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LE analysis on unsaturated slope stability with introduction of nonlinearity of soil strength

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Abstract. Based on the effective stress principle, a new formula for shear strength of unsaturated soil is derived under the general nonlinear Mohr-Coulomb (M-C) strength criterion to improve the classical strength criterion of unsaturated soil. Meanwhile, the simple irrigation model under steady seepage is adopted to obtain the distribution of the matrix suction or the degree of saturation (DOS) above the groundwater table in the slope. Then, combined with the improved strength criterion of unsaturated soil and the simple irrigation model under steady seepage, the limit equilibrium (LE) solutions for the unsaturated slope stability are established according to the global LE conditions of the entire sliding body with assumption of the stresses on the slip surface. Compared to the classical strength criterion of unsaturated soil, not only the cohesion soil but also the internal friction angle is affected by the matric suction or the DOS in the improved strength criterion. Moreover, the internal friction angle related to the matric suction has the nonlinear characteristics, particularly for a small of the matric suction. Thereafter, the feasibility of the present method is verified by comparison and analysis on some slope examples. Furthermore, stability charts are also drawn to quickly analyze the unsaturated slope stability.

Keywords: unsaturated slope; DOS; nonlinear M-C strength criterion; LE analysis; stability charts

1. Introduction

Groundwater, especially due to its position in a shallow region belowground, is an important factor that induces slope instability. In recent years, climatic change has led to a rise in sea level and increased rainfall in local land areas. Vahedifard *et al.* (2016) pointed out that these changes trend directly or indirectly result in substantial and unprecedented varying the degree of saturation (DOS) within the unsaturated zone, which can lead to the failure of manufactured or natural slopes. Meanwhile, during the rainy season, the same situation also arises in mountain slopes (Zhan *et al.* 2007, Hamdhan and Schweiger 2013, Kim and Jeong 2017, Tang *et al.* 2018). Therefore, a stability analysis of an unsaturated slope has become a hot topic in the field of geotechnical engineering.

With the existence of groundwater, the slope geotechnical body above the groundwater is in an unsaturated state (Ray *et al.* 2010, Yeh *et al.* 2015, Urciuoli *et al.* 2016). If the slip surface of slope passes through the unsaturated zone, the matric suction would act on the slip surface to affect the shear strength of the soil (Uchaipichat 2010, Estabragh and Javadi 2012, Lin *et al.* 2018). Hence, for an unsaturated slope, the effects of the matric suction or the DOS on slope stability should be considered. In fact, the martic suction or the DOS affects the stability of the unsaturated slope by enhancing or weakening the strength of the unsaturated soil. Thereby, to analyze the stability of an unsaturated slope, the strength criterion of the unsaturated soil should be adopted. At present, the classical strength criterion of the unsaturated soil from Fredlund *et al.* (1978 and 1996) was obtained by modifying the linear Mohr-Coulomb (M-C) strength criterion, which did not consider the nonlinear characteristic of the strength parameters. Meanwhile, the existing classical strength criterion of the unsaturated soil has failed to reflect the influence of the DOS on the strength parameters. Thus, an important work is to describe the shear failure behavior of the unsaturated soil by incorporating the nonlinear strength criterion. Moreover, the relationship between the strength parameters of the unsaturated soil and the DOS also needs to be established for studying the stability of unsaturated slope.

Currently, with the modified M-C strength criterion for the unsaturated soil, researchers had established the limit equilibrium (LE) solutions for analyses on the stability of a unsaturated slope (Babu and Murthy 2005, Zhang *et al.* 2014, Sun *et al.* 2015). Moreover, other researchers obtained the numerical results of the unsaturated slope stability (Tsiampousi *et al.* 2013, Xiong *et al.* 2014, Wang *et al.* 2015, Qi and Vanapalli 2016, Mahmood *et al.* 2016, Liu *et al.* 2016, Cho 2016) and studied the effects of the unsaturated soil's matric suction and pore-air pressure on slope stability. Recently, Vahedifard *et al.* (2016) deduced an analytical solution for the LE analysis of the unsaturated slope stability based on the effective stress principle with the introduction of the theoretical calculation formula of the unsaturated soil. By applying the effective stress principle, the improved strength criterion of the unsaturated soil can

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be deduced, whereas Vahedifard *et al.* (2016) did not carry out this work. Meanwhile, the Vahedifard's method could not analyze stability of the unsaturated slope with the composite slip surface (or the non-logarithmic spiral slip surface) (Deng *et al.* 2019a) under the complex condition for the reason that this method is only suitable for the logarithmic spiral slip surface. In addition, the Vahedifard's method can not solve the stability of the unsaturated slope under the nonlinear strength criterion.

Whether it is an unsaturated slope or a general slope, the LE method has been widely used by engineers to analyze the slope stability (Deng *et al.* 2019b, c). For the LE method, it is established on basic of the simple theory and has also an easy-to-use formula for calculating the slope factor of safety (FOS) (Deng *et al.* 2019d). However, the traditional LE method still has shortcomings. For example, in the traditional LE method, because the results of stresses on the slip surface are not directly given, it is difficult to incorporate the nonlinear strength criterion (Morgenstern and Price 1965). As a result, the applicability of the traditional LE method to a slope stability analysis is limited. If the stresses on the slip surface can be reasonably assumed, the distribution of stresses would be obtained by solving the LE conditions before the slope FOS is calculated (Deng *et al.* 2015 and 2016). Then, with adoption of the stresses on the slip surface, it is convenient to analyze the slope stability using the nonlinear strength criterion. Moreover, with the assumption of stresses, the divided slices used in traditional LE method would be no longer needed and the global LE conditions for the entire sliding body can be established so that a rigorous solution is easily obtained. Thereby, the LE solution derivation becomes simple. Meanwhile, if the established LE solution is a linear equation from the assumption of stresses, the iterative calculation of FOS in the traditional LE method such as the Morgenstern-Price (M-P) method would be avoided and a faster calculation speed would be obtained.

In this work, based on the effective stress principle, the formula for the shear strength of an unsaturated soil is derived under the general nonlinear M-C strength criterion to improve the strength criterion of the unsaturated soil. The improved strength criterion of the unsaturated soil can not only consider the nonlinear characteristic of the strength parameters, but also reflect the influence of the DOS on the shear strength of the unsaturated soil. Thereafter, to obtain the distribution of the matrix suction or the DOS above the groundwater table in the slope, the simple irrigation model under the steady seepage is adopted. Then, combined with the improved strength criterion of the unsaturated soil and the simple irrigation model under the steady seepage, the model for stability analysis of the unsaturated slope is established on basic of the assumption of the stresses on the slip surface. In the model of slope stability based on stress analysis, the LE stability of the unsaturated slope would be solved according to the global LE conditions of the entire sliding body. By comparison and analysis on some slope examples, the feasibility of the present method is verified. Stability charts are also drawn to quickly analyze the unsaturated slope stability. Thus, these studies offer an effective and reliable method for solving the LE stability of an unsaturated slope.

2. Improved strength criterion of unsaturated soil with consideration of nonlinearity of soil strength

2.1 Derivation of calculation formula for shear strength of unsaturated soil under the general nonlinear M-C strength criterion

Under matric suction, the soil above the groundwater table is commonly under an unsaturated state. For the unsaturated soil, the shear strength of the unsaturated soil should be used to describe its failure sate. At present, the classical strength criterion of the unsaturated soil from Fredlund *et al.* (1978 and 1996) was obtained by modifying the linear M-C strength criterion. The nonlinearity of the strength parameters for the unsaturated soil was less reported. The pervious experiments have proved that the soil strength has the nonlinear characteristics (Gao *et al.* 2016, Zhang *et al.* 2019). Hence, the strength parameters of the unsaturated soil should also have the nonlinear characteristics despite the strength of the unsaturated soil is affected by the matric suction.

Fig. 1 shows the stress analysis of the soil in the unsaturated zone above the groundwater table. In Fig. 1, S is the DOS and $(u_a - u_w)$ is the martic suction. Based on the Terzaghi's effective stress principle, the normal stress (σ) of the unsaturated soil consists of the following components as (Lu and Likos 2006)

$$\sigma = \sigma' + u_a + \sigma_s \tag{1}$$

where σ' is the effective normal stress, u_a is the pore-air pressure for generally u_a = 0 kPa as the atmospheric pressure, and σ_s is the suction stress.

For the suction stress σ_s in Eq. (1), its closed-form expression from Lu *et al.* (2010) with the use of van Genuchten's (1980) soil water characteristic curve (SWCC) is

$$\sigma_s = -\frac{Se}{\alpha}\left(Se^{\frac{n}{1-n}} - 1\right)^{\frac{1}{n}} \quad (0 \le Se \le 1) \tag{2}$$

where Se is the effective DOS, α is the fitting parameter of unsaturated soil, which approximates the inverse of air-entry pressure and typically varies between 0 and 0.5 kPa^{-1}, and n is anther fitting parameter of unsaturated soil, which related to the breadth of pore size of soil with the range of 1.1 ~ 8.5.

Meanwhile, by applying the relationship of the matric suction (u_a - u_w) and the effective DOS (Se) expressed in van Genuchten's (1980) SWCC, the suction stress (σ_s) can be also calculated as

$$\sigma_s = -\frac{(u_a - u_w)}{\{1 + [\alpha(u_a - u_w)]^n\}^{\frac{n-1}{n}}} \tag{3}$$

In Eq. (2), the effective DOS has the relationship with the DOS as

$$S = Sr + Se(1 - Sr) \tag{4}$$

where Sr is the residual DOS, which refers to water content due to soil particle hydration with a relatively small value, and here Sr is assumed to be zero.

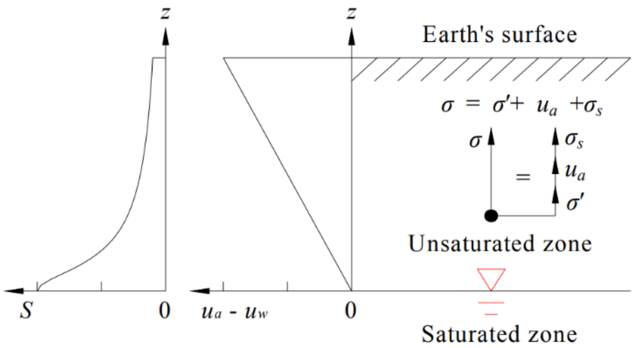


Fig. 1 Stress analysis of soil in unsaturated zone above groundwater table

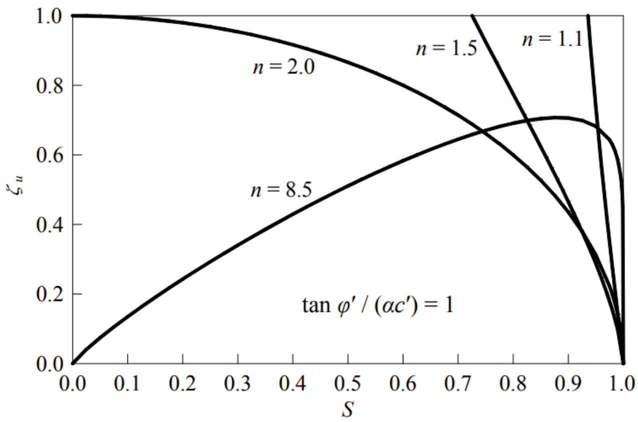


Fig. 2 Relationship of magnification coefficient of cohesion with pore-water DOS in unsaturated slope subject to the linear M-C strength criterion

Substituting Eq. (3) into Eq. (2), the following formula can be then obtained as

$$\sigma_s = -\frac{1}{\alpha}\frac{S-S_r}{1-S_r}\left[\left(\frac{S-S_r}{1-S_r}\right)^{\frac{n}{1-n}} - 1\right]^{\frac{1}{n}} \tag{4}$$

According to Eq. (1), the effective normal stress can be obtained. Then, based on the stress state of the unsaturated soil, the shear strength of the unsaturated soil would be got. For the soil, its destruction is usually due to the shear failure and is subject to the M-C strength criterion. Furthermore, since the shear strength of the soil generally exhibits the nonlinear characteristics and the linear strength criterion is a special case of the general nonlinear strength criterion, the general nonlinear M-C strength criterion is used here to describe the failure behavior of the soil. Thereby, using the effective normal stress (Lu and Godt 2013), the shear strength (τ_f) of the soil within the general nonlinear M-C strength criterion can be calculated as

$$\tau_f = c_0(1+\frac{\sigma'}{\sigma_t})^{\frac{1}{m}} \tag{5}$$

where c_0 is the initial cohesion with $c_0 \geq 0$,σ_t is the uniaxial tensile strength with $\sigma_t \geq 0$, and m is the non-linear parameter with $m \geq 1$.

For $m = 1$ in Eq. (5), it represents the linear M-C strength criterion, and then its simplified expression is given as $\tau_f = c' + \sigma'\tan\varphi'$, where $c' = c_0$, $\varphi' = \arctan(c_0 / \sigma_t)$, c' is the effective cohesion, and φ' is the effective internal friction angle.

Substituting Eq. (3) into Eq. (5), the formulas for the shear strength of unsaturated soil can be obtained as

$$\tau_f = c + \sigma\tan\varphi \tag{6}$$

where c is the cohesion of the unsaturated soil and φ is the internal friction angle of the unsaturated soil.

In Eq. (6), c and φ is calculated by

$$c = c_0\left(1+\frac{\sigma}{\sigma_t}+\zeta_u\right)^{\frac{1-m}{m}} \times \left[1+(1-\frac{1}{m})\frac{\sigma}{\sigma_t}+\zeta_u\right] \tag{7a}$$

$$\varphi = \arctan\left[\frac{c_0}{m\sigma_t}\left(1+\frac{\sigma}{\sigma_t}+\zeta_u\right)^{\frac{1-m}{m}}\right] \tag{7b}$$

where ζ_u is a dimensionless variable (or called the magnification coefficient) to reflect the effect of the matric suction (u_a - u_w) on the strength parameters of the unsaturated soil.

For the variable ζ_u in Eq. (7), it is calculated as

$$\zeta_u = -\frac{u_a}{\sigma_t} + \frac{(u_a - u_w)}{\sigma_t\{1+[\alpha(u_a-u_w)]^n\}^{(n-1)/n}} \tag{8}$$

In addition, substituting Eqs. (2) and (4) into Eq. (5), the same formulas for the shear of unsaturated soil as Eq. (6) can be also obtained. Moreover, the variable ζ_u would reflect the effect of the DOS on the strength parameters of the unsaturated soil with the given following formulas (i.e., Eq. (9)).

When $n \geq 1.1$ and $n < 2$, ζ_u is calculated as

$$\zeta_u = \frac{1}{\alpha\sigma_t}\left[\left(\frac{1-S_r}{S-S_r}\right)^{\frac{n(2-n)}{n-1}} - \left(\frac{S-S_r}{1-S_r}\right)^n\right]^{\frac{1}{n}} - \frac{u_a}{\sigma_t} \tag{9a}$$

When $n = 2$, ζ_u is calculated as

$$\zeta_u = \frac{1}{\alpha\sigma_t}\sqrt{1-\left(\frac{S-S_r}{1-S_r}\right)^2} - \frac{u_a}{\sigma_t} \tag{9b}$$

When $n > 2$, ζ_u is calculated as

$$\zeta_u = -\frac{u_a}{\sigma_t} + \frac{1}{\alpha\sigma_t}\left[\left(\frac{S-S_r}{1-S_r}\right)^{\frac{n(n-2)}{n-1}} - \left(\frac{S-S_r}{1-S_r}\right)^n\right]^{\frac{1}{n}} \tag{9c}$$

From Eq. 9(a), it can be seen that when $n \geq 1.1$ and $n < 2$, ζ_u would tends to infinity if $S = Sr$. According to Eq. (9c), the obtained ζ_u is the same for two cases of $S = Sr$ and $S = 1$ when $n > 2$.

Because the matric suction (u_a - u_w) has a functional relationship with the DOS (S), the calculated variable ζ_u

from Eq. (8) is equivalent to that from Eq. (9). Furthermore, if the matric stress (u_a - u_w) or the DOS is got, the strength parameters of unsaturated soil could be subsequently calculated, and then the shear strength of unsaturated soil would be obtained.

In Eqs. (7a-b) for m =1 (i.e., the linear M-C strength criterion), Eq. (7a) is simplified as $c = c'(1 + \zeta_u)$ and Eq. (7b) is simplified as $\varphi = \varphi'$. In other words, under the linear strength criterion, the matric suction or the DOS only affects the soil cohesion. In fact, the classical modified M-C strength criterion for the unsaturated soil also considers that the soil internal friction angle under the linear M-C strength criterion is not affected by the martic suction or the DOS. Then, selecting $m = 1$, $u_a = 0$ kPa, $Sr = 0$, and $[\tan\varphi' / (\alpha c')] = 1$ in Eq. (9), the curves of S versus ζ_u for n = 1.1, 1.5, 2.0, and 8.5 are drawn in Fig. 2, which explains that the cohesion of the unsaturated soil varies by a change in the DOS. For $n \le 2$, the cohesion of the unsaturated soil decreases with an increase in S, but for $2 < n \le 8.5$, it first increases and subsequently decreases.

2.2 Discussion from comparison with the classical modified M-C strength criterion for unsaturated soil

In the mechanical analysis of the unsaturated soil, the classical modified M-C strength criterion for unsaturated soil from Fredlund *et al.* (1978 and 1996) is widely used, and its formula is

$$\tau_f = c' + (\sigma - u_a)\tan\varphi' + (u_a - u_w)\tan\varphi^{\mathrm{b}} \tag{10}$$

where φ^{b} is the friction angle with respect to changes in (u_a - u_w) when (σ - u_a) is held constant.

Then, Eq. (10) is converted into

$$\tau = c'\left(1 - \frac{u_a}{c'}\tan\varphi' + \frac{u_a - u_w}{c'}\tan\varphi^{\mathrm{b}}\right) + \sigma\tan\varphi' \tag{11}$$

By comparing Eq. (11) with Eqs. (6) and (7) under the linear M-C strength criterion (i.e., m = 1), the calculation formulas for $\tan\varphi^{\mathrm{b}}$ in the improved strength criterion of the unsaturated criterion would be obtained as

$$\tan\varphi^{\mathrm{b}} = \frac{1}{\{1 + [\alpha(u_a - u_w)]^n\}^{(n-1)/n}}\tan\varphi' \tag{12}$$

From Eq. (12), it can be seen that the internal friction angle φ^{b} in the improved strength criterion of the unsaturated criterion is a function of the matric suction (u_a - u_w), which is different from the assumption that φ^{b} is a constant in the classical modified M-C strength criterion for unsaturated soil. Here, let $\lambda_u = 1 / \{1 + [\alpha(u_a - u_w)]^n\}^{(n-1)/n}$, and the curves of λ_u vs. (u_a - u_w) are drawn in Fig. 3. Fig. 3 shows that λ_u decreases with the increase of (u_a - u_w). Moreover, λ_u is significantly affected by (u_a - u_w) when (u_a - u_w) < 40 kPa, while the curve of λ_u vs. (u_a - u_w) tends to be gentle and λ_u approaches the constant when (u_a - u_w) > 40kPa. Thus, compared to the classical modified M-C strength criterion for the unsaturated soil, the improved strength criterion of the unsaturated soil can reflect the nonlinear characteristics of φ^{b} under the small value of (u_a - u_w). Additionally, $\lambda_u \le 1$ in Fig. 3, which is to say that $\varphi^{\mathrm{b}} \le \varphi'$. Meanwhile, λ_u is close to zero when n = 8.5, and λ_u = 1 when α = 0.

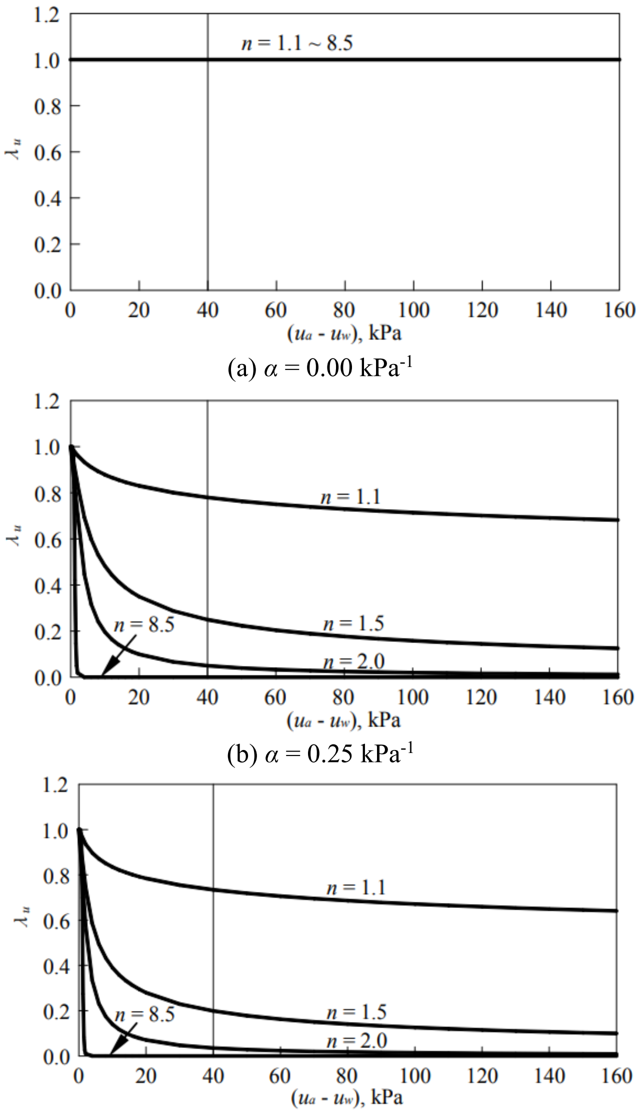


Fig. 3 Curve of λ_u versus (u_a - u_w) under the linear M-C strength criterion

2.3 Calculation of matric stress or DOS in unsaturated soil

As shown in Fig. 4 under the steady seepage, the groundwater moves vertically downward due to the simultaneous action of the gravity and the matric suction. Then, Yeh (1989) addressed a simple infiltration model, which is also introduced by Lu and Likos (2004) to analyze the main vertical flow of the pore water under its gravity and matrix suction. In Yeh's model, the matric suction (u_a - u_w) can be given as

$$u_a - u_w = -\frac{1}{\alpha}\ln\left[\left(1 + \frac{q}{ks}\right)e^{-\alpha\gamma_w(z-f)} - \frac{q}{ks}\right] \tag{13}$$

where γ_w is the unit weight of water with γ_w = 10 kN / m^3, q is the vertical specific discharge, ks is the saturated

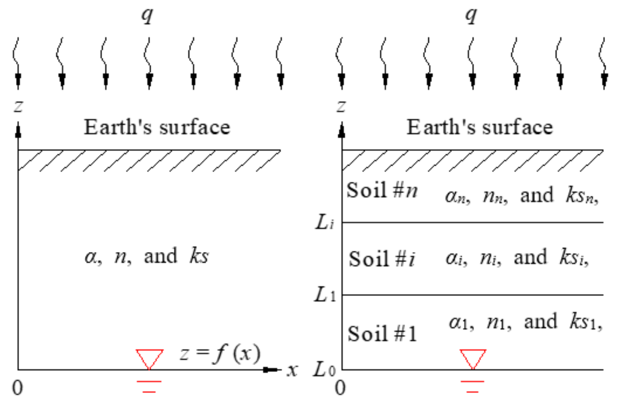


Fig. 4 Simple irrigation model for calculation of matric suction or DOS above groundwater table

hydraulic conductivity, $(z - f)$ represents the vertical distance of the calculated point from the groundwater table, z is the z-axis coordinate of the calculated point above the groundwater table, and f is the equation of the groundwater table.

From the Yeh's infiltration model of Eq. (13), the effective DOS (S_e) can be solved by combing with Eqs. (2), (3) and (13), where the intermediate variable (i.e., σ_s) is calculated by substituting Eq. (13) into Eq. (3). Then, the effective DOS (S_e) is expressed as

$$Se=\left\{\left[\frac{-\ln[(1+\frac{q}{ks})e^{-\alpha\gamma_w(z-f)}-\frac{q}{ks}]}{S_e\left(1+\{-\ln[(1+\frac{q}{ks})e^{-\alpha\gamma_w(z-f)}-\frac{q}{ks}]\}^n\right)^{\frac{n-1}{n}}}\right]^n+1\right\}^{\frac{1-n}{n}} \quad (14)$$

Eq. (14) is an implicit formula for calculating the effective DOS. Therefore, the loop iteration of Se is needed to solve it. Thereupon, the initial effective DOS (i.e., $Se^{(0)}$) is given to replace Se in the right side of Eq. (14) and here $Se^{(0)} = 0.5$. Then, a new Se can be obtained from Eq. (14). If $|Se - Se^{(0)}| \leqq \varepsilon_s$ (here $\varepsilon_s = 0.001$), the obtained value would be the final results. Otherwise, let $Se^{(0)} = Se$, and S_e is recalculated. Thereafter, substituting the effective DOS solved from Eq. (14) into Eq. (4), the DOS would be obtained.

When the geotechnical body above the groundwater table is composed of the multi-layer soils, the Yeh's infiltration model can also be used to solve the matric suction or the DOS of the unsaturated soil at different positions in the multi-layer soils. For the geotechnical body composed of the multi-layer soils above the groundwater table, the calculation of matric stress should satisfy the condition of the continuous suction head at the interface of two adjacent layers. Thus, the calculation formula of the matric suction $(u_a - u_w)_i$ for one point in i-th layer soil above the groundwater table is

$$(u_a-u_w)_i=-\frac{1}{\alpha_i}\ln\left[\frac{f_i}{ks_i}e^{-\alpha_i\gamma_w(z-f)}-\frac{q}{ks_i}\right] \quad (15)$$

where f_i ($i \geq 1$ and $i \leq n$) is the parameter solved by the boundary condition with the given following formulas, α_i is the unsaturated soil parameter in the i-th layer, and ks_i is the saturated hydraulic conductivity of the soil in the i-th layer.

When $i = 1$, f_i is calculated as

$$f_1=q+ks_1 \quad (16a)$$

When $i \geq 2$, f_i is calculated as

$$f_i=\left[q+ks_i\left(-\frac{q}{ks_{i-1}}+\frac{f_{i-1}}{ks_{i-1}}e^{-\alpha_{i-1}\gamma_w L_{i-1}}\right)^{\alpha_i/\alpha_{i-1}}\right]e^{\alpha_i\gamma_w L_{i-1}} \quad (16b)$$

where L_{i-1} is the vertical distance of the boundary line between i-th layer and $(i-1)$-th layer from the groundwater table, and $L_0 = 0$ when $i = 1$.

Thereafter, the matric suction $(u_a - u_w)_i$ obtained by Eq. (15) is substituted into Eq. (3) to get the suction stress. Then, substituting the got suction stress into Eq. (2), the calculation formula of the effective DOS (Se_i) for one point in i-th layer soil above the groundwater table is derived as

$$Se_i=\left\{\left[\frac{-\ln[\frac{f_i}{ks_i}e^{-\alpha_i\gamma_w(z-f)}-\frac{q}{ks_i}]}{S_e\left(1+\{-\ln[\frac{f_i}{ks_i}e^{-\alpha_i\gamma_w(z-f)}-\frac{q}{ks_i}]\}^{n_i}\right)^{\frac{n_i-1}{n_i}}}\right]^{n_i}+1\right\}^{\frac{1-n_i}{n_i}} \quad (17)$$

where n_i is the unsaturated soil parameter in the i-th layer.

Eq. (17) is also an implicit equation, and it is solved in the same way as Eq. (14).

3. LE stability analysis on unsaturated slope

Fig. 5 shows the stability analysis model of an unsaturated slope with slope height H. In Fig. 1, A and B are the lower and upper sliding points of the slip surface, respectively. Establishing the xz axis coordinate system with the slope toe as the origin, the equations for the slope surface, slip surface, and groundwater table are given as $z = g(x)$, $z = s(x)$, and $z = f(x)$, respectively. If a vertical tensile crack appears on the slope top, the slope would slide along the curve AE, where E is the point on the lower edge of the vertical tensile. The crack has a horizontal distance l_w from the slope vertex and the depth z_w.

As the mentioned above, with the given unsaturated soil parameters (α, n, q / ks, and u_a), the strength parameters (by Eq. (7)) or the shear strength (by Eq. (6)) of unsaturated soil at one point above the groundwater table in the slope can be calculated under the known normal stress. Furthermore, after obtaining the shear strength of the unsaturated soil on the slip surface, the calculation formula of the FOS for the unsaturated slope can be established on basic of the LE analysis on the slope sliding body.

In the LE calculation methods, if the normal stress on slip surface could be known, solving a stability problem would be then straightforward. Meanwhile, the nonlinear strength criterion expressed with the normal stress is also easy to be used for derivation of the LE equations. Fredlund *et al.* (1999) had the suggestion of taking the normal stress

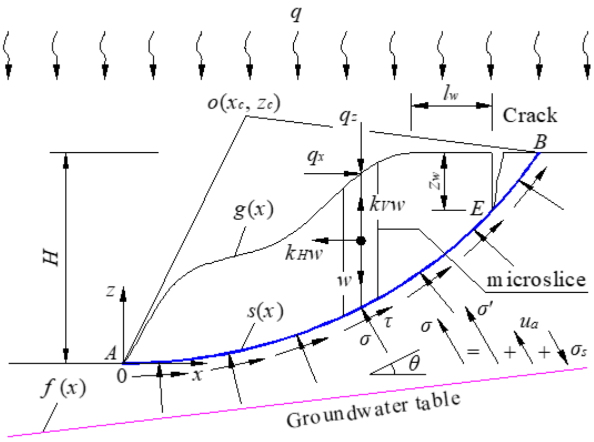


Fig. 5 Stability analysis model of an unsaturated slope

from the finite element (FE) and using it in LE. However, it is still a challenge to calculate the normal stress within the LE frame and then analyze the slope stability. Here, a good approach is adopted to assume the normal stress on the slip surface by linearly amending the initial normal stress, which is obtained from the traditional LE methods. Meanwhile, the variables used in the assumption of the normal stress would be solved according to the following global mechanical equilibrium conditions of the entire sliding body. Thereby, the above work would ensure that the calculated normal stress is in a reasonable range.

To obtain the formula of the initial normal stress on slip surface, the vertical micro-slice with a width dx in the sliding body is selected to analyze the forces acting on it. As shown in Fig. 5, the external forces acting on the vertical microslice include wdx (the gravity force), k_Hwdx (the vertical seismic force), k_Vwdx (the horizontal seismic force), q_xdx (the external load along x-direction on slope surface), q_zdx (the external load along z-direction on slope surface), $\sigma dx/\cos\theta$ (the normal force on slip surface), and $\tau dx/\cos\theta$ (the shear force on slip surface), where k_V and k_H are the vertical and horizontal seismic force coefficients, respectively, σ and τ are the normal and shear stress on slip surface, respectively, and θ is the horizontal inclination angle of tangent to the slip surface of micro-slice. Among the traditional LE methods, the simplest formula of the normal stress on slip surface is obtained from the Swedish method, which gets the explicit solution of the normal stress by neglecting the increment of inter-slice forces. Thus, the normal stress on slip surface is assumed on basic of the initial normal stress form the Swedish method's solution, and it is calculated as

$$\sigma = \lambda_1 \sigma_0 \tag{18a}$$

$$\sigma_0 = \frac{[(1-k_V)w + q_z] - (k_H w - q_x)s_x}{1 + s_x^2} \tag{18b}$$

where σ_0 is the initial normal stress, obtained from the force equilibrium conditions of all forces on the vertical micro-slice used in the Swedish method, λ_1 is a dimensionless variable to amend the initial normal stress, s_x is the first derivative of the slip surface equation $s(x)$ with respect to the x-axis, and $s_x = \tan\theta$.

It is well known that compared to the LE M-P method and others traditional LE methods, the Swedish method gets the non-rigorous solution on the slope stability as it neglects the increment of inter-slice forces on the stresses on slip surface despite satisfying all the mechanical equilibrium conditions. According to Eq. (18a), the calculation of the normal stress in the present method indirectly considers the effect of the inter-slice forces with the introduction of the variable λ_1. Thus, the obtained normal stress in the present method is more reasonable than that of the Swedish method. Meanwhile, by comparison and analysis on some examples with the traditional LE methods, such as the M-P method, the research from Deng *et al.* (2015 and 2016) had verified the rationality of the normal stress expressed by Eq. (18a). Moreover, compared with the traditional LE methods, an advantage for the present method is to solve slope stability based on the global LE conditions without the use of the slices. Furthermore, it is also easier to realize slope stability with the nonlinear strength criterion in the present method than the traditional LE methods.

For the stresses on slip surface, except for the normal stress, the shear stress also needs to be determined. One fact is that the slope FOS is defined as the ratio of the resisting force on the slip surface to the sliding force when the slope is in the LE state. Thus, for a vertical micro-slice, the FOS can be further simplified as the ratio of the shear strength on the slip surface to the shear stress. Thereby, using Eq. (6), the shear stress (τ) on the slip surface can be calculated as

$$\tau = \frac{c}{F_s} + \sigma \frac{\tan\varphi}{F_s} \tag{19}$$

where F_s is the slope FOS.

Eq. (19) shows that the shear stress is a function of the normal stress on the slip surface. Furthermore, according to the formulas by substituting Eq. (18a) into Eq. (19), it can be known that the shear stress is related to the initial normal stress. Then, with substitution of Eq. (7a) into Eq. (7b), the shear stress can be assumed as

$$\tau = \lambda_2 \tau_{01} + \lambda_3 \tau_{02} \tag{20a}$$

$$\tau_{01} = c_0 \left(1 + \lambda_1 \frac{\sigma_0}{\sigma_t} + \zeta_u\right)^{\frac{1-m}{m}} \times \left[1 + \lambda_1 (1 - \frac{1}{m}) \frac{\sigma_0}{\sigma_t} + \zeta_u\right] \tag{20b}$$

$$\tau_{02} = \frac{c_0}{m\sigma_t} \left(1 + \lambda_1 \frac{\sigma_0}{\sigma_t} + \zeta_u\right)^{\frac{1-m}{m}} \sigma_0 \tag{20c}$$

where λ_2 and λ_3 are both the dimensionless variables to amend the shear stress.

After the abovementioned stresses (σ and τ) on the slip surface are assumed, the stability of the unsaturated soil slope can be solved according to the global LE conditions of the entire sliding body. Then, the force equilibrium conditions in the x- and z-directions, and the moment equilibrium condition of all forces about the point O with the coordinates (x_c, z_c) in a sliding body can be respectively determined as

$$\int(-\sigma s_x+\tau-k_H w+q_x)dx=0 \tag{21a}$$

$$\int[\sigma+\tau s_x-(1-k_V)w-q_z]dx=0 \tag{21b}$$

$$\begin{aligned}&\int[(-\sigma s_x+\tau)(z_c-s)+(\sigma+\tau s_x)(x-x_c)]dx-\\&\int[(1-k_V)w+q_z](x-x_c)dx+\\&\int\left\{k_H w[z_c-\frac{1}{2}(f+g)]-q_x(z_c-g)\right\}dx=0\end{aligned} \tag{21c}$$

Substituting Eqs. (21a-b) into Eq. (21c) and simplifying, the formula can be obtained as

$$\begin{aligned}&\int[\sigma(ss_x+x)+\tau(xs_x-s)]dx-\\&\int\left\{[(1-k_V)w+q_z]x-\frac{1}{2}k_H w(f+g)+q_x g\right\}dx=0\end{aligned} \tag{22}$$

Eq. (22) explains that the position of the moment center point O of the sliding body has no effect on the establishment of the LE equations.

Substituting Eqs. (18a) and (20a) into Eqs. (21a-b) and (22), respectively, the following linear equations for variables λ_1, λ_2, and λ_3 can be obtained as

$$\sum_{j=1}^{3}a_{ij}\lambda_j=b_i\quad(i=1,2,3) \tag{23a}$$

$$a_{11}=-\int\sigma_0 s_x dx \tag{23b}$$

$$a_{12}=\int\tau_{01}dx \tag{23c}$$

$$a_{13}=\int\tau_{02}dx \tag{23d}$$

$$b_1=\int(k_H w-q_x)dx \tag{23e}$$

$$a_{21}=\int\sigma_0 dx \tag{23f}$$

$$a_{22}=\int\tau_{01}s_x dx \tag{23g}$$

$$a_{23}=\int\tau_{02}s_x dx \tag{23h}$$

$$b_2=\int[(1-k_V)w+q_z]dx \tag{23i}$$

$$a_{31}=\int\sigma_0(ss_x+x)dx \tag{23j}$$

$$a_{32}=\int\tau_{01}(xs_x-s)dx \tag{23k}$$

$$a_{33}=\int\tau_{02}(xs_x-s)dx \tag{23l}$$

$$\begin{aligned}b_3=&\int[(1-k_V)w+q_z]xdx-\\&\int\left\{\frac{1}{2}k_H w(f+g)+q_x g\right\}dx\end{aligned} \tag{23m}$$

Solving Eq. (23a), the variables (λ_1, λ_2, and λ_3) are obtained. Then, substituting these variables into Eqs. (18a) and (20a), the stresses (σ and τ) on the slip surface are calculated. However, since λ_1 is included in the formulas of τ_{01} and τ_{02}, it is necessary to determine λ_1 before the parameters a_{ij} containing two components τ_{01} and τ_{02} in Eqs. (23c, d, g, h, k and l) could be calculated. Thereby, the loop iteration calculation of the variable λ_1 is required to solve Eq. (23a).

According to the above analysis, an initial value of λ_1 is given as $\lambda_1^{(0)}$, and $\lambda_1^{(0)}=1$ for the first time. Thereafter, with substitution of $\lambda_1^{(0)}$, τ_{01} (Eq. (20b)) and τ_{02} (Eq. (20c)) are got, and later a_{ij} in Eqs. (23b-m) are calculated. Then, solving Eq. (23a), λ_2, λ_3, and a new λ_1 would be obtained. If $|\lambda_1-\lambda_1^{(0)}|\leqq\varepsilon$ (here $\varepsilon=0.001$), the obtained values for λ_1, λ_2, and λ_3 would be the final results. Otherwise, let $\lambda_1^{(0)}=\lambda_1$, and $\lambda_1\sim\lambda_3$ are recalculated.

Different from Deng *et al.* (2015 and 2016), which used the Taylor series to expand the nonlinear strength criterion and approximately linearizes the nonlinear strength criterion to solve the slope stability, this work analyzes the slope stability under the nonlinear strength criterion by applying the iterative calculation of the variable λ_1 so the results of the present method is more rigorous.

After the stress solutions are obtained, the strength parameters of unsaturated soil (c and φ) would be calculated using Eqs. (7a-b). Furthermore, the slope FOS can be solved according to its definition. As stated above, the FOS of a slope can be solved using the ratio of the total resisting force along the entire slip surface to the total sliding force, and its formula is given as

$$F_s=\frac{\int[(c+\lambda_1\sigma_0\tan\varphi)/\cos\theta]dx}{\int[(\lambda_2\tau_{01}+\lambda_3\tau_{02})/\cos\theta]dx} \tag{21}$$

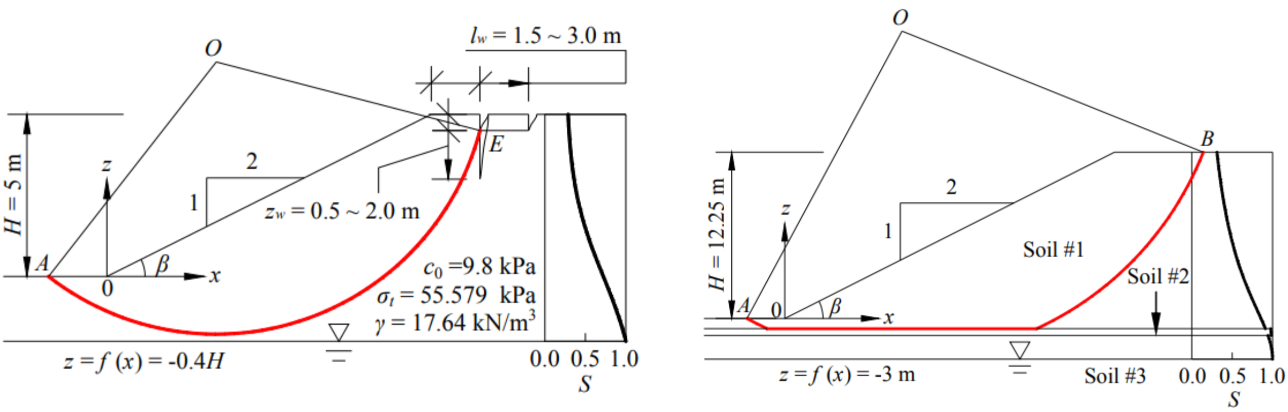
4. Comparison and analysis on slope examples

Slope example 1: a homogeneous slope, as an example from Vahedifard *et al.* (2016), is given with a slope height H and slope angle β. This slope subject to the linear M-C strength criterion has the natural soil unit weight (γ) and the strength parameters c' and φ'. The groundwater table is near the slope toe with an equation of $f(x)=0$ m, which is obtained by establishing the xz coordinate system with the slope toe as the origin. When the soil is above the groundwater table, the unsaturated soil parameters are α, n, q/k_s, and u_a, where $q/k_s=0$ and $u_a=0$ kPa. Then, based on different combinations of unsaturated soil parameters, 25 cases shown in Table 1 are adopted. In these 25 cases, when the minimum slope FOS is given to be 1.000, Vahedifard *et al.* (2016) calculated the slope stability number N ($N=c'/(\gamma H)$). In this work, using the parameters from the example of Vahedifard *et al.* (2016), the slope stability is again analyzed with the present method to solve the minimum slope FOS, and the results are listed in Table 1.

Table 1 shows that the difference of the results between the Vahedifard's method and the present method is less than 14%, thus illustrating the feasibility of the present method.

Table 1 Calculated results in slope example 1

Case	Slope height	Slope angle	Unit weight of soil	Strength parameters of soil subject to the linear M-C strength criterion		Parameters of unsaturated soil				Calculated results		
										Vahedifard *et al.* (2016)	Present method	
	H (m)	β (°)	γ (kN/m^3)	c' (kPa)	φ' (°)	α (kPa^{-1})	n	u_a (kPa)	q/k_s	Logarithmic spiral slip surface	Circular slip surface	Non-circular slip surface
1	10	90	18	6.565	30	0.0500	1.1	0	0	1.000	1.125	1.114
2	10	90	18	5.644	30	0.0250	1.1	0	0	1.000	1.131	1.131
3	10	90	18	5.384	30	0.0200	1.1	0	0	1.000	1.133	1.134
4	10	90	18	4.916	30	0.0125	1.1	0	0	1.000	1.136	1.140
5	10	90	18	4.746	30	0.0100	1.1	0	0	1.000	1.138	1.141
6	10	90	18	18.652	30	0.0500	2.0	0	0	1.000	1.047	1.057
7	10	90	18	13.628	30	0.0250	2.0	0	0	1.000	1.073	1.075
8	10	90	18	11.895	30	0.0200	2.0	0	0	1.000	1.083	1.090
9	10	90	18	8.569	30	0.0125	2.0	0	0	1.000	1.105	1.113
10	10	90	18	7.327	30	0.0100	2.0	0	0	1.000	1.115	1.123
11	10	90	18	21.383	30	0.0500	2.5	0	0	1.000	1.034	1.033
12	10	90	18	15.764	30	0.0250	2.5	0	0	1.000	1.058	1.064
13	10	90	18	13.515	30	0.0200	2.5	0	0	1.000	1.069	1.081
14	10	90	18	9.118	30	0.0125	2.5	0	0	1.000	1.099	1.119
15	10	90	18	7.475	30	0.0100	2.5	0	0	1.000	1.112	1.131
16	10	90	18	24.397	30	0.0500	4.0	0	0	1.000	1.021	1.026
17	10	90	18	19.155	30	0.0250	4.0	0	0	1.000	1.036	1.058
18	10	90	18	16.325	30	0.0200	4.0	0	0	1.000	1.048	1.079
19	10	90	18	9.620	30	0.0125	4.0	0	0	1.000	1.090	1.128
20	10	90	18	7.051	30	0.0100	4.0	0	0	1.000	1.111	1.157
21	10	90	18	25.492	30	0.0500	8.5	0	0	1.000	1.018	1.025
22	10	90	18	21.619	30	0.0250	8.5	0	0	1.000	1.023	1.056
23	10	90	18	18.965	30	0.0200	8.5	0	0	1.000	1.031	1.072
24	10	90	18	9.802	30	0.0125	8.5	0	0	1.000	1.078	1.171
25	10	90	18	5.781	30	0.0100	8.5	0	0	1.000	1.117	1.221



(a) Slope example 2 (b) Slope example 3

Fig. 6 Unsaturated slope examples

The main reason for the above difference is from the adopted slip surface in the two methods. Only the logarithmic spiral slip surface could be used for the Vahedifard's method. However, the present method is established without requirement on the type of slip surface. In Table 2, the circular and non-circular (Deng *et al.* 2017) results of the present method are listed.

Duncan *et al.* (2014) stated that the logarithmic spiral is

Table 2 Calculated results in slope example 2

Case	Slope height	Slope angle	Unit weight of soil	Strength parameters of soil subject to the general nonlinear M-C strength criterion			Parameters of unsaturated soil					Position of vertical tensile crack		Calculated results	
	H (m)	β (°)	γ (kN/m^3)	c_0 (kPa)	σ_t (kPa)	m	α (kPa^{-1})	n	u_a (kPa)	q/ks	Sr	l_w (m)	z_w (m)	M-P method	Present method
1	5	26.565	17.64	9.8	55.579	1.0	0.05	2	0	0	0	1.5	0.5	1.523	1.513
2	5	26.565	17.64	9.8	55.579	1.0	0.05	2	0	0	0	1.5	1.0	1.502	1.488
3	5	26.565	17.64	9.8	55.579	1.5	0.05	2	0	0	0	1.5	0.5	—	1.198
4	5	26.565	17.64	9.8	55.579	1.5	0.05	2	0	0	0	1.5	1.0	—	1.173
5	5	26.565	17.64	9.8	55.579	2.0	0.05	2	0	0	0	1.5	0.5	—	1.066
6	5	26.565	17.64	9.8	55.579	2.0	0.05	2	0	0	0	1.5	1.0	—	1.042
7	5	26.565	17.64	9.8	55.579	1.0	0.05	2	0	0	0	3.0	0.5	1.487	1.476
8	5	26.565	17.64	9.8	55.579	1.0	0.05	2	0	0	0	3.0	1.0	1.476	1.373
9	5	26.565	17.64	9.8	55.579	1.5	0.05	2	0	0	0	3.0	0.5	—	1.152
10	5	26.565	17.64	9.8	55.579	1.5	0.05	2	0	0	0	3.0	1.0	—	1.134
11	5	26.565	17.64	9.8	55.579	2.0	0.05	2	0	0	0	3.0	0.5	—	1.017
12	5	26.565	17.64	9.8	55.579	2.0	0.05	2	0	0	0	3.0	1.0	—	1.000

Table 3 Parameters in slope example 3

Soil	Unit weight of soil	Strength parameters of soil subject to the linear M-C strength criterion		Strength parameters of soil subject to the general nonlinear M-C strength criterion			Parameters of unsaturated soil					Vertical distance from water table
	γ (kN/m^3)	c' (kPa)	φ' (°)	c_0 (kPa)	σ_t (kPa)	m	α (kPa^{-1})	n	u_a (kPa)	q/ks	Sr	L (m)
#1	18.84	—	—	28.5	78.303	m_1	0.02	2	0	0	0	0.00
#2	18.84	0	10	—	—	—	0.01	2	0	0	0	1.75
#3	18.84	—	—	28.5	78.303	m_3	0.02	2	0	0	0	2.25

Table 4 Calculated results in slope example 3

Case	Slope height	Slope angle	Nonlinear parameter for soil		Calculated results	
	H (m)	β (°)	m_1	m_3	M-P method	Present method
1	12.25	26.565	1.0	1.0	1.516	1.544
2	12.25	26.565	1.2	1.2	—	1.389
3	12.25	26.565	1.4	1.4	—	1.290
4	12.25	26.565	1.6	1.6	—	1.219
5	12.25	26.565	1.8	1.8	—	1.166
6	12.25	26.565	2.0	2.0	—	1.125

the most critical failure surface for the homogenous slope. Moreover, with the use of a logarithmic spiral slip surface in the LE method (e.g., Vahedifard *et al.* 2016), the stresses related to the normal stress would produce no moment about the center of the spiral so that no additional assumptions are required to establish the moment equilibrium for solving the unique variable (i.e., FOS). Meanwhile, despite only the moment equilibrium is adopted to solve the slope stability in the logarithmic spiral method, the force equilibrium is also satisfied implicitly (Baker *et al.* 1983, Zhang *et al.* 2016). However, the Vahedifard's method is not suitable to be used with other slip surfaces (e.g., the circular or arbitrary curved slip surface) for the reason that it is useful for only the logarithmic spiral slip surface to analyze slope stability without the requirement of solving the stresses related to the normal stress. In fact, the slip surface would exhibit the non-logarithmic spiral characteristic in the actual slope under the complex conditions, such as the composite slip surface with the interface of weak layer (slope example 3). In addition, without the need of solving the normal stress on the logarithmic spiral slip surface, the Vahedifard's method is difficult to be used to analyze slope stability with application of the nonlinear strength criterion expressed by the normal stress (slope examples 2 and 3). Compared to the Vahedifard's method, the present method can be applied with the circular or non-circular slip surface (including the logarithmic spiral) and can also be used to solve the slope

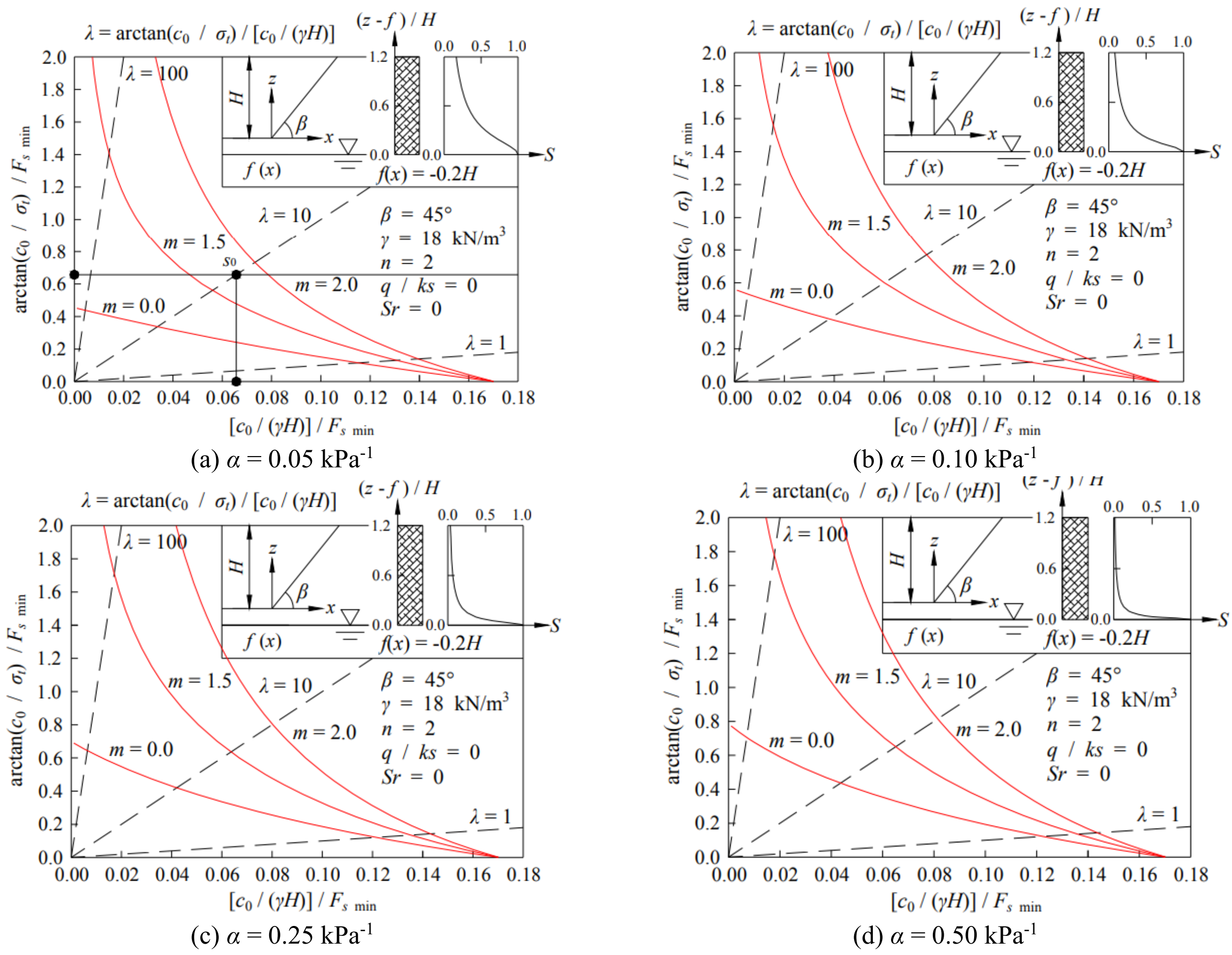


Fig. 7 Stability charts of unsaturated slope

stability under the nonlinear strength criterion.

Slope example 2: another homogeneous slope is given by Rocscience Inc. (Rocscience Inc. 2003). The slope parameters (H and β), the strength parameters of soil, and the unsaturated soil parameters are listed in Table 2. The groundwater table has an equation of $f(x)$ = -0.4H with the slope toe as the origin in Fig. 6(a). Here, the vertical tensile crack is considered on the slope top with a horizontal distance l_w from the slope vertex and the depth z_w. Then, selecting different parameters related to the position of the crack, the stability of the unsaturated slope under the general nonlinear M-C strength criterion is analyzed, and the results are listed in Table 2. In Table 2, the results of the present method are compared with that of the traditional LE M-P method when m = 1.0 (i.e., the slope subject to the linear M-C strength criterion). In addition, the curve of the DOS distribution along the z-axis was drawn in the slope and the critical circular slip surface was also plotted for the slope when l_w = 1.5 m, h_w = 0.5 m, and m = 1.0 in Fig. 6(a).

Slope example 3: a slope with a weak interlayer is also provided by Rocscience Inc. (Rocscience Inc. 2003). In this slope, the weak interlayer is located below the slope toe with the vertical distance of 0.75 m, and its thickness is 0.5 m. The slope parameters (H and β), the strength parameters of the soil, and the unsaturated soil parameters are listed in Table 3. The groundwater table has an equation of $f(x)$ = -3 m with the slope toe as the origin in Fig. 6(b). Here, the geotechnical body above the groundwater table in the slope is composed of three-layer soils, and the matric suction or the DOS in the slope would be solved according to the section 2.3. Moreover, the slope tends to slide along the surface of a weak interlayer for the reason of its low shear strength. Thus, when the circular slip surface is below the weak interlayer, it would be replaced by the surface of weak interlayer, thereby forming a composite slip surface. Then, under the general nonlinear M-C strength criterion with the variation of the nonlinear parameter m, the slope stability is analyzed, and the calculated results are shown in Table 3. In Fig. 6(b), the curve of the DOS distribution along the z-axis was drawn in the slope and the critical circular slip surface was also plotted for the slope when m_1 = 1.6 and m_3 = 1.6.

From Tables 2 and 3, it can be obtained that (1) the feasible of the present method is explained by less than 2% difference with the LE M-P method; (2) the minimum slope FOS decreases slightly with the increase of the depth of the crack or its horizontal distance from the slope vertex; and (3) the nonlinear parameter m has an important impact on the slope stability and the slope stability would become worse with increase of m.

5. Stability charts of unsaturated slope

To facilitate the engineering design of an unsaturated slope, the stability charts under the general nonlinear M-C strength criterion are drawn and displayed in Fig. 7. In Fig. 7, the curves of [arctan (c_0 / σ_t) / $F_{s\_\min}$] versus [(c_0 / $F_{s\_\min}$) /(γH)] for the slope in the critical state (Sun and Zhao 2013) are obtained by analyzing the stability of the unsaturated slope, where $F_{s\_\min}$ represents the minimum FOS for the slope in the critical state and γ =17.8 kN/m^3. Moreover, u_a = 0 kPa, q / ks = 0, and Sr = 0 for the unsaturated soil.

Here, four cases of DOS distribution are considered to represent the saturated state of the slope from wet (α = 0.05 kPa^{-1} in Fig. (6a)) to dry (α = 0.50 kPa^{-1} in Fig. (6d)) by increasing the parameter α when the parameter n = 2. Meanwhile, the equation of groundwater table is set to be $f(x)$ = -0.2H with the slope toe as the origin in Cartesian coordinate system and the slope angle β = 45°. Certainly, different slope angle β, different groundwater table, and different parameter n could be selected to generate the same stability charts with Fig. 7. For these charts in Fig. 7, the dotted line represents the value of the parameter λ (λ = [arctan (c_0 / σ_t)] / [c_0 / (γH)]), and the solid line reflects the relationship among the minimum slope FOS ($F_{s\_\min}$), the slope parameter (H), and the strength parameters (c_0, σ_t, and m) for the slope under the critical LE state. According to the relationship reflected by the solid line, the minimum slope FOS can be calculated and the slope parameters (such as H and β) can be also designed for the unsaturated slope with a certain requirement for safety.

The application of these charts would be illustrated by Fig. 7(a) (here β = 45°, α = 0.05 kPa^{-1}, n = 2, and $f(x)$ = -0.2H). After the slope parameters (H), the unit weight of the soil (γ), and the strength parameters (c_0 and σ_t) of the soil are given, the parameter λ can be calculated. Then, according to the given nonlinear parameter m (such as m > 1.5 and m < 2.0), the point s_0 (e.g., s_0 is located on the dotted line of λ = 10) is determined by the linear interpolation in Fig. 7(a). Thereby, based on the longitudinal or the transverse coordinates of the obtained point s_0, the minimum FOS for the slope can be solved. For the slope with the certain safety requirement (for example, the minimum slope FOS should be $F_{s\_\min}$), the slope parameter (H or β) can be designed to satisfy this requirement. According to the given strength parameters of the soil and the designed minimum slope FOS, the horizontal line with the longitudinal coordinates of [arctan (c_0 / σ_t) / $F_{s\_\min}$] can be plotted in Fig. 7(a). Then, based on the given nonlinear parameter m (such as m > 1.5 and m < 2.0), the point s_0 is determined by the linear interpolation method in Fig. 7(a). Thus, the slope height is designed from the transverse coordinates of the obtained point s_0.

6. Conclusions

For the unsaturated soil, the formula for the shear strength of unsaturated soil under the general nonlinear M-C strength criterion is derived on basic of the effective stress principle to improve the classical strength criterion of the unsaturated soil. Meanwhile, the simple irrigation model under the steady seepage is adopted to obtain the distribution of the matric suction or the DOS above the groundwater table in the slope. Then, combined with the improved strength criterion of the unsaturated soil and the simple irrigation model under the steady seepage, the LE solutions for analyzing the unsaturated slope stability are established according to the global LE conditions of the entire sliding body with the stress assumptions on the slip surface. Thereafter, by comparison and analysis of some selected slope examples, the feasibility of the present method is verified. Furthermore, the stability charts of the unsaturated slope are drawn. In addition, the following conclusions were obtained as:

(1) Compared to the classical strength criterion of the unsaturated soil from Fredlund *et al.*, the nonlinear characteristics of the internal friction angle related to the matric suction under the small value of the matric suction can be reflected in the improved strength criterion.

(2) Under the nonlinear strength criterion, not only the cohesion of the unsaturated soil but also the internal friction angle of the unsaturated soil is affected by the matric suction or the DOS.

(3) The nonlinear parameter m in the strength criterion has an important impact on the slope stability, and the minimum slope FOS decreases slightly with the increase of the depth of the vertical tensile crack or its horizontal distance from the slope vertex.

(4) The stability charts are convenient for engineers to further design the unsaturated slope and also quickly determine the stability of the unsaturated slope.

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