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Effect of time harmonic sources on transversely isotropic thermoelastic thin circular plate

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Abstract. The present research deals with the deformation in transversely isotropic thin circular thermoelastic rotating plate due to time-harmonic sources. Frequency effect in the presence of rotation and two temperature is studied under thermally insulated as well as isothermal boundaries. The Hankel transform technique is used to find a solution to the problem. The displacement components, stress components, and conductive temperature distribution with the radial distance are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. Some specific cases are also figured out from the current research.

Keywords: transversely isotropic; thin circular plate; Hankel transform; time harmonic sources

1. Introduction

Thick plate, thin plate, and a membrane have a large difference according to their structure. Thin circular plates and membranes are widely used in pressure sensors, microphones, loudspeakers, gas flow meters, optical telescopes, solar powers, radio and radar antennae, and many other devices. Plate theories are beneficial for designs and analyses of these devices. Zhao (2008) described the flexural properties of a plate which were influenced by its thickness. According to Zhao (2008), plates can be classified into three categories: membranes, thin plates, and thick plates depending upon the aspect ratio. The aspect ratio is defined as a/h, where a is diameter of a plate and h is the thickness of plate. The plates with aspect ratio, $a/h \geq 80\ldots100$ are referred as membranes. It is termed as thin plate with aspect ratio as $8\ldots10 \leq a/h \leq 80\ldots.100$. Moreover, if $a/h \leq 8\ldots10$, the plate is termed as thick plate.

Deshmukh *et al.* (2005, 2006) considered the inverse problem of transient heat conduction in a thin finite circular plate with integral transform technique and the thin circular plate for unknown heating temperatures in the form of Bessel functions and with integral techniques. Kanoria *et al.* (2011) studied the thermoelastic response of fiber-reinforced thin circular disc with three-phase lag due to axisymmetric thermoelastic loading. Gaikwad *et al.* (2005, 2012, 2016) deliberated the circular plate for known interior temperature under Steady-state field, a thin circular plate due to uniform internal energy generation using Hankel transform technique for its solution and the inverse problem of thermoelasticity in a thin isotropic circular plate. Khobragade *et al.* (2012) examined thermal deflection of a thin circular plate using boundary conditions of radiation type. Mahmoud (2012) had considered the impact of rotation, relaxation times, magnetic field, gravity field and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lame's potential techniques. Mahmoud (2012) considered the impact of rotation, relaxation times, magnetic field, gravity field and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lame's potential techniques. Varghese *et al.* (2014) studied thermoelastic deformation with annular heat supply on a thin circular plate. Keivani *et al.* (2014) discussed the forced vibration problem of a Euler-Bernoulli beam with a semi-infinite field by considering it a BVP in the frequency domain. Alzahrani (2016) investigated 2D generalized magneto-thermoelasticity of a fiber-reinforced anisotropic material under GN theory- III type. Tripathi *et al.* (2017a, b) investigated a quasi-static uncoupled theory of thermoelasticity based on time-fractional heat conduction equation for a thin circular plate and studied a thin hollow circular disk with quasi static uncoupled theory of thermoelasticity with the time-fractional derivative of order alpha subjected to a time-dependent heat flux. Vinyas *et al.* (2017) discovered a multiphysics behaviour of magneto-electro-elastic (MEE) cantilever beam using thermo-mechanical loading. Moreover, Kumar *et al.* (2017a) investigated the homogeneous isotropic thermoelastic thick circular plate with dual-phase lags and two temperatures. Ezzat and El-Bary (2017a) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space based on a memory-dependent derivative. Ahire *et al.* (2019) studied a

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problem of thermal stresses in circular plate due to internal moving heat sources with integral transform technique. Despite this, several researchers worked on different theory of thermoelasticity as Marin (1994, 1999), Abbas & Youssef (2009), Mohamed *et al.* (2009), Palani and Abbas (2009),Othman and Abbas (2012), Zenkour & Abbas (2014), Sharma *et al.* (2015), Kumar *et al.* (2017, 2016a), Marin (2016), Ezzat *et al.* (2012), Abbas (2007, 2014, 2015), Ezzat *et al*. (2017), Othman & Marin (2017), Akbaş (2017), Ozdemir (2018), Taleb *et al.* (2018), Houari *et al.* (2018), Heydari (2018), Chauthale *et al*. (2017) , Marin and Craciun (2016, 2017), Lata and Kaur (2018) , Marin *et al.* (2017, 2018), Lata and Kaur (2019a, b, c, d), and Kaur and Lata (2019a, b) and Liu *et al.* (2019). Inspite of all these efforts, no attempt has been made for thermoelasticity of thin circular plate with rotation and time-harmonic source.

In this paper, we have attempted to study the deformation in transversely isotropic thin circular plate due to isothermal/thermally insulated boundaries with rotation and time-harmonic source. The Laplace and Hankel transform has been used to obtain the general solution of the field equations. The analytical expressions of stresses, conductive temperature, displacement components are computed in transformed domain. However, the resulting quantities are obtained in the physical domain by using numerical inversion technique. Some particular cases are also discussed.

2. Basic equations

Following Lata *et al.* (2019c) and Abd-Alla & Alshaikh (2015) the constitutive relations and field equations for an anisotropic thermoelastic medium in absence of body forces and heat sources are

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \quad (1)$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,j} = \rho\ddot{u}_i, \quad (2)$$

$$K_{ij}\varphi_{,ij} = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho C_E\dot{T}\right) + \beta_{ij}T_0\dot{e}_{ij}, \quad (3)$$

where

$$T = \varphi - a_{ij}\varphi_{,ij},\ \beta_{ij} = C_{ijkl}\alpha_{ij},\quad e_{ij} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right).\ i = 1,2,3 \quad (4)$$

We should note that for G-L theory the thermal relaxation time must satisfy the relation $\tau_1 \geq \tau_0 > 0$and following Kumar *et al.* (2016b), equation of motion for a uniformly rotating medium with an angular velocity $\mathbf{\Omega}$ is

$$t_{ij,j} + F_i = \rho\{\ddot{u}_i + (\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}))_i + (2\mathbf{\Omega}\times\dot{\boldsymbol{u}})_i\}, \quad (5)$$

Here $C_{ijkl}$are elastic parameters and having symmetry ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$). The symmetries in C_{ijkl} is due to

- The stress tensor is symmetric, which is only possible if ($C_{ijkl} = C_{jikl}$).
- If a strain energy density symmetry exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$.
- From stress tensor and elastic stiffness, tensor symmetries infer ($C_{ijkl} = C_{ijlk}$) and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$.

3. Formulation of the problem

We consider a transversely isotropic thin circular plate of thickness 2b occupying the space D defined by $0 \leq r \leq \infty, -b \leq z \leq b$. Thin plates are usually characterized by the ratio a / b (the ratio between the length of a side, a, and the thickness of the material, b, falling between the values of 8 and 80 as mentioned by Ventsel *et. al.* (2001). Let the plate be subjected to axisymmetric heat supply and thermomechanical load applied into its inner boundary having initially undisturbed state at a uniform temperature T_0. We use plane cylindrical co-ordinates (r, θ, z) with the centre of the plate as the origin.

As the problem considered is plane axisymmetric, $(u, v, w, and\ \varphi)$ are independent of θ. We restrict our analysis to a two-dimension problem with $\vec{\boldsymbol{u}} = (u, 0, w)$. Also applying the transformation

$$x' = x' = x\cos\phi + y\sin\phi,\ y' = -x\sin\phi + y\cos\phi,\ z' = z. \quad (6)$$

where ϕ is the angle of rotation in x-y plane, on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperatures and with energy dissipation, to obtain

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}u\right) + C_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + C_{44}\frac{\partial^2 u}{\partial z^2} + C_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \quad (7)$$

$$(C_{11}+C_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + C_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right), \quad (8)$$

$$(K_1)\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) + (K_3)\frac{\partial^2\varphi}{\partial z^2} = T_0\left(\beta_1\frac{\partial\dot{u}}{\partial r} + \beta_3\frac{\partial\dot{w}}{\partial z}\right) + \rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\left\{\dot{\varphi} - a_1\left(\frac{\partial^2\dot{\varphi}}{\partial r^2} + \frac{1}{r}\frac{\partial\dot{\varphi}}{\partial r}\right) - a_3\frac{\partial^2\dot{\varphi}}{\partial z^2}\right\}. \quad (9)$$

In above equations, we use the contracting subscript notations (1→ 11, 2→ 22, 3→ 33, 5→ 23, 4→ 13, 6→ 12) to relate C_{ijkl} to C_{mn}. Also a_1 and a_3 are two temperature parameters.

For axisymmetric problem following Dhaliwal (1980) and Lata *et al.* (2019c) Constitutive relations are

$$\begin{aligned} t_{rr} &= c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T,\\ t_{zr} &= 2c_{44}e_{rz}\\ t_{zz} &= c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T,\\ t_{\theta\theta} &= c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T, \end{aligned} \quad (10)$$

where

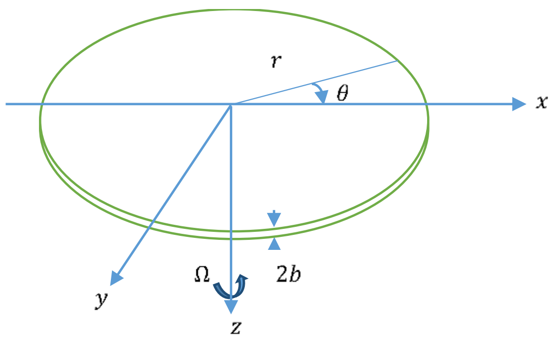

Fig. 1 Geometry of the problem

$$e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right),$$

$$e_{rr} = \frac{\partial u}{\partial r},$$

$$e_{\theta\theta} = \frac{u}{r},$$

$$e_{zz} = \frac{\partial w}{\partial z},$$

$$T = \varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2},$$

$$\beta_{ij} = \beta_i\delta_{ij}, \qquad K_{ij} = K_i\delta_{ij},$$

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3,$$

$$\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3.$$

To facilitate the solution, the following dimensionless quantities are introduced

$$r' = \frac{r}{L},\ z' = \frac{z}{L}, t' = \frac{c_1}{L}t, u' = \frac{\rho c_1^2}{L\beta_1 T_0}u, w' = \frac{\rho c_1^2}{L\beta_1 T_0}w, T' = \frac{T}{T_0}, t'_{zr} = \frac{t_{zr}}{\beta_1 T_0}, t'_{zz} = \frac{t_{zz}}{\beta_1 T_0}, \Omega' = \frac{L}{c_1}\Omega, \varphi' = \frac{\varphi}{T_0}, a'_1 = \frac{a_1}{L^2}, a'_3 = \frac{a_3}{L^2}. \tag{11}$$

where $c_1^2 = \frac{c_{11}}{\rho}$, and L is a constant of dimension of length.

Assume the time-harmonic behaviour as

$$(u, w, \varphi)(r, z, t) = (u, w, \varphi)(r, z)e^{i\omega t}. \tag{12}$$

Using the dimensionless quantities defined by (11) in equations (7)-(9) and after that suppressing the primes and applying Hankel transforms defined by

$$\tilde{f}(\xi, z, \omega) = \int_0^\infty f^*(r, z, \omega)rJ_n(r\xi)dr. \tag{13}$$

On the resulting quantities, we obtain

$$(\zeta_1 + \delta_2 D^2)\tilde{u} + [\zeta_2 D - \zeta_3]\tilde{w} + (-\zeta_5 D^2 + \zeta_4)\tilde{\varphi} = 0, \tag{14}$$

$$(\zeta_2 D + \zeta_3)\tilde{u} + (\delta_3 D^2 + \zeta_6)\tilde{w} + (\zeta_7 D + \zeta_8 D^3)\tilde{\varphi} = 0, \tag{15}$$

$$\zeta_9\tilde{u} + \zeta_{10}D\tilde{w} + (\zeta_{11} + D^2\zeta_{12})\tilde{\varphi} = 0, \tag{16}$$

where

$$\delta_1 = \frac{c_{13}+c_{44}}{c_{11}},\ \delta_2 = \frac{c_{44}}{c_{11}},\ \delta_3 = \frac{c_{33}}{c_{11}},\ \delta_4 = \frac{\beta_1^2 T_0}{\rho},\ \delta_5 = \frac{\beta_1\beta_3 T_0}{\rho},\ \delta_6 = \rho C_E C_1^2, D \equiv \frac{d}{dz}\ .$$

and

$$\zeta_1 = -\xi^2 + \omega^2 + \Omega^2,$$

$$\zeta_2 = \delta_1\xi,$$

$$\zeta_3 = 2\Omega\omega \mathrm{i},$$

$$\zeta_4 = \xi(1 + a_1\xi^2)(1 + \tau_1 i\omega),$$

$$\zeta_5 = a_3\xi(1 + \tau_1 i\omega),$$

$$\zeta_6 = -\delta_2\xi^2 + \omega^2 + \Omega^2,$$

$$\zeta_7 = -\frac{\beta_3}{\beta_1}(1 + a_1\xi^2)(1 + \tau_1 i\omega),$$

$$\zeta_8 = \frac{\beta_3}{\beta_1}a_3(1 + \tau_1 i\omega)$$

$$\zeta_9 = \delta_4\omega i\xi,$$

$$\zeta_{10} = -\delta_5\omega i,$$

$$\zeta_{11} = -\delta_6(\omega i - \tau_0\omega^2)(1 + a_1\xi^2) - \xi^2 K_1,$$

$$\zeta_{12} = \left(K_3 + a_3\delta_6(\omega i - \tau_0\omega^2)\right).$$

Using the dimensionless quantities defined by (11) in equations (10) and after that suppressing the primes and applying Hankel transforms defined by (13) we have

$$\widetilde{t_{zz}} = \frac{C_{13}}{C_{11}}\xi\tilde{u} + \delta_3 D\tilde{w} - \frac{\beta_3}{\beta_1}(1 + a_1\xi^2 - a_3 D^2)\tilde{\varphi}, \tag{17}$$

$$\widetilde{t_{rz}} = \delta_2 D\tilde{u} - \xi\delta_2\tilde{w}, \tag{18}$$

$$\widetilde{t_{rr}} = -\xi\tilde{u} + \frac{C_{12}\xi}{C_{11}}\tilde{u} + \frac{C_{13}}{C_{11}}D\tilde{w} - (1 + a_1\xi^2 - a_3 D^2)\tilde{\varphi}. \tag{19}$$

The non-trivial solution of (14)-(16) by eliminating $\tilde{u}$, $\tilde{w}$, and $\tilde{\varphi}$ yields

$$AD^6 + BD^4 + CD^2 + E = 0, \tag{20}$$

where

$$A = \delta_2\delta_3\zeta_{12} - \delta_2\zeta_{10}\zeta_8,$$

$$B = \zeta_1\zeta_{12}\delta_3 - \zeta_1\zeta_{10}\zeta_8 + \delta_2\delta_3\zeta_{11} + \delta_2\zeta_{12}\zeta_6 - \delta_2\zeta_{10}\zeta_7 - \zeta_2^2\zeta_{12} - \zeta_2\zeta_9\zeta_8 + \zeta_5\zeta_{10}\zeta_2 - \zeta_5\zeta_9\delta_3,$$

$$C = \delta_3\zeta_1\zeta_{11} + \delta_2\zeta_6\zeta_{11} + \zeta_1\zeta_6\zeta_{12} - \zeta_1\zeta_{10}\zeta_7 - \zeta_2^2\zeta_{11} + \zeta_2\zeta_7\zeta_9 - \zeta_5\zeta_6\zeta_9 + \zeta_4\zeta_2\zeta_{10} - \delta_3\zeta_4\zeta_9 + \zeta_3^2\zeta_{12},$$

$$E = \zeta_6\zeta_1\zeta_{11} - \zeta_4\zeta_6\zeta_9.$$

The solutions of the equation (20) can be written in the form

$$\tilde{u} = \sum A_i(\xi, \omega)cosh(q_i z), \tag{21}$$

$$\tilde{w} = \sum d_i A_i(\xi, \omega)cosh(q_i z), \tag{22}$$

$$\tilde{\varphi} = \sum l_i A_i(\xi, \omega)cosh(q_i z), \tag{23}$$

where $A_{i,} i = 1, 2, 3$ being arbitrary constants, $\pm q_i (i = 1,2,3)$ are the roots of the equation (20) and d_i and l_i are given by

$$d_i = \frac{(\zeta_2\zeta_{12} - \zeta_8\zeta_9)q_i^3 + \zeta_3\zeta_{12}q_i^2 + (\zeta_2\zeta_{11} - \zeta_7\zeta_9)q_i + \zeta_3\zeta_{11}}{(-\zeta_8\zeta_{10} + \delta_3\zeta_{12})q_i^4 + (\delta_3\zeta_{11} + \zeta_6\zeta_{12} - \zeta_7\zeta_{10})q_i^2 + \zeta_6\zeta_{11}}, \tag{24}$$

$$l_i = \frac{(-\zeta_9\delta_3+\zeta_1\zeta_{10})q_i^2+\zeta_3\zeta_{10}q_i-\zeta_8\zeta_9}{(-\zeta_8\zeta_{10}+\delta_3\zeta_{12})q_i^4+(\delta_3\zeta_{11}+\zeta_6\zeta_{12}-\zeta_7\zeta_{10})q_i^2+\zeta_6\zeta_{11}}. \quad (25)$$

Also, from (17)-(19) and (21)-(23) we have

$$\widetilde{t_{zz}} = \sum A_i(\xi,\omega)\eta_i cosh(q_i z) + \sum \mu_i A_i(\xi,\omega)\sinh(q_i z), \quad (26)$$

$$\widetilde{t_{rz}} = \sum A_i(\xi,\omega)M_i cosh(q_i z) + \sum N_i A_i(\xi,\omega)\sinh(q_i z), \quad (27)$$

$$t_{rr} = \sum A_i(\xi,\omega)R_i cosh(q_i z) + \sum S_i A_i(\xi,\omega)\sinh(q_i z). \quad (28)$$

where

$$\eta_i = \frac{C_{13}}{C_{11}}\xi - \frac{\beta_3}{\beta_1} l_i(1 + a_1\xi^2 - a_3 q_i^2),$$

$$R_i = -\xi + \frac{C_{12}\xi}{C_{11}} - (1 + a_1\xi^2 - a_3 q_i^2),$$

$$S_i = \frac{C_{13}}{C_{11}} d_i q_i,$$

$$\mu_i = \delta_3 d_i q_i,$$

$$M_i = \delta_2 d_i \xi,$$

$$N_i = \delta_2 q_i, i = 1,2,3.$$

4. Boundary conditions

We consider a cubical thermal source and normal force following Kar and Kanoria (2011) of unit magnitude along with vanishing of tangential stress components at the stress-free surface at z = ±b. Mathematically, these can be written as

$$h_1\frac{\partial\varphi}{\partial z} + h_2\varphi = \pm g_o F(r,z), \quad (29)$$

$$t_{zz}(r,z,t) = f(r,t), \quad (30)$$

$$t_{rz}(r,z,t) = 0. \quad (31)$$

Here, $h_2 \to 0$ corresponds to thermally insulated boundaries, $h_1 \to 0$ corresponds to isothermal boundaries. Using the dimensionless quantities defined by(11) on the equations (29)-(31) and taking Hankel and Laplace transform of the resulting equations and then using (26)-(27) and (21)-(23) yields

$$\sum A_i(\xi,\omega)\, l_i(h_1 q_i + h_2)\sinh(q_i z) = \pm g_o\tilde{F}(\xi,z), \quad (32)$$

$$\sum A_i(\xi,\omega)\eta_i cosh(q_i z) + \sum \mu_i A_i(\xi,\omega)\sinh(q_i z) = \tilde{f}(\xi,\omega), \quad (33)$$

$$\sum A_i(\xi,\omega)M_i cosh(q_i z) + \sum N_i A_i(\xi,\omega)\sinh(q_i z), = 0. \quad (34)$$

Solving the equations (21)-(23) with the aid of (32)-(34) and also solving (17)-(19), we obtain

$$\tilde{u} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1\vartheta_1 + \chi_2\vartheta_2 - \chi_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\vartheta_1 - \chi_5\vartheta_2 + \chi_6\vartheta_3\}, \quad (35)$$

$$\tilde{w} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 d_1\vartheta_1 + \chi_2 d_2\vartheta_2 - \chi_3 d_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 d_1\vartheta_1 - \chi_5 d_2\vartheta_2 + \chi_6 d_3\vartheta_3\}, \quad (36)$$

$$\tilde{\varphi} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 l_1\vartheta_1 + \chi_2 l_2\vartheta_2 - \chi_3 l_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 l_1\vartheta_1 - \chi_5 l_2\vartheta_2 + \chi_6 l_3\vartheta_3\}, \quad (37)$$

$$\widetilde{t_{zz}} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 G_4 + \chi_2 G_5 - \chi_3 G_6\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 G_4 - \chi_5 G_5 + \chi_6 G_6\}, \quad (38)$$

$$\widetilde{t_{zr}} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 G_7 + \chi_2 G_8 - \chi_3 G_9\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 G_7 - \chi_5 G_8 + \chi_6 G_9\}, \quad (39)$$

$$\widetilde{t_{rr}} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 G_{10} + \chi_2 G_{11} - \chi_3 G_{12}\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 G_{10} - \chi_5 G_{11} + \chi_6 G_{12}\}, \quad (40)$$

where

$$G_i = l_i(h_1 q_i + h_2)\theta_i,$$

$$G_{i+3} = \eta_i\vartheta_i + \mu_i\theta_i,$$

$$G_{i+6} = N_i\theta_i + M_i\vartheta_i,$$

$$G_{i+9} = S_i\theta_i + R_i\vartheta_i, i = 1,2,3.$$

$$\Delta = G_1\chi_4 - G_2\chi_5 + G_3\chi_6,$$

$$\Delta_1 = -\tilde{f}(\xi,s)\chi_1 + g_o\tilde{F}(\xi,z)\chi_4,$$

$$\Delta_2 = \tilde{f}(\xi,s)\chi_2 - g_o\tilde{F}(\xi,z)\chi_5,$$

$$\Delta_3 = -\tilde{f}(\xi,s)\chi_3 + g_o\tilde{F}(\xi,z)\chi_6,$$

$$\chi_1 = [G_2G_9 - G_8G_3],$$

$$\chi_2 = [G_1G_9 - G_7G_3],$$

$$\chi_3 = [G_1G_8 - G_2G_7],$$

$$\chi_4 = [G_5G_9 - G_8G_6],$$

$$\chi_5 = [G_4G_9 - G_6G_7],$$

$$\chi_6 = [G_4G_8 - G_5G_7],$$

$$\vartheta_i = \cosh(q_i z), \theta_i = \sinh(q_i z), i = 1,2,3$$

5. Applications

For constant load and heat source which decays moving away from the centre of the thin circular plate in the radial direction as well as along the axial directions

$$f(r,t) = H(\alpha - r)e^{i\omega t}, F(r,z) = \frac{1}{\sqrt{r^2+z^2}} \quad (41)$$

where $H(\alpha - r)$ is the Heaviside function. Applying Hankel Transform, on Equations (41), gives

$$\tilde{f}(\xi,\omega) = \frac{\alpha J_1(\xi\alpha)}{\xi}e^{i\omega t}, \tilde{F}(\xi,z) = \frac{e^{-\xi|z|}}{\xi} \quad (42)$$

6. Inversion of the transforms

To find the solution of the problem in physical domain, invert the transforms in equations (35)-(40) by inverting the Hankel transform using

$$f^*(r,z,s) = \int_0^\infty \xi \tilde{f}(\xi,z,s) J_n(\xi r) d\xi \quad (45)$$

The last step is to calculate the integral in Eq. (45). The method for evaluating this integral by using Romberg's integration with adaptive step size is described in Press *et al.* (1986).

7. Particular cases

(i) If the coupling constant is taken zero i.e., $\delta_4 = 0\ and \delta_5 = 0$ then (32)-(37) gives results for a transversely isotropic thermoelastic thin plate for uncoupled generalized thermoelasticity with two relaxation times.

8. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of rotation in different forms of boundary conditions as mentioned in applications in the above part, we now present some numerical results. Cobalt material is chosen for the purpose of numerical calculation, which is transversely isotropic. The physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal and Singh (1980) is given by

$$c_{11} = 3.07 \times 10^{11} Nm^{-2},$$
$$c_{12} = 1.650 \times 10^{11} Nm^{-2},$$
$$c_{13} = 1.027 \times 10^{10} Nm^{-2},$$
$$c_{33} = 3.581 \times 10^{11} Nm^{-2}$$
$$c_{44} = 1.510 \times 10^{11} Nm^{-2},$$
$$C_E = 4.27 \times 10^2 JKg^{-1} deg^{-1},$$
$$\beta_1 = 7.04 \times 10^6 Nm^{-2} deg^{-1}, \rho = 8.836 \times 10^3 Kgm^{-3}$$
$$\beta_3 = 6.90 \times 10^6 Nm^{-2} deg^{-1},$$
$$K_1 = 0.690 \times 10^2 Wm^{-1} Kdeg^{-1},$$
$$K_3 = 0.690 \times 10^2 Wm^{-1} K^{-1},$$
$$L = 1, \tau_1 = \tau_0 = 1, b = 0.01m$$

The values of normal force stress t_{zz}, tangential stress t_{zr}, radial stress t_{rr} and conductive temperature φ for a transversely isotropic thermoelastic solid with two temperature and thermal relaxation times is presented graphically to show the impact of frequency.

• The solid black line with centre symbol square corresponds to thermal insulated boundaries and $\omega = 0.5$.

• The solid red line with centre symbol circle corresponds to thermally insulated boundaries and $\omega = 1.0$.

• The solid green line with centre symbol triangle corresponds to isothermal boundaries and $\omega = 0.5$.

• The solid blue line with centre symbol diamond corresponds to isothermal boundaries and $\omega = 1.0$.

Fig. 2 shows the variations of displacement component u with radius r for constant load and heat source. In the initial range of radius r, there is a sharp increase in the value of displacement component u for $\omega = 0.5$ and then oscillates with smaller amplitude for thermally insulated boundary conditions as compared to that of an isothermal boundary. However, for $\omega = 1.0$, there is a sharp decrease in the value of displacement component u for $\omega = 0.5$ and then oscillates with smaller amplitude for thermally insulated boundary conditions as compared to that of isothermal boundary. Moreover, away from source applied, it follows oscillatory behaviour near the zero value. We can see that the frequency of time-harmonic source has significant effect on the displacement component

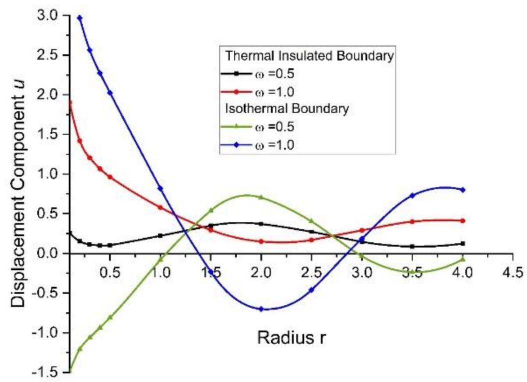

Fig. 2 variations of displacement component u with radius r

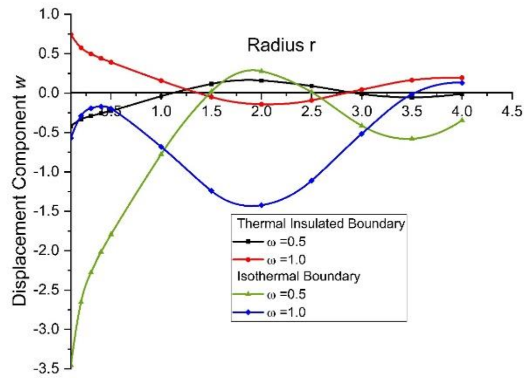

Fig. 3 variations of displacement component w with radius r

Fig. 3 illustrates the variations of displacement component w with radius r for constant load and heat source. In the initial range of radius r, there is a decrease in the value of displacement component for all the cases. However, for thermal insulated boundary and rotation, the displacement component varies more as compared to isothermal boundaries, with and without rotation. Moreover, away from source applied, it follows opposite oscillatory behaviour near the zero value. We can see that the rotation have significant effect on the displacement component in all the cases as there are more variations in w in case of rotation, it behaves opposite for thermal insulated and isothermal boundary conditions.

Fig. 4 illustrates the variations of conductive temperature $\boldsymbol{\varphi}$ with radius r for constant load and heat source. In the initial range of radius r, there is a sharp increase in the value of $\boldsymbol{\varphi}$ for all the cases for the isothermal boundary. However, for thermal insulated boundary conductive temperature $\boldsymbol{\varphi}$ decreaseswith radius r.

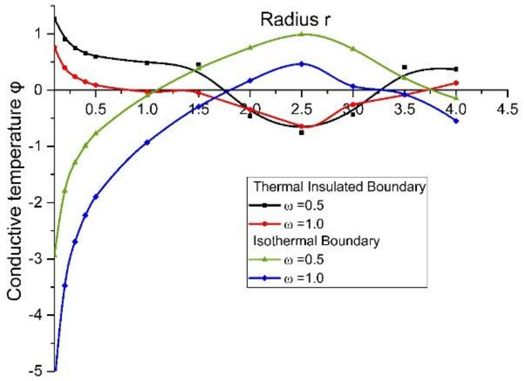

Fig. 4 variations of Conductive temperature $\boldsymbol{\varphi}$ with radius r

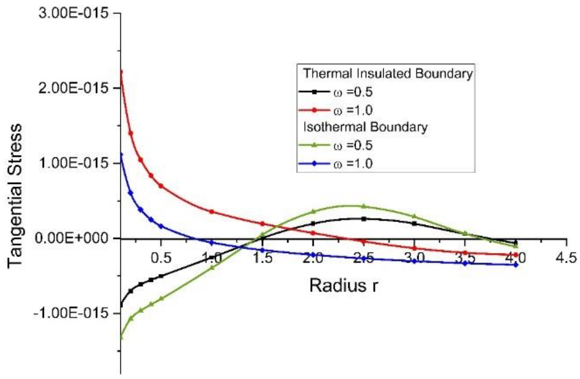

Fig. 5 variations of the tangential stress component $\boldsymbol{t_{zr}}$ with radius r

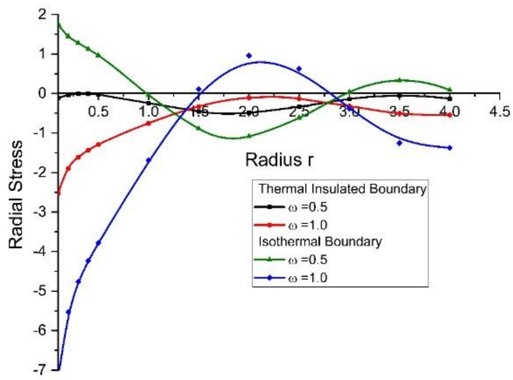

Fig. 6 variations of radial stress with radius r

Moreover, away from source applied, it follows opposite oscillatory behaviour near the zero value.

Fig. 5 illustrates the variations of the tangential stress component $\boldsymbol{t_{zr}}$ with radius r for constant load and heat source. In the initial range of radius r, tangential stress component $\boldsymbol{t_{zr}}$ decreases for $\omega = 1.0$ whereas for $\omega = 0.5$ its value increases for both types of boundary conditions with a difference of amplitude. Moreover, away from source applied, it follows opposite oscillatory behaviour near the zero value

Fig. 6 illustrates the variations of radial stress t_{rr} with radius r for constant load and heat source. For thermally insulated boundaries, the radial stress varies less as compared to isothermal boundary for both with $\omega = 0.5$ and $\omega = 1.0$. Moreover, away from source applied, it follows oscillatory behaviour near the zero value.

9. Conclusions

In this paper, we have discussed the thermoelastic problem for a transversely isotropic thin circular plate with rotation, two temperature and with two relaxation time of generalized thermoelasticity with a time-harmonic source in the context of GL theory. Thermally insulated and isothermal boundary cases for circular edges are considered and temperature is maintained on upper and lower surface of the circular thin plate. The finite Hankel transform technique is used to obtain numerical results.

In the present research article, conductive temperature, displacement, and stresses along with rotation, two temperature, relaxation time with time-harmonic source and thermally insulated and isothermal boundary for constant load and heat source have been outlined. Since the thickness of plate is very small, the series solution given here will be definitely convergent. The temperature, displacement and thermal stresses that are obtained can be applied to the design of pressure sensors, microphones, gas flow meters, optical telescopes, radar antennae and many other devices structures or machines in engineering applications.

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