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Footing settlement formula based on multi-variable regression analyses

Murat Hamderi*

Faculty of Engineering, Turkish-German University, Istanbul, Turkey

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Abstract. The formulas offered so far on the settlement of raft footings provide only a rough estimate of the actual settlement. One of the best ways to make an accurate estimation is to conduct 3-dimensional finite element analyses. However, the required procedure for these analyses is comparatively cumbersome and expensive and needs a bit more expertise. In order to address this issue, in this study, a raft footing settlement formula was developed based on ninety finite element model configurations. The formula was derived using multi-parameter exponential regression analyses. The settlement formula incorporates the dimensions and the elastic modulus of a rectangular raft, vertical uniform pressure and soil moduli and Poisson's ratios up to 5 layers. In addition to this, an equation was offered for the estimation of average deflection of the raft. The proposed formula was checked against 3 well-documented case studies. The formula that is derived from 3D finite element analyses is useful in optimising the raft properties.

Keywords: footing; raft formula; DIANA; settlement; PLAXIS 3D

1. Introduction

For the last 70 years, numerous methods have been proposed for the settlement estimation of shallow foundations. The leading ones can be categorized as follows: 1-) Elastic methods (Janbu *et al.* 1956, Grioud 1972, Das 1983, Mayne and Poulos 1999, Bowles 1987, Mohamed *et al.* 2013), 2-) SPT-based methods (Terzaghi and Peck 1948, Meyerhof 1956, Burland and Burbidge 1985, ErzÍn and Gul 2013), 3-) CPT-based methods (De Beer 1965, Schmertmann 1970, Berardi *et al.* 1991, Mir *et al.* 2017). In an attempt to evaluate the reliability of the available methods, Lutenegger and Degroot (1995) calculated the settlement of the North Footing at the Texas A & M University and compared the results with the measured data (reported by Briaud and Gibbons 1994). Lutenegger and Degroot (1995) used 24 different SPT-based and 7 different CPT-based methods to calculate the settlement. The settlement calculations included the load level, which produced 25 mm settlement during the test. They reported that using SPT-based methods, the settlement resulted in an average value of 42 mm, whereas this was 53 mm for CPT-based methods. For SPT-based and CPT-based methods, the average deviation in settlement results were 67% and 113%, respectively. Such deviations indicate a major problem with accuracy. The settlement of the North footing reported by Briaud and Gibbons (1994) has also been calculated with several different methods within the scope of this study (Section 2.3.3). Das and Nagaratnam (2007) also reported an overview on the current settlement formulas, and concluded that the current formulas generally overestimate settlements, whereas they underestimate the allowable pressures.

Instead of these empirical and analytical methods, nowadays, engineers are practicing 3-dimensional finite element (3D FE) methods to enhance their settlement prediction (Lee *et al*. 2015, Anil, *et al.* 2017). On the other hand, our experience shows that establishing 3D FE models is expensive and needs a bit more expertise. As a result, old empirical and analytical methods are still widely used in practice, which result in less economical design outputs.

In order to address this issue, in this study, a raft footing settlement formula was developed based on about a ninety finite element model configurations created in DIANA FE program (Release 9.5). The raft footing settlement formula incorporates the dimensions and the elastic modulus of a rectangular raft, vertical uniform pressure, soil moduli and Poisson's ratios up to 5 layers. The reliability of the offered formula was checked against another 3D FE program (PLAXIS 3D) and also against 3 well-documented case studies.

2. The creation of settlement database from finite element models

In order to derive a settlement formula with high estimation capabilities, it is required that sufficient number of FE model configurations are incorporated. To do this, input parameters should be included at certain intervals within a smartly chosen range (Table 1, column 3-4). For example, the soils with a modulus less than 15 MPa were not incorporated in the data pool, due to the fact that they would behave quite nonlinear more than the Mohr-Coulomb material model would handle. The intervals were small in the relatively nonlinear range, (15, 20, 20, 30, 40, 45, 50, 60, 75, 80, 85, 90, 100 MPa), whereas the intervals were

∗Corresponding author, Associate Professor
E-mail: hamderi@tau.edu.tr

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Table 1 The range of the input parameters incorporated in the FE models

No	Description	Sym.	Range/Values	Unit
1	width of raft @ x direction	wi_x	(3-54), 3, 5, 10, 15, 20, 30, 36, 40, 49.3, 50, 54	m
2	width of raft @ y direction	wi_y	(3-50), 3, 5, 10, 15, 17.8, 20, 21, 24, 27, 30, 33.5, 40, 50	m
3	Soil moduli of the first 20 m	E_1,E_2,E_3,E_4	(15-600), 15, 20, 23, 25, 30, 40, 50, 60, 75, 90, 100, 200, 300, 500, 600	MPa
4	Soil modulus below 20 m	E_5	(15-600), 15 , 20, 30, 40, 45, 50, 60, 75, 80, 85, 90, 100, 150, 200, 300, 500, 550, 600	MPa
5	Distributed vertical load	ld	(10-800), 10, 25, 50, 90, 100, 134, 160, 200, 220, 400, 800	kPa
6	20 m depth to bedrock distance	bed	(30-100), 30, 50, 70, 75, 100	m
7	raft thickness	th	(0.5-3.0), 0.5, 0.9, 1, 1.5, 2.5, 3.0	m
8	elas. mod. of raft	E_{raft}	(10-50), 10, 15, 20, 25, 30, 40, 50	GPa
9	Poisson's ratio of the first 20 m	po_1, po_2, po_3, po_4	(0.2 -0.45), 0.2, 0.25, 0.3, 0.35, 0.4, 0.45	-
10	Poisson's ratio below 20 m	po_5	(0.2 -0.45), 0.2, 0.25, 0.3, 0.35, 0.4, 0.45	-

Table 2 The description of the material input parameters incorporated in the FE models

Material	Mat. Model	Density (g/cm^3)	Frict. Angle (°)	Cohe. (kPa)	Mod. of Elast.	Poisson's ratio
Soil	Mohr-Coulomb	1.7	32	0.5	Table 1	0.2-0.45
Raft Concrete	Linear-Elastic	2.4	-	-	Table 1	0.2

greater in the relatively linear range (150, 200, 300, 500, 550, 600 MPa). In general, reasonable values were selected for the lower and upper bounds. By applying these approaches to all parameter intervals, 90 FE model configurations were obtained. In each configuration, at least one parameter was changed. The varying model parameters were basically the width and length (wi_x, wi_y) and thickness (th) of the rectangular raft, applied uniform load (*ld*), soil moduli and Poisson's ratios up to 5 layers (E_1, E_2, E_3, E_4, E_5, po_1, po_2, po_3, po_4, po_5,) and the elastic modulus of concrete raft (E_c). The range of parameters incorporated in the finite element model is given in Table 1. Some of the parameters that were fixed for all finite element configurations are given in Table 2.

The general geometrical configuration of the finite element model and details of the rectangular raft are given in Fig. 1. The mesh was built from higher-order tetrahedral elements with mid-side nodes. It should be noted that the sum of the thicknesses of the first four layers is 20 m.

2.1 The output of the finite element runs

A sample settlement output obtained from the FE program is shown in Fig. 2. The settlement values at the centre and at the corner of the raft were used as an input for the settlement formula derivation.

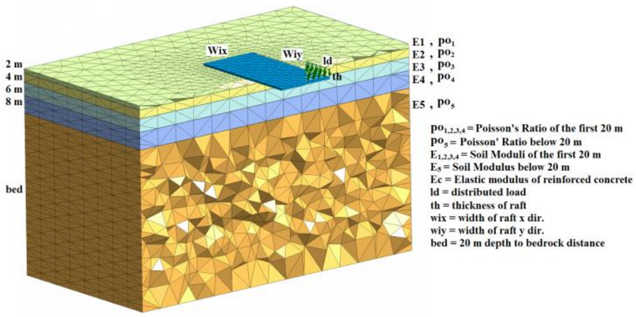

Fig. 1 An example of a FE configuration with a 20 m × 40 m size raft

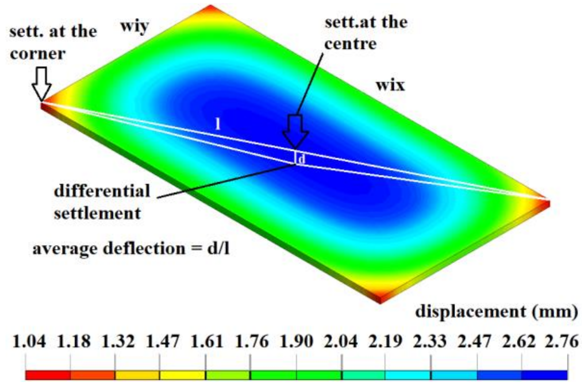

Fig. 2 A sample settlement output of the 20 m × 40 m raft

Table 3 The unitless fitting coefficients of the settlement formula

@centre	a	b	c	d	e
	0.4387	-0.1073	-0.1996	-0.2258	-0.2287
	f	g	h	i	j
	-0.1874	1.0214	0.0957	-0.1338	-0.0616
	k	l	m	n	o
	-0.0566	-0.0475	-0.0446	-0.0347	-0.0645
@corner	a	b	c	d	e
	0.0908	-0.1512	-0.2484	-0.4621	0.0681
	f	g	h	i	j
	-0.2209	1.0225	0.1734	0.2824	0.0483
	k	l	m	n	o
	-0.2144	-0.0353	-0.0219	-0.0195	-0.0763

2.2 The derivation of the settlement formula

The settlement dataset obtained from the finite element runs include the values of the settlement at the centre and at the corner of the raft. The derived settlement formula is given as follows

$$S_n = S_{bn}\cdot\left(\frac{wi_x.wi_y}{u_1}\right)^{a_n}\cdot\left(\frac{E_1}{u_2}\right)^{b_n}\cdot\left(\frac{E_2}{u_3}\right)^{c_n}\cdot\left(\frac{E_3}{u_4}\right)^{d_n}\cdot$$
$$\left(\frac{E_4}{u_5}\right)^{e_n}\cdot\left(\frac{E_5}{u_6}\right)^{f_n}\cdot\left(\frac{ld}{u_7}\right)^{g_n}\cdot\left(\frac{bed}{u_8}\right)^{h_n}\cdot\left(\frac{th}{u_9}\right)^{i_n}\cdot\left(\frac{Ec}{u_{10}}\right)^{j_n}\cdot \tag{1}$$
$$\cdot\left(\frac{po_1}{u_{11}}\right)^{k_n}\cdot\left(\frac{po_2}{u_{12}}\right)^{l_n}\cdot\left(\frac{po_3}{u_{13}}\right)^{m_n}\cdot\left(\frac{po_4}{u_{14}}\right)^{n_n}\cdot\left(\frac{po_5}{u_{15}}\right)^{o_n}$$

where S_n and S_{bn} are the calculated and the base settlement in meters, respectively. The symbols a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n, j_n, k_n, l_n, m_n, n_n and o_n, are *unitless fitting coefficients* (see Table 3). The physical meaning of these coefficients are explained in Section 3. On the other hand, u_1, u_2 .... u_{15} are *the fitting constants with units* (see Table 4). These constants were included to establish dimension compatibility. The formula is capable of calculating settlement at the centre (S_{centre}) or at the corner (S_{corner}) of the raft depending on the fitting coefficient sets used. The indices of n=0 and n=1 stand for the centre and corner settlements, respectively.

The average deflection of the raft can be calculated with the following formula

$$Average\ Deflection = \frac{(S_{centre} - S_{corner})}{\left(\left(\frac{wi_x}{2}\right)^2 + \left(\frac{wi_y}{2}\right)^2\right)^{0.5}} \qquad (2)$$

2.3 The performance check of the formula against Case Studies

In this section the proposed footing settlement formula will be checked against 3 case studies in which the footing settlements are calculated using various methods such as the settlement formula offered by Das (1983) and FE models created in DIANA FE (DIANA FE User's Manual 2014) and Plaxis 3D (Plaxis 3D user's Manual 2013). Settlement can be calculated at the centre of a rectangular flexible area using the following formula given by Das (1983)

$$Settlement = \frac{q_s B(1 - v^2)}{E} I_s \qquad (3)$$

where, q_s = the applied uniform load, B = the width of the rectangular raft, υ = the Poisson's ratio, I_s = settlement influence factor at the centre of a rectangular raft and E = the average soil modulus up to 2B or 3B depth. I_s can be calculated using the following formula given by Giroud (1968)

$$I_s = \frac{2}{\pi}\left[ln\left(\left(\frac{L}{B}\right) + \sqrt{1 + \left(\frac{L}{B}\right)^2}\right) + \left(\frac{L}{B}\right) ln\left(\frac{1 + \sqrt{1 + \left(\frac{L}{B}\right)^2}}{\left(\frac{L}{B}\right)}\right)\right] \qquad (4)$$

where, L = length of the rectangular raft. The FE models of the case studies were quite similar to the ones explained in the earlier section. The general views of the created meshes are given in Fig. 3

2.3.1 Case Study 1 by Kay and Cavagnaro (1983)

Kay and Cavagnaro (1983) reported the settlement of the Savings Bank building located in the city of Adelaide, South Australia. The settlement was monitored over a 2-year period. The 13-storey building was founded on a 33.5 m × 39.5 m rectangular raft. The thickness of the raft was 0.9 m. The final dead load of the building was 134 kPa.

The building was seated on 14 m-thick green-gray clay underlain by sandstone bedrock. It is reported that three Down Hole Plate Tests were performed to determine the soil modulus of the 14 m clay layer. Based on the reported data, the average soil modulus for the 14 m clay layer turned out to be 48.3 MPa. In addition, in the reference study, a soil modulus of 500 MPa is recommended for the sandstone bedrock. The soil profile and other details are

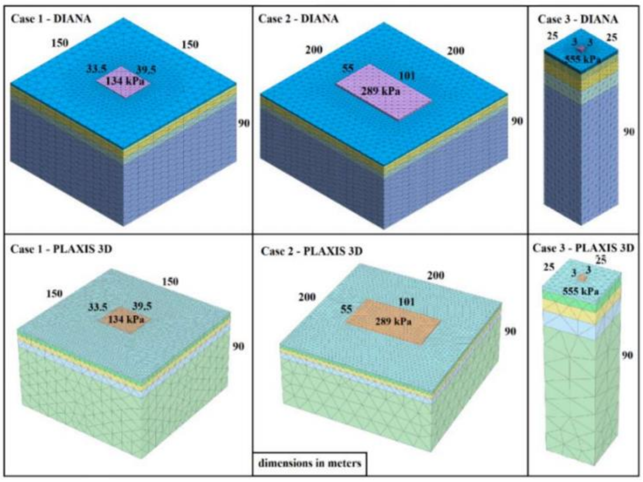

Fig. 3 The general views of the meshes created in DIANA FE and PLAXIS 3D

Table 4 The fitting constants of the settlement formula (with units)

Constant	Constant value centre (n=0)	Constant value corner (n=1)	Unit
S_{bn}	0.1294	0.0870	[m]
u_1	400	400	[m^2]
u_2, u_3, u_4, u_5, u_6	10000	10000	[kN/m^2]
u_7	100	100	[kN/m^2]
u_8	30	30	[m]
u_9	1	1	[m]
u_{10}	25000	25000	[kN/m^2]
u_{11}, u_{12}, u_{13}, u_{14}, u_{15}	0.35	0.35	-

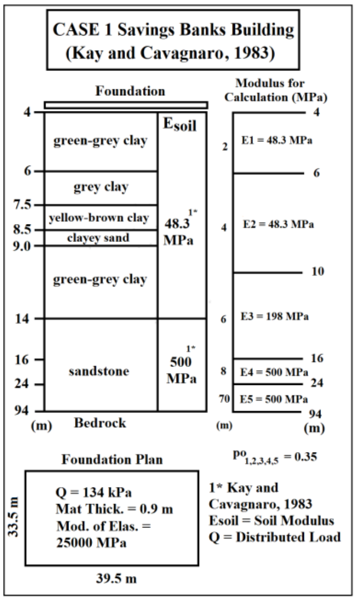

Fig. 4 The summary of parameters used for Case Study 1 by Kay and Cavagnaro (1983)

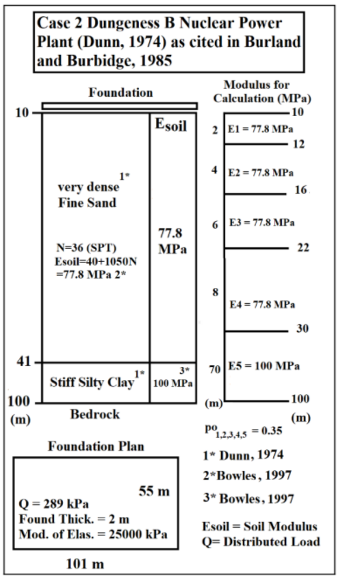

Fig. 5 The summary of parameters used for Case Study 2 by Dunn, 1974 as cited in Burland and Burbidge 1985

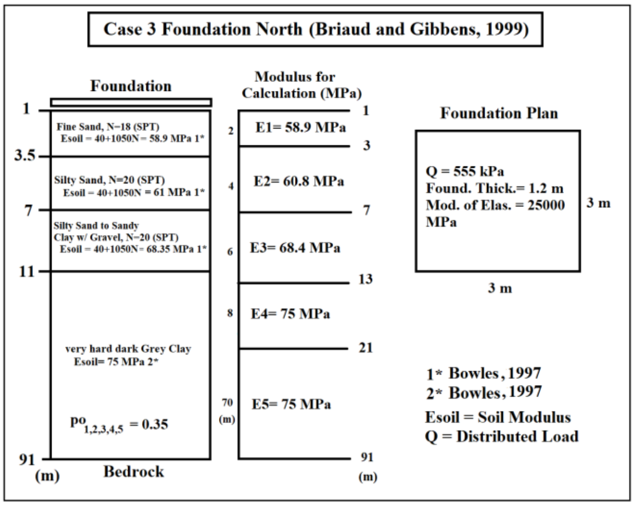

Fig. 6 The summary of parameters used for Case Study 3 by Briaud and Gibbens (1999)

summarized in Fig. 4. The soil modulus set (E1, E2, E3, E4 and E5) required for the settlement estimation was interpolated from these reported values (Fig. 4). In this case study, the soil modulus of E3 layer is 198 MPa. This value was calculated by averaging soil moduli of green-grey clay and sandstone, namely E3= (48.3 MPa × 4 m + 500 MPa × 2 m) / (4+2) m = 198 MPa. Additionally, a constant Poisson's ratio value of 0.35 was incorporated.

2.3.2 Case Study 2 by Dunn (1974) as cited in Burland and Burbidge (1985)

Dunn (1974) reported the settlement of Dungeness B Nuclear Power Plant in Kent, England (as cited in Burland and Burbidge 1985). The building was founded on a 55 × 101 m rectangular raft. The thickness of the raft was not mentioned in the reference studies, therefore the raft thickness was assumed 3 m (based on the author's past design experience on Nuclear Power Plant foundations). The final dead load of the building was reported as 289 kPa.

The power plant was seated on a 31 m-thick dense fine beach sand layer overlying stiff silty clays. SPT count for the dense beach clay was reported as 36. By using the formula recommended by Bowles (1997), the soil modulus value became 77.8 MPa for dense fine beach (Fig. 5). In addition, a soil modulus of 100 MPa was designated for the underlying stiff silty clay using the table of range of soil modulus recommended by Bowles (1997) (Fig. 5). A constant Poisson's ratio value of 0.35 was incorporated.

2.3.3 Case Study 3 by Briaud and Gibbons (1999)

Briaud and Gibbens (1999) (also in FHWA-RD-97-068 1997) reported the load-settlement tests of five square footings ranging in sizes between 1 and 3 m. This case study included the North Foundation Load Test, which incorporated a reinforced concrete raft with 3 m × 3 m × 1.2 m side lengths. During the test, a maximum load of 10.5 MN was applied until 150 mm settlement developed. For our case study, the settlement value of 25 mm was focused, which occurred under the distributed load of 555 kPa.

The test footing was seated on a 10 m-thick sand layer overlying stiff silty clays. The SPT counts for the sand layer were reported to be in the range of 18-20. Using the formula recommended by Bowles (1997), the soil moduli values of the sand were calculated to be in the range of 58.9-68.35 MPa (Fig. 6). In addition, a soil modulus of 75 MPa was designated for the underlying very hard dark Grey Clay using the range of soil modulus table reported in Bowles, 1997 (Fig. 6). A constant Poisson's ratio value of 0.35 was incorporated.

2.3.4 Evaluation of Case Study Settlements Using Different Methods

For evaluation purposes, the deviation of settlement results were determined by comparing the measured values with the calculated ones. The settlements of the rafts were calculated using:

1-) The formula offered in this study,

2-) The Das (1983) – Giroud (1968) formula combination,

3-) The FE program PLAXIS 3D,

4-) The FE program DIANA FE.

In Case 1, where there is only a small amount of settlement occurred (0.017 m), the FE programs PLAXIS 3D and DIANA FE and the offered formula were able to predict the settlement with a deviation of 18-19% (Table 5). On the other hand, the Das-Giroud formula combination was able to predict the settlement with a deviation of 38%. The high deviation in Das-Giroud formula combination is attributed to the fact that the formula given in Eq. (1) incorporates an averaged soil modulus.

Case 2 includes a large raft with one of its sides longer than 100 meters. Due to its large size, the final measured settlement turned out to be quite high, 0.131 m. The deviations (Dev) of settlements in Case 2 are as follows: $Dev_{OfferedFormula}$=18%, Dev_{DIANA}=26% and $Dev_{PLAXIS\ 3D}$=33%. This order seems to be odd because the offered formula has a better accuracy than the FE program that is derived from (DIANA FE). This result is attributed to the

Table 5 The details of the settlement calculations using different methods

	2	3	4	5	6	7	8	9	10	11	12
Case No	Title	B1 (wix) (m)	B2 (wiy) (m)	Bottom (bed) (m)	mat thickness (th) (m)	E1 (kPa)	E2 (kPa)	E3 (kPa)	E4 (kPa)	E5 (kPa)	Load (ld) (kPa)
1	Saving Bank Building (Kay and Cavagnaro1983)	39.5	33.5	70	0.9	48300	48300	198000	500000	500000	134
2	Burland 32 (Dunn 1974)	101	55	70	3	77800	77800	77800	77800	100000	289
3	Foundation North (Briad and Gibbens 1999)	3	3	70	1.2	58900	61000	68350	75000	75000	555

13	14	15	16	17	18	19	20	21	22	23	24	25
Case No	Title	Econc (Ec) (MPa)	po$_{avg}$	Settl. Measured (m)	Settlement calculated with (m):				% deviation in results with:			
					The Formula	Das and Giroud's Formula	Plaxis 3D	DIANA FE	The Formula	Das and Giroud's Formula	Plaxis 3D	DIANA FE
1	Saving Bank Building (Kay and Cavagnaro1983)	25000	0.35	0.017	0.0200	0.0105	0.0200	0.0203	17.7%	38.0%	17.6%	19.4%
2	Burland 32 (Dunn 1974)	25000	0.35	0.131	0.1547	0.2164	0.1738	0.1646	18.1%	65.2%	32.7%	25.6%
3	Foundation North (Briad and Gibbens 1999)	25000	0.35	0.025	0.0241	0.0220	0.0280	0.0259	3.6%	11.9%	12.0%	3.6%
			Average % Deviation---->						13.2%	38.4%	20.8%	16.2%

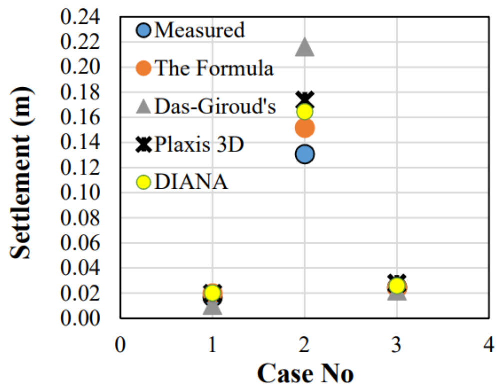

Fig. 7 The plot of calculated settlements using different methods

fact that elasto-plastic model (Mohr-Coulomb) incorporated in both FE programs have a limited accuracy in such large strains. Since the formula data pool did not include any combinations beyond the raft side length of 54 m, the formula behaved more elastic and gave less settlement (Please see Table 1, row 1-2 for maximum side length).

From the geotechnical engineering point of view, the deviation range between 18%-33% is still acceptable for such a large settlement value (0.131 m). Considering that the Das-Giroud formula combination could calculate the settlement with much larger deviation (65%), FE methods are still the best choices for large settlements.

Case 3 includes a relative small raft size of 3 m × 3 m. For this case, the deviation of the formula and DIANA FE were both 3.6%. The deviation of Das-Giroud's formula combination and PLAXIS 3D were the same and 12%. One should notice that the prediction accuracy in this case even with the Das-Giroud's formula combination was quite high. Since the soil moduli of the layers were quite uniform, incorporating an average soil modulus was not a great disadvantage this time (see Table 5, columns 7-11). The plot of settlement values are demonstrated in Fig. 7.

3. Physical Meanings of Input Parameters of the Formula

In this section, a demonstration will be performed on how strongly input parameters such as wix-wiy, ld, th are related to settlement. To explain this relation, the settlement formula (Eq. (1)) should be rewritten in the form of $S = S_b.x_1^{t1}.x_2^{t2} \dots x_{15}^{t15}$. We can find the influence coefficient between x_1 and S by filtering out the effect of $x_2 \dots .x_{15}$ parameters. In other words, various values to x_1 should be assigned while keeping the values of $x_2 \dots .x_{15}$ parameters constant. As a basic example, the average soil modulus E_{soil} will be brought into focus for the case where $E_{soil}=E_1=E_2=E_3=E_4=E_5$. Secondly, a certain FE model configuration will be designated as a base-line system. The input parameters of the base-line system are given in Table 6, row 5.

From the base-line system, some other combinations can be derived, each time a certain input parameter is changed. For this case, E_{soil} is the input parameter under focus (see Table 6, column 10). Subsequently, the settlement is calculated for these combinations. One further step is to normalize the settlement and E_{soil} with those of the base-line system (Table 6, columns 14-15). By doing so, E_{soil} vs. settlement plots will yield values around 1. For example, the plot in Fig. 8(a)) demonstrates the change of normalized total settlement vs. normalized average soil modulus, E_{soil}.

If some trend curves in the form of $y = x^t$ are associated with these plots, the exponent (t) becomes -0.95. Hamderi (2018) called this, "influence coefficient". From the

Table 6 The base-line system and the influence coefficient determination for Esoil

	1	2	3	4	5	6	7	8	9	10	11	12	12	13	14	15
1	wix (m)	wiy (m)	bed (m)	th (m)	E1 (MPa)	E2 (MPa)	E3 (MPa)	E4 (MPa)	E5 (MPa)	Esoil avg (MPa)	ld (kPa)	Ec (MPa)	po_{avg}	Total Settl. (m)	norml. Esoil	norml. Settl.
2	20	20	30	1	20	20	20	20	20	20	100	30000	0.35	0.067	0.4	2.4
3	20	20	30	1	30	30	30	30	30	30	100	30000	0.35	0.045	0.6	1.6
4	20	20	30	1	40	40	40	40	40	40	100	30000	0.35	0.035	0.8	1.2
5	20	20	30	1	50	50	50	50	50	50	100	30000	0.35	0.028	1.0	1.0
6	20	20	30	1	60	60	60	60	60	60	100	30000	0.35	0.024	1.2	0.8
7	20	20	30	1	70	70	70	70	70	70	100	30000	0.35	0.020	1.4	0.7
8	20	20	30	1	80	80	80	80	80	80	100	30000	0.35	0.018	1.6	0.6

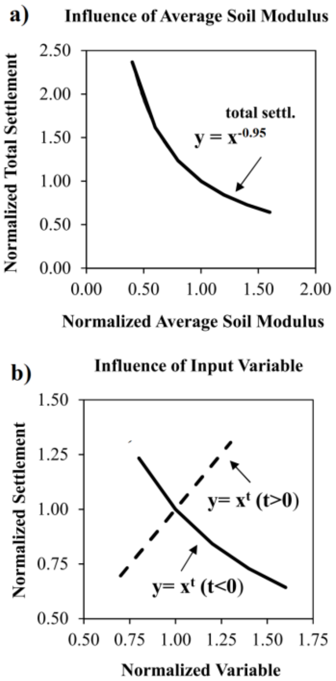

Fig. 8 (a) The variation of normalized total settlement vs. normalized average soil modulus and their trend curves and (b) The general shape of normalized settlement vs. normalized input parameter curves

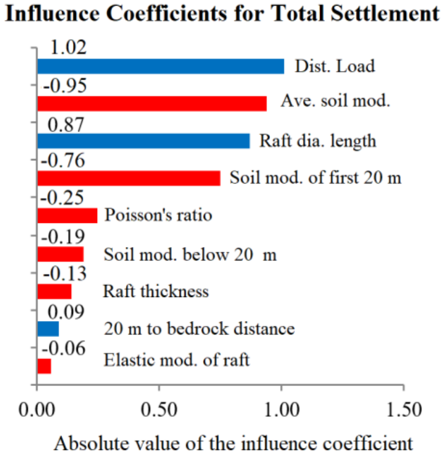

Fig. 9 The influence coefficients for total settlement (Blue and red colours indicate positive and negative effects on total settlement)

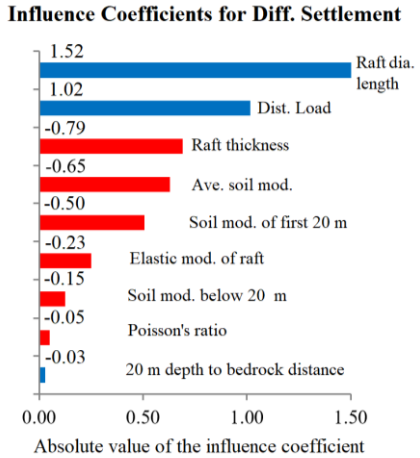

Fig. 10 The influence coefficients for differential settlement (Blue and red colours indicate positive and negative effects on diff. settlement)

mathematical point of view, the absolute value of “*t*” of the function $y = x^t$ indicates how strongly x and y are related (in this case x=E_{soil}, y=Settlement). The schematic of the $y = x^t$ function is given for different signs of t in Fig. 8(b). The positive value “*1*„ for “*t*” indicates a direct proportionality between *x* and *y* whereas the negative value “-1” indicates an inverse proportionality. In our case, t =-0.95 indicates a slightly inversed proportionality between E_{soil} and *S*.

The influence coefficients of all input parameters for the total and differential settlement cases are given in Figs. 9 and 10, respectively. The input parameters that are directly and inversely related to settlement are given in blue and red colours, respectively.

According to Fig 9. distributed load has the greatest influence on total settlement ($|t_{ld}|$=1.02). This result makes sense due to the fact that it indicates a direct proportionality between load and total settlement ($y \approx x^1$). Another prediction can be made for the overall modulus of elasticity of the system. The overall modulus of elasticity should be inversely proportional with total settlement ($y=x^{-1}$). In our system, there are two different types of materials that are defined with modulus of elasticity values: soil and raft. It turns out that the sum of the influence values of these materials t_{Esoil} +t_{Eraft} = (-0.95) + (-0.06) equals to -1.01. As expected, this result demonstrates an inverse proportionality between modulus of elasticity and total settlement.

However, it is important to mention that the influence of elasticity of raft on total settlement is very small (($|t_{Eraft}|$=0.06). On the other hand, the influence of soil modulus of the first 20 m soil section ($|t_{Esoil<20m}|$=0.76) is much greater than the section below 20 m: ($t_{Esoil>20}|$=0.19). In addition, another influential parameter is the raft diagonal length (($|t_{diag.length}|$=0.87). In other words, greater the raft size is, greater is the total settlement. Beside above parameters, raft thickness and 20-m-depth-to-bedrock distance are relatively less influential on total settlement. The influence coefficients for these parameters are $|t_{th}|$=0.13, $|t_{bed}|$=0.09, respectively. In addition, the influence coefficient of Poisson's ratio on total settlement ($|t_{Poisson's}|$) is 0.25.

In terms of differential settlement, the most influential parameters are the raft diagonal length and the distributed load. The influence coefficients for these are $|t_{diag.length}|$=1.52 and $|t_{ld}|$=1.02, respectively. In reality, the foundation engineers usually cannot control these parameters, because they appear as an inevitable outcome of the architectural and structural design. On the other hand, raft thickness is quite controllable. Considering that the influence of raft thickness on differential settlement is quite high ($|t_{th}|$=0.79), the raft thickness can be adjusted to optimise differential settlement.

The influence of average soil modulus on differential settlement is also high ($|t_{Esoil}|$=0.65). Especially, the soil modulus of the first 20 m is very influential on differential settlement ($|t_{Esoil<20}|$=0.50). In contrast, below the depth of 20 m, the soil modulus influence on differential settlement is quite low $|t_{Esoil>20}|$=0.15.

The elastic modulus of the raft has a medium influence on differential settlement ($|t_{Ec}|$=0.23). Within all these input parameters, 20-m-depth to bedrock distance has the least influence on differential settlement ($|t_{bed}|$=0.03). In addition, the influence coefficient of Poisson's ratio on differential settlement ($|t_{Poisson's}|$) is 0.05.

4. Conclusions

In this study, a comprehensive raft footing formula was offered, which incorporates the dimensions and the elastic modulus of a rectangular raft, vertical uniform pressure, soil moduli up to 5 layers. The following conclusions can be drawn from the study:

• The formula includes all the main parameters that influence the settlement of raft footings.

• The formula is 3D FE-based; therefore the average prediction performance of the formula is considerably high.

• An additional equation (Eq. (2)) was offered for average raft deflection. Using this equation, engineers may iterate through the thickness of the raft to optimise the raft deflection.

• "An influence coefficient" term was introduced to demonstrate the rate of influence of each parameter in the formula on settlement. In a descending order, the most influential parameters on total settlement turned out to be the distributed load, raft diagonal length, soil modulus of the first 20 meters, raft thickness and elastic modulus of the raft (|tld|=1.02, |tdiag.length|=0.87, |tEsoil<20|=0.76, |tth|=0.13, |tEc|=0.06). Of these input parameters, the ones we have a control on are the soil modulus of the first 20 m and the thickness of the raft. For example, using the formula, an engineer can find out, how much soil improvement to be made in the first 20 meters to satisfy the allowable total settlement criteria (Formula accepts the soil modulus input along these depths: 2 m, 6 m, 14 m, 20 m). In addition, it is also possible to manipulate the raft thickness to optimise the raft settlement.

• Because the settlement of rafts is primarily dependent on the layers in close vicinity, 4 out of 5 pre-assigned layers were devoted to the first 20 m, whereas only one layer was assigned to the depths below 20 m. This approach is considered reasonable, because the soil modulus influence coefficient of the first 20 is quite high |tEsoil<20|=0.76, whereas this influence coefficient is much smaller for the layers below 20 (|tEsoil>20|=0.19) (Fig. 9).

• The settlement formula adopts the linear elastic perfectly-plastic Mohr-Coulomb model. In this model, the soil modulus is constant, which is considered as a disadvantage in calculating large deformations. On the other hand, there are also other material models such as Modified-Mohr-Coulomb model, in which the soil modulus is updated based on the stress and strain level. In other words, the soil modulus is variable throughout the soil mesh. However, such an advanced material model has not been adopted in this study, because it was not possible to unite the spatially varying soil moduli into a single $\left(\frac{E_1}{u_2}\right)^{b_n}$ term as it was done in the current formula.

• Influence coefficients were also offered for differential settlement. In a descending order, the most influential parameters on differential settlement are the raft diagonal length, distributed load, raft thickness, soil modulus of the first 20 meters and the elastic modulus of the raft (|tdiag.length|=1.52, |tld|=1.02, |tth|=0.79, |tEsoil<20|=0.50, |tEc|=0.23). In terms of differential settlement, the most influential parameter that we have a control on is the raft thickness. Engineers can manipulate the raft thickness to optimise the differential settlement. In addition, soil improvement within the first 20 m depth is also effective in reducing the differential settlement.

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