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The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate

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Abstract. In this research, a simple quasi 3D hyperbolic shear deformation model is employed for bending and dynamic behavior of functionally graded (FG) plates resting on visco-Pasternak foundations. The important feature of this theory is that, it includes the thickness stretching effect with considering only 4 unknowns, which less than what is used in the First Order Shear Deformation (FSDT) theory. The visco-Pasternak's foundation is taken into account by adding the influence of damping to the usual foundation model which characterized by the linear Winkler's modulus and Pasternak's foundation modulus. The equations of motion for thick FG plates are obtained in the Hamilton principle. Analytical solutions for the bending and dynamic analysis are determined for simply supported plates resting on visco-Pasternak foundations. Some numerical results are presented to indicate the effects of material index, elastic foundation type, and damping coefficient of the foundation, on the bending and dynamic behavior of rectangular FG plates.

Keywords: vibration; bending; FGM; plate; visco-pasternak foundations; quasi-3D HSDT

1. Introduction

Functionally graded materials (FGMs) are a type of composites whose properties vary from surface to surface and thus eliminate the stress concentration encountered in laminated composites. A typical FGM consists of a mixture of two material phases, for example a metal and ceramic. The cause of the growing use of FGM in various industries such as automotive, aerospace, civil and mechanical engineering structures is that their material characteristics can be adapted to different applications (Reddy 2000, Qian and Batra 2005, Eltaher *et al.* 2013, Tounsi *et al.* 2013, Bousahla *et al.* 2014, Akbaş 2015, Arefi 2015, Pradhan and Chakraverty 2015, Ebrahimi and Dashti 2015, Larbi Chaht *et al.* 2015, Kar and Panda 2015, Yahia *et al.* 2015, Daouadji and Hadji 2015, Ahouel *et al.* 2016, Barati and Shahverdi 2016, Bounouara *et al.* 2016, Boukhari *et al.* 2016, Avcar 2015, 2016 and 2019, Bellifa *et al.* 2017a, Bouafia *et al.* 2017, Sekkal *et al.* 2017a, Benadouda *et al.* 2017, Fourn *et al.* 2018, Zouatnia *et al.* 2018, Bourada *et al.* 2018, Belabed *et al.* 2018, Karami *et al.* 2018a, Ahmed *et al.* 2019, Hussain and Naeem 2019, Bourada *et al.* 2019, Tlidji *et al.* 2019, Karami and Karami 2019).

Plates resting on an elastic base can be found in various fields of structural engineering. A two-parameter model of Pasternak (1954) taking into account the shear deformation between springs was proposed to describe the interaction between the plate and the foundation. The Winkler model (Winkler 1867) is a special case of the Pasternak model by setting the shear modulus to zero. Another type of foundations are those that consider viscoelastic damping (Kerr 1964).

Some research papers have investigated the mechanical response of the FG plate based on visco or elastic Pasternak foundation. Huang *et al.* (2008) employed a 3D theory of elasticity to examine FG plates on elastic foundation. Malekzadeh (2009) used the 3D elasticity theory to study the dynamic response of FG plates resting on an elastic foundation. Amini *et al.* (2009) investigated the 3D vibration behavior of FG plates resting on an elastic foundation. Lü *et al.* (2009) provided and exact solutions for dynamic response of FG thick plates resting on Winkler-Pasternak elastic foundation. By employing the parabolic shear deformation model, Baferani *et al.* (2011) developed an accurate method for vibration of FG thick plates resting on elastic foundation. Fallah *et al.* (2013) examined the vibration response of FG plates supported by elastic foundation utilizing the extended Kantorovich technique together with infinite power series method. Sheikholeslami and Saidi (2013) used the quasi 3D HSDT to study the free vibration behavior of simply supported FG plates resting on elastic foundation. Sobhy (2013) studied the dynamic and

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buckling responses of exponentially graded material sandwich plate resting on Winkler–Pasternak elastic foundation. Bouderba *et al.* (2013) investigated the thermomechanical bending response of FG thick plates resting on Winkler-Pasternak elastic foundations. An analytical formulation based on the first-order shear deformation plate theory is proposed by Yaghoobi and Yaghoobi (2013) to examine the thermo-mechanical buckling of symmetric sandwich plates with FG face sheets resting on two-parameter elastic foundation. Meziane *et al.* (2014) presented an efficient and simple refined theory for buckling and dynamic of exponentially graded sandwich plates resting on elastic foundation. Zidi *et al.* (2014) investigated the bending behavior of FG plate resting on elastic foundation and subjected to hygro-thermo-mechanical loading. By using strain gradient theory and Euler–Bernoulli beam model, Zeighampour and Beni (2015) studied the vibration of axially functionally graded nanobeam resting on visco-Pasternak foundation. Beldjelili *et al.* (2016) studied hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Kolahchi *et al.* (2016) analyzed dynamic stability of temperature-dependent FG CNT-reinforced visco-plates resting on orthotropic elastomeric medium. Attia *et al.* (2018) presented a refined four variable plate theory for thermoelastic analysis of FG plates resting on variable elastic foundations. Kadari *et al.* (2018) analyze the buckling of orthotropic nanoscale plates resting on elastic foundations. Bakhadda *et al.* (2018) analyzed dynamic and bending response of carbon nanotube-reinforced composite plates with elastic foundation. Chaabane *et al.* (2019) presented an analytical study of bending and dynamic responses of FG beams resting on elastic foundation. Avcar and Mohammed (2018) analyzed the free vibration of FG beams resting on Winkler-Pasternak foundation. Arani and Kiani (2018) presented a nonlinear free and forced vibration analysis of microbeams resting on the nonlinear orthotropic visco-Pasternak foundation with different boundary conditions. Recently, Arshid *et al.* (2019) studied the effect of porosity on free vibration of SPFG circular plates resting on visco-Pasternak elastic foundation based on CPT, FSDT and TSDT.

The aim of this work is to propose a simple quasi-3D theory with only four unknowns for bending and dynamics of FG thick plates resting on a visco-Pasternak foundation. The kinematics is chosen based on a hyperbolic distribution of axial and vertical displacements across the thickness. The visco-Pasternak's foundation is taken into account by adding the impact of damping to the usual foundation model which characterized by the linear Winkler's modulus and Pasternak's foundation modulus. Due to the lack of any study on the mechanics of FG plates resting on visco-Pasternak foundation, it is hoped that the present study may be employed as a benchmark for future works of such structures.

2. Mathematical formulations

Consider a FG plate as shown in Fig. 1 with a thickness

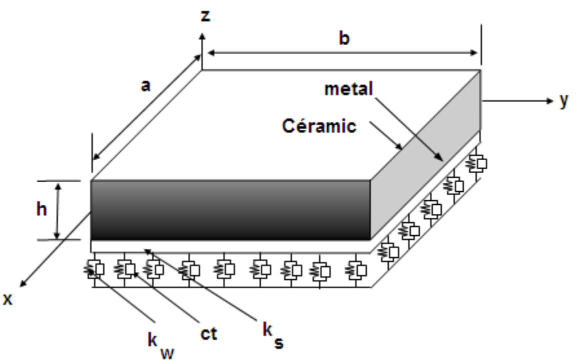

Fig. 1 Geometry and coordinates of the FG plate resting on visco-Pasternak foundation

h, length a and width b. The material characteristics change within the thickness with a power law distribution, which is presented below (Belabed *et al.* 2014, Hebali *et al.* 2014, Zidi *et al.* 2014 and 2017, Belkorissat *et al.* 2015, Zemri *et al.* 2015, Bourada *et al.* 2015, Al-Basyouni *et al.* 2015, Mahi *et al.* 2015, Houari *et al.* 2016, Hachemi *et al.* 2017, Abdelaziz *et al.* 2017, Meksi *et al.* 2019)

$$P(z) = (P_c - P_m)V_c + P_m \tag{1}$$

where (P_c) and (P_m) are the Young's moduli (E), Poisson's ratio (ν) and mass density (ρ) of ceramic and metal materials located at the top and bottom surfaces, respectively. The volume fraction of ceramic material V_c is given as follows

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^p \tag{2}$$

where p is the gradient index, which is positive.

2.1 Kinematics and strains

The displacement field satisfying the condition of zero transverse shear stresses on the top and bottom surfaces of the plate, is expressed as follows (Sekkal *et al.* 2017b, Benchohra *et al.* 2018, Bouhadra *et al.* 2018, Abualnour *et al.* 2018, Boukhlif *et al.* 2019, Boutaleb *et al.* 2019, Zaoui *et al.* 2019, Bendaho *et al.* 2019, Khiloun *et al.* 2019)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\theta(x,y,t) \tag{3c}$$

The coefficients k_1 and k_2 depends on the geometry. It can be seen that the kinematics in Eq. (3) introduces only four unknowns (u_0, v_0, w_0 and θ) with considering the thickness stretching effect.

In this work, the present quasi-3D HSDT is obtained by setting

$$f(z)=-\left[\frac{3\pi\, z}{2}sech^2\left(\frac{1}{2}\right)\right]+\frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right) \tag{4a}$$

$$g(z)=\frac{2}{15}\frac{df}{dz} \tag{4b}$$

The strain-displacement relations, based on this method, are given as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \tag{5a}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=f'(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}+g(z)\begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix}, \tag{5b}$$

$$\varepsilon_z=g'(z)\,\varepsilon_z^0 \tag{5c}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{6a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\displaystyle\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\displaystyle\int\theta\,dy\end{Bmatrix}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial\theta}{\partial y}\\ \dfrac{\partial\theta}{\partial x}\end{Bmatrix},\quad \varepsilon_z^0=\theta, \tag{6b}$$

$$g'(z)=\frac{dg(z)}{dz} \tag{6c}$$

The integrals employed in the above equations shall be resolved by a Navier type method and can be given as follows

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\ \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y},\ \int\theta\,dx=A'\frac{\partial\theta}{\partial x},\ \int\theta\,dy=B'\frac{\partial\theta}{\partial y} \tag{7}$$

where the coefficients A' and B' are considered according to the type of solution utilized, in this case via Navier method. Therefore, A', B', k_1 and k_2 are given as follows

$$A'=-\frac{1}{\alpha^2},\quad B'=-\frac{1}{\beta^2},\ k_1=-\alpha^2,\ k_2=-\beta^2 \tag{8}$$

where α and β are defined in expression (19).

The linear constitutive relations are given below

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{66}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \tag{9}$$

where C_{ij} are the three-dimensional elastic constants defined by

$$C_{11}=C_{22}=C_{33}=\frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)} \tag{10a}$$

$$C_{12}=C_{13}=C_{23}=\frac{\nu E(z)}{(1-2\nu)(1+\nu)} \tag{10b}$$

$$C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)} \tag{10c}$$

2.2 The visco-Winkler-Pasternak foundations

The two-parameter Pasternak model is the most natural extension of the Winkler model (the one-parameter). It takes into account a shear interaction between the spring elements by connecting the ends of the springs to a structure of an incompressible shear layer. The present FG plate is supported by a homogeneous three-parameter viscoelastic foundation. This latter is characterized by the linear Winkler module k_w, the Pasternak foundation module (shear) k_s and the damping coefficient c_t of the viscoelastic medium. Considering the unglued contact between the FG plate and the support, the interaction follows the three-parameter visco-Pasternak type foundation model such as (Zenkour 2016).

$$R_f=\left(k_w-k_s\nabla^2+c_t\frac{\partial}{\partial t}\right)w \tag{11}$$

where w is the transverse displacement and ∇^2 is the Laplacian. If the foundation is modeled as the visco-Winkler foundation, the coefficient k_s in Eq. (11) is zero.

2.3 Equations of motion

Considering the kinematic of the proposed simple quasi-3D model in Eq. (3) and employing the Hamilton's principle, the equations of motion of FG plates resting on visco-Pasternak foundation can be obtained as

$$\delta u_0:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_0}{\partial x}-J_1A'k_1\frac{\partial\ddot{\theta}}{\partial x} \tag{12a}$$

$$\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y}-J_1B'k_2\frac{\partial\ddot{\theta}}{\partial y} \tag{12b}$$

$$\delta w_0:\ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+q-R_f= I_0\ddot{w}_0+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_0 -J_2A'k_1\frac{\partial^2\ddot{\theta}}{\partial x^2}+J_2B'k_2\frac{\partial^2\ddot{\theta}}{\partial y^2}+J_1^s\ddot{\theta} \tag{12c}$$

$$\begin{aligned}\delta\theta:\ & -k_1A'\frac{\partial^2 M_x^s}{\partial x^2}-(k_1A'+k_2B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} -k_2B'\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}+k_2A'\frac{\partial Q_{xz}^s}{\partial x}\\ & +k_2B'\frac{\partial Q_{yz}^s}{\partial y}-N_z = J_1^S\ddot{w}_0+K_2^S\ddot{\theta} -J_1k_1A'\frac{\partial \ddot{u}_0}{\partial x}+J_2k_1A'\frac{\partial^2\ddot{w}_0}{\partial x^2}\\ & -K_2A'^2k_1^2\frac{\partial^2\ddot{\theta}}{\partial x^2}-J_1B'k_2\frac{\partial \ddot{v}_0}{\partial y} +J_2B'k_2\frac{\partial^2\ddot{w}_0}{\partial y^2}-K_2B'^2k_2^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\end{aligned} \tag{12d}$$

In the above equations dot above each parameter denotes partial differentiating with respect to time. Also the stress resultants (N, M, Q, S and N_z) and the mass inertias (I_i, J_i, J^S_i, K_i, KSi) are as follows

$$\left(N_i,M_i^b,M_i^s\right)=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz\ ,\ (i=x,y,xy),\qquad N_z=\int_{-h/2}^{h/2}g'(z)\sigma_z dz \tag{13a}$$

$$S=\left\{S_{xz}^s,S_{yz}^s\right\},\ \left(Q_{xz}^s,Q_{yz}^s\right)=\int_{-h/2}^{h/2}f'\left(\tau_{xz},\tau_{yz}\right)dz,\qquad \begin{Bmatrix}L\\L^a\\R\\R^a\end{Bmatrix}=\int_{-h/2}^{h/2}\lambda(z)\begin{Bmatrix}1\\z\\f(z)\\g'(z)\dfrac{1-\nu}{\nu}\end{Bmatrix}g'(z)dz \tag{13b}$$

$$\left(I_0,I_1,J_1,J_1^s,I_2,J_2,K_2,K_2^s\right)=\int_{-h/2}^{h/2}\left(1,z,f,g,z^2,z\,f,f^2,g^2\right)\rho(z)dz \tag{13c}$$

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}= \int_{-h/2}^{h/2}\lambda(z)\left(1,z,z^2,f(z),z\ f(z),f^2(z)\right)\begin{Bmatrix}\dfrac{1-\nu}{\nu}\\1\\\dfrac{1-2\nu}{2\nu}\end{Bmatrix}dz \tag{13d}$$

$$\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right) \tag{13e}$$

$$\left(F_{44}^s,X_{44}^s,A_{44}^s\right)=\left(F_{55}^s,X_{55}^s,A_{55}^s\right)\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\upsilon)}\left(f(z),f(z)g(z),g^2(z)\right)dz \tag{13f}$$

Substituting Eqs. (13) into (12) and using stress–strain relations, the governing equations of motion are obtained as

$$\begin{aligned}& A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0\\ & -B_{11}d_{111}w_0-\left(B_{12}+2B_{66}\right)d_{122}w_0\\ & +\left(B_{66}^s\left(k_1A'+k_2\,B'\right)+B_{12}^sk_2\,B'\right)d_{122}\theta\\ & +B_{11}^sk_1A'\,d_{111}\theta+Ld_1\theta=I_0\ddot{u}_0\\ & -I_1\,d_1\ddot{w}_0+J_1\,A'k_1d_1\ddot{\theta}\end{aligned} \tag{14a}$$

$$\begin{aligned}& A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0\\ & -B_{22}\,d_{222}w_0-\left(B_{12}+2B_{66}\right)d_{112}w_0\\ & +\left(B_{66}^s\left(k_1A'+k_2\,B'\right)+B_{12}^sk_1\,A'\right)d_{112}\theta\\ & +B_{22}^sk_2\,B'\,d_{222}\theta+Ld_2\theta=I_0\ddot{v}_0-I_1\,d_2\ddot{w}_0\\ & +J_1\,B'k_2\,d_2\ddot{\theta},\end{aligned} \tag{14b}$$

$$\begin{aligned}& B_{11}\,d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0\\ & +\left(B_{12}+2B_{66}\right)d_{112}v_0+B_{22}\,d_{222}v_0\\ & -D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\ & -D_{22}\,d_{2222}w_0+D_{11}^sk_1\,A'd_{1111}\theta\\ & +\left((D_{12}^s+2D_{66}^s)(k_1A'+k_2B')\right)d_{1122}\theta\\ & +D_{22}^sk_2B'd_{2222}\theta+L^a(d_{11}\theta+d_{22}\theta)\\ & +q-R_f=I_0\ddot{w}_0+I_1\left(d_1\ddot{u}_0+d_2\,\ddot{v}_0\right)\\ & -I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)\\ & +J_2\left(k_1A'\,d_{11}\ddot{\theta}+k_2\,B'\,d_{22}\ddot{\theta}\right)+J_1^{st}\ddot{\theta},\end{aligned} \tag{14c}$$

$$\begin{aligned}& -\left(B_{12}^sk_2B'+B_{66}^s(k_1A'+k_2B')\right)d_{122}u_0\\ & -\left(B_{12}^sk_1A'+B_{66}^s\,(k_1A'+k_2B')\right)d_{112}v_0\\ & -B_{22}^sk_2B'd_{222}v_0+D_{11}^sk_1A'd_{1111}w_0\\ & +\left((D_{12}^s+2D_{66}^s)(k_1A'+k_2B')\right)d_{1122}w_0\\ & +D_{22}^sk_2B'd_{2222}w_0-k_1A'\,B_{11}^sd_{111}u_0\\ & -H_{11}^s(k_1A')^2\,d_{1111}\theta-H_{22}^s(k_2B')^2\,d_{2222}\theta\\ & -\left(2H_{12}^sk_1A'k_2B'+(k_1A'+k_2B')^2\,H_{66}^s\right)d_{1122}\theta\\ & +((k_1A')^2\,F_{55}^s+2k_1A'X_{55}^s+A_{55}^s)d_{11}\theta\\ & +2R(k_1A'd_{11}\theta+k_2B'd_{22}\theta)-L(d_1u_0+d_2v_0)\\ & +L^a(d_{11}w_0+d_{22}w_0)-R^a\theta-N_z=\\ & J_1\left(k_1A'd_1\ddot{u}_0+k_2B'd_2\ddot{v}_0\right)+J_1^s\ddot{w}_0\\ & -J_2\left(k_1A'd_{11}\ddot{w}_0+k_2B'd_{22}\ddot{w}_0\right)+K_2^S\ddot{\theta}\\ & -K_2\left((k_1A')^2\,d_{11}\ddot{\theta}+(k_2B')^2\,d_{22}\ddot{\theta}\right)\end{aligned} \tag{14d}$$

where d_{ij} , d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i \partial x_j},\ d_{ijl}=\frac{\partial^3}{\partial x_i \partial x_j \partial x_l} \tag{15a}$$

$$\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial \ddot{w}_0}{\partial y}-J_1 B' k_2\frac{\partial \ddot{\theta}}{\partial y} \tag{15b}$$

$$d_{ijlm}=\frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\ (i,j,l,m=1,2). \tag{15c}$$

The in-plane normal and shear stresses (σ_x, σ_y and τ_{xy}) can be determined accurately by the constitutive relations (9) for FG plates. However, if the transverse shear stresses (τ_{yz} and τ_{xz}) are computed from the constitutive relations (9), they may not respect the boundary conditions at the upper and lower surfaces of the plate. So these stresses are determined by integrating the equilibrium equations of 3D elasticity with respect to thickness coordinate as (Younsi *et al.* 2018)

$$\tau_{xz}=-\int_{-h/2}^{z}\left(\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}\right)dz+C_1(x,y) \tag{16a}$$

$$\tau_{yz}=-\int_{-h/2}^{z}\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}\right)dz+C_2(x,y) \tag{16b}$$

where C_i (i=1, 2) are constants and determined by the following boundary conditions at the upper and lower surfaces of the plate

$$\tau_{xz}\big|_{z=\pm h/2}=0\ ,\quad \tau_{yz}\big|_{z=\pm h/2}=0 \tag{17}$$

3. Exact solution for a simply supported plate

The Navier solution procedure is employed to determine the analytical solutions for which the displacement variables are written as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions (Bennoun *et al.* 2016, Draiche *et al.* 2016, Ait Atmane *et al.* 2017, Bellifa *et al.* 2017b,Besseghier *et al.* 2017, Kaci *et al.* 2018, Bouadi *et al.* 2018, Adda Bedia *et al.*, 2019, Draoui *et al.* 2019).

$$\begin{Bmatrix} u_0\\ v_0\\ w_0\\ \theta \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}e^{i\omega t}\cos(\alpha x)\sin(\beta y)\\ V_{mn}e^{i\omega t}\sin(\alpha x)\cos(\beta y)\\ W_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y)\\ X_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{18}$$

where ω is the frequency of free vibration of the plate, $\sqrt{i}=-1$ the imaginary unit.

With

$$\alpha=m\pi/a \quad and \quad \beta=n\pi/b \tag{19}$$

The transverse load q is also expressed in the double-Fourier sine series as

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin(\alpha x)\sin(\beta y) \tag{20}$$

For the case of a sinusoidally distributed load, the coefficient q_{mn}=q_0 and m=n=1. However, In the case of a load uniformly distributed (UDL), we have

$$q_{mn}=\frac{16 q_0 ab}{\alpha\beta},\quad m,n=1,3,5,...$$

Substituting Eqs. (20) and (18) into Eq. (14), the analytical solutions can be obtained by

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{12} & a_{22} & a_{23} & a_{24}\\ a_{13} & a_{23} & a_{33} & a_{34}\\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} -\omega^2\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14}\\ 0 & m_{22} & m_{23} & m_{24}\\ m_{13} & m_{23} & m_{33} & m_{34}\\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right)\cdot\begin{Bmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\\ \theta \end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ q_{mn}\\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{aligned}
a_{11}&=\left(A_{11}\lambda^2+A_{66}\mu^2\right)\\
a_{12}&=a_{21}=\lambda\ \mu\ \left(A_{12}+A_{66}\right)\\
a_{13}&=a_{31}=-\lambda\ [B_{11}\lambda^2+(B_{12}+2B_{66})\ \mu^2]\\
a_{14}&=a_{41}=\lambda\ ((k_2 B' B_{12}^S+(k_1 A'+k_2 B')B_{66}^S{}')\mu^2\\
&\quad+k_1 A' B_{11}^S\lambda^2-L13)
\end{aligned} \tag{22a}$$

$$\begin{aligned}
a_{22}&=\left(A_{66}\lambda^2+A_{22}\mu^2\right)\\
a_{23}&=a_{32}=-\mu\ [(B_{12}+2B_{66})\ \lambda^2+B_{22}\mu^2]\\
a_{24}&=a_{42}=k_2 B'\ \mu^3 B_{22}^S+k_1\ A'\ \lambda^2\mu\ B_{12}^S\\
&\quad+k_2\ B'\ \lambda^2\mu\ B_{66}^S-\mu E_{23}+k_1 A'\lambda^2\ \mu\ B_{66}^S
\end{aligned} \tag{22b}$$

$$\begin{aligned}
a_{33}&=D_{11}\lambda^4+2(D_{12}+2D_{66})\lambda^2\mu^2+D_{22}\mu^4\\
&\quad-K_x^s\lambda^2-K_y^s\mu^2+kw-ct*\omega\\
a_{34}&=a_{43}=-k_1 A'\lambda^2\mu^2 D_{12}^S-k_2 B'\lambda^2\mu^2 D_{12}^S\\
&\quad-k_1 A'\lambda^4 D_{11}^S-k_2 B'\mu^4 D_{22}^S+\mu^2 L_{23}^a\\
&\quad-2k_2 B'\lambda^2\mu^2 D_{66}^S+\lambda^2 L_{13}^a\\
&\quad-2\ k_1 A'\lambda^2\mu^2 D_{66}^S
\end{aligned} \tag{22c}$$

$$\begin{aligned}
a_{44}&=k_1 A'\left(-\lambda^2 R_{13}+k_1 A'\lambda^4 H_{11}^S+k_2 B'\lambda^2\mu^2 H_{12}^S\right)\\
&\quad-k_2 B'\mu^2 R_{23}+\lambda^2 A_{55}^S+R^a\\
&\quad+k_2 B'\left(-\mu^2\ R23+k_1 A'\lambda^2\mu^2 H_{12}^S+k_2 B'\mu^4 H_{22}^S\right)\\
&\quad-k_2 B'\left(-\mu^2\ k_2 B' F_{44}^S-\mu^2 X_{44}^S\right)\\
&\quad+\mu^2 k_2 B' X_{44}^S-k_2 A'\left(-\lambda^2 k_1 A' F_{55}^S-\lambda^2 X_{55}^S\right)\\
&\quad+\left(k_1 A'+k_2 B'\right)\left(k_2 B'\lambda^2\mu^2\ H_{66}^S+k_1 A'\lambda^2\mu^2\ H_{66}^S\right)\\
&\quad+\mu^2 A_{44}^S-k_1 A'\lambda^2 R_{13}+\lambda^2 k_1 A' X_{55}^S
\end{aligned} \tag{22d}$$

Table 1 Material properties used for FG plates

Properties	Metal	Ceramic	Ceramic
	Al	Al_2O_3	ZrO_2
E (GPa)	70	380	211
v	0.3	0.3	0.3
ρ (kg/m^3)	2702	3800	4500

Table 2 Comparison of non-dimensional deflection and stresses of isotropic square plate (a/h= 10) subjected to a UDL

Theory	$\hat{w}(a/2,b/2,0)$	$\hat{\sigma}_x(h/2)$	$\hat{\sigma}_y(h/2)$	$\hat{\tau}_{xy}(h/2)$	$\bar{\tau}_{xz}(0,b/2,0)$	$\bar{\tau}_{yz}(a/2,0,0)$
Shimpi *et al.* (2003)	4.625	0.307	0.307	0.195	0.505	0.505
Srinivas *et al.* (1970)	4.639	0.290	0.290	/	0.488	/
Benahmed *et al.* (2017)	4.633	0.302	0.302	0.197	0.481	0.502
Present	4.622	0.304	0.304	0.195	0.482	0.482

Table 3 Comparison of non-dimensional deflection $D10^3w(0.5a,0.5b,z=0)/qa^4$ of simply supported isotropic thin square plate under UDL (a/h=100)

K_W	K_S	Present				Benahmed *et al.* (2017)	3D (Huang *et al.* 2008)
		$\bar{c}_t=1$	$\bar{c}_t=0.1$	$\bar{c}_t=0.05$	$\bar{c}_t=0$	$\bar{c}_t=0$	$\bar{c}_t=0$
1	1	1.6879	3.4213	3.6228	3.8489	3.8490	3.8546
	3^4	0.6008	0.7430	0.7528	0.7628	0.7628	0.7630
	5^4	0.1106	0.1148	0.1151	0.1153	0.1153	0.1153
3^4	1	1.5449	2.9018	3.0468	3.2066	3.2067	3.2105
	3^4	0.5808	0.7133	0.7223	0.7316	0.7316	0.7317
	5^4	0.1099	0.1140	0.1143	0.1145	0.1145	0.1145
5^4	1	0.9700	1.4041	1.4391	1.4758	1.4759	1.4765
	3^4	0.4722	0.5588	0.5645	0.5703	0.5703	0.5704
	5^4	0.1052	0.1090	0.1093	0.1095	0.1095	0.1095

Table 4 Comparison of non-dimensional deflection $D10^3w(0.5a,0.5b,z=0)/qa^4$ of a uniformly loaded simply supported homogeneous square plate on a visco-Winkler foundation (a/h=100)

K_w	Present				Benahmed *et al.* (2017)	Benyoucef *et al.* (2010)	3D Huang *et al.* (2008)	Lam *et al.* (2000)
	$\bar{c}_t=1$	$\bar{c}_t=0.1$	$\bar{c}_t=0.05$	$\bar{c}_t=0$	$\bar{c}_t=0$	$\bar{c}_t=0$	$\bar{c}_t=0$	$\bar{c}_t=0$
1	1.7264	3.5777	3.7982	4.0470	4.0472	4.0530	4.0530	4.0520
3^4	1.5773	3.0143	3.1708	3.3440	3.3440	3.3480	3.3490	3.3470
5^4	0.98291	1.4308	1.4672	1.5053	1.5050	1.5060	1.5070	1.5060

$$m_{11}=m_{22}=I_0,\quad m_{12}=m_{21}=0,\\ m_{13}=-\lambda I_1,\\ m_{14}=m_{41}=k_1\,A'\lambda J_1 \tag{22e}$$

$$m_{23}=-\mu I_1,\quad m_{24}=k_2\,B'\mu J_1 \tag{22f}$$

$$m_{33}=\left(I_0+I_2\left(\lambda^2+\mu^2\right)\right) \tag{22g}$$

$$m_{43}=J_{S1}+J_2\left(-A'\,k_1\,\lambda^2-B'\,k_2\,\mu^2\right)\\ m_{44}=K_2\left(A'^2\,k_1^2\,\lambda^2+B'^2\,k_2^2\,\mu^2\right)+K_2^s \tag{22h}$$

4. Numerical results and discussion

In this section, various numerical examples are provided and discussed to show the effect of visco-Pasternak foundation on the bending and free vibration responses of FG thick plates. Two types of FG plate are considered: Al/Al_2O_3 plates and Al/ZrO_2plate. The used material properties are given in Table 1. Numerical results are given in terms of non-dimensional stresses, displacements and frequencies. Here the non-dimensional parameters are defined as

$$\hat{w}=\frac{100E}{q_0hS^4}w\left(\frac{a}{2},\frac{b}{2},\bar{z}\right) \tag{23}$$

$$
\begin{aligned}
&\hat{\sigma}_x = \frac{1}{q_0 S^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},\bar{z}\right)\\
&\bar{\sigma}_y = \frac{1}{q_0 S}\sigma_y\left(\frac{a}{2},\frac{b}{2},\bar{z}\right)\\
&\hat{\tau}_{xy} = \frac{1}{q_0 S^2}\tau_{xy}\left(0,0,\bar{z}\right)\\
&\bar{\sigma}_x = \frac{h^2}{q_0 a^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{-h}{2}\right)\\
&\bar{\tau}_{yz} = \frac{1}{q_0 S}\tau_{yz}\left(\frac{a}{2},0,\bar{z}\right)\\
&\bar{\tau}_{xz} = \frac{1}{q_0 S}\tau_{xz}\left(0,\frac{b}{2},\bar{z}\right),\bar{z} = \frac{z}{h}, S = a/h\\
&\bar{\sigma}_z = \frac{1}{q_0}\sigma_z\left(\frac{a}{2},\frac{b}{2},z\right)\\
&\bar{\tau}_{xy} = \frac{h^2}{q_0 a^2}\tau_{xy}\left(0,0,\frac{-h}{2}\right)\\
&\tau^{*}_{xy} = \frac{1}{10 q_0}\tau_{xy}\left(0,0,\frac{-h}{3}\right),\bar{c}_t = \frac{a^4 c_t}{10^3 D_c}\\
&\tau^{*}_{xz} = \frac{1}{10 q_0}\tau_{xz}\left(0,\frac{b}{2},0\right)\\
&\bar{u} = \frac{100D}{q_0 a^4}u\left(0,\frac{b}{2},\frac{-h}{2}\right)\\
&\bar{w} = \frac{100D}{q_0 a^4}w\left(\frac{a}{2},\frac{b}{2},0\right), D = \frac{E h^3}{12(1-v^2)}\\
&K_w = \frac{k_w a^4}{h^3}, K_s = \frac{k_s a^2}{h^3 v}\\
&\bar{\omega} = \omega\ \frac{a^2}{h}\sqrt{\rho_m / E_m}\\
&\hat{\omega} = \omega h\sqrt{\rho_m / E_m}, \tilde{\omega} = \omega\ \frac{a^2}{\pi^2}\sqrt{\rho_c h / D_c}\\
&D_C = E_C h^3 / 12(1-v^2)
\end{aligned}
\qquad (23)
$$

4.1 Bending analysis

In the first example, the deflections and dimensionless stresses of an isotropic square plate (a/h = 10) subjected to a UDL are compared in Table 2 with those given by the quasi-3D solutions of Shimpi *et al.* (2003), the exact solution realized by Srinivas *et al* (1970) and the quasi-3D hyperbolic theory of Benahmed *et al.* (2017). It is clear that the results are in good agreement. It is noticed that the proposed theory use only four variables contrary to that of Benahmed *et al.* (2017) where five variables are employed.

To demonstrate the effect of damping coefficients and to validate the present formulation for plates resting on an elastic foundation, the results for dimensionless deflections of a thick isotropic plate are compared with results published previously. Indeed, in the second example, Table 3 presents the center deflections of a uniformly loaded homogeneous square plate simply supported on a visco-Pasternak foundation. The results are compared with those of Benahmed *et al.* (2017) and of 3D solution (Huang *et al.* 2008) in the case where the viscosity effect is neglected ($\bar{c}_t = 0$). It is observed that the results agree closely. However, it is seen that the deflection of the plate is very sensitive to the inclusion of the viscosity effect. The deflections are decreasing with the increase of the parameters K_w, K_S and $\bar{c}_t$.

In the third example, the deflections of a uniformly loaded homogeneous simply supported square plate resting on a visco-Winkler foundation are presented in Table 4. The results obtained in the case where the damping coefficient $\bar{c}_t$ is zero, are compared with those given by Benahmed *et al.* (2017), Benyoucef *et al.* (2010), Lam *et al.* (2000) and Kobayashi and Sonoda (1989). It is observed that when the viscosity term is omitted, a good agreement between the results is remarked. However, the introduction of the viscosity term reduce the deflections because of the damping effect.

Table 5 provides identical results as those presented in

Table 5 Comparison of non-dimensional deflection $D10^3 w(0.5a,0.5b,z=0)/qa^4$ of a uniformly loaded simply supported homogeneous square plate on a visco-Winkler foundation (a/h=100)

K_w	Present				Benahmed *et al.*	Zenkour and Sobhy (2012)	Buczkowsi and Torbacki (2001)
	$\bar{c}_t$=1	$\bar{c}_t$=0.1	$\bar{c}_t$=0.05	$\bar{c}_t$=0	$\bar{c}_t$=0	$\bar{c}_t$=0	$\bar{c}_t$=0
0	1.7354	3.6185	3.8441	4.0990	4.1026	4.1149	4.1197
1 ^{4}	1.7334	3.6100	3.8346	4.0881	4.0917	4.1039	4.1088
3 ^{4}	1.5829	3.0374	3.1963	3.3723	3.3747	3.3813	3.3855
5 ^{4}	0.9841	1.4352	1.4719	1.5103	1.5107	1.5094	1.5114
10 ^{4}	0.1052	0.1104	0.1107	0.1111	0.1110	0.1108	0.1096
15 ^{4}	0.0194	0.0196	0.0196	0.0196	0.0196	0.0196	0.0191

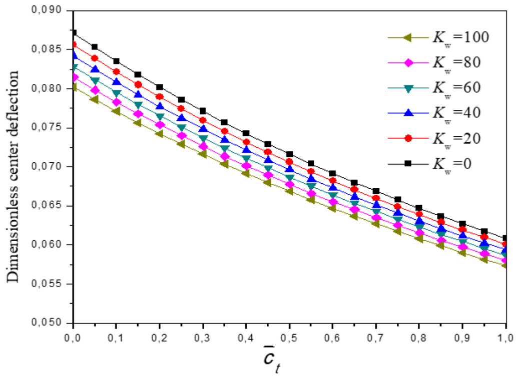

Fig. 2 Effect of damping coefficient and Winkler modulus parameter on the dimensionless center deflection of a square FG plate (P=2, a/h=10, K_w=100 and q_0=100)

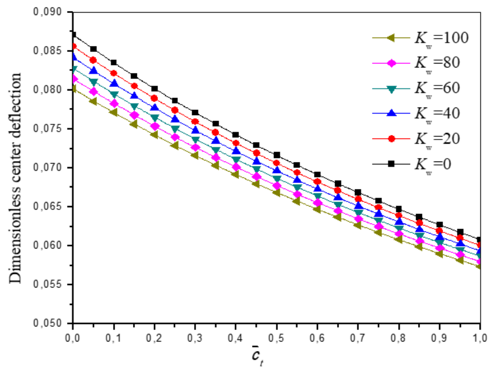

Fig. 3 Effect of damping coefficient and Pasternak shear modulus parameter on the dimensionless center deflection of a square FG plate (P=2, a/h=10, K_w=100 and q_0=100)

Table 6 Comparison of the displacements and stresses of simply supported Al/Al_2O_3 rectangular plate under UDL (a/h=10 and b=3a)

P	K_w	K_s	Theory		$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
0.50	0	0	Thai and Choi (2011)		0.34910	1.93450	0.23370	0.09410	—
			Zenkour and Sobhy (2013)		0.34919	1.93441	0.23372	0.09415	7.68354
			Benahmed *et al.* (2017)		0.33498	1.90215	0.23941	0.09007	7.56253
			Present	$\bar{c}_t = 0$	0.33266	1.89754	0.23949	0.08909	7.29110
				$\bar{c}_t = 0.05$	0.24844	1.40701	0.17507	0.07311	5.68108
				$\bar{c}_t = 0.1$	0.19788	1.11261	0.13656	0.06296	4.72287
				$\bar{c}_t = 1$	0.04387	0.21940	0.02178	0.02429	1.85994
	100	0	Thai and Choi (2011)		0.33580	1.85900	0.22420	0.09160	—
			Zenkour and Sobhy (2013)		0.33586	1.85907	0.22424	0.09167	7.42978
			Benahmed *et al.* (2017)		0.32246	1.82955	0.22989	0.08774	7.31675
			Present	$\bar{c}_t = 0$	0.32025	1.82528	0.22998	0.08679	7.05312
				$\bar{c}_t = 0.05$	0.24138	1.36585	0.16968	0.07173	5.54669
				$\bar{c}_t = 0.1$	0.19334	1.08621	0.13312	0.06202	4.63731
				$\bar{c}_t = 1$	0.04367	0.21824	0.02164	0.02422	1.85608
	100	100	Thai and Choi (2011)		0.30120	1.66400	0.19990	0.08500	—
			Zenkour and Sobhy (2013)		0.30131	1.66399	0.19989	0.08503	6.76069
			Benahmed *et al.* (2017)		0.28991	1.64138	0.20536	0.08151	6.66745
			Present	$\bar{c}_t = 0$	0.28799	1.63788	0.20549	0.08061	6.42471
				$\bar{c}_t = 0.05$	0.22239	1.25585	0.15541	0.06785	5.17559
				$\bar{c}_t = 0.1$	0.18090	1.01433	0.12387	0.05932	4.39218
				$\bar{c}_t = 1$	0.04303	0.21500	0.02133	0.02396	1.83369
2	0	0	Thai and Choi (2011)		0.65640	3.22660	0.43950	0.17660	—
			Zenkour and Sobhy (2013)		0.65655	3.22672	0.43961	0.17666	6.91072
			Benahmed *et al.* (2017)		0.60340	3.07560	0.44695	0.16202	6.79513
			Present	$\bar{c}_t = 0$	0.60313	3.07536	0.44701	0.16175	6.75113
				$\bar{c}_t = 0.05$	0.38874	1.95902	0.27832	0.12047	4.65711
				$\bar{c}_t = 0.1$	0.28605	1.42479	0.19814	0.09882	3.66764
				$\bar{c}_t = 1$	0.05385	0.22483	0.02237	0.03431	1.44650
	100	0	Thai and Choi (2011)		0.61560	3.02180	0.41050	0.16900	—
			Zenkour and Sobhy (2013)		0.61576	3.02190	0.41060	0.16906	6.53895
			Benahmed *et al.* (2017)		0.56771	2.88981	0.41881	0.15538	6.44548
			Present	$\bar{c}_t = 0$	0.56747	2.88959	0.41887	0.15512	6.40087
				$\bar{c}_t = 0.05$	0.37337	1.87902	0.26628	0.11734	4.50827
				$\bar{c}_t = 0.1$	0.27761	1.38091	0.19158	0.09695	3.58688
				$\bar{c}_t = 1$	0.05360	0.22358	0.02220	0.03420	1.44382
	100	100	Thai and Choi (2011)		0.51860	2.53640	0.34230	0.15010	—
			Zenkour and Sobhy (2013)		0.51872	2.53642	0.34233	0.15020	5.63882
			Benahmed *et al.* (2017)		0.48189	2.44460	0.35187	0.13875	5.59033
			Present	$\bar{c}_t = 0$	0.48171	2.44449	0.35193	0.13850	5.54537
				$\bar{c}_t = 0.05$	0.33374	1.67428	0.23589	0.10882	4.10927
				$\bar{c}_t = 0.1$	0.25501	1.26488	0.17461	0.09162	3.35459
				$\bar{c}_t = 1$	0.05273	0.22023	0.02198	0.03378	1.42062

Table 6 Continued

P	K_w	K_s	Theory		$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
	0	0	Thai and Choi (2011)		0.78020	3.85060	0.52230	0.21030	—
			Zenkour and Sobhy (2013)		0.78046	3.85174	0.52237	0.21044	6.14557
			Benahmed *et al.* (2017)		0.72061	3.69376	0.53104	0.19389	6.03129
			Present	$\bar{c}_t = 0$	0.72351	3.68911	0.53087	0.19475	6.48378
				$\bar{c}_t = 0.05$	0.43526	2.18731	0.30643	0.13877	4.22659
				$\bar{c}_t = 0.1$	0.31049	1.53808	0.21017	0.11178	3.26465
				$\bar{c}_t = 1$	0.05594	0.22524	0.02075	0.03775	1.29481
5	100	0	Thai and Choi (2011)		0.72300	3.56200	0.48160	0.19960	—
			Zenkour and Sobhy (2013)		0.72323	3.56296	0.48167	0.19975	5.75485
			Benahmed *et al.* (2017)		0.66999	3.42857	0.49132	0.18445	5.66241
			Present	$\bar{c}_t = 0$	0.67277	3.42466	0.49124	0.18529	6.08398
				$\bar{c}_t = 0.05$	0.41606	2.08736	0.29157	0.13477	4.07783
				$\bar{c}_t = 0.1$	0.30060	1.48662	0.20258	0.10951	3.18893
				$\bar{c}_t = 1$	0.05568	0.22397	0.02058	0.03763	1.29246
	100	100	Thai and Choi (2011)		0.59220	2.90460	0.38970	0.17400	—
			Zenkour and Sobhy (2013)		0.59231	2.90518	0.38971	0.17410	4.84302
			Benahmed *et al.* (2017)		0.55294	2.81786	0.40060	0.16159	4.79288
			Present	$\bar{c}_t = 0$	0.55541	2.81539	0.40066	0.16236	5.14294
				$\bar{c}_t = 0.05$	0.36739	1.83624	0.25483	0.12402	3.68152
				$\bar{c}_t = 0.1$	0.27430	1.35211	0.18327	0.10303	2.96777
				$\bar{c}_t = 1$	0.05472	0.22067	0.02047	0.03713	1.12450

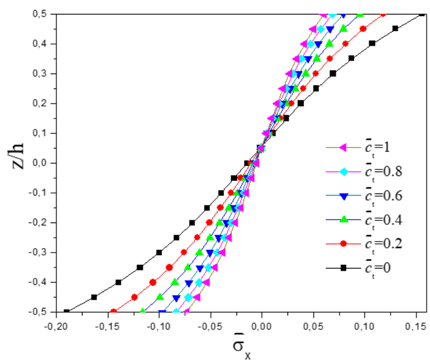

Fig. 4 Variation of dimensionless axial stress ($\bar{\sigma}_x$) within-the-thickness of a square FG plate (p=2, a/h=10, K_w=100, Ks=10 and q_0=100) for different values of damping coefficient

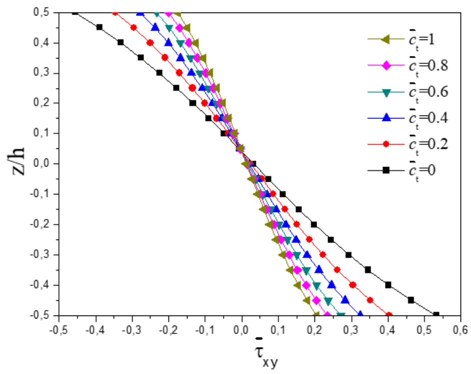

Fig. 5 Variation of dimensionless shear stress ($\bar{\tau}_{xy}$) within-the-thickness of a square FG plate (p=2, a/h=10, K_w=100, Ks=10 and q_0=100) for different values of damping coefficient

Table 4 but for a/h=20. The computed results are compared with those given by Benahmed *et al.* (2017), Zenkour and Sobhy (2012) and Buczkowski and Torbacki (2001). In the case where the damping coefficient $\bar{c}_t$ is zero, an excellent agreement is demonstrated between the different theories for all values of the Winkler coefficient K_w. Also, the consideration of the damping effect reduce the deflections.

Figs. 2 and 3 show the deflection $\bar{w}$ of the plate versus the damping coefficient $\bar{c}_t$ for different values of the foundation stiffness of FG square plate (p=2). It is observed that the increase of damping coefficient $\bar{c}_t$ leads to a decrease of the center deflection of the FG plate. Furthermore, it is seen from Figs. 2 and 3 that as the K_w or K_s increase the center deflection of the FG plate decreases.

Table 6 presents a comparison of displacement and stresses of the proposed theory with those of Benahmed *et al.* (2017), Thai and Choi (2011) and Zenkour and Sobhy (2013). The results are provided for FG Al/Al_2O_3 rectangular plate subjected to a uniformly distributed load

Table 7 Effect of the gradient index and visco-Pasternak's parameters on the dimensionless deflection and stresses of FG rectangular plate under sinusoidal load. (a/h=10, b=2a and q_0=100)

P	K_w	K_s	Theory		$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}^*_{xy}$	$-\bar{\tau}^*_{xz}$
0	0	0	Bouderba *et al.* (2013)		0.68131	0.42424	0.86240	-0.39400
			FSPT[(a)]		0.68135	0.42148	0.86459	-0.30558
			CPT[(a)]		0.65704	0.42148	0.86459	—
			Benahmed *et al.* (2017)		0.67669	0.44410	0.85538	-0.38933
			Present	$\bar{c}_t = 0$	0.67359	0.44504	0.84127	-0.37717
				$\bar{c}_t = 0.05$	0.57709	0.38128	0.72075	-0.32314
				$\bar{c}_t = 0.1$	0.50478	0.33351	0.63043	-0.28264
				$\bar{c}_t = 1$	0.15505	0.10244	0.19365	-0.08682
	100	0	Bouderba *et al.* (2013)		0.40523	0.25233	0.51296	-0.23435
			FSPT[(a)]		0.40525	0.25070	0.51426	-0.18175
			CPT[(a)]		0.39652	0.25437	0.52183	—
			Benahmed *et al.* (2017)		0.40481	0.26567	0.51170	-0.23290
			Present	$\bar{c}_t = 0$	0.40362	0.26667	0.50410	-0.22600
				$\bar{c}_t = 0.05$	0.36686	0.24239	0.45819	-0.20542
				$\bar{c}_t = 0.1$	0.33624	0.22215	0.41994	-0.18827
				$\bar{c}_t = 1$	0.13436	0.08877	0.16781	-0.07524
	0	100	Bouderba *et al.* (2013)		0.08365	0.05209	0.10589	-0.04838
			FSPT[(a)]		0.08366	0.05175	0.10615	-0.03752
			CPT[(a)]		0.08328	0.05342	0.10959	—
			Benahmed *et al.* (2017)		0.08413	0.05522	0.10635	-0.04841
			Present	$\bar{c}_t = 0$	0.08405	0.05553	0.10497	-0.04706
				$\bar{c}_t = 0.05$	0.08233	0.05440	0.10283	-0.04610
				$\bar{c}_t = 0.1$	0.08068	0.05331	0.10077	-0.04518
				$\bar{c}_t = 1$	0.05930	0.03918	0.07407	-0.03321
	100	100	Bouderba *et al.* (2013)		0.07720	0.04807	0.09772	-0.04464
			FSPT[(a)]		0.07720	0.04775	0.09796	-0.03462
			CPT[(a)]		0.07688	0.04932	0.10116	—
			Benahmed *et al.* (2017)		0.07765	0.05096	0.09815	-0.04468
			Present	$\bar{c}_t = 0$	0.07758	0.05126	0.09689	-0.04344
				$\bar{c}_t = 0.05$	0.07611	0.05029	0.09506	-0.04262
				$\bar{c}_t = 0.1$	0.07466	0.04807	0.09329	-0.04183
				$\bar{c}_t = 1$	0.05601	0.03700	0.06995	-0.03136
0.50	100	100	Bouderba *et al.* (2013)		0.07873	0.04579	0.08173	-0.03807
			FSPT[(a)]		0.07873	0.04546	0.08187	-0.02984
			CPT[(a)]		0.07846	0.04693	0.08451	—
			Benahmed *et al.* (2017)		0.07918	0.04873	0.08026	-0.03822
			Present	$\bar{c}_t = 0$	0.07910	0.04906	0.07927	-0.03714
				$\bar{c}_t = 0.05$	0.07758	0.04812	0.07775	-0.03642
				$\bar{c}_t = 0.1$	0.07470	0.04935	0.07628	-0.03573
				$\bar{c}_t = 1$	0.05680	0.03523	0.05692	-0.02666
1	100	100	Bouderba *et al.* (2013)		0.07932	0.04489	0.07305	-0.03502
			FSPT[(a)]		0.07932	0.04458	0.07321	-0.02716
			CPT[(a)]		0.07907	0.04604	0.07561	—
			Benahmed *et al.* (2017)		0.07976	0.04789	0.07120	-0.03525

Table 7 Continued

P	K_w	K_s	Theory		$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}^*_{xy}$	$-\bar{\tau}^*_{xz}$
1	100	100	Present	$\bar{c}_t = 0$	0.07970	0.04818	0.07042	-0.03428
				$\bar{c}_t = 0.05$	0.07815	0.04724	0.06905	-0.03361
				$\bar{c}_t = 0.1$	0.07666	0.04634	0.06774	-0.03297
				$\bar{c}_t = 1$	0.05710	0.03452	0.05045	-0.02456
2	100	100	Bouderba *et al.* (2013)		0.07976	0.04460	0.06719	-0.03222
			FSPT[a]		0.07975	0.04430	0.06740	-0.02435
			CPT[a]		0.07950	0.04581	0.06969	—
			Benahmed *et al.* (2017)		0.08020	0.04758	0.06530	-0.03244
			Present	$\bar{c}_t = 0$	0.08015	0.04779	0.06471	-0.03185
				$\bar{c}_t = 0.05$	0.07859	0.04686	0.06344	-0.03123
				$\bar{c}_t = 0.1$	0.07709	0.04596	0.06223	-0.03063
				$\bar{c}_t = 1$	0.05734	0.03419	0.04629	-0.02278
5	100	100	Bouderba *et al.* (2013)		0.08015	0.04574	0.06413	-0.02992
			FSPT[a]		0.08014	0.04546	0.06440	-0.02205
			CPT[a]		0.07989	0.04710	0.06672	—
			Benahmed *et al.* (2017)		0.08063	0.04860	0.06275	-0.03005
			Present	$\bar{c}_t = 0$	0.08059	0.04875	0.06227	-0.02965
				$\bar{c}_t = 0.05$	0.07901	0.04779	0.06105	-0.02907
				$\bar{c}_t = 0.1$	0.07749	0.04687	0.05987	-0.02851
				$\bar{c}_t = 1$	0.05756	0.03482	0.04447	-0.02118
∞	100	100	Bouderba *et al.* (2013)		0.08119	0.05056	0.05815	-0.02657
			FSPT[a]		0.08119	0.05023	0.05829	-0.02060
			CPT[a]		0.08099	0.05196	0.06030	—
			Benahmed *et al.* (2017)		0.08172	0.03035	0.05845	-0.02660
			Present	$\bar{c}_t = 0$	0.08166	0.03053	0.05771	-0.02587
				$\bar{c}_t = 0.05$	0.08004	0.02992	0.05656	-0.02536
				$\bar{c}_t = 0.1$	0.07848	0.02934	0.05546	-0.02486
				$\bar{c}_t = 1$	0.05810	0.02172	0.04106	-0.01841

Table 8 Comparison of the first three non-dimensional frequencies $\tilde{\omega}/\pi^2$ ($\tilde{\omega}=\omega a^2\sqrt{\rho_m h/D_m}$) of simply supported isotropic square plate (a/h=5 and K_s=10)

$\tilde{\omega}_{mn}$	Theory		K_w=0	K_w=10	K_w=10^2	K_w=10^3
$\tilde{\omega}_{11}$	Zhou *et al.* (2004)		2.2334	2.2539	2.4300	3.7111
	Matsunaga (2000)		2.2334	2.2539	2.4300	3.7112
	Sheikholeslami and Saidi (2013)		2.2334	2.2539	2.4300	3.7111
	Benahmed *et al.* (2017)		2.2383	2.2590	2.4377	3.7726
	Present	$\bar{c}_t = 0$	2.2469	2.2678	2.4485	3.8093
		$\bar{c}_t = 0.05$	2.2989	2.3194	2.4964	3.8402
		$\bar{c}_t = 0.1$	2.3498	2.3699	2.5433	3.8708
		$\bar{c}_t = 1$	3.1274	3.1425	3.2752	4.3860
$\tilde{\omega}_{12}$	3D Zhou *et al.* 2004		4.4056	4.4150	4.4986	5.2285
	HSDPT Matsunaga 2000		4.4056	4.4150	4.4986	5.2285
	Sheikholeslamia nd Saidi (2013)		4.4056	4.4150	4.4986	5.2285
	Benahmed *et al.* (2017)		4.4220	4.4317	4.5182	5.2959

Table 8 Continued

$\tilde{\omega}_{mn}$	Theory		K_w=0	K_w=10	K_w=10^2	K_w=10^3
$\tilde{\omega}_{12}$	Present	$\bar{c}_t = 0$	4.4464	4.4564	4.5458	5.3582
		$\bar{c}_t = 0.05$	4.4714	4.4814	4.5703	5.3790
		$\bar{c}_t = 0.1$	4.4964	4.5063	4.5947	5.3997
		$\bar{c}_t = 1$	4.9234	4.9325	5.0134	5.7602
$\tilde{\omega}_{13}$	3D Zhou *et al.* 2004		7.2436	7.2487	7.2948	7.7191
	HSDPT Matsunaga 2000		7.2436	7.2488	7.2948	7.7191
	Sheikholeslamia nd Saidi (2013)		7.2436	7.2488	7.2948	7.7191
	Benahmed *et al.*		7.2864	7.2919	7.3412	7.8096
	Present	$\bar{c}_t = 0$	7.3280	7.3339	7.3862	7.8904
		$\bar{c}_t = 0.05$	7.3426	7.3485	7.4007	7.9040
		$\bar{c}_t = 0.1$	7.3572	7.3630	7.4151	7.9175
		$\bar{c}_t = 1$	7.6144	7.6201	7.6704	8.1571

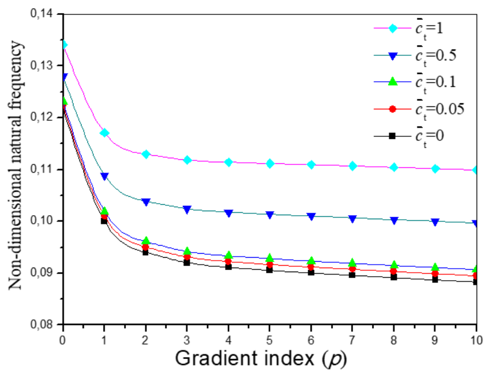

Fig. 6 The variation of non-dimensional fundamental natural frequency $\hat{\omega}$ versus the gradient index p for different values of damping coefficient (a/b=1, a/h=10, K_w=100 and K_s=10)

and for different visco-Pasternak's parameters K_w, K_s and $\bar{c}_t$. The results obtained in the case where the damping effect is omitted, demonstrate that the results predicted by Benahmed *et al.* (2017), Thai and Choi (2011) and Zenkour and Sobhy (2013) overestimates the displacements and stresses, and this is due to the thickness stretching influence, which is not considered in the theories developed by these two references. The consideration of the damping effect decreases the displacement and stresses.

In Table 7, the obtained results are compared with those predicted by Benahmed *et al.* (2017) and Bouderba *et al.* (2013) for different values of the power index p and visco-Pasternak's parameters K_w, K_s and $\bar{c}_t$. When the damping coefficient $\bar{c}_t$ is omitted, an excellent agreement is shown for all values of the gradient index p and foundation parameters K_w and K_s. In addition, it is seen that the deflection and stresses are reduced with the consideration of the elastic foundations K_w and K_s. As the damping coefficient $\bar{c}_t$ increases, deflection and stresses become smaller.

Fig. 4 presents the variations of the axial stress $\bar{\sigma}_x$ within the thickness of FG square plate for different values of the damping coefficient $\bar{c}_t$. It is observed that the maximum compressive stresses occur at a point near the upper surface and the maximum tensile stresses occur, of course, at a point near the lower surface of the FG plate. It is remarked that normal stress are very sensitive to the inclusion of the viscoelastic term near the external surfaces of the plate.

Fig. 5 shows the variation of the dimensionless shear stress $\bar{\tau}_{xy}$ within the thickness of a square FG plate for different values of the viscosity term $\bar{c}_t$. The results reveal that the maximum compressive stresses occur at a point near the upper surface and the maximum tensile stresses occur at a point near the lower surface of the FG plate. Again, the in-plane shear stress are very sensitive to the inclusion of the viscoelastic term near the external surfaces of the plate.

4.2 Vibration analysis

In order to prove the accuracy of the proposed theory in predicting the dynamic response of plates, several examples are presented and discussed in this section.

Table 8 shows the comparison between the first three non-dimensional frequencies of simply supported square plate resting on visco-Pasternak foundation calculated employing the current theory and those given by Zhou *et al.* (2004), Matsunaga (2000), Sheikholeslamia and Saidi (2013) and Benahmed *et al.* (2017). It is seen that, the results of the present theory that considers the stretching effect are in excellent agreement with those predicted by other theories. However, the inclusion of the viscosity effect increases the vales of frequencies. Thus, this effect makes the plate rigid.

The non-dimensional natural frequency $\hat{\omega}$ of FG square plate versus the visco-Pasternak's parameters K_w, K_s and $\bar{c}_t$, gradient index and thickness–length ratio are presented in Table 9. The obtained results are in good agreement with those of given by other theories. Again, the consideration of the viscosity term makes the plate rigid.

Fig. 6 present the variation of the non-dimensional fundamental natural frequency of simply supported square FG plates versus the gradient index for various values of the

Table 9 The non-dimensional natural frequency $\hat{\omega}=\omega h\sqrt{\rho_m / E_m}$ of square FG plate versus the visco-Pasternak's parameters, gradient index and thickness–length ratio

P	Theory	$\bar{c}_t$	kw=0,ks=0				kw=0,ks=100				kw=100,ks=0				kw=100,ks=100			
			h/a															
			0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2	0.05	0.1	0.15	0.2
0	Ref[a]	0	0.0291	0.1135	0.2459	0.4169	0.0405	0.1593	0.3487	0.5988	0.0298	0.1163	0.2521	0.4281	0.0410	0.1613	0.3531	0.6070
	Ref[b]	0	0.0291	0.1134	0.2454	0.4154	0.0406	0.1599	0.3515	0.6080	0.0298	0.1162	0.2519	0.4273	0.0411	0.1619	0.3560	0.6162
	Ref[c]	0	0.0291	0.1136	0.2461	0.4174	0.0406	0.1594	0.3492	0.6011	0.0298	0.1164	0.2524	0.4286	0.0411	0.1614	0.3537	0.6089
	Present	0	0.0291	0.1138	0.2467	0.4185	0.0406	0.1596	0.3501	0.6040	0.0298	0.1165	0.2529	0.4298	0.0411	0.1615	0.3545	0.6119
		0.05	0.0293	0.1145	0.2483	0.4214	0.0407	0.1600	0.3512	0.6060	0.0300	0.1172	0.2545	0.4326	0.0412	0.1620	0.3556	0.6139
		0.1	0.0295	0.1152	0.2498	0.4242	0.0408	0.1605	0.3523	0.6080	0.0301	0.1179	0.2560	0.4354	0.0413	0.1625	0.3567	0.6158
		1	0.0324	0.1269	0.2766	0.4724	0.0430	0.1692	0.3718	0.6425	0.0330	0.1294	0.2822	0.4825	0.0435	0.1711	0.3759	0.6500
1	Ref[a]	0	0.0222	0.0870	0.1891	0.3222	0.0377	0.1482	0.3236	0.5509	0.0233	0.0911	0.1983	0.3383	0.0384	0.1506	0.3288	0.5598
	Ref[b]	0	0.0227	0.0891	0.1939	0.3299	0.0382	0.1517	0.3365	0.5876	0.0238	0.0933	0.2036	0.3476	0.0388	0.1542	0.3422	0.5978
	Ref[c]	0	0.0226	0.0883	0.1918	0.3264	0.0380	0.1497	0.3295	0.5699	0.0236	0.0924	0.2011	0.3431	0.0386	0.1521	0.3349	0.5794
	Present	0	0.0226	0.0884	0.1922	0.3272	0.0380	0.1498	0.3303	0.5733	0.0236	0.0925	0.2015	0.3439	0.0386	0.1523	0.3358	0.5830
		0.05	0.0228	0.0894	0.1945	0.3315	0.0381	0.1504	0.3317	0.5757	0.0238	0.0935	0.2037	0.3480	0.0387	0.1529	0.3371	0.5854
		0.1	0.0231	0.0904	0.1969	0.3357	0.0383	0.1510	0.3331	0.5781	0.0241	0.0944	0.2059	0.3520	0.0389	0.1535	0.3385	0.5878
		1	0.0273	0.1073	0.2350	0.4040	0.0410	0.1617	0.3569	0.6203	0.0282	0.1107	0.2426	0.4176	0.0415	0.1640	0.3620	0.6292
2	Ref[a]	0	0.0202	0.0789	0.1711	0.2906	0.0373	0.1463	0.3180	0.5370	0.0214	0.0836	0.1817	0.3092	0.0380	0.1489	0.3236	0.5460
	Ref[b]	0	0.0209	0.0819	0.1778	0.3016	0.0380	0.1508	0.3351	0.5861	0.0221	0.0867	0.1889	0.3219	0.0386	0.1535	0.3412	0.5970
	Ref[c]	0	0.0207	0.0807	0.1748	0.2965	0.0376	0.1483	0.3265	0.5650	0.0218	0.0854	0.1855	0.3158	0.0383	0.1509	0.3323	0.5752
	Present	0	0.0207	0.0807	0.1748	0.2966	0.0376	0.1483	0.3269	0.5673	0.0218	0.0854	0.1856	0.3159	0.0383	0.1510	0.3328	0.5777
		0.05	0.0210	0.0819	0.1776	0.3015	0.0378	0.1490	0.3284	0.5699	0.0221	0.0865	0.1881	0.3206	0.0384	0.1516	0.3342	0.5802
		0.1	0.0213	0.0831	0.1803	0.3064	0.0379	0.1496	0.3298	0.5725	0.0224	0.0877	0.1907	0.3252	0.0386	0.1522	0.3356	0.5828
		1	0.0260	0.1021	0.2234	0.3837	0.0408	0.1610	0.3552	0.6173	0.0270	0.1059	0.2319	0.3989	0.0414	0.1634	0.3606	0.6268

[a]Sheikholeslami and Saidi (2013), [b] Baferani *et al.* (2011), [c]Benahmed *et al.* (2017)

damping coefficient. It is observed that the increase of the gradient index makes the plate flexible. However, the increase of the damping coefficient makes the plate rigid.

5. Conclusions

A simple quasi-3D hyperbolic shear deformation model for bending and dynamic behavior of FG plates resting on visco-Pasternak foundations is presented. The model contains only four unknown variables, satisfies the zero traction boundary conditions at the plate's surfaces without requiring a shear correction factor and considers the thickness stretching effect. Thus, a considerably lower computational time is reached. The accuracy of the proposed formulation is proved by comparing it with existing solutions, and excellent agreement was observed in all cases where the viscosity effect is omitted. The inclusion of this effect makes a plate stiffer, and hence leads to a reduction of deflection and an increase of frequency. An improvement of present formulation will be considered in the future work to consider the thermal or magnetic effects or other types of materials (Panjehpour *et al.* 2014ab, Attia *et al.* 2015, Hamidi *et al.* 2015, Bousahla *et al.* 2016, Bouderba *et al.* 2016, Panjehpour *et al.* 2016, Karami *et al.* 2017, Fahsi *et al.* 2017, Chikh *et al.* 2017, El-Haina *et al.* 2017, Menasria *et al.* 2017, Klouche *et al.* 2017, Khetir *et al.* 2017, Mouffoki *et al.* 2017, Karami *et al.* 2018bcd, Yazid *et al.* 2018, Youcef *et al.* 2018, Cherif *et al.* 2018,Zine *et al.* 2018, Mokhtar *et al.* 2018, Karami *et al.* 2019a, b, c, Berghouti *et al.* 2019,Semmah *et al.* 2019).

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