Geomechanics and Engineering, *Vol. 17, No. 6 (2019) 565-572*
DOI: https://doi.org/10.12989/gae.2019.17.6.565

Evaluation of soil spatial variability by micro-structure simulation

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(Received November 1, 2018, Revised March 16, 2019, Accepted March 17, 2019)

Abstract. Spatial variability is an inherent characteristic of soil, and auto-correlation length (ACL) is a very important parameter in the reliability or probabilistic analyses of geotechnical engineering that consider the spatial variability of soils. Current methods for estimating the ACL need a large amount of laboratory or in-situ experiments, which is a great obstacle to the application of random field theory to geotechnical reliability analysis and design. To estimate the ACL reasonably and efficiently, we propose a micro-structure based numerical simulation method. The quartet structure generation set algorithm is used to generate stochastic numerical micro-structure of soils, and scanning electron microscope test of soil samples combined with digital image processing technique is adopted to obtain parameters needed in the QSGS algorithm. Then, 2-point correlation function is adopted to calculate the ACL based on the generated numerical micro-structure of soils. Results of a case study shows that the ACL can be estimated efficiently using the proposed method. Sensitivity analysis demonstrates that the ACL will become stable with the increase of mesh density and model size. A model size of 300 × 300 with a grid size of 1 × 1 is suitable for the calculation of the ACL of clayey soils.

Keywords: auto-correlation length (ACL); quartet structure generation set (QSGS); scanning electron microscope (SEM); digital image processing (DIP); numerical simulation; spatial variability; correlation function

1. Introduction

Soil is formed by the process of weathering, erosion, removal and deposition of rock. Due to the difference of formation condition and sedimentary history, soil has spatial variability, which means that soil properties at any two different points are different but they are correlated (Degroot and Baecher 1993). Knowledge of soil's spatial variability or auto-correlation is very important to many geotechnical engineering problems, such as the reliability analysis of slope stability (Babu and Murthy 2005), seepage analysis of a dam (Tan *et al*. 2011, Chan and Low 2012) and the determination of characteristic values of soil properties for reliability-based design (Orr 2017).

Spatial variability of soil can be modeled using random field theory, in which the spatial variability of soil is described by auto-correlation function and auto-correlation length (Cornell 1971, Wu 1974; Vanmarcke 1977, Kulllawy 1992, Degroot 1996, Lacasse and Nadim 1997, Phoon and Kulhawy 1999, Uzielli *et al*. 2005, Srivastava and Babu 2011, Stuedlein *et al*. 2012, Lombardi *et al*. 2017). The auto-correlation function describes the reduction mode of the auto-correlation with distance between spatial points. The auto-correlation length (ACL) is a critical distance within which soil properties at two points are deemed as correlated. Otherwise, they are considered as independent (Onyejekwe *et al*. 2016). Because the type of auto-correlation function has much smaller influence on the reliability of geotechnical engineering compared to the influence of the auto-correlation length. (Salgado and Kim 2014) evaluation of the auto-correlation length of soils is a key task in geotechnical reliability analysis and reliability-based design.

Due to the importance of the ACL, many studies have been made to evaluate the ACLs of soil properties, including the physical parameters (Phoon and Kulhawy 1999), shear strength parameters (Degroot and Baecher 1993, Haldar and Babu 2009, Matteo *et al*. 2013), and hydraulic parameters (Gupta *et al*. 2006, Wang *et al*. 2007, Moradi *et al*. 2016). Many studies evaluate the ACLs based on laboratory or in-situ experiments. However, the measuring of soil properties are very time-consuming (especially for unsaturated soil), and a large number of tests are required for calculating the ACL (Falchetto *et al*. 2014). The huge amount of laboratory or in-situ experiments is a great obstacle to the application of random field theory to geotechnical reliability analysis and design.

Evaluating the ACL using a micro-structure method combined with digital image processing (DIP) can reduce the amount of laboratory or in-situ experiments (Berryman 1985). Moon *et al*. (2014) adopted the DIP technique to investigate the micro-structure of asphalt mixture images, and then evaluated the ACL of the three-phrase random heterogeneous mixture by using *n*-point correlation functions. Wang *et al*. (2007) proposed a quartet structure generation set (QSGS) method, which can generate the micro-structures of porous media very well. Although the QSGS method is better than most other micro-structure generation methods, there is not a clear instruction for the determination of the parameters needed in QSGS algorithm

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http://www.techno-press.org/?journal=gae&subpage=7
ISSN: 2005-307X (Print), 2092-6219 (Online)

(Wang *et al*. 2007).

The aim of this paper is to estimate the auto-correlation length of soils using a micro-structure based numerical method. We proposed a method to determine the QSGS parameters based on scanning electron microscope (SEM) test and digital image processing (DIP) technique. After obtaining the QSGS parameters of soils, we generated the stochastic porous micro-structures of soils using the QSGS algorithm, and 2-point correlation function was used to calculate the ACL for the numerical micro-structure of soils. A detail description of the proposed method is presented in Section 2, followed by a case study in Section 3. And then, discussions and conclusions are presented in Section 4 and 5, respectively.

2. Evaluation of ACL by micro-structure simulation

There are mainly two steps for evaluating the ACL of soils using the proposed micro-structure numerical simulation. The first step is to generate a numerical micro-structure of soils by the QSGS algorithm, and the second step is to calculate the ACL of soils according to the concept of n-point correlation function.

2.1 Generation of numerical micro-structure of soils

2.1.1 Quartet structure generation set (QSGS) algorithm

Several methods have been proposed to generate the random porous media, among which quartet structure generation set (QSGS) algorithm is very suitable for modeling the micro-structure of soils, which is a mixture of solid particles and pores (Tacher *et al*. 1997, Pilotti 1998, Wang and Pan 2007, Griffiths *et al*. 2012). The QSGS algorithm is a multi-parameter random generation-growth method. Considering soil particle as a growing phase and pore as a non-growing phase (Wang and Pan 2007), the QSGS algorithm for generating a two-dimensional stochastic micro-structure of soils can be summarized as follows.

(1) Select the parameters needed in QSGS algorithm (i.e., soil porosity n_p, core distribution probability P_c, and directional growth probability P_{di}). The detail method for the determination of n_p, P_c, and P_{di} will be described in Section 2.1.2.

(2) Designate mesh density and model size for the numerical simulation. For example, a square model with side length of 200 and grid side length of 1 is used here for the purpose of illustration. Therefore, the number of grids (n_x, n_y) along the x- and y-direction of a two-dimensional coordinate system are both 200 in this example.

(3) Designate each grid in the two-dimensional coordinate system as solid core or pore by assigning a uniformly distributed random number between 0 and 1 to that grid. Grids with random numbers less than the core distribution probability P_c are chosen as initial solid cores (growing phase), and the other grids in the grid system are chosen as pores (non-growing phase).

(4) Expand each solid core of the growing phase into their neighboring grids along eight directions shown in Fig. 1. The expansion process can be performed by assigning a uniformly distributed random number between 0 and 1 to each of the eight neighbor grids for each solid core. The neighboring grid in direction i (i = 1, 2, …, 8) will become a solid particle if the random number in this direction is less than the corresponding directional growth probability P_{di}.

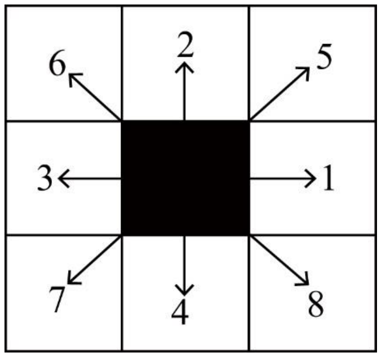


Fig. 1 Eight growth directions of a grid in 2-dimensional coordinate system

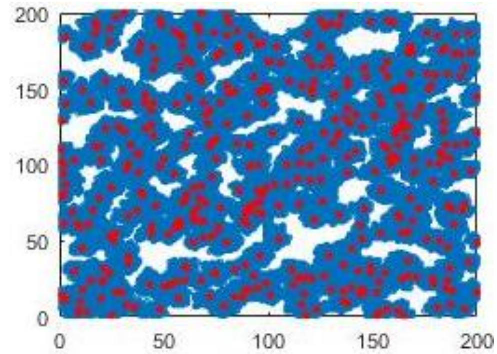


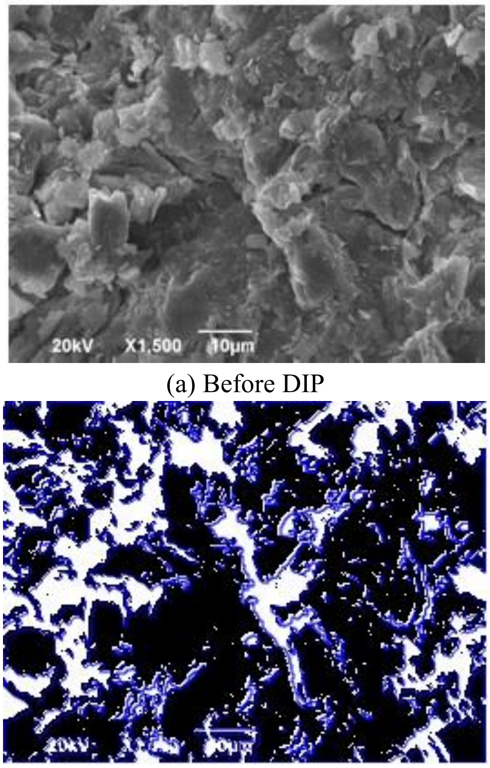
Fig. 2 A random realization of soil micro-structure (n_p = 0.39, P_c = 0.01, P_{d14} = 0.15 and P_{d58} = 0.25P_{d14}. The red point, the blue part, and the white part represents initial solid cores, soil particles, and pores, respectively)

(5) Repeat the growing process of Step 4 until the volume fraction of growing phrase is larger than the given value of the fraction of solid particles (i.e., $1-n_p$), or the volume fraction of non-growing phrase is less than or equal to the given porosity n_p.

Note that the formation process of the growing phrase (soil particles) is stochastic in the QSGS algorithm, so the micro-structure generated using this method is also stochastic. A random realization of soil micro-structure with n_p = 0.39, P_c = 0.01, P_{d14} = 0.15 and P_{d58} = 0.25P_{d14} is shown in Fig. 2, in which the red point represents initial solid cores, the blue part represents soil particles, and the white part represents pores. It is apparent that the numerical models can reflect the stochastic characteristics of soil particles and pores. The generated soil model is very similar to the realistic micro-structure of soils. Hence, the QSGS algorithm is widely used in the numerical modeling of soil micro-structure.

2.1.2 Estimation of QSGS parameters based on SEM and DIP

As described in Section 2.1.1, three parameters (porosity n_p, core distribution probability P_c, and directional growth



(a) Before DIP

(b) DIP for calculating the properties of soil particles

Fig. 3 SEM image of a soil sample (The black parts with blue boundaries are soil particles)

probability P_{di}) are needed in the QSGS algorithm for generating the numerical micro-structure of soils. Considering scanning electron microscope (SEM) test can show soil micro-structure clearly, and digital image processing (DIP) technique has been widely used to analyze the micro-structure of porous material (Falchetto *et al*. 2014), we propose a SEM-based digital image processing (DIP) technique for estimating the three QSGS parameters. The main steps are as follows.

(1) Carry out SEM tests for soil samples to obtain SEM images.

(2) Conduct digital image processing for the SEM images by using some software. The Image Processing Toolbox™ of Matlab is used in this paper because of the powerful programming function of Matlab (Velasquez *et al*. 2010). By converting SEM image to binary image, an $M\times N$ digital image matrix whose element values are ones or zeros can be obtained, where one and zero represent soil particle and pore, respectively. Furthermore, the connected components (soil particles) in the binary image and the properties of soil particles (such as the areas, centroids, equivalent diameters, and perimeters) in the binary image can be easily obtained. The length unit for the binary image is pixel. A SEM image of a soil sample before and after digital image processing are shown in Fig. 3(a) and 3(b), respectively.

(3) Estimate n_p. Soil porosity n_p can be calculated according to its definition using Eq. (1)

$$n_p = 1 - A_s/(M\times N) \tag{1}$$

where A_s is the summation of the areas of soil particles, and $M\times N$ is the total area of the digital image matrix.

(4) Estimate P_c. Core distribution probability P_c is the probability of a grid to become a initial core of the growing phase. Based on this definition, P_c can be estimated using Eq. (2)

$$c_d = N_c/(M\times N) \tag{2}$$

where N_c is the number of initial solid cores (i.e., the number of the connected components in the binary image). Theoretically, the core distribution probability P_c should be less than or equal to the volume fraction of soil particles (Wang *et al*. 2007), and the value of P_c controls the size of soil particles. A greater value of P_c leads much more solid particles and much smaller average size of solid particles to be generated within a given area (Wang and Pan 2007).

(5) Estimate P_{di}. Directional growth probability P_{di} is the probability for a non-growing grid cell to expand into its neighboring cell in the ith (i = 1, 2, …, 8) direction to become part of the growing phase. $P_{d14} : P_{d58} = 4$ is widely assumed for generating an isotropic porous material (He and Luo 1997, Takashi and Abe 1997, Wang and Pan 2007, Wang *et al*. 2007). Similar to the estimation of 2-point correlation function, which will be described in Section 2.2.1, we proposed a method for estimating P_{d14} as follows.

For a binary digital image shown in Fig. 4 (the black grids represent soil particles and the white grids represent pores), designate randomly two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ along direction angle θ (θ = 0°, 90°, 180°, and 270° for i = 1, 2, 3, and 4, respectively) with a distance of $P_1P_2 = r$. The distance r is assumed to be a uniformly distributed random number whose value is in the range of zero and the maximum value of the equivalent diameter of soil particles. If the two points P_1 and P_2 both lies in the solid phase in the binary digital image, let $N_h = N_h + 1$, where N_h is a counter with an initial value of zero. Repeat this simulation for N_s times. If the value of N_s is very large (e.g., 10^6), the directional growth probability P_{di} can be estimated using Eq. (3)

$$P_{di} = N_h/N_s \tag{3}$$

2.2 Calculation of auto-correlation length of soils

2.2.1 2-point correlation function

The correlation between different parts of a heterogeneous material can be represented by n-point correlation functions, such as 2-point or 3-point correlation function (Velasquez *et al*. 2010, Falchetto *et al*. 2014, Moon *et al*. 2014). The n-point correlation function measures the probability of finding n points of a heterogeneous material located on the same phase of that material. For example, the 2-point correlation function (S_2) of a two-phase porous material measures the probability that two points separated by a certain distance (r) are both located in the solid phase or pore phase. Wang (2017) demonstrated that the difference of the ACLs evaluated using the 2-point and 3-point correlation function were negligible, so only the 2-point correlation function is adopted in this paper.

The estimation of the 2-point correlation function can

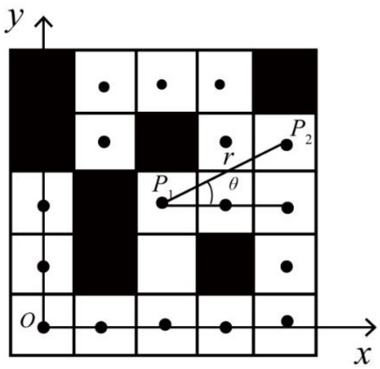


Fig. 4 Schematic diagram for estimating directional growth probability parameter (P_{di})

also be performed based on Fig. 4. However, Fig. 4 is now considered as a micro-structure numerical model of a heterogeneous soil. From Fig. 4, the relationship between the 2-point correlation function S_2 and the distance r can be obtained at several different distance by an iterative algorithm. The steps for estimating the 2-point correlation function ($S_2(r)$) is as follows.

(1) Designate the range of distance r and the step length dr. The range of distance r is suggested to increase from $r_{\min} = 0$ to $r_{\max} = \min\{M, N\}/2$, where M and N is the number of pixels along x and y coordinate of the numerical model, respectively (Moon *et al*. 2014). The step length dr determines the number of distance ($\underline{N}_r$) in the range of [$r_{\min}$, $r_{\max}$] and the smoothness of the $S_2(r)$ curve. A smaller step length corresponds to a smoother $S_2(r)$ curve.

(2) For the ith iteration, let $r = r_{\min} + dr * (i-1)$. Designate randomly two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ along direction angle θ with a distance of $P_1P_2 = r$. The direction angle θ is a uniformly distributed random number whose value lies between 0° and 360°. If points P_1 and P_2 both lies in the same phase in the numerical model, let $N_h = N_h + 1$, where N_h is a counter with an initial value of zero.

(3) Repeat Step 2 for N_s times, where N_s is very large number (e.g., 10^6). Then, the 2-point correlation function for distance r can be estimated as follows

$$S_2(r) = N_{\mathrm{hits}}/N_{\mathrm{s}} \tag{4}$$

(4) Repeat Step 2~3 for all values of r in the range of [$r_{\min}$, $r_{\max}$]. Then, the relationship between S_2 and r at N_r different values of r can be estimated.

2.2.2 Auto-correlation length

Based on the generated micro-structure model, the auto-correlation function of both the solid phase and the pore phase can be computed. Because many soil properties (such as hydraulic conductivity, saturation and density) are correlated with soil porosity, only the correlation function (S_2) of the pore phase were calculated. And then, the 2-point auto-correlation function (R_2) of pore phase can be calculated by Eq. (5) as follows (Li *et al*. 2009)

$$R_2(r) = \frac{S_2(r) - n_{\mathrm{p}}^2}{n_{\mathrm{p}} - n_{\mathrm{p}}^2} \tag{5}$$

Table 1 Relationship between scale of ACL and auto-correlation model parameter

Model	Expression	ACL
SNX	$R(r) = \exp(-r/b)$	$2b$
SQX	$R(r) = \exp[-(r/b)2]$	$\pi^{0.5}b$
CSX	$R(r) = \exp(-r/b)\cos(r/b)$	b
SMK	$R(r) = (1+r/b)\exp(-r/b)$	$4b$
BIN	$R(r) = 1-r/b\ (r < b);\ R(r) = 0\ (r \geq b)$	b

By curve fitting for the data points of [$R_2(r)$, r] using some auto-correlation functions, the ACLs can be evaluated easily from the curve fitting parameters of auto-correlation functions. Five popularly used auto-correlation models (single exponential (SNX), squared exponential (SQX), cosine exponential (CSX), second-order Markov (SMK), and binary noise (BIN)) are listed in Table 1 (Jaksa *et al*. 1997, Phoon *et al*. 2003). For each correlation function model, parameter b can be obtained by curve fitting, and then the corresponding value of ACL can be estimated according to column 3 in Table 1.

3. Case study

Five clay soil samples were sampled in an engineering site in Hefei, China. These soil samples were yellowish-brown clay which were located 10 m below the ground level. The basic soil properties are listed in Table 2, where w_0 is water content, ρ is density, G_s is soil specific gravity, n_p is porosity, and I_p is plasticity index.

After carrying out the SEM tests for each soil sample, the three QSGS parameters and their statistics (mean and coefficient of variation (*COV)*) are listed in Table 3. It can be seen that the COVs of parameters n_p, P_c and P_{d14} are all less than 0.2, which means that the variation of these parameters is small. Therefore, the means of parameters n_p, P_c and P_{d14} listed in Table 3 were adopted for generating the

Table 2 Basic properties of soil samples

No.	w_0 (%)	ρ (g.cm^{-3})	G_s	n_p	I_p
1#	25.1	1.96	2.71	0.42	28.3
2#	24.0	2.01	2.75	0.41	30.7
3#	22.8	2.05	2.65	0.37	22.0
4#	22.6	2.03	2.64	0.38	31.0
5#	23.3	2.00	2.66	0.39	33.7

Table 3 Values of QSGS parameters

No.	n_p	P_c	P_{d14}
1#	0.42	0.008	0.122
2#	0.41	0.010	0.160
3#	0.37	0.008	0.119
4#	0.38	0.010	0.144
5#	0.39	0.008	0.185
Mean	0.39	0.01	0.15
COV	0.05	0.10	0.19

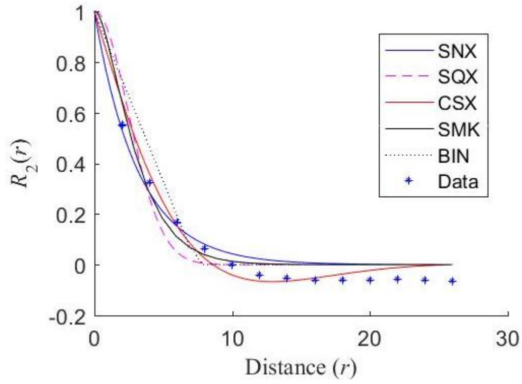


Fig. 5 Curve fitting using five auto-correlation models

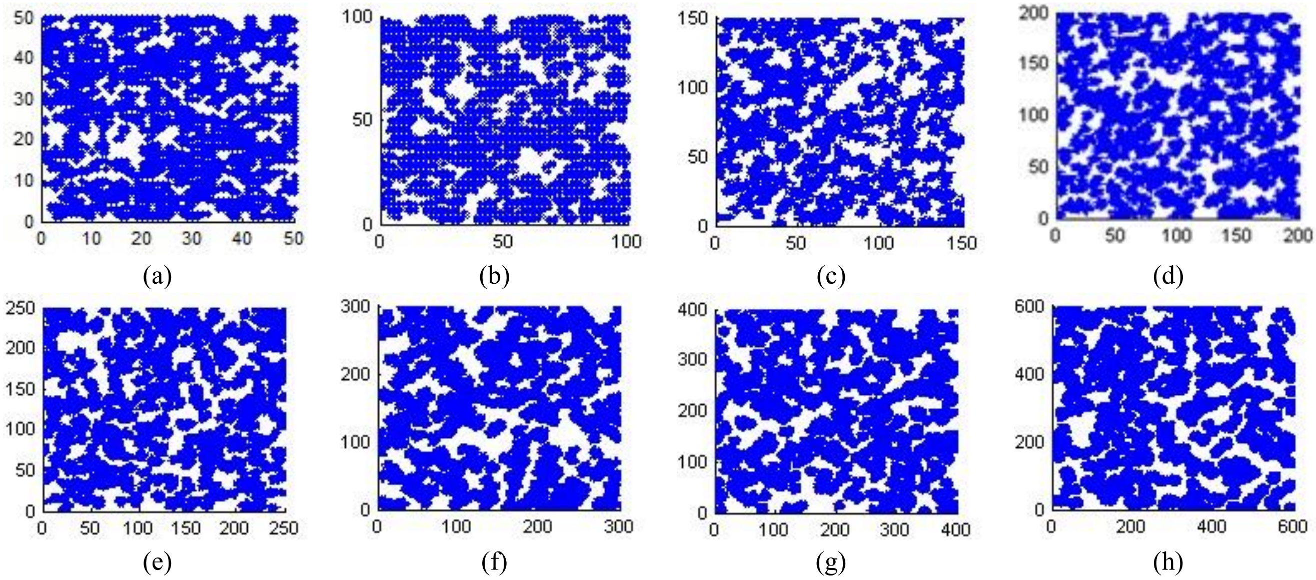


Fig. 6 Soil structures with different number of grids: (a) 50×50, (b)100×100, (c)150×150, (d)200×200, (e)250×250, (f)300×300, (g)400×400 and (h)600×600

numerical micro-structure model of the clay. As suggested by Wang *et al*. (2007), we assumed $P_{d58} = 0.25P_{d14}$ for generating isotropic micro-structure of soils. For the numerical realization of soil micro-structure shown in Fig. 2, the data points of [$R_2(r)$, r] and the fitted curves using the five auto-correlation models listed in Table 1 are shown in Fig. 5. The ACLs and the corresponding determination coefficients (R^2) are listed in Table 4. It can be seen that the value of R^2 are all greater than 0.93 for the five auto-correlation models, which means that all the five auto-correlation models are suitable for calculating the ACLs of soils. The COV of the ACLs using the five auto-correlation models is 0.10, which is very small. This represents that the type of auto-correlation functions has little influence on the values of ACLs of soils.

4. Discussions

The ACL estimated by proposed micro-structure simulation may be influenced by some factors such as mesh density and model size of the numerical model. Therefore, eight groups of mesh densities and eight groups of model sizes were analyzed, respectively. In each case, the mean values of parameters n_p, P_c and P_{d14} listed in Table 3 and $P_{d58} = 0.25P_{d14}$ are used for generating the micro-structures by using the QSGS algorithm.

4.1 Influence of mesh density

To study the influence of mesh density on the ACLs of soils, a square numerical model of side length 200 with eight mesh densities were studied. The numbers of grids of these models are 50×50, 100×100, 150×150, 200×200, 250×250, 300×300, 400×400, 600×600, respectively. Correspondingly, the side length of each grid is 4, 2, 1.33, 1, 0.8, 0.67, 0.5, and 0.33, respectively. The micro-structures generated by the QSGS algorithm with different mesh densities are shown in Fig. 6. We can see that the difference among the images of micro-structures of sparse mesh densities (such as Fig. 7(a)-7(d)) are obvious. With

Table 4 Auto-correlation lengths of soils

Model	SNX	SQX	CSX	SMK	BIN	Mean	COV
ACL	7.71	7.92	7.91	6.70	9.00	7.85	0.10
R^2	0.93	0.93	0.94	0.98	0.93	/	/

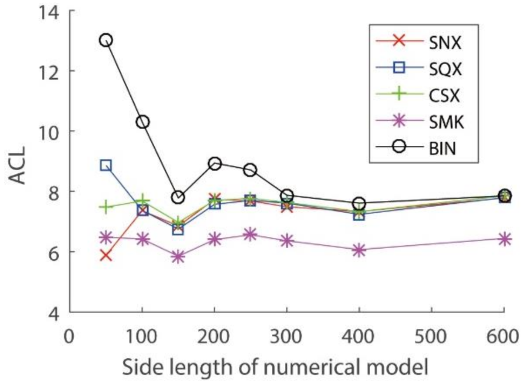


Fig. 7 Variation of ACL with different mesh densities

(a) (b) (c) (d) (e)

Fig. 8 Soil structures with different model sizes: (a) 100 × 100, (b) 200 × 200, (c) 300 × 300, (d) 400 × 400 and (e) 600 × 600

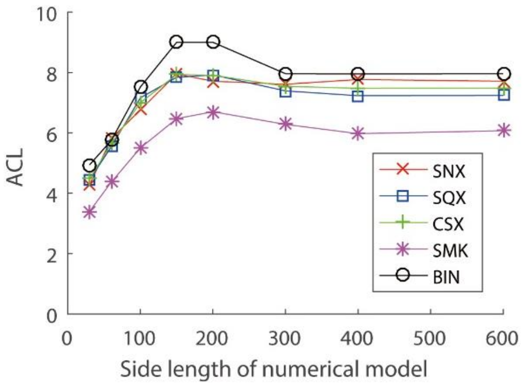


Fig. 9 Variation of ACL with different model sizes

the increase of mesh density, the difference among the images of different micro-structures (such as Fig. 7(d)-7(h)) become negligible.

The ACLs evaluated from Fig. 6 are shown in Fig. 7 for different mesh densities. The variation of ACLs with mesh density is a little large for all the five auto-correlation models when the mesh density is sparse (number of grids on each side of the numerical model is less than 200, or side length of each grid is greater than 1). With the increase of mesh density, the variation of ACLs becomes weaker and the values of ACLs tend to be stable.

4.2 Influence of model sizes

Based on the study of the influence of mesh density on the ACL of soils, the mesh density of side length of 1 was used for investigating the influence of model size on the ACL of soils. Then, eight model sizes (30×30, 60×60, 100×100, 150×150, 200×200, 300×300, 400×400, 600×600) with this mesh density were generated using the QSGS algorithm. Five of these numerical micro-structure models are shown in Fig. 8. We can find there are no obvious difference among these micro-structures because all these models are generated using the same QSGS parameters.

The ACLs estimated from the eight numerical models with different model sizes are shown in Fig. 9. The ACLs increase quickly when the model length is small, and then they decrease a little until the model length is 300. The ACLs become nearly stable when the model length is larger than 300. Therefore, a model size of 300 × 300 with a grid size of 1 × 1 is suitable for the calculation of the ACL of soil property. The stable value of the ACL is about 7.3 for the five auto-correlation models.

5. Conclusions

A micro-structure based simulation method is proposed for estimating the auto-correlation length (ACL) of soils. The proposed method combines the scanning electron microscope (SEM) test and the digital image processing (DIP) technique for obtaining the QSGS parameters, and the QSGS algorithm is used to generate the micro-structure of soils. And then, the auto-correlation length (ACL) of soils can be estimated by 2-point correlation function method.

The results in this paper show that the proposed method can generate reasonably the micro-structures of soils, and the ACL of soils can be computed easily based on the generated micro-structures of soils. A model size of 300 × 300 with a grid size of 1 × 1 is suitable for the calculation of the ACL of clay soil. However, further studies to confirm or amend this finding are warranted.

Acknowledgements

The financial supports from the National Natural Science Foundation (41572282) are greatly acknowledged.

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