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Bending moments in raft of a piled raft system using Winkler analysis

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Abstract. Bending moments in the raft of a pile raft system is affected by pile-pile interaction and pile-raft interaction, amongst other factors. Three-Dimensional finite element program has to be used to evaluate these bending moments. Winkler type analysis is easy to use but it however ignores these interactions. This paper proposes a very simplified and novel method for finding bending moments in raft of a piled raft based on Winkler type where raft is supported on bed of springs considering pile-pile and pile-raft interaction entitled as "Winkler model for piled raft (WMPR)"
The pile and raft spring stiffness are based on load share between pile and raft and average pile raft settlement proposed by Randolph (1994). To verify the results of WMPR, raft bending moments are compared with those obtained from PLAXIS 3D software. A total of sixty analysis have Performed varying different parameters. It is found that raft bending moments obtained from WMPR closely match with bending moments obtained from PLAXIS 3D. A comparison of bending moments ignoring any interaction in Winkler model is also made with PLAXIS-3D, which results in large difference of bending moments. Finally, bending moment results from eight different methods are compared with WMPR for a case study. The WMPR, though, a simple method yielded comparable raft bending moments with the most accurate analysis.

Keywords: pile-raft interaction; pile-pile interaction; Winkler model; bending moment; piled raft foundation

1. Introduction

Nowadays combined piled raft foundation (CPRF) is very useful type of foundation for tall buildings and has been used for various famous tall buildings like Burj Khalifa, the world tallest building. In CPRF both piles and raft play a vital role, especially in stiff clayey soil profile where bearing capacity is good for raft to contribute as well as piles can mobilize their full capacity under a small settlement (Poulos 2001a). Load sharing percentages of raft and piles in piled raft system is also very complex (Ko *et al.* 2018). In conventional designs, contribution of raft in CPRF is usually ignored which results in uneconomical design. In fact, raft takes its share of total load, thus reducing the number of piles required. In the CPRF system, both piles and raft contribute substantially to increase the overall robustness of the system like the thickness of raft reduces the differential settlement as well as to satisfy punching shear requirements (Poulos 2001b), piles can be used to reduce overall settlement and to increase the capacity (Capacity and settlement based Design approach, CSBD), to minimize differential settlement (Differential settlement based Design approach, DSBD) and can be used as a stress reducer in raft (Raft based Design approach, RBD) (Mandolini *et al.* 2013). Concentrated arrangement of piles, also minimize bending moments in raft considerably (Ghiasi and Moradi 2018). Increasing raft thickness in a piled raft system can reduce differential settlement effectively (Fattah *et al.* 2013).

However, designing a pile raft system has not yet been part of any building code, and pile-raft system is designed conventionally by assuming only piles take the total load.

Various methods have been developed for the geotechnical design of pile-raft system. These methods include Randolph method (1994), Poulos-Davis-Randolph (PDR) method (1994), Modified PDR method (2000) and Burland Approach (1995).

For structure design of CPRF, the methods of analysis vary from very simple to sophisticated. FEM software's like FLAC (2002), PLAXIS 3D (2012) and ELPLA (2018) can be use but most of the structural engineers do not have access to this sophisticated software. Approximate computer-based methods for structure design involves "strip on springs" approach proposed by Poulos and "plate on springs" approach proposed by Clancy and Randolph. Another approach proposed by Nguyen (2013) in SAP 2000 can be used for analysis but it involves a trial and error procedure for adjusting each spring in raft mesh and pile spring, which makes the method computationally prohibitive. Some structural engineers find stiffness's of pile and raft from FEM software's and use these stiffness's as a Winkler springs to find bending moments in raft of piled raft. More often, structural engineers perform CPRF analysis using discrete springs for raft and piles ignoring interaction between pile-raft and pile-pile. Discrete springs concept is based on Winkler model which cannot take interaction effects and every spring in Winkler model is acting in isolation without affecting the neighboring springs. Moreover, Winkler springs have the limitation to model differential settlement (Chang *et al.* 2018). Interaction effects between pile-raft and pile-pile will

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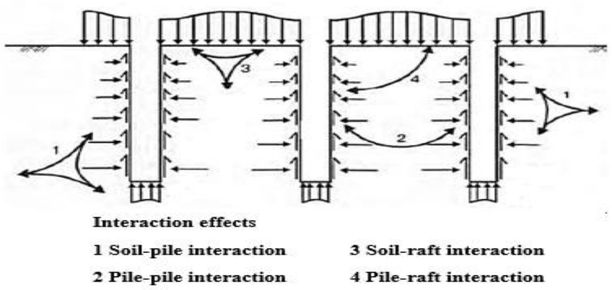


Fig. 1 Interaction effects in CPRF

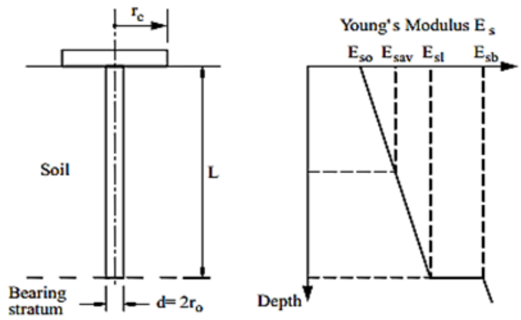


Fig. 2 Equivalent raft concept (Randolph 1994)

reduces stiffness's of the piled raft system, resulting in high bending moments. Using Winkler springs concept and ignoring interaction factors, analysis of piled raft will underestimate bending moment as well as settlements.

Interaction factors affects response of piled raft system largely. There are mainly four interaction effects which is shown in Fig. 1, in which pile-pile and pile-raft interaction factors are important and must be taken into account otherwise it will affect bending moment and settlements of the CPRF. Pile-pile interaction can be defined as additional settlement of a pile caused by an adjacent pile while pile-raft interaction can define as superposing the displacement field of raft caused by a pile supporting the raft. For calculating pile-pile interaction effects Poulos and Davis (Poulos and Davis 1980) approach can be used while for pile-raft interaction approach of Randolph (Randolph 1994) can be used.

This paper proposes a computationally simple method to find bending moments in raft of a CPRF. The method uses Winkler formulation that takes into account effects of pile-raft and pile-pile interaction. The proposed model named Winkler model for piled raft (WMPR) is based on Randolph (1994) method that approximates the load taken by piles and rafts in CPRF and pile raft settlement. The spring stiffness for raft and piles thus determined can be used in any structure analysis program to solve for the raft bending moments. The raft bending moments of sixty different CPRF systems obtained from WMPR are compared with 3-D finite element analysis carried out in PLAXIS-3D software. The results show a good agreement between the simple and more sophisticated analysis. Since the proposed method of WMPR is based on Randolph (1994), a brief description of this method is provided in subsequent section.

2. Randolph Method (1994)

This method is based on compatibility of both raft and piles, which means average settlement of raft and pile is identical. The combined pile raft stiffness (Kpr) is given as shown in Eq. (1).

$$K_{pr}=\frac{(K_p+K_r(1-2\alpha_{rp}))}{(1-\alpha_{rp}^2 * \frac{K_r}{K_p})} \tag{1}$$

where K_p is pile group stiffness obtained from (Fleming *et al.* 1992), Kr is raft stiffness alone calculated from (Poulos and Davis 1974) and and αrp is pile-raft interaction factor given by (Randolph 1994). Ratio of loads taken by raft (Pr) and piles (Pp) is given by Eq. (2).

$$\frac{\mathrm{Pr}}{\mathrm{Pt}}=\frac{\mathrm{Pr}}{\mathrm{Pr}+\mathrm{Pp}}=\frac{(1-\propto_{\mathrm{rp}})\mathrm{Kr}}{\mathrm{Kp}+(1-\propto_{\mathrm{rp}})\mathrm{Kr}} \tag{2}$$

The average pile raft settlement can be calculated using Eq. (3).

$$\Delta=\frac{\mathrm{Pt}}{\mathrm{Kpr}} \tag{3}$$

where Pt=Pr + Pp.

α_{rp} can be calculated using Eq. (4).

$$\propto_{rp}=1-\frac{\ln(\frac{r_c}{r_p})}{\zeta} \tag{4}$$

where ζ=ln(r_m/r_p), rm={0.25+x[2.5r(1-u)-0.25]*L}, x=E_{sl}/E_{sb}, r=E_{sav}/E_{sl}, r_c= average radius of pile cap which is equal to total area of Raft divide by number of Piles, r_m=maximum radius of influence of Raft, r_o= radius of pile, L= length of pile, E_{sl}= soil Young's modulus at pile tip, E_{sb}=soil Young's modulus below pile tip at bearing stratum and E_{sav}= average soil Young's modulus along pile shaft. All parameters are shown in below in Fig. 2.

2.1 Stiffness of raft and piles

Initial stiffness's of raft and piles are required while using the WMPR method. To find raft stiffness, a simple solution proposed by Poulos and Davis (1974) which is briefly described below. For axial stiffness of raft, Eq. (5) is used.

$$Kr=\frac{\mathrm{P}}{\rho_z} \tag{5}$$

where *Kr* is the axial stiffness of raft, P is the applied load and ρ_z is the vertical displacement of the raft. ρ_z is obtained using Eq. (6)

$$r_z=\frac{P_{av}\ a}{E_s}\mathrm{I_p} \tag{6}$$

where $P_{av}=P/\pi a^2$ is the average stress on the raft, "*a*" is the equivalent radius of raft, Es is the elastic modulus of soil and I_p is the influence factor which can be found from the following Fig. 3.

For estimating single pile stiffness kp, Randolph (1994) developed an equation which is very suitable because it can

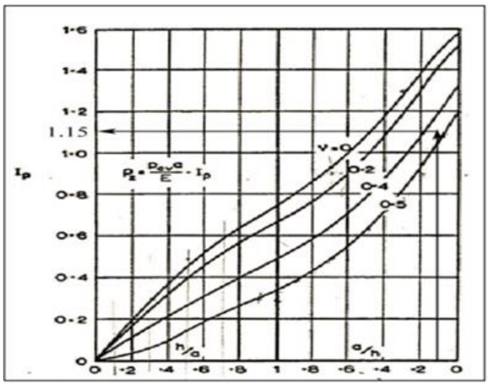


Fig. 3 Influence factors for displacement of rigid circle (Poulos and Davis 1974)

take variation of several parameters and can be applicable for many situations. Randolph formulation for kp is shown below in Eq. (7).

$$k_p = \frac{P_t}{w_t} = \frac{\frac{4\eta}{(1-\nu)\xi} + \rho \frac{2\pi}{\zeta} \frac{\tanh(\mu l)}{\mu l} \frac{l}{r_0}}{1 + \frac{1}{\pi\lambda} \frac{4\eta}{(1-\nu)\xi} \frac{\tanh(\mu l)}{\mu l} \frac{l}{r_0}} * (G_l r_0) \tag{7}$$

where ζ, rm, x, r are defined in Eq. (4).

$$h = \frac{r_b}{r_o}, \qquad l = \frac{E_p}{G_l}, \qquad ml = \sqrt{\frac{2}{\zeta\lambda\left(\frac{l}{r_o}\right)}}$$

where P_t is applied load on pile, l is pile length, G_l is soil shear modulus at depth l, w_t is settlement of pile, r_o is pile radius and r_b is pile radius at pile base, E_p is Elasticity modulus of pile materials.

To find pile group stiffness, Poulos (2000) developed a very simple but approximate approach given below in Eq. (8).

$$Kp = k_p * \sqrt{n} \tag{8}$$

where Kp is the pile group stiffness, k_p is the single pile stiffness and n is total number of piles in the pile group.

3. Winkler model for Piled raft (WMPR)

Pile-raft and pile-pile interactions reduces the overall stiffness of the piled raft stiffness. Winkler springs in structural analysis programs has to be reduced to take into account interaction factors. To take into account interaction effects in Winkler model Randolph (1994) method can be used to calculate raft and pile spring stiffness. The following steps are used to calculate raft and pile spring stiffness for the proposed WMPR method:

1) Find the average piled raft settlement (**Δ**) from equation-3.

2) Calculate load taken by each pile (p_p) as $p_p = \frac{P_p}{N}$, where N is the number of piles in CPRF.

3) Stiffness of single pile (kp′) as $Kp' = p_p/\Delta$

4) Similarly, modulus of subgrade reaction of raft (kr′) kr′= Pr/ **Δ** /A, where A is the raft area.

The above steps are also shown in a flow diagram in Fig. 4 above.

While finding the values of K_{pr} using equation-1, pile raft interaction factor is incorporated using Eq. (4). Similarly, to account pile-pile interaction in WMPR method, Eq. (7) is used to find pile group stiffness.

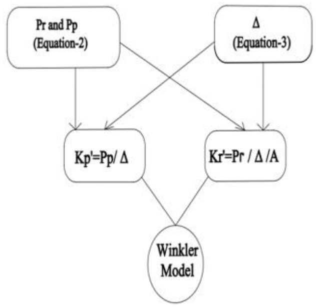


Fig. 4 Flow diagram for WMPR

Table 1 Cases selected for verification

Raft thickness [m]	Es[MPa]	Ec [MPa]	Raft dimensions [LXB] [m]	Total no. of analysis
0.3	80,60,30	30000	5x5, 10x10, 15x15, 20x20	
0.5	80,60,30	30000	5x5, 10x10, 15x15, 20x20	
0.8	80,60,30	30000	5x5, 10x10, 15x15, 20x20	60
1.2	80,60,30	30000	5x5, 10x10, 15x15, 20x20	
1.5	80,60,30	30000	5x5, 10x10, 15x15, 20x20	

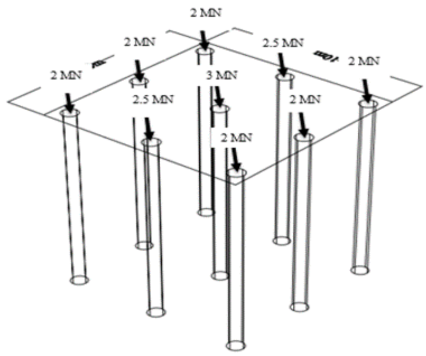


Fig. 5 Typical piled raft layout and applied load (10×10 m)

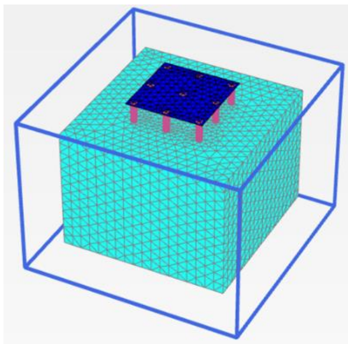


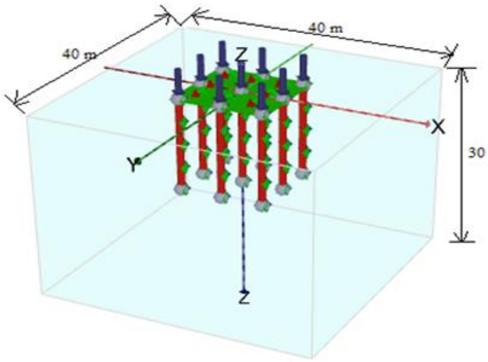
Fig. 6 Meshed model in PLAXIS 3D (Top soil is hidden for clarification)

Table 2 Basic properties of soil and pile materials

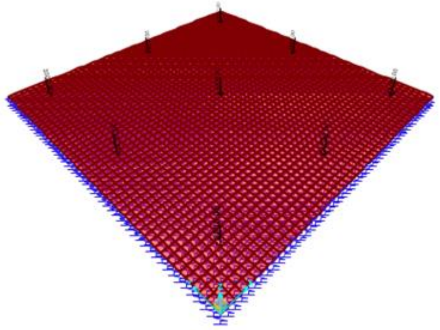
Properties	Soil	Pile materials (Concrete)
Poison's ratio of pile, u	0.3	0.2
Cohesion	50 [KPa]	-
Angle of internal friction	0	-
Unit weight	g_{sat} = 20 [KN/m^3] g_{unsat}=19 [KN/m^3]	25 [KN/m^3]

Table 3 Springs values based on proposed method for 10×10 m piled raft

E_s of soil [MPa]	Pile Spring [KN/m]	Raft Spring [KN/m]
30	67787.49	56.06
60	129192.71	114.79
80	166888.93	155.5



(a)



(b)

Fig. 7 Typical model in (a) PLAXIS 3D and (b) SAP 2000

3.1 Bending moments comparison

A total of 60 piled raft cases are analyzed using WMPR. Different parameters considered for analysis are provided in Table 1. Pile length and diameter considered are 15 m and 0.5 m, respectively. Number of piles selected is 9. The applied load and layout for 10x10 m piled raft is shown in Fig. 5. Meshed model of PLAXIS 3D is shown in Fig. 6. Typical model in PLAXIS 3D and SAP 2000 is also shown in Fig. 7. Model dimension selected for all cases as 40 m (X), 40 m (Y) and 30 m (Z, thickness of soil layer) as shown in Fig. 7(a). Maximum positive and negative moments are compared with the results of PLAXIS 3D.

Basic properties of soil and pile material are shown in Table 2.

The comparison of positive and negative maximum bending moments obtained from WMPR and PLAXIS 3D are shown from Figs. 8 to 13. Figures also include results from Winkler analysis that ignores the pile-raft and pile-pile interaction.

Values of raft and pile springs calculated for a typical 10x10 m piled raft of 0.5 m raft thickness is shown in Table 3. These values were calculated using equation 1 to 6 and WMPR method. The raft was meshed using iterative practice starting from 2×2 m mesh size down to 0.2×0.2 m. At mesh size of 0.2×0.2 m, bending moments in raft converged.

The results of WMPR closely match those obtained

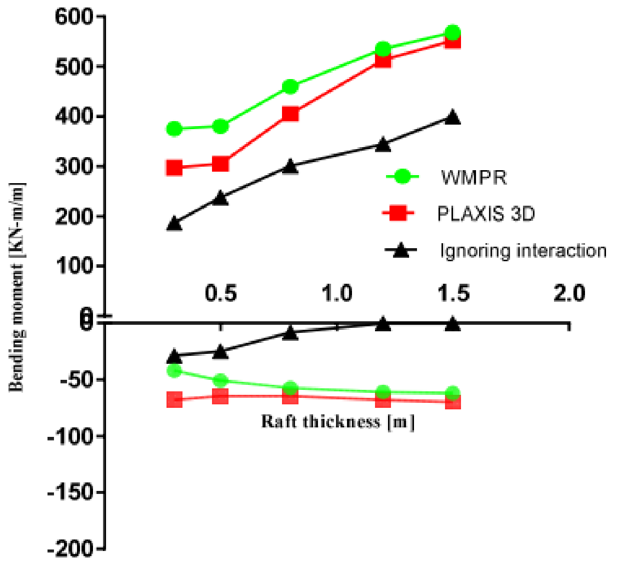


Fig. 8 Maximum (+ & -) moment comparison for different cases of 10x10 m Piled raft, Es=80 MPa

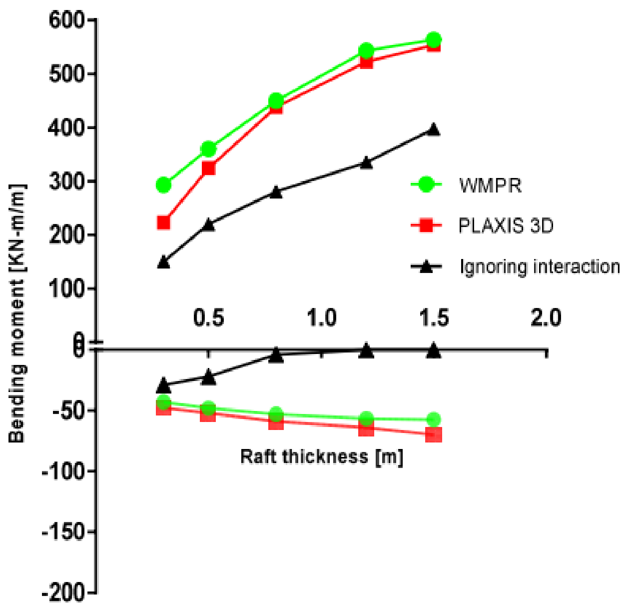


Fig. 9 Maximum (+ & -) moment comparison for different cases of 10x10 m Piled raft, Es=60 MPa

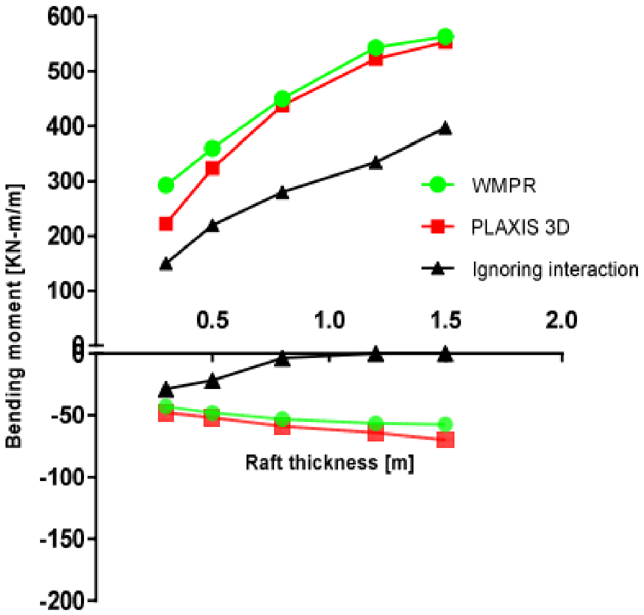


Fig. 10 Maximum (+ & -) moment comparison for different cases of 10x10 m Piled raft, Es=30 MPa

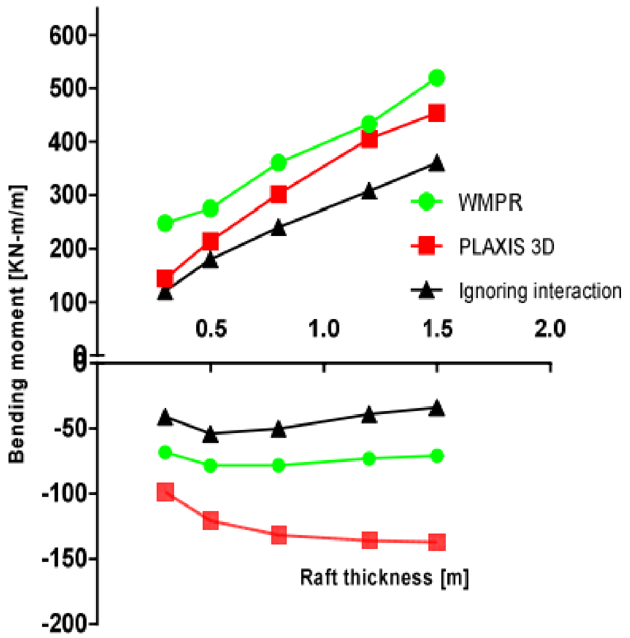


Fig. 12 Maximum (+ & -) moment comparison for different cases of 15x15 m Piled raft, Es=60 MPa

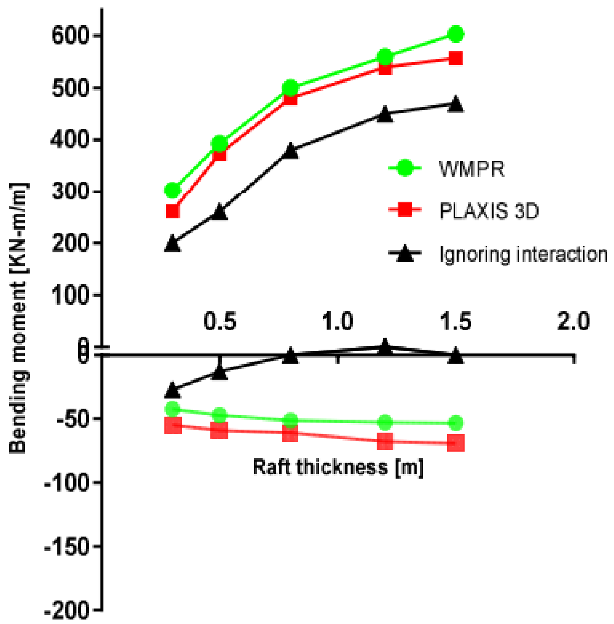


Fig. 11 Maximum (+ & -) moment comparison for different cases of 15x15 m Piled raft, Es=80 MPa

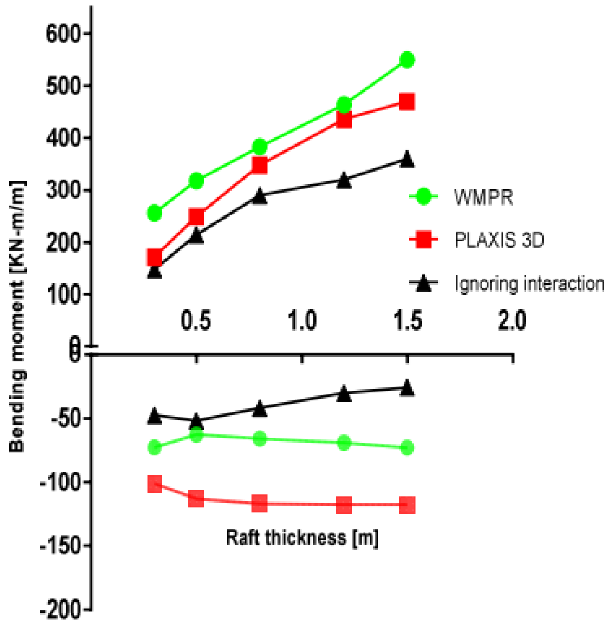


Fig. 13 Maximum (+ & -) moment comparison for different cases of 15x15 m Piled raft, Es=30 MPa

from more sophisticated analysis of PLAXIS-3D. The WMPR based bending moments consistently show slightly higher bending moments compared to those from PLAXIS-3D, which would result is slightly more conservative raft reinforcement. Whereas, results obtained from Winkler Analysis ignoring interaction result in very low bending moments. Which means ignoring interaction factors will underestimate bending moment in raft and can result an unsafe design.

At smaller values of soil elastic modulus, the difference between WMPR and PLAXIS 3D is more as compared to larger values of soil elastic modulus. Similarly, the difference becomes more at smaller thickness of raft as compared to larger thickness of raft.

It can be observed from all figures that increase in raft thickness cause a drastic increase in bending moment of raft. It can also be concluded from figures that raft on stiff soil profile will have less bending moment as compared to raft on soft soil profile with low elastic modulus. The load taking percentage of raft also increases with increase in its dimension and elastic modulus of soil. This shows the suitability of piled raft foundation system on stiff soil profile to resist high vertical loads.

Percentage result difference of WMPR and PLAXIS 3D for positive bending moments are shown in Table 4 below:

Table 4 Percentage difference of WMPR and PLAXIS 3D for +ive bending moment

Thickness (m)	Percentage difference between WMPR and PLAXIS 3D					
	10×10 m Piled raft			15×15 m Piled raft		
	Es=80 MPa	Es=60 MPa	Es=30 MPa	Es=80 MPa	Es=60 MPa	Es=30 MPa
0.3	20.8%	24%	13.7%	27.3%	31.2%	33.1%
0.5	19.7%	9.9%	4.9%	28.4%	22.1%	21.8%
0.8	12%	2.8%	3.9%	8.5%	16.3%	9.2%
1.2	4.1%	3.8%	3.6%	1.8%	6.40%	6%
1.5	2.9%	1.8%	7.7%	13.7%	12.6%	14.6%

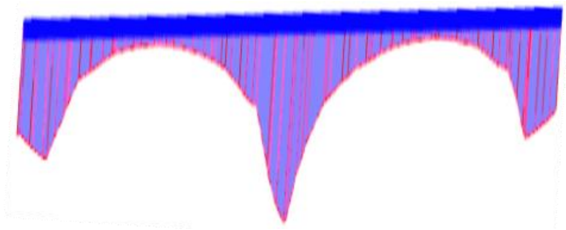


Fig. 14 PLAXIS 3D typical cross section for bending moment

Fig. 15 WMPR typical cross section for bending moment (SAP 2000)

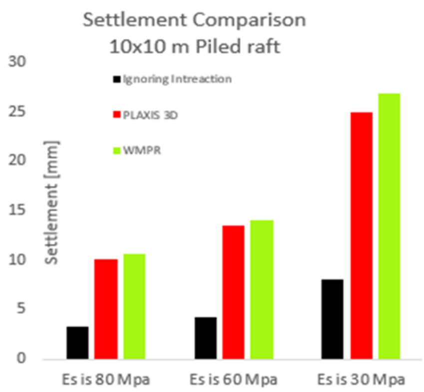


Fig. 16 Settlement comparison of WMPR, PLAXIS 3D and ignoring interaction

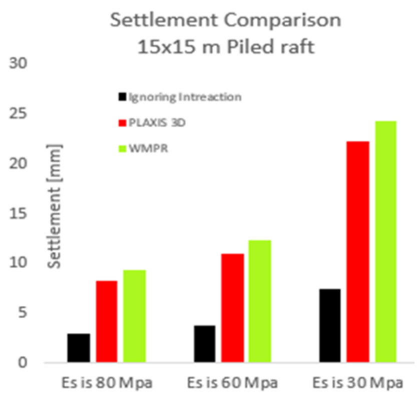


Fig. 17 Settlement comparison of WMPR, PLAXIS 3D and ignoring interaction

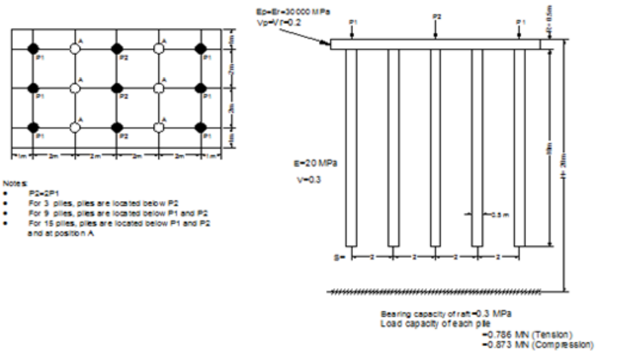


Fig. 18 Poulos hypothetical 10×6 m piled raft with 9 piles

Table 3 Summary of Poulos results along with current analysis (Poulos 2001)

Method	Central Settlement [mm]	Corner Pile Settlement [mm]	Maximum Raft Moment [MN-m/m]	Percentage of Load Taken by Pile	Reference
Poulos-Davis Randolph	36.8	-	-	77	
GARP5	34.2	26	0.684	65.1	
GASP	33.8	22	0.563	65.5	
Burland	33.8	29.7	0.688	65.5	
FLAC 2-D	65.9	60.5	0.284	79.5	
FLAC 3-D: Moments directly from output stresses			0.326		(Poulos 2001)
FLAC 3-D: Moments from extrapolated stresses			0.421		
FLAC 3-D: Moments from displacements			0.484		
Proposed WMPR	35.14	28.73	0.64	79.04	Current Analysis
PLAXIS 3D	39.6	32.7	0.61	65	
Ignoring interactions	10.2	6.9	0.46	73.9	

Bending moment profile obtained from WMPR and PLAXIS-3D is also shown in Figs. 14 and 15, which shows approximately same bending moment pattern. The case considered for below moment pattern is 15×15 m piled raft of thickness 1.2 m with 9 piles and Es is 80 MPa.

3.2 Settlement comparison

Settlement was also calculated and compared for all cases using PLAXIS 3D, WMPR and ignoring interaction. For 10×10 m and 15×15 m piled raft with raft thickness of 1.2 m, settlement results are shown in Figs. 16 and 17. As illustrated in figures, settlement calculated with PLAXIS 3D and WMPR are matching closely. When interaction factors are ignored, the settlement results are too much small, as compared to other two methods. Therefore, ignoring interaction are also underestimating settlement results which can lead to create serious issues in structures after construction.

4. Proposed WMPR comparison with Poulos example

Proposed Winkler method for piled raft has also validated with the analysis performed by Poulos (Poulos,2001) on simple hypothetical piled raft of 10x6 m with 9 piles as shown in Fig. 18. Poulos compared different methods in terms of settlement, load sharing percentages and maximum raft bending moment as shown in Table 3. Additionally, included in the Table-3 are the results obtained from WMPR, PLAXIS 3D, and Winkler method ignoring interaction. Proposed simplified WMPR method shows a very good agreement with all parameters given in Table 3.

5. WMPR under different loading condition

The most ideal loading condition for CPRF system is point loads above piles in case, where piles are located under columns.

Poulos (2001) shows four conditions in which pile may be provided below column. It includes when maximum moment and shear demand exceeds the capacity of raft, demand contact pressure exceeds the bearing capacity of soil and when local settlement underneath column is more than allowable. To check the reliability of WMPR method analysis were also performed for all cases of 15x15 m piled raft under uniformly distributed load of 80 KN/m^2 using WMPR method and PLAXIS 3D. Moment pattern changes completely under this loading condition as compared to point loads for small thicknesses of raft. It is very important during analysis to idealize the most applicable condition of loading. The analysis performed shows approximately same percentage difference of moment as under point loads.

6. Conclusions

Use of CPRF is very advantageous because contribution of raft to support the total applied load is also considered as against the conventional design where only piles are considered to transfer the total load. Though yet not part of any building code, a number of methods ranging from simple to more sophisticated have been developed to analyze and design the CPRF both geotechnically and structurally. This research work was focused on developing a simple analysis tool for structure engineers to find bending moments in raft of a CPRF and yet accurate enough to yield comparable results with the more sophisticated 3D finite element analysis. The proposed method called the Winkler Model for Pile Raft (WMPR) use the raft and pile spring stiffness that accounts for pile-pile and pile-raft interaction based on Randolph (1994) method. The results of WMPR are solved for sixty different CPRF systems and maximum positive and negative raft bending moments compared with PLAXIS-3D results. It was found that the WMPR gives relatively conservative values of positive raft bending moments, while in case of negative moments, WMPR method gives 5% less value from PLAXIS-3D throughout all the analyzed cases. In addition, it was also found that ignoring interactions will result in very low bending moments as well as in small settlement which will ultimately lead to unsafe design. It is recommended to incorporate interaction factors during analysis and design of CPRF. The proposed WMPR is recommended for use in ordinary low-rise buildings, and can also be used for preliminary design of important high-rise structures.

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