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Risk assessment of karst collapse using an integrated fuzzy analytic hierarchy process and grey relational analysis model

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Abstract. A karst collapse, as a natural hazard, is totally different to a normal collapse. In recent years, karst collapses have caused substantial economic losses and even threatened human safety. A risk assessment model for karst collapse was developed based on the fuzzy analytic hierarchy process (FAHP) and grey relational analysis (GRA), which is a simple and effective mathematical algorithm. An evaluation index played an important role in the process of completing the risk assessment model. In this study, the proposed model was applied to Jiaobai village in southwest China. First, the main controlling factors were summarized as an evaluation index of the model based on an investigation and statistical analysis of the natural formation law of karst collapse. Second, the FAHP was used to determine the relative weights and GRA was used to calculate the grey relational coefficient among the indices. Finally, the relational sequence of evaluation objects was established by calculating the grey weighted relational degree. According to the maximum relational rule, the greater the relational degree the better the relational degree with the hierarchy set. The results showed that the model accurately simulated the field condition. It is also demonstrated the contribution of various control factors to the process of karst collapse and the degree of collapse in the study area.

Keywords: karst collapse; risk assessment; evaluation index; fuzzy analytic hierarchy process (FAHP); grey relational analysis (GRA)

1. Introduction

Guizhou is located on the Yunnan-Guizhou Plateau in south China, which is a typical karst landform. The special geological structure creates a number of geological hazards, such as landslides, debris flows, and karst collapses. With the intensification of human activities, karst collapse has become a serious threat to human life and economic development (Jiang *et al*. 2012, He *et al*. 2010). Therefore, how to realize and evaluate a karst collapse has received increasing attention. A karst collapse is different from an ordinary collapse. A karst collapse is a general term for the collapse of a roof rock mass in a karst area due to cave expansion into the lower part of a rock mass, or the collapse of the overlying soil layer over a soil cave roof due to an imbalance in natural or human factors (Zhou *et al*. 2015, Kaufmann *et al*. 2014, 2018). Karst collapses have caused large economic losses in many places around the world. They are closely related to the intensity of human activities and have typical time-controlled characteristics due to the differences in their spatial positions, geological background conditions, and other influencing factors. (Galve *et al*. 2009a, b). Due to the influence of human activities, many hidden caves have formed under the stimulation of various external factors. Karst collapse is a complex process, which is controlled by multiple factors that need to be considered. Hence, a comprehensive analysis and quantitative evaluation of the factors influencing karst collapse play a significant role in the risk assessment of this hazard.

There have been many studies of this problem, including an analysis of the collapse mechanism and the classification of collapses into categories such as gravity action, suffosion action, vacuum suction action, and impact burst action (Jiang *et al*. 2012, Papadopoulou-Vrynioti *et al*. 2013, Farrant *et al*. 2008, Gómez-Ortiz *et al*. 2012, Lin *et al*. 2018). Using this classification, it is relatively easy to clearly distinguish which factors are key to a specific collapse and to quickly understand the process of its formation. However, a karst collapse can be so complicated that one mechanism alone cannot explain it. A karst collapse is often a process of multi-mechanism interaction. Therefore, the factors affecting the collapse are complex and changeable. In consideration of this problem, unified indices are required to conduct a risk assessment of karst collapse, which include more controlling factors. In recent years, many researchers have used statistical methods, a geographic information system (GIS), artificial neural network (ANN), or the analytic hierarchy process (AHP) to predict and evaluate karst collapse (He *et al*. 2014, Jiang *et al*. 2003, Taheri *et al*. 2015, Oh *et al*. 2010, Kim *et al*. 2009). The use of these quantitative mathematical methods

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Fig. 1 Plane form of a karst collapse

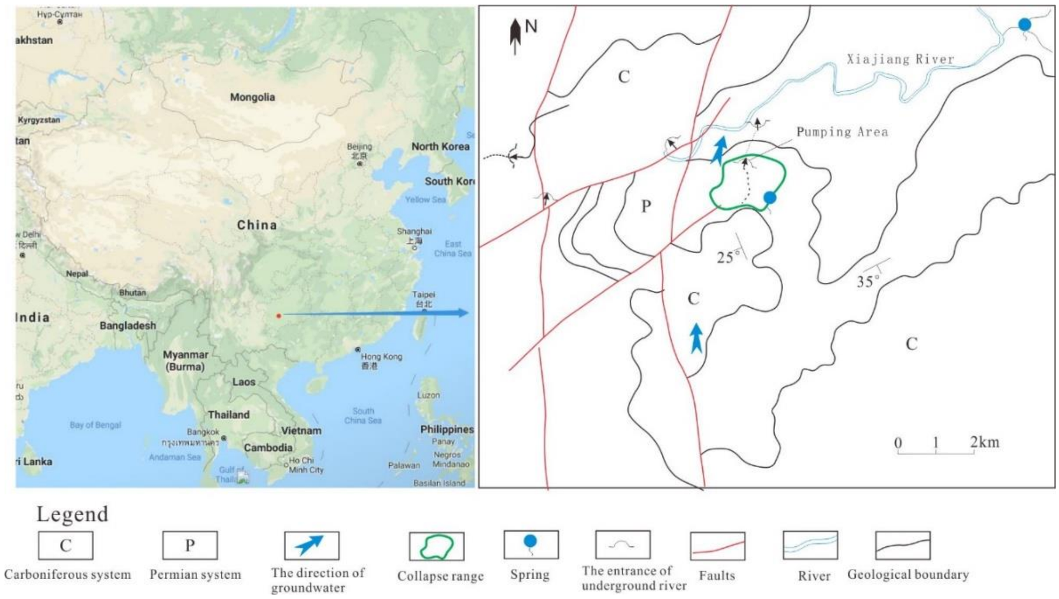

Fig. 2 Location and hydrogeological map of the study area

has enabled progress to be made in the risk assessment of karst collapse. However, each of these methods had its limitations in terms of practical application, mainly due to two aspects. The first is the choice of influencing factors and the second is the choice of methods. The Fisher discriminant construction is a linear function. The ANN needs a large amount of sample data and network parameters must be subjectively designed. A GIS requires high levels accuracy and a large quantity of data, which can prevent its use in evaluations. Therefore, for the analysis of the cause of karst collapses, the determination of the main controlling factors and the prediction of future developments and trends, many space-time variation factors must be comprehensively considered at the same time. Moreover, the geological environment has experienced a long history of evolution. The regional variations of various factors that can predict karst collapse have very complicated initial conditions, in addition to the heterogeneity of the geological body itself. The uncertainty and variability of human activities make it difficult to accurately determine such complex environmental conditions. The huge data acquisition required, the difficulty of determining the initial and boundary conditions of the geological environment, and the huge computational complexity has limited the application of quantitative prediction theory in studies of karst collapse.

In recognition of these problems, this study attempted to determine the main factors controlling karst collapse in a typical karst area. The aim of the study was to enable appropriate methods to be used to evaluate the risk of a karst collapse and then take effective measures to prevent it. In this study, the causes of a collapse were analyzed and the controlling factors affecting the collapse were identified. Human factors were not included because the influence of human factors is objectively reflected by natural factors. Furthermore, the evaluation of karst collapse is a decision system with nonlinear characteristics influenced by multiple factors. In this system, the controlling factors of the karst collapse are initially known but are not independent. There is a relationship between them that is not clear. Hence, it is a grey relation. The fuzzy analytic hierarchy process (FAHP) is one of the most popular multicriteria-decision making methods (Lu. 2002, Kahraman *et al*. 2004, Gao *et al*. 2012, Isalou *et al*. 2013). It can deal with fuzzy and uncertain decision-making problems, as well as having the advantages of being easy to use and having a superior flexibility compared to AHP. Moreover, it makes up for the shortcoming of a grey relational analysis (GRA) in calculating weights. The GRA method is a very active branch of grey system theory. Its basic concept is to determine whether the relationship between different sequences is close according to the geometric shape of a sequence of curves. The method is based on the grey relational analysis model proposed by Professor Deng Julong (Deng 1989, Wei *et al*. 2010, Yuan *et al*. 2016). To summarize, the study developed a risk assessment model using the FAHP and GRA methods. Its effectiveness was verified by applying it to a village in Guizhou, China.

2. Study area

The karst collapse area investigated in this study is in a karst depression in the Jiaobai Village, Guizhou Province, China. The depression is about 1.1 km northwest of the village, with an area of about 0.77 km^2. The depression is flat and is used as farmland by the villagers. Because of a drought in the summer of 2011, the residents pumped groundwater to irrigate cultivated land and for use as a domestic water supply, resulting in many collapse points in the cultivated land, as shown in Fig. 1.

The collapse area is a landform prone to soluble depression and has an elevation of 930~940 m. The terrain is flat, but the surrounding area is mountainous. The height difference between the depression and the surrounding mountains is about 180 m and the slope is 20~50°. The Quaternary deposits in the collapse area form a double-layer structure. The upper part is cultivated soil, with a thickness of 0~0.3 m, while the lower part is silty clay and laterite, with a thickness of 0.5~2 m, and the underlying bedrock is a medium thickness Carboniferous limestone, with a tendency to the south and dip angle of about 30~40°. The collapse area has a complicated geological structure. In the west of the depression there are four faults running in a westerly direction for 0.5~5 km. The development of the regional joints and fractures that are shown in Fig. 2 has formed two groups of faults running in the north-south and east-west directions.

3. Methods

The recent results of fuzzy mathematics research were used to determine the index weight. The principle was to extend the AHP to the fuzzy environment and therefore conduct a fuzzy analytic hierarchy process (FAHP) (Naghadehi *et al*. 2009). The grey system can solve the uncertainties that arise from the use of incomplete information, because it refers to a system where part of the information is known and part of the information is unknown. (Deng. 1989, Kuo *et al*. 2008). Given the nonlinear complex system of a karst collapse, the spatial conditions, overburden conditions, and hydrodynamic conditions have been determined. However, the degree of closeness between these factors is uncertain. A karst collapse risk assessment is considered to be a grey system and the grey incidence evaluation method can be used to evaluate the risk. The application of GRA can effectively deal with the uncertain relationship between various factors and is an effective solution. Finally, a karst collapse evaluation model for Guizhou was established in combination with the FAHP and the GRA.

3.1 Fuzzy analytic hierarchy process

3.1.1 Fuzzy complementary judgment matrix

In the FAHP, a comparison between two factors is expressed quantitatively by the importance of one factor over the other. The fuzzy judgment matrix $A = (a_{ij})n \times n$ is obtained if it has the following properties.

$$a_{ii} = 0.5, i = 1,2, \dots, n.$$

$$a_{ij} + a_{ji} = 1, i, j = 1,2, \dots, n.$$

This judgment matrix is a fuzzy complementary judgment matrix.

A quantitative description of the relative importance of a criterion is usually given by a 0.1~0.9 scale method. $a_{ii} = 0.5$ indicates that factors are equally important compared to themselves; if $a_{ij} \in [0.1,0.5)$, the factor x_j is more important than x_i; if $a_{ij} \in (0.5,0.9]$, the factor x_i is more important than x_j. In this study, the fuzzy complementary judgment matrix of the criterion hierarchy was established based on the above principles using two or more expert scoring methods.

3.1.2 The weighting process

A practical formula is used to determine the weight of the fuzzy complementary judgment matrix. The formula can reflect the fuzzy consistency judgment matrix and its judgment information, meanwhile it is convenient and quick to calculate the weight. The formula is as follows.

$$w_i = \frac{\sum_{i=1}^{n} a_{ij} - 1 + \frac{n}{2}}{n(n-1)}, i = 1,2, \dots, n. \tag{1}$$

3.1.3 Consistency judgment

The consistency of the matrix should be checked to ensure the rationality of the weights. The principle of consistency is tested in the following ways.

Definition 1, the set matrices$A = (a_{ij})n \times n$ and $B = (b_{ij})n \times n$ are fuzzy judgment matrices. $I(A,\ B)$ is the consistency index of A and B.

$$w_i = \frac{\sum_{i=1}^{n} a_{ij} - 1 + \frac{n}{2}}{n(n-1)}, i = 1,2, \dots, n. \tag{2}$$

Definition 2, set $\mathrm{W} = (W_1, W_2, \dots W_n)^T$ is the weight vector of the fuzzy judgement matrix A, where $\sum_{i=1}^{n} W_i = 1, W_i \geq 0 (i = 1,2, \dots n)$ and $W_{ij} = \frac{W_i}{W_i + W_j} (i, j = 1,2, \dots n)$, which is the n-matrix. $W^* = (W_{ij})n \times n$ is the eigenmatrix of the matrix A. For a decision maker, when the consistency index $I(A,\ W^*) \leq t$, it is considered that the judgment matrix is satisfactory. At the same time, the smaller the t, the higher the consistency of the fuzzy judgment matrix. According to previous research, t = 0.1 was considered reasonable in this study (Deng 1989, Kuo *et al.* 2008).

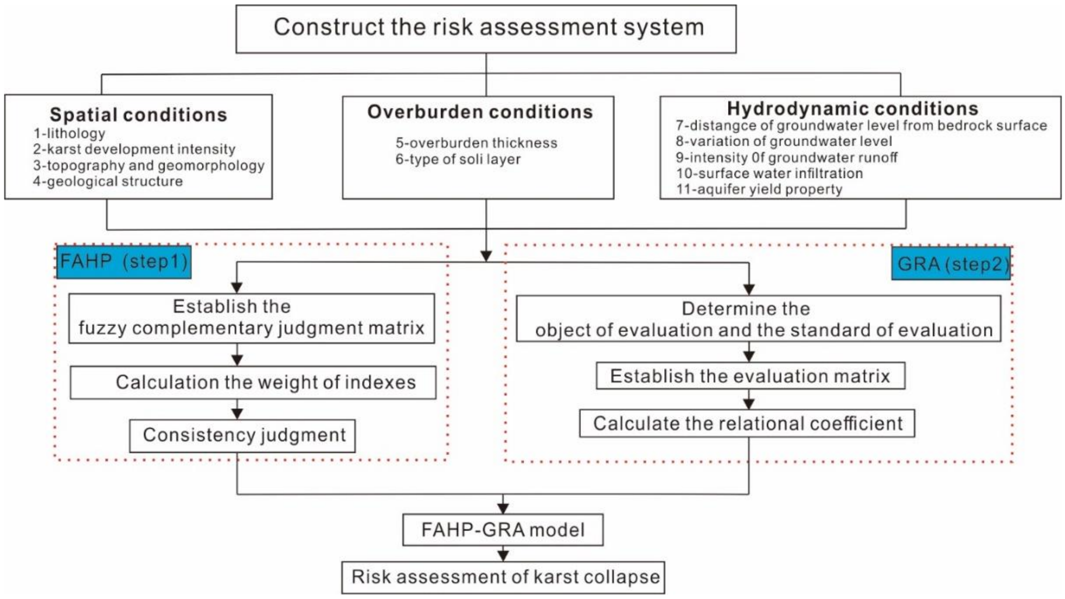

Fig. 3 Flowchart of the methodology used in this study

3.2 Grey relational analysis

3.2.1 The object of evaluation and the standard of evaluation

The reference sequence (evaluation standard) is $x_j^0 = \{x_j^0(1), x_j^0(2), \ldots, x_j^0(n)\}$. The comparison sequence (evaluation object) is $x_i^0 = \{x_i^0(1), x_i^0(2), \ldots, x_i^0(n)\}$. In the determination of the comparative sequence, the measured value of the index is used to determine the quantitative indices. The quantified qualitative indices are determined by the actual situation of the study area, and are based on the above definition to establish classification criteria for the qualitative indices. The evaluation matrix is dimensionless by the initial value, which forms the following matrix.

$$\begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{mn} \end{bmatrix}$$

The dimensionless treatment is conducted by comparing the quantized value with the average value of the standard value of each level. The formula is as follows.

$$x(k) = \frac{x^0(k)}{\frac{1}{n}\sum_{j=1}^{n} x_j^0(k)} \tag{3}$$

where $x(k)$ is the dimensionless value of the sample to be evaluated; $x^0(k)$ is the actual value of each evaluation factor for the sample to be evaluated; and $x_j^0(k)$ is standard value of each evaluation index in the evaluation classification standard; $k = 1,2 \ldots n$.

The formula for calculating the relational coefficient is as follows.

$$\xi_i(k) = \frac{\min min|x_i(k)-x_j(k)|+\rho maxmax|x_i(k)-x_j(k)|}{|x_i(k)-x_j(k)|+\rho maxmax|x_i(k)-x_j(k)|} \tag{4}$$

where $\xi_i(k)$ is the relational coefficient; ρ is distinguishing coefficient; Normally, $\rho = 0.5$; $k = 1,2, \ldots, m$; $k = 1,2 \ldots n$.

3.2.2 Grey weighted relational degree

Considering the difference in weights between factors the grey weighted relational degree is calculated based on the weight of each index determined by FAHP.

$$r_i = \sum_{k=1}^{n} w_k \xi_i(k) \tag{5}$$

where w_k is used to determine the weight of each impact index by FAHP and r_i is the grey weighted relational degree of the evaluation object to the ideal object; $i = 1,2, \ldots, m$; $k = 1,2 \ldots n$.

A detailed flowchart of the methodology used in this study is shown in Fig. 3.

4. Determination of the controlling factors

Considering the formation conditions of a karst collapse, the main controlling factors were extracted as evaluation indices after a comprehensive analysis. These were: lithology (Li), karst development intensity (KDI), topography and geomorphology (TG), geological structure (GS), overburden thickness (OT), type of soil layer (TSL), distance of groundwater level from bedrock surface (DGL), variation of groundwater level (VGL), intensity of groundwater runoff (IGR), surface water infiltration (SWI), and aquifer yield property (AYP). It was considered that Li, KDI, TG, and GS could reflect the spatial conditions, while OT and TSL reflected overburden conditions, and, DGL, VGL, IGR, SWI, and AYP reflected hydrodynamic conditions.

4.1 Formation conditions

Three conditions must be simultaneously met for the formation of a karst collapse: spatial conditions (SC),

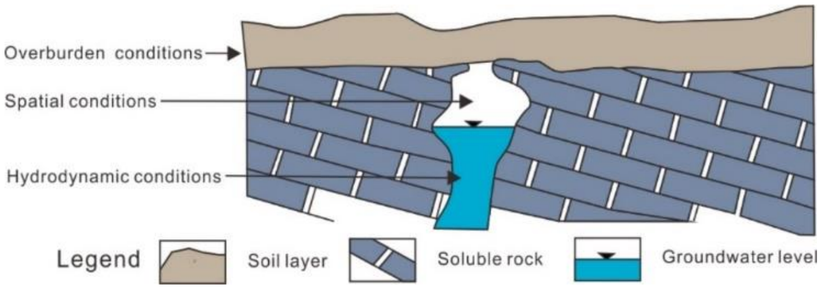

Fig. 4 Formation conditions of a karst collapse

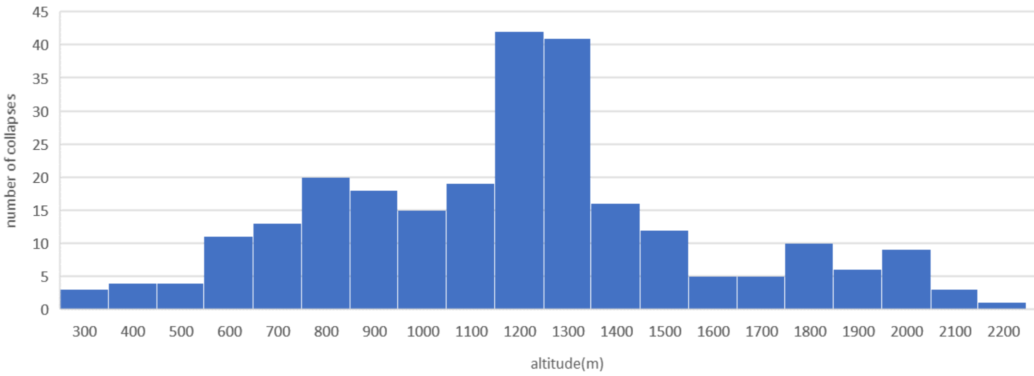

Fig. 5 Relationship between the distribution of karst collapses and terrain in Guizhou (Zhang *et al*. 2007)

overburden conditions (OC), and hydrodynamic conditions (HC), as shown in Fig. 4. Initially, karst caves or soil cavities provide storage places or channels for groundwater recharge, runoff, discharge, and collapse material. Furthermore, the bedrock is covered with a certain thickness of soil (or rock with poor integrity) and the soil can provide the anti-collapse force during the collapse process.

4.1.1 Spatial conditions

A karst landscape needs a long time and specific geological conditions to form, which restricts the development and distribution of cave-fissures. Generally, they are mainly distributed within the tectonic fracture zone, the extensional fracture zone along the fold axis, the distribution area of the thick layer of soluble rock, and the contact zone with non-soluble rock. The degree of development of karst features and caves are direct factors that determine the collapse of a karst ground surface. On the one hand, a karst cave and fissure can result in an incompleteness in the rock mass structure and form local instability. On the other hand, it provides sufficient spatial conditions for the movement of groundwater and collapsed materials. Normally, the more developed the karst, the better the opening of the caverns and the larger the scale of the caves, and therefore the more serious the karst collapse could be.

4.1.2 Overburden conditions

Karst collapse is a phenomenon caused by the rock and soil of a cave roof under the action of various collapse factors. It can be divided into two types in terms of roof composition. One is a roof that consists of the bedrock, which is called a rock collapse. The other is a soil layer collapse, which is mainly due to loose Quaternary sediments. The lithology and thickness of the collapsed roof determines the resistance ability of the collapsed roof. In southern China, the collapse of the soil layer accounts for 96.7% of the total number of karst collapses.

4.1.3 Hydrodynamic conditions

The groundwater dynamic conditions provide the force that causes a collapse. Groundwater runoff is concentrated and intense in the seepage field of a karst system. Sharp changes in groundwater level can produce a collapse in sensitive areas through actions such as reservoir storage (discharge), irrigation leakage, severe drought, mine drainage, and strong pumping.

4.2 Factors controlling a karst collapse

The main controlling factors of a karst collapse were determined based on the analysis of the formation conditions in combination with the geological investigation in Guizhou. The main natural factors can be summarized in terms of six aspects: topography and geomorphology, stratigraphic lithology, karst development, geological structure, the soil layer, and groundwater. An additional eleven factors were selected as evaluation indices.

4.2.1 Topography and geomorphology

In terms of the geomorphology, a karst collapse develops in karst depressions, karst valleys, soluble basins, karst residual hills, and plains of a solitary peak (Zhang *et al*. 2007). In such areas it is easy to form a depression, groundwater skylight, funnel, or ponor, all of which are beneficial to the collection of surface water and the recharge of groundwater. Peaks, marshland, valleys, and basins will influence the development density of karst collapses in different landforms. In terms of the topography, karst collapses are mostly distributed in low-lying areas and the

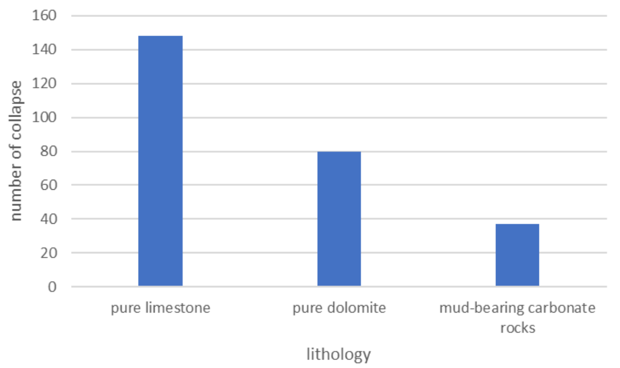

Fig. 6 Relationship between the distribution of karst collapses and lithology (Lei *et al*. 2002)

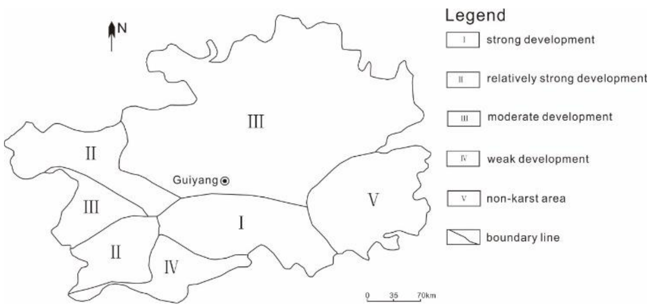

Fig. 7 Map of karst development intensity in Guizhou (Ding *et al.* 2017)

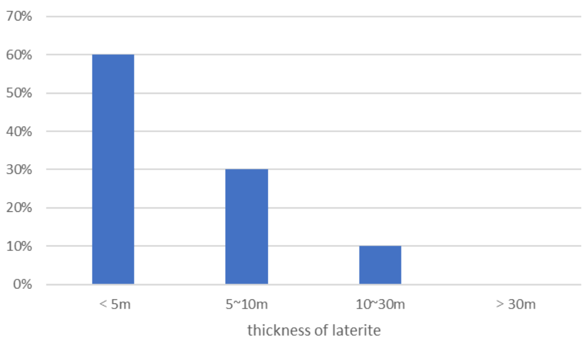

Fig. 8 The relationship between karst collapses and the thickness of laterite (Liao 2004)

transition zones between high and low terrain. The relationship between the number of karst collapses and altitude is shown in Fig. 5. The figure shows that most karst collapses occur at about 1200 m height above sea level. Karst collapses in Guizhou are mostly distributed in the range of 600~1500 m above sea level; hence, TG should be considered one of the controlling factors.

4.2.2 Stratigraphic lithology

Guizhou Province, which is located in the karst region of East Asia, has the most complex, most complete, and largest distribution of karst landforms in the region. Generally, the density of karst collapses is closely related to the degree of karst landform development, which in turn is dependent on lithology (Lei *et al.* 2002). The relationship between the number of karst collapses and the lithology of the stratum is shown in Fig. 6. It can be seen that pure limestone develops more karst collapses than other lithologies. This is because limestone provides an abundance of $CaCO_3$ for karst development. As a result, Li has an important role in karst collapses and should be considered one of the controlling factors.

4.2.3 Karst development

Karst landforms are widely distributed throughout Guizhou Province, with the area of soluble rock accounting for 69.1% of the total area (Duan *et al*. 2016). According to the lithologic characteristics of karst formation, the exposed area, and structural conditions, karst development can be divided into five regions: strong development, relatively strong development, moderate development, weak development, and non-karst area. Fig. 7 shows that in the different regions of Guizhou, the intensity of karst development is different. In the area where the groundwater alternates strongly, the strong karst development intensifies the alternation of groundwater, which leads to a stronger development of karst. The foundation of a karst collapse is the KDI, which should therefore be considered one of the controlling factors.

4.2.4 Geological structure

The geological structure controls the development of a karst collapse, with most collapses occurring in areas with gentle topography (Papadopoulou-Vrynioti *et al.* 2013). Karst collapses in Guizhou occurred in 193 locations in strata with dip angles less than 30° (72.8% of the total) and 72 locations in strata with dip angles less than 10° (27.2% of the total). Karst collapses are often distributed in faulted structural zones, especially inextensible and tension-torsional fault zones in soluble rocks. Due to rock fragmentation and the special development of fissures, these sites are beneficial for the storage and migration of groundwater. Therefore, GS should be considered one of the controlling factors.

4.2.5 Soil layer

Fig. 8 shows the relationship between the distribution of karst collapses and the thickness of the soil layer. It was found that the number of karst collapses decreased with an increase in soil thickness, and nearly 60% of karst collapses occurred in areas where the thickness of the soil layer was less than 5 m. About 30% of karst collapses occurred over an area of 5~10 m. They rarely occurred when the thickness of the soil layer was more than 30 m. Different types of soil have different properties in terms of their physical mechanics. Therefore, OT and TSL are closely related to karst collapses and are two essential controlling factors.

4.2.6 Groundwater

The influence of groundwater on karst collapse is mainly manifested in the burial conditions and dynamic changes of groundwater (He *et al.* 2003). The influence of groundwater burial conditions is mainly due to the depth of groundwater and the type of water-bearing medium. Karst collapse can easily occur in areas with a shallow groundwater depth. The type of water-bearing medium reflects the intensity of groundwater runoff. According to the spatial characteristics of the water-bearing medium, karst groundwater in Guizhou could be divided into three

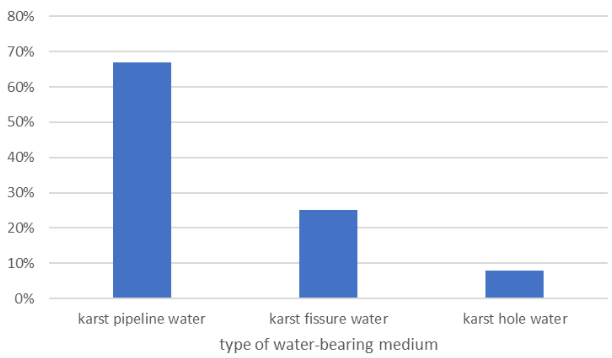

Fig. 9 Relationship between the type of water-bearing medium and karst collapses (Ding *et al.* 2017)

Table 1 Evaluation index and grade division

Control factors		Risk grade			
		Low (I)	Moderate (II)	High (III)	Very high (IV)
KC	LI	Argillaceous carbonate rock	Interbedded carbonate and clastic rocks	Dolomite	Limestone
	KDI	Weakly developed	Relatively developed	Developed	Extremely developed
	TG	Peak	Marsh land	Karst valley	Karst basin
	GS (Fault)	1	2	3	4
OC	OT (m)	50	30	10	5
	TSL	Clay	Laterite	Silty	Sandy
HC	DGL(m)	5	3	2	1
	VGL(m)	0.5	1	1.5	2
	IGR	Weak	Moderate	Strong	Very strong
	SWI	0.06	0.12	0.18	0.2
	AYP (L/s·m)	0.01	0.1	1	5

types: karst pipeline water, karst fissure water, and karst hole water. The relationship between the type of water-bearing medium and karst collapse is shown in Fig. 9.

Groundwater plays an important role in the whole collapse process, because it not only forms caves but also provides the collapse force in the later stage. Groundwater is therefore extremely important, with all five of DGL, VGL, IGR, SWI, and AYP being significant controlling factors.

4.3 Evaluation index and grade division

Based on the investigation and analysis of karst collapses in Guizhou Province, the risk of karst collapse could be divided into four grades: low, moderate, high, and very high. The evaluation indices and classification criteria are shown in Table 1, and refer to industry specifications, empirical values, and existing research results (Duan *et al.* 2016, Dai *et al.* 2010, Ding *et al.* 2017). For example, according to the results of this investigation, about 90% of karst collapses occurred in areas with a soil layer of less than 10 m, and only 10% occurred in areas where the depth of the soil layer was 10~30 m. Very few collapses occurred in areas where the depth of the soil layer was more than 30 m. On this basis, the thickness of the soil layer could be divided into four grades of 30~50, 10~30, 10~5, and <5 m. Eventually, the corresponding relationship between quantitative or qualitative influencing factors and the risk grade of a karst collapse was established.

Unlike the quantitative index, there was no definite numerical interval for each grade in the qualitative index and the dividing line was not clear. Therefore, the fuzzy index of the qualitative index was evaluated by the classification method. According to certain criteria, the indices were classified into four grades: excellent, good, bad, and very bad, and the corresponding grade scores were 0.1 (I), 0.40 (II), 0.70 (III), and 1.0 (IV). The higher the score, the greater the risk. Then, according to the actual situation in the evaluation area, the corresponding indices were quantified according to the grade score.

5. Results and discussion

5.1 Results

According to the analysis of the main controlling factors affecting karst collapses, this research project could be expressed as a model with three levels. The evaluation of a karst collapse was the ultimate outcome, and was considered the target layer of the model (level A). The spatial, overburden, and hydrodynamic conditions determined the potential for a collapse, but the mode of influence of these specific factors had to be evaluated and this was considered an intermediate link to solve the problem, i.e., the criterion layer of the model (level B). Each index of the main controlling factors constitutes the decision level (level C) of the model. Through the evaluation of the problem at this level, the required solution is finally achieved as a model outcome.

In this step, the weight of the criterion hierarchy was determined. For the spatial, overburden, and hydrodynamic conditions, the weighted fuzzy complementary judgment matrix was obtained by comparing and judging each factor by two (or more) field experts. Let the weighted fuzzy complementary judgment matrix given by the expert be B.

$$B_1 = \begin{bmatrix} 0.5 & 0.6 & 0.6 \\ 0.4 & 0.5 & 0.4 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}$$

According to Eq. (1), the calculated weight vector is $W_1 = [0.3667 \ \ 0.3 \ \ 0.3333]$.

The eigenmatrix of the judgment matrix B_1 is

$$W_1^* = \begin{bmatrix} 0.5 & 0.55 & 0.5239 \\ 0.45 & 0.5 & 0.4737 \\ 0.4761 & 0.5263 & 0.5 \end{bmatrix}$$

According to Eq. (2), the compatibility of B_1 and W_1^* is $I(A_1, W_1^*) = 0.09 \le 0.1$. Then, the fuzzy judgment matrix B_1 is satisfactory and consistent. Moreover, the distribution of the weight set is reasonable. Using the same method calculates the weight of the decision level at the same time. Table 2 gives the weight of each index.

5.1.1 Dimensionless data

The comparison sequence is the numerical value of the

Table 2 Calculated index weights

Target hierarchy	Criterion hierarchy	Weight	Consistency test	Index hierarchy	Weight	Total weight	Rank	Consistency test
Risk assessment of karst collapse	SC	0.3667	0.09	Li	0.2667	0.0978	3	0.096
				TG	0.2333	0.0856	6	
				KDI	0.2583	0.0947	4	
				GS	0.2417	0.0886	5	
	OC	0.3		OT	0.55	0.165	1	0.075
				TSL	0.45	0.135	2	
	HC	0.3333		DGL	0.185	0.0617	10	0.097
				VGL	0.24	0.0800	7	
				IGR	0.205	0.0683	8	
				WI	0.19	0.0633	9	
				AYP	0.18	0.0600	11	

indices and the reference sequence is the grade standard. According to Eq. (3), the dimensionless data can be calculated as follows:

$$x_1 = \begin{bmatrix} 0.0418 & 0.1674 & 0.2929 & 0.4184 & 0.4184 \\ 0.0418 & 0.1674 & 0.2929 & 0.4184 & 0.1674 \\ 0.4184 & 1.6736 & 2.9289 & 4.1841 & 2.5105 \\ 0.4840 & 0.8368 & 1.2552 & 1.6736 & 1.6736 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 4.9068 & 2.9441 & 0.9814 & 0.4907 & 0.3827 \\ 0.0098 & 0.0393 & 0.0667 & 0.0981 & 0.0785 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 4.182 & 2.5092 & 1.6728 & 0.8364 & 1.7564 \\ 0.4182 & 0.8364 & 1.2546 & 1.6728 & 1.5055 \\ 0.0836 & 0.3346 & 0.5855 & 0.8364 & 0.5855 \\ 0.0502 & 0.1004 & 0.1506 & 0.1840 & 0.1673 \\ 0.0084 & 0.0836 & 0.8364 & 4.1820 & 0.1673 \end{bmatrix}$$

5.1.2 Relational coefficient

According to Eq. (4), the relational coefficient can be calculated as follows

$$\xi_1 = \begin{bmatrix} 0.7360 & 0.8065 & 0.8929 & 1.0000 \\ 0.8928 & 1.0000 & 0.8929 & 0.8065 \\ 0.3333 & 0.5555 & 0.7143 & 0.3846 \\ 0.4679 & 0.5556 & 0.7143 & 1.0000 \end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} 0.3351 & 0.4714 & 0.7948 & 0.9594 \\ 0.9756 & 0.9881 & 1.0000 & 09966 \end{bmatrix}$$

$$\xi_3 = \begin{bmatrix} 0.4528 & 0.7273 & 0.9600 & 0.6857 \\ 0.6486 & 0.7500 & 0.8889 & 0.9426 \\ 0.8000 & 0.8889 & 1.0000 & 0.8889 \\ 0.9449 & 0.9677 & 0.9917 & 0.9917 \\ 0.9266 & 0.9600 & 0.7500 & 0.3333 \end{bmatrix}$$

5.1.3 Grey weighted relational degree

Combined with the weight of each index determined by FAHP, the weighted relational degree was calculated using Eq. (5),

$$r_1 = [0.2667 \;\; 0.2333 \;\; 0.2583 \;\; 0.2417] \times \begin{bmatrix} 0.7360 & 0.8065 & 0.8929 & 1.0000 \\ 0.8928 & 1.0000 & 0.8929 & 0.8065 \\ 0.3333 & 0.5555 & 0.7143 & 0.3846 \\ 0.4679 & 0.5556 & 0.7143 & 1.0000 \end{bmatrix} [0.6040 \;\; 0.7262 \;\; 0.8036 \;\; 0.7959]$$

In addition,

$$r_2 = [0.6233 \;\; 0.7039 \;\; 0.8871 \;\; 0.9761],$$

$$r_3 = [0.7498 \;\; 0.8534 \;\; 0.9194 \;\; 0.7837].$$

According to Eq (5), the relational degree of karst collapse in the study area is

$$r = [0.3667 \;\; 0.3 \;\; 0.3333] \times \begin{bmatrix} 0.6040 & 0.7262 & 0.8036 & 0.7959 \\ 0.6233 & 0.7039 & 0.8871 & 0.9761 \\ 0.7498 & 0.8534 & 0.9194 & 0.7837 \end{bmatrix} = [0.6584 \;\; 0.7619 \;\; 0.8672 \;\; 0.8459]$$

5.1.4 Model results

The model results showed that for the spatial conditions the maximum grey weighted relational degree was $r_{1max} = 0.8036$, which indicated a high risk grade. In the same way, $r_2 = [0.6233 \;\; 0.7039 \;\; 0.8871 \;\; 0.9761]$, the maximum grey weighted relational degree was $r_{2max} = 0.9761$, which indicated that the risk grade for the overburden conditions was very high, and $r_3 = [0.7498 \;\; 0.8534 \;\; 0.9194 \;\; 0.7837]$, the maximum grey weighted relational degree was $r_{3max} = 0.9194$, which indicated that the risk grade for the hydrodynamic conditions was high. The result of the comprehensive evaluation was $r_{max} = 0.8672$. Therefore, the risk grade of a karst collapse in the study area was high.

According to these results, the risk grade in the karst collapse area was high, indicating that collapses were likely. The results of the calculation indicated that the risk in this area was close to grade IV, indicating that the potential for a karst collapse was high. In terms of single factors, many were determined to be grade IV. Considering the relational degree of each single condition, the weight of the overburden conditions was lower than that of the spatial and hydrodynamic conditions, but the relational degree of each condition was high, extending to grade IV. This shows that the overburden conditions in this area were poor, which was related to the geographical location of the study area. Because the study area was located in a depression and laterite area with a binary soil structure, the soil layer was very thin.

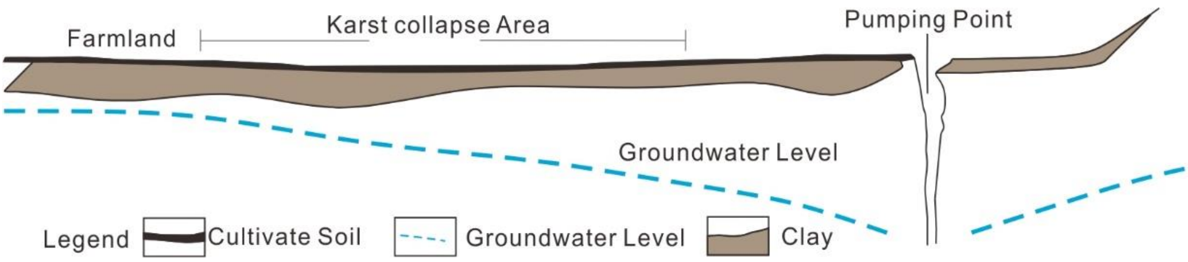

Fig. 10 Karst collapse profile

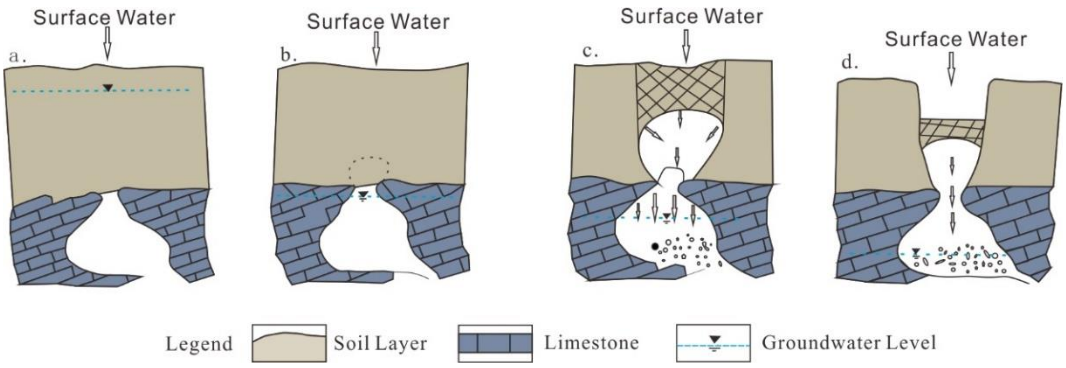

Fig. 11 The karst collapse formation process

5.1.5 Collapse mechanism in the study area

The collapse area is a karst depression and the topography is relatively flat. The underground karst pipeline is therefore more developed and the groundwater depth is relatively shallow in this area. As shown in Fig. 10, during the drought period, the groundwater level decreases because the local residents frequently pump groundwater. However, pumping groundwater leads to an increase in the hydraulic gradient and flow velocity of groundwater. Therefore, under the action of seepage force, soil particles are removed in the loose overburden layer and the cave fissure. Gradual erosion and emptying leads to the embryonic form of a karst cave developing. The long-term fluctuation of the groundwater level controls this process.

A greater abundance of groundwater will result in faster water flows. The hydraulic slope will then increase. This phenomenon could result in a greater probability of soil cavity formation and increase the potential for collapses in the contact zone between rock and soil. The pumping of groundwater artificially changes the hydrogeological conditions in the contact zone. A greater hydraulic slope is formed near the pumping well, which strengthens the potential erosion by groundwater in the soil layer.

Fig. 11 shows the formation process of a karst collapse. In Fig. 11 (a, b), with the formation of soil caves, the groundwater level fluctuates with surface water infiltration and the pumping of groundwater. The reasons for this are as follows. (1) The upper soil layer is continuously washed away, dispersed, and eventually completely removed by groundwater. (2) With groundwater pumping for surface irrigation, the groundwater level will decrease. The surface irrigation constantly recharges into the ground from the surface. Therefore, the submersible erosion of the capped soil is strong. In addition, the decrease in the groundwater level gradually produces a vacuum with negative pressure. The volume of the soil cave increases under the combined effect of the external pressure and internal suction on the capped soil as shown in Fig. 11(c). The collapse force eventually exceeds the anti-collapse force. Figure 11(d) shows that the capped soil is unstable. The whole formation process can be completely explained by the functions of the space, overburden, and hydrodynamic conditions.

5.2 Discussion

In previous studies of karst collapses, several mathematical models have been developed to conduct risk assessments. However, each of these methods has its own limitations in terms of their practical application, mainly for two reasons. Most studies only considered the local study area when considering the controlling factors. This has resulted in some potential key factors being overlooked, limiting the application of these studies. At the same time, some quantitative evaluation methods cannot adequately reflect the relationships between the control factors. To better reflect the mechanisms involved and assess the risk of a karst collapse, appropriate controlling factors and effective methods need to be considered.

In this study, the controlling factors of a karst collapse were determined by a geological investigation and statistical data. A number of factors play a significant role in determining whether and how a karst collapse happens. Based on the results obtained in this study, together with a comprehensive analysis of a number of records of karst collapse, the controlling factors were analyzed and their formation conditions were summarized as follows.

First, it is necessary to identify the topography and geomorphology to determine the development of surface karst and the intensity of karst collapses. Specific topography is not only conducive to surface water collection, but also the recharge of groundwater. In mountainous areas, with strong geological activity, the rock is broken and the hydraulic slope is large, enabling karst landforms to develop. Therefore, particular attention needs to be given to these areas.

Second, stratigraphic lithology also plays an important role. Rocks, especially pure limestone, provide the material basis of karst development.

Third, karst development needs to be considered. The intensity of karst development determines the density of collapse development, and karst development provides the spatial conditions for collapse formation. In a strong karst development area, underground karst pipelines and caves are also extremely well developed. This provides the basis of the two-layer spatial structure of a karst collapse.

Fourth, the geological structure plays a critical role in karst development. If the geological structure is not well developed and the rock structure is intact, the degree of karst development will be reduced.

Fifth, the soil layer could have a significant effect. Different types and thicknesses of soil have different physical and mechanical properties. The soil layer essentially functions as a protective layer, i.e., it provides the anti-collapse force. The thicker the soil layer, the greater the anti-collapse force, and the smaller the soil permeability coefficient, the lower the hydraulic relation between surface water and groundwater.

The sixth factor is groundwater. On one hand, in a karst area dominated by pipeline flow, the groundwater runoff is strong and the amplitude of the groundwater dynamic variation is large, with changes in the water level often being intense. With a sudden rise and fall in groundwater levels, it is relatively easy for a karst collapse to occur. On the other hand, in a karst area dominated by fissure flow, the groundwater has a relatively uniform water volume and level, the groundwater is restricted by the water-bearing space, the recharge and runoff are relatively slow, and it is not easy to produce a karst collapse. In the karst groundwater recharge area, the groundwater depth is generally small, and the groundwater dynamics are strongly affected by atmospheric precipitation, especially in the region dominated by pipeline flow. The soil layer is therefore prone to subsurface erosion and the formation of soil holes. In contrast, in the drainage area, especially the canyon area, although the groundwater runoff is strong, a karst collapse will not develop due to the large groundwater depth. Groundwater plays an important role in the formation of a karst collapse, with groundwater providing the force that causes a collapse.

Importantly, FAHP and GRA will not only enable the risk assessment of a karst collapse to be conducted, but they also indicate the effect and influence of various controlling factors. Table 2 shows how each of these factors plays a decisive role in the formation of a karst collapse according to the value of their weight in the analysis, and indicates that the spatial and hydrodynamic conditions are more important than the overburden conditions. From the analysis of the formation process of a karst collapse, the spatial conditions represent the initiation of the collapse, while the hydrodynamic conditions are the dynamic conditions of the late collapse. The weight of both conditions was slightly higher than the overburden conditions, and the overall weight distribution was considered reasonable. The weight of each controlling factor is shown in Table 2. It was found that the OT and TSL contributed greatly to the formation of a collapse because they played a protective role. At the same time, the weights of other controlling factors were roughly equal, which indicated that these factors play important roles in the formation of a collapse. These factors were classed among the material and dynamic conditions for the formation of a collapse, which indicates that the contribution of these control factors was the same and was indispensable in the early and late stages of a collapse. From the distribution of the overall weights, there is a need to focus on improving the strength of the overburden conditions to achieve collapse prevention.

6. Conclusions

A risk assessment model was developed to study karst collapses based on an analysis of the formation conditions of karst collapses and the actual geological characteristics of Guizhou Province. Before constructing the evaluation index to apply to an FAHP and GRA model, it was crucial to determine the controlling factors. A geological investigation and statistical data collection were necessary to determine and analyze the controlling factors. A risk evaluation model and classification standard for karst collapses were established quantitative indices. A case study was conducted to examine the applicability of a quantitative model of karst collapses. Several issues were investigated regarding the formation of karst collapses and how to quantitatively evaluate these hazardous geological events.

FAHP is one of the most popular multi-criteria decision-making methods and provides a convenient method for calculating the weight of selected factors. GRA is a very active branch of grey system theory, which can effectively deal with the uncertain relationship between various factors. Hence, the FAHP-GRA model was suitable for karst collapse assessments and could solve the complicated geological phenomenon of multi-factorial influences. In this study, the selected controlling factors of a collapse were considered reasonable. Moreover, FAHP-GRA has a strong potential for application. The model is practicable for risk assessments, while further studies of how to forecast a karst collapse are expected.

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References

Dai, J. and Lei, M. (2018), “Standard guide for karst collapse investigation and its technical essential”, *Environ. Earth Sci.*, **77**(4), 133. https://doi.org/10.1007/s12665-018-7288-7.

Deng, J.L. (1989), *Introduction To Grey System Theory*, Huazhong University of Science and Technology Press, Wuhan, China.

Ding, J.P., Chu, X.W., Duan, X.Q., Liang, F. and Wang, Z.M. (2017), *Karst Collapse in Guizhou*, Geological Publishing House, Beijing, China.

Duan, X.Q., Chu, X.W. and Bo, L. (2016), "Risk prediction and evaluation of the karst collapse based on the set pair mechanism analysis", *J. Safety Environ.*, **16**(4), 72-76. https://doi.org/ 10.13637/j.issn.1009-6094.2016.04.014.

Farrant, A.R. and Cooper, A.H. (2008), "Karst geohazards in the UK: The use of digital data for hazard management", *Quart. J. Eng. Geol. Hydrogeol.*, **41**(3), 339-356. https://doi.org/ 10.1144/1470-9236/07-201.

Galve, J.P., Gutiérrez, F., Lucha, P., Guerrero, J., Bonachea, J., Remondo, J. and Cendrero, A. (2009), "Probabilistic sinkhole modelling for hazard assessment", *Earth Surface Proc. Landforms*, **34**(3), 437-452. https://doi.org/ 10.1002/esp.1753.

Galve, J.P., Gutiérrez, F., Remondo, J., Bonachea, J., Lucha, P. and Cendrero, A. (2009), "Evaluating and comparing methods of sinkhole susceptibility mapping in the Ebro valley evaporite karst (NE Spain)", *Geomorphology*, **111**(3), 160-172. https://doi.org/ 10.1016/j.geomorph.2009.04.017.

Gao, L. and Hailu, A. (2012), "Ranking management strategies with complex outcomes: an AHP-fuzzy evaluation of recreational fishing using an integrated agent-based model of a coral reef ecosystem", *Environ. Model. Software*, **31**(7), 3-18. https://doi.org/ 10.1016/j.envsoft.2011.12.002.

Gómez-Ortiz, D. and Martín-Crespo, T. (2012), "Assessing the risk of subsidence of a sinkhole collapse using ground penetrating radar and electrical resistivity tomography", *Eng. Geol.*, **149**(4), 1-12. https://doi.org/ 10.1016/j.enggeo.2012.07.022.

He, K.Q., Du, R.L. and Jiang, W.F. (2010), "Contrastive analysis of karst collapses and the distribution rules in northern and southern China", *Environ. Earth Sci.*, **59**(6), 1309-1318. https://doi.org/ 10.1007/s12665-009-0118-1.

He, K.Q., Liu, C.L. and Wang, S.J. (2003), "Karst collapse related to over-pumping and a criterion for its stability", *Environ. Geol.*, **43**(6), 720-724. https://doi.org/ 10.1007/s00254-002-0669-x.

He, S.N., Zhou, G.Q., Li, Y.F. and Chen, K.H. (2014), "The studying on relationship between collapse points and geological factors based on GIS: a case study of Laibin, Jili in Guangxi", *Geomat. Spatial Inform. Technol.*, **37**(02), 38-42. (in Chinese)

Isalou, A.A., Zamani, V. Shahmoradi, B. and Alizadeh, H. (2013), "Landfill site selection using integrated fuzzy logic and analytic network process (F-ANP)", *Environ. Earth Sci.*, **68**(6), 1745-1755. https://doi.org/ 10.1007/s12665-012-1865-y.

Jiang, X.Z. (2003), "Application of GIS to the assessment of surfaces collapse hazards in china", *Acta Geosicientia Sinica*, **24**(5), 469-473 (in Chinese).

Jiang, X.Z., Lei, M.T. and Guan, Z.D. (2012), "Characterization criteria of karst collapse hazard on groundwater fluctuations in Qingyun village, Guigang, Guangxi, China", *Chin. J. Undergr. Sp. Eng.*, **72**(1), 43-43 (in Chinese).

Kahraman, C., Cebeci, U. and Ruan, D. (2004), "Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey", *Int. J. Prod. Econ.*, **87**(2), 171-184. https://doi.org/ 10.1016/S0925-5273(03)00099-9.

Kaufmann, G. (2014), "Geophysical mapping of solution and collapse sinkholes", *J. Appl. Geophys.*, **111**, 271-288. https://doi.org/10.1016/j.jappgeo.2014.10.011.

Kaufmann, G., Romanov, D., Tippelt, T., Vienken, T., Werban, U., Dietrich, P., Mai, F. and Boerner, F. (2018), "Mapping and modelling of collapse sinkholes in soluble rock: The Munsterdorf site, northern Germany", *J. Appl. Geophys.*, **154**, 64-80. https://doi.org/10.1016/j.jappgeo.2018.04.021.

Kim, K.D., Lee, S. and Oh, H.J. (2009), "Prediction of ground subsidence in Samcheok City, Korea using artificial neural networks and GIS", *Environ. Geol.*, **58**(1), 61-70. https://doi.org/ 10.1007/s00254-008-1492-9.

Kuo Y.Y., Yang T.H. and Huang G.W. (2008), "The use of grey relational analysis in solving multiple attribute decision-making problems", *Comput. Indust. Eng.*, **55**(1), 80-93. https://doi.org/ 10.1016/j.cie.2007.12.002.

Lei, M.T., Jiang, X.Z. and Yu, L. (2002), "New advances in karst collapse research in china", *Environ. Geol.*, **42**(5), https://doi.org/462-468. 10.1007/s00254-001-0506-7.

Liao, Y.L. (2004), *Carbonate Laterite in Guizhou*, Guizhou People's Publishing House, Guiyang, China.

Lin, P., Li, S.C., Xu, Z.H., Huang, X., Pang, D.D., Wang, X.T. and Wang, J. (2018), "Location determining method of critical sliding surface of fillings in a karst cave of tunnel", *Geomech. Eng.*, **16**(4), 415-421. https://doi.org/10.12989/gae.2018.16.4.415.

Lu, Y.J. (2002), "Weight calculation method of fuzzy analytical hierarchy process", *Fuzzy Syst. Math.*, **2002**(02), 79-85. (in Chinese)

Naghadehi, M.Z., Mikaeil, R. and Ataei, M. (2009), "The application of fuzzy analytic hierarchy process (FAHP) approach to selection of optimum underground mining method for Jajarm Bauxite Mine, Iran", *Expert Syst. Appl.*, **36**(4), 8218-8226. https://doi.org/10.1016/j.eswa.2008.10.006.

Oh, H.J. and Lee, S. (2010), "Assessment of ground subsidence using GIS and the weights-of-evidence model", *Eng. Geol.*, **115**(1), 36-48. https://doi.org/10.1016/j.enggeo.2010.06.015.

Papadopoulou-Vrynioti, K., Bathrellos, G.D., Skilodimou, H.D., Kaviris, G. and Makropoulos, K. (2013), "Karst collapse susceptibility mapping considering peak ground acceleration in a rapidly growing urban area", *Eng. Geol.*, **158**(8), 77-88. https://doi.org/77-88. 10.1016/j.enggeo.2013.02.009.

Taheri, K., Gutierrez, F., Mohseni, H., Raeisi, E. and Taheri, M. (2015), "Sinkhole susceptibility mapping using the analytical hierarchy process (AHP) and magnitude–frequency relationships: A case study in Hamadan province, Iran", *Geomorphology*, **234**, 64-79. https://doi.org/10.1016/j.geomorph.2015.01.005.

Wei, G.W. (2010), "A method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting", *Knowledge-Based Syst.*, **23**(3), 243-247. https://doi.org/ 10.1016/j.knosys.2010.01.003.

Yuan, Y.C., Li, S.C., Zhang, Q.Q., Li, L.P., Shi, S.S. and Zhou, Z.Q. (2016), "Risk assessment of water inrush in karst tunnels based on a modified grey evaluation model: Sample as Shangjiawan Tunnel", *Geomech. Eng.*, **11**(4), 493-513. https://doi.org/10.12989/gae.2016.11.4.493.

Zhang, L.F., Zeng, X.S., Yao, Y.S. and Liao, W.L. (2007), "Review on karst collapse in china", *Chin. J. Geol. Hazard Control*, **18**(3), 126-130. https://doi.org/10.16031/j.cnki.issn.1003-8035.2007.03.022.

Zhou, Z.Q., Li, S.C., Li, L.P., Shi, S.S. and Xu, Z.H. (2015), "An optimal classification method for risk assessment of water inrush in karst tunnels based on grey system theory", *Geomech. Eng.*, **8**(5), 631-647. https://doi.org/10.12989/gae.2015.8.5.631.

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