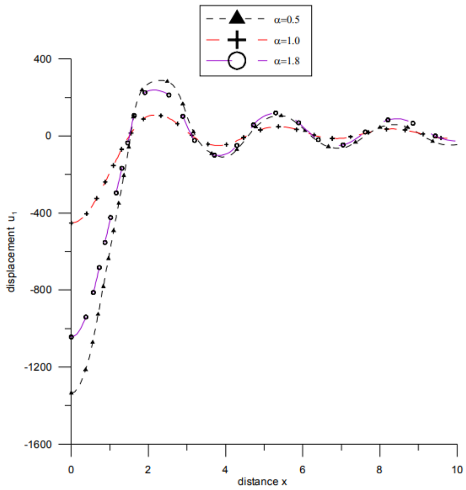

Fig. 16 Variation of tangential displacement u with distance x (uniformly distributed thermal force)

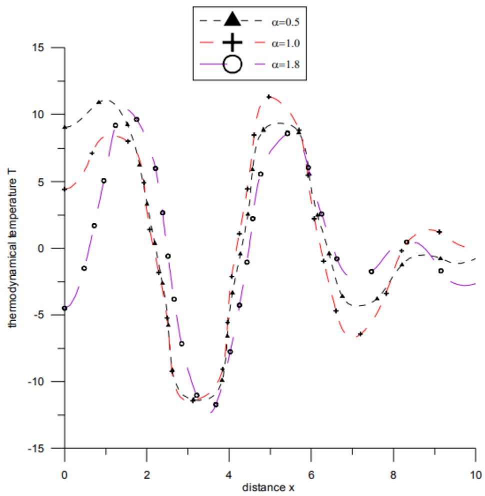

Fig. 19 Variation of Temperature change T with distance x (uniformly distributed thermal source)

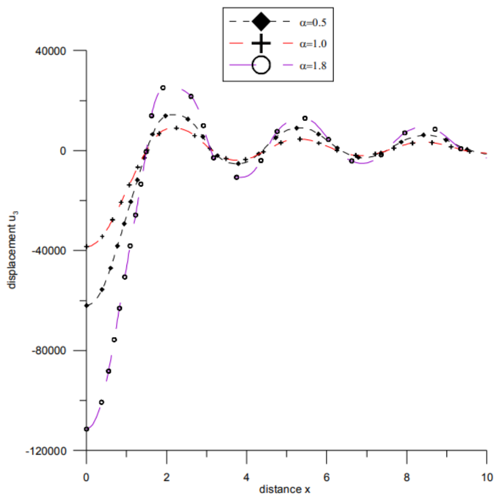

Fig. 17 Variation of normal displacement w with distance x (uniformly distributed thermal force)

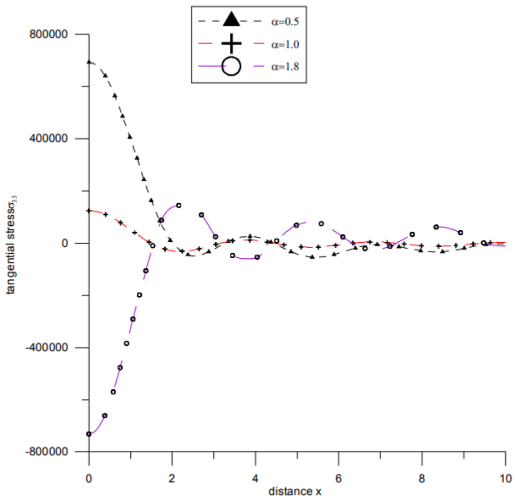

Fig. 20 Variation of tangential stress σ_{33} with distance x (uniformly distributed thermal source))

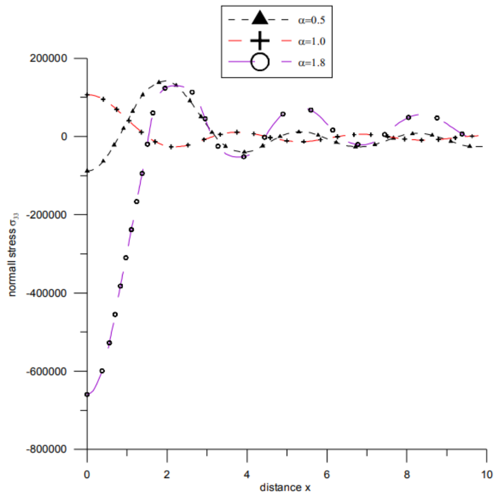

Fig. 18 Variation of normal stress σ_{33} with distance x uniformly distributed thermal source)

variation of tangential and normal displacement is same i.e., both starts from minimum to maximum value for α=0.5, 1.0, 1.8 respectively and shows an oscillatory behavior for all three cases weak, strong and normal conductivity and meet each other at some point. In Fig. 18 magnitude of normal stress first decreases then increases for α=1.8 and 1.0 also for α=0.5 at the boundary where $y = 0$ it varies from maximum to minimum and there comes a point where all the three curves intersect. Fig. 19 shows the variation of temperature change with distance$x$.

It is clear from the graph that it shows an oscillatory nature. Like in all cases Fig. 20 shows the variation of tangential stress with distance x. It can be seen that for α=0.5 it varies from maximum to minimum value and for α=1.0 it is just opposite i.e., from minimum to maximum value and shows an oscillatory nature for both. For α=1.8 behaviour is same as for α=1.0 and all the three curves intersect each other at some point.

10. Conclusions

From the above discussion, we find that change in the value of fractional parameter produces significant effect on the various components in orthotropic thermoelastic medium with and without energy dissipation in generalized thermoelasticity with three phase lag model. The problem is useful as an improvement in the field of generalized thermoelasticity. According to this theory, we have given a classification to all the materials according to its fractional parameter where this parameter becomes new indicator of its ability to conduct the thermal energy. Fractional order thermoelasticity has great importance in dynamical problems of solid mechanics and structural mechanics.

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