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A new formulation for calculation of longitudinal displacement profile (LDP) on the basis of rock mass quality

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Abstract. Longitudinal Displacement Profile (LDP) is an appropriate tool for determination of the displacement magnitude of the tunnel walls as a function of the distance to the tunnel face. Some useful formulations for calculation of LDP have been developed based on the monitoring data on site or by 3D numerical simulations. However, the presented equations are only based on the tunnel dimensions and for different quality of rock masses proposed a unique LDP. In the present study, it is tried to present a new formulation, for calculation of LDP, on the basis of Rock mass quality. For this purpose, a comprehensive numerical simulation program was developed to investigate the effect of rock mass quality on the LDP. Results of the numerical modelling were analyzed and the least square technique was used for fitting an appropriate curve on the derived data from the numerical simulations. The proposed formulation in the present study, is a logistic function and the constants of the logistic function were predicted by rock mass quality index (GSI). Results of this study revealed that, the LDP curves of the tunnel surrounded by rock masses with high quality (GSI>60) match together; because the rock mass deformation varies over an elastic range.

Keywords: longitudinal displacement profile (LDP); logistic function; least square technique; rock mass quality

1. Introduction

One of the most important issues in the design of tunnel support system is determination of the tunnel surrounding rock mass deformation due to the excavation. The amount of the deformation, before the support installation, significantly influences on the applied load on the support of tunnel. If no displacement is assumed to occur, the loads acting on the support will be unrealistic and overpredicted. On the other hand, a large amount of relaxation in the tunnel surrounding rock may be led to an underestimation of the loads acting on the support.

In a two-dimensional program, it is difficult to quantify the real relaxation, because this depends on the distance behind the face at which the support is installed (Itasca, 2009).

In other to determine the allowable unsupported span for the installation of support, it is necessary to establish the Longitudinal Displacement Profile (LDP) for the tunnel (Vlachopoulos and Diederichs 2009). LDP is a graphical view of radial displacement in the rock mass surrounding of an advancing tunnel versus distance to the tunnel face, and usually depicted for an unsupported tunnel section, behind and ahead of the tunnel face, along the tunnel axis (Alejano *et al.* 2012).

LDP is one of the three basic components of the Convergence-Confinement Method (CCM) and provides an accurate insight into how quickly the support begins to interact with the rock-mass behind the face of the tunnel (Carranza-Torres and Fairhurst 2000).

Application of LDP is not limited to the CCM and in two-dimensional numerical analysis of tunnels can be used for calculation of appropriate relaxation, but a three-dimensional analysis is necessary to precisely determine this profile. Therefore, if only two-dimensional numerical model is available or if an analytical convergence-confinement solution is to be used, it is more practical to use an analytical function for calculation of the LDP (Vlachopoulos and Diederichs 2009).

LDP strictly depend on the characteristics of the rock mass to be excavated, but most of the proposed equation for calculation of the LDP are only based on the tunnel dimensions (Alejano *et al.* 2012). Therefore, in the present study, it is tried to present a new formulation, based on the characteristics of the tunnel surrounding rock mass, for determination of the LDP.

2. Analytical functions of LDP

Determination of the optimum distance of supports from the tunnel face needs an accurate description of the LDP. Fig. 1 depicts a longitudinal cross-section of an unsupported circular tunnel with radius of R in the vicinity of the face. At a distance x from the face the radial displacement is u_r. When the distance x is large enough the maximum radial deformation u_r^m, happens. Also, a part of the displacement happens in unexcavated area and becomes zero at some finite distance ahead of the face.

Various equations were proposed for calculation of the LDP based on the different procedures.

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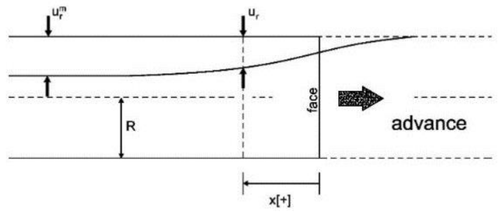


Fig. 1 Longitudinal cross-section of a tunnel

Based on the finite element analyses, Panet and Guenot (1982) presented the following expression for determination of radial displacement of tunnel wall

$$\frac{u_r}{u_r^m} = 0.28 + 0.72\left[1 - \left(\frac{0.84}{0.84 + x/R}\right)^2\right] \quad (1)$$

Corbetta *et al.* (1991) suggested another empirical equation in a different form as follows

$$\frac{u_r}{u_r^m} = 0.29 + 0.71\left\{1 - \exp\left[-1.5(x/R)^{0.7}\right]\right\} \quad (2)$$

Based on the research by Panet (1995), the following equation was presented

$$\frac{u_r}{u_r^m} = 0.25 + 0.75\left[1 - \left(\frac{0.75}{0.75 + x/R}\right)^2\right] \quad (3)$$

The mentioned equations are only valid for the region behind the face (x>0)

Hoek (1999) proposed an empirical equation based on the data measured by Chern *et al.* (1998) in a tunnel in Mingtam Power Cavern project

$$\frac{u_r}{u_r^m} = \left[1 + \exp\left(\frac{-x/R}{1.10}\right)\right]^{-1.7} \quad (4)$$

Eq. (4) is valid for the regions ahead of the face (x>0) and behind the face (x<0) of the tunnel.

In all of the above equations there is no parameter which represent the tunnel surrounding rock mass quality and the ratio of $\frac{u_r}{u_r^m}$ is only correlated with $\frac{x}{R}$. Therefore, these equations are only valid when the tunnel surrounding rock masses deform in the range of elastic deformations.

Unlu and Gercek (2003) evaluated the effect of Poisson's ratio on the radial displacement of the tunnel surrounding medium and proposed a dual function for calculation of the normalized radial displacements occurring in the vicinity of the face. They believed that LDP does not follow a unique continuous function, and then suggested Eqs. (5) and (6) for ahead and behind of the tunnel face, respectively.

$$U_a = U_o + A_a[1 - \exp(-B_a X^*)] \text{ for } X^* \leq 0 \quad (5)$$

$$U_b = U_o + A_b\left\{1 - \left[\frac{B_b}{(B_b + X^*)}\right]^2\right\} \text{ for } X^* \geq 0 \quad (6)$$

$$U_o = -0.22\,\nu - 0.19 \quad (7)$$

$$\begin{cases} A_b = -0.22\nu + 0.81, & B_b = 0.39\nu + 0.65 \\ A_a = -0.22\nu - 0.19, & B_a = 0.73\nu + 0.81 \end{cases} \quad (8)$$

where u_0 is the radial displacement at face $X^* = 0, X^* = \frac{x}{R}, U_0 = \frac{U_r^0}{U_r^m}, U$ is the radial displacement at the desired distance X from the face, U_r^m is the maximum radial displacement, and R is the tunnel radius.

If an elastic-base equation is used to calculate LDP and the result is implemented in the CCM, the recommended time for support installation will not be the real-time and then induced stress in the support system was incorrectly estimated (Alejano *et al.* 2012). Therefore, Vlachopoulos and Diederichs (2009) presented a new formulation for the LDP calculation that takes into account the significant influence of plastic radius.

The displacement at the tunnel face u_{if} is calculated from the following equation derived by Vlachopoulos and Diederichs

$$u_{if} = \left(\frac{u_{im}}{3}\right) e^{-0.15(r_{pm}/r_0)} \quad (9)$$

where u_{im} is the maximum displacement which occurs at r_{pm}.

The tunnel wall displacement ahead of the face (x<0) is

$$u_i = \frac{u_{if}}{u_{im}} . e^{x/r_0} \quad (10)$$

The tunnel wall displacement behind the face (x>0) is

$$u_i = 1 - \left(1 - \frac{u_{if}}{u_{im}}\right) . e^{(-3x/r_0)/(2r_{pm}/r_0)} \quad (11)$$

Alejano *et al.* (2012) emphasized that the plastic zone around excavations tends to be larger for strain softening (SS) rock masses compared to elastic perfect plastic (EPP) rock masses and tried to develop the Vlachopoulos and Diederichs (2009) approach to SS behavior.

The equations presented by Vlachopoulos and Diederichs (2009) and Alejano *et al.* (2012) are really useful for calculation of LDP in plastic media, but estimation of the plastic radius around an opening is very difficult and significantly related to the chosen constitutive model for the tunnel surrounding rock mass. However, Alejano *et al.* (2012) tried to propose an equation for estimation of the plastic radius around a tunnel, but it is an approximate estimation and limited to strain-softening media in average quality rock masses.

Komurlu *et al.* (2015) investigated the effect of horizontal in situ stress on failure mechanism and plastic zone around underground openings and showed that prediction of the plastic zone radius around underground excavation is complicated and many simplification is necessary for a safe estimation.

Whereas the qualitative description of rock masses and subsequent correlation to establish engineering quantities is inevitable in design of underground rock structures, it is so applicable to propose a new formulation for calculation of LDP based on the general rock mass quality. In the present study, it is tried to suggest a more simple and accurate equation for calculation of LDP based on GSI and in a wide range of rock mass quality.

3. Numerical modelling

Numerical simulation is one of the well-known methods for calculation of LDP in different conditions. Some of the proposed analytical formulation for calculation of LDP are on the basis of some numerical models which developed in different geotechnical conditions. For example, Vlachopoulos and Diederichs (2009) have proposed a robust formulation based on the ultimate plastic radius around the tunnel. Also Sadeghiyan *et al.* (2016) suggested the concept of Longitudinal Convergence Profile (LCP) as alternative to LDP. Then, a series of 3-dimensional numerical modeling was developed to evaluate the effect of modulus of elasticity, cohesion and internal friction angle of soil on the LCP.

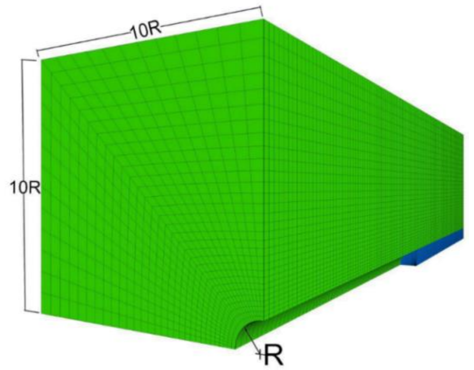


Fig. 2 Domain geometry of the developed numerical model

In this study, extensive numerical models of a circular tunnel on the basis of different geotechnical and geometrical conditions was established and analyzed. Fig. 2 shows the three-dimensional domain geometry of the developed numerical models. The model represents only a quarter of the area around the excavation due to the symmetry. Also the dimension of the models were determined based on the tunnel dimensions. As shown in Fig. 2, D is diameter of the tunnel and the width and length of the models are equal to 5D and 10D, respectively. The mechanical behavior of the tunnel surrounding rock was described by an elastic-perfectly plastic material with a Mohr-Coulomb constitutive model.

In the next step, the LDPs corresponding to the developed models were derived. For this purpose, the half-length of the tunnel is excavated and vertical displacements of the tunnel crown in the longitudinal profile is recorded. In the next, a comprehensive set of data were prepared to distinguish different LDP in various geo-mechanical conditions.

3.1 Effect of rock mass quality on LDP

The qualitative description of rock masses by means of classification systems and subsequent correlation to establish engineering quantities or design parameters has become one of the integral parts of rock engineering design (Aydan *et al.* 2014). Therefore it will be useful to propose an analytical equation based on the rock mass quality parameters for calculation of LDP. A number of empirical indices have been developed for determination of rock mass quality such as RQD, RMR, Q, GSI and etc. However, the GSI system is a proper index within the scope of our study, as it reflects general rock mass quality (joint density and joint strength behavior) in a simple way and so enables us to easily assign a realistic model to the rock mass and estimate its parameters (Alejano *et al.* 2012). As mentioned before, the previous proposed equations can precisely estimate the LDP only in elastic medium, and they are inapplicable in weak rock. For evaluation of the effect of rock mass quality on the LDP, twenty different geotechnical conditions corresponding to different GSI were assigned to numerical models. Table 1 shows the geotechnical properties of rock mass with different GSI in the range of 5-100. The uniaxial compressive strength of the intact rock is equal to 45 MPa and the empirical equations proposed by Hoek and brown (1997) used for estimation of rock mass properties. The rock mass modulus of deformation is given by

Table 1 Mechanical properties of rock mass with different GSI

GSI	E_m (GPa)	C(MPa)	ϕ (degree)	σ_t (MPa)
100	119.29	8.80	47.00	5.63
95	89.46	6.40	48.00	3.86
90	67.08	4.66	49.00	2.65
85	50.30	3.42	49.00	1.82
80	37.72	2.52	50.00	1.25
75	28.29	1.89	49.00	0.85
70	21.21	1.44	49.00	0.59
65	15.91	1.12	49.00	0.40
60	11.93	0.90	48.00	0.28
55	8.95	0.73	47.00	0.19
50	6.71	0.62	46.00	0.13
45	5.03	0.53	44.00	0.09
40	3.77	0.46	43.00	0.06
35	2.83	0.40	41.00	0.04
30	2.12	0.35	39.00	0.03
25	1.59	0.31	37.00	0.02
20	1.19	0.26	35.00	0.01
15	0.89	0.22	32.00	0.01
10	0.67	0.17	29.00	0.01
5	0.50	0.12	25.00	0.00

$$E_m(GPa) = \sqrt{\frac{\sigma_{ci}}{100}}.10^{((GSI-10)/40)} \quad (12)$$

where E_m is rock mass deformation modulus and σ_{ci} is uniaxial compressive strength of the intact rock. The other

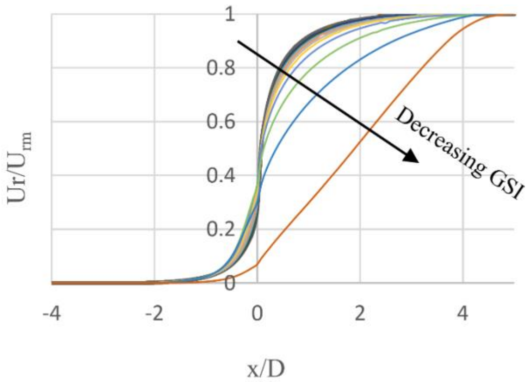


Fig. 3 Comparison of LDPs extracted from the numerical models with different GSI

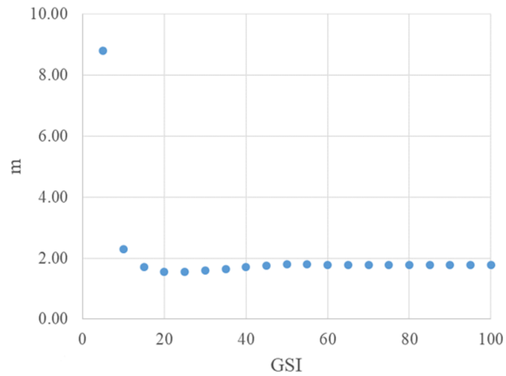
Table 2 Descriptive information of the modeling data at different condition of rock mass quality, tunnel dimension and overburden depth

Variable	Minimum	Mean	Maximum	SE Mean	St Dev
Tunnel Diameter (m)	6.00	8.14	12.00	0.39	2.74
Overburden Depth (m)	50.00	113.27	200.00	7.68	53.79
GSI	5.00	45.61	100.00	4.34	30.39
UCS (MPa)	45.00	53.06	90.00	2.27	15.87
m (Hoek-Brown's constant)	8.00	9.39	16.00	0.40	2.78
Cohesion (MPa)	0.08	1.50	8.80	0.29	2.00
Deformation Modulus of rock mass (GPa)	0.50	19.36	119.29	4.12	28.84
Tensile strength (MPa)	0.00	17.70	836.70	17.10	119.40
Friction Angle (Deg)	22.00	34.52	50.00	1.33	9.34

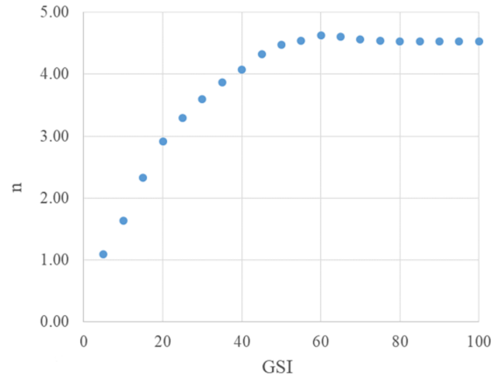
required equations for estimation of Mohr-Coulomb parameters is available in the reference of Hoek *et al.* (2002).

The numerical model of a circular tunnel with diameter of 8 m and overburden depth of 100 m was developed to evaluate the tunnel convergence in different geotechnical condition according to Table 1. Variation of the vertical displacement at the tunnel crown versus distance from the face was plotted for the medium with different GSI in Fig. 3. It should be noted that the vertical displacement of the tunnel crown normalized with the maximum vertical displacement in longitudinal profile and the distance from the face normalized with the tunnel diameter.

As shown in Fig. 3, the quality of rock mas significantly influences on LDP and using of a single equation for both strong and weak rocks lead to an inaccurate estimation of LDP. However, in order to suggest a robust equation for estimation of LDP, it is necessary to develop much more numerical models at different conditions of tunnel dimension, overburden depth and rock strength. Table 2 lists statistical information of the extra data regarding the different conditions of numerical modeling. SE mean represents standard error of the mean and St Dev indicates standard deviation of the parameters.

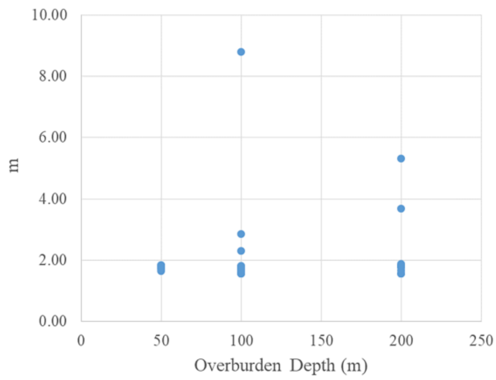


(a)

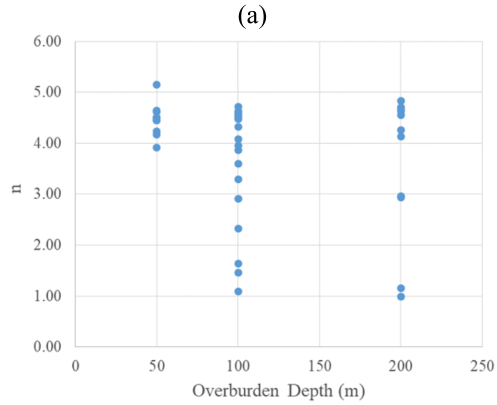


(b)

Fig. 4 Variation of (a) m and (b) n with GSI



(a)

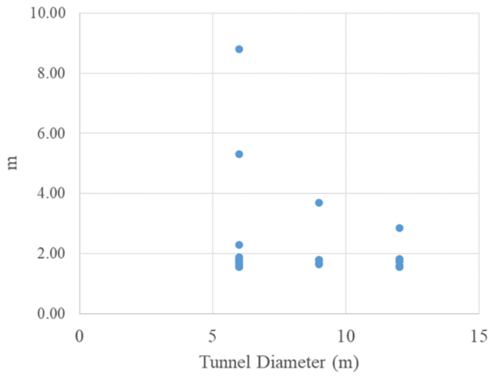


(b)

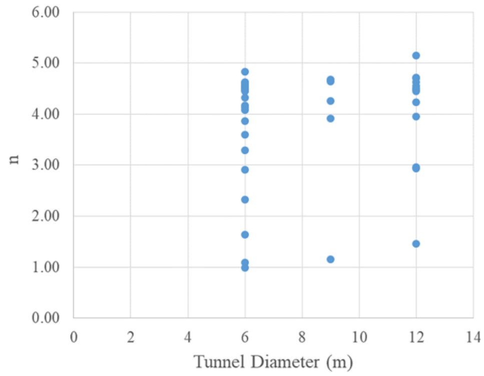
Fig. 5 Variation of (a) m and (b) n with overburden depth

4. Statistical analysis

A comprehensive statistical analysis was conducted on the data derived from the numerical simulations. The LDP values were extracted for tunnel crown and it is tried to



(a)



(b)

Fig. 6 Variation of (a) m and (b) n with tunnel diameter

establish an analytical equation for calculation of LDP at different geo-mechanical conditions. For this purpose, fitting techniques, including mathematical equations and nonparametric methods, were used to model the derived data. Curve fitting is the process of introducing mathematical relationships between dependent and independent variables in the form of an equation for a given set of data. The appropriate models for curve fitting of LDP data are S-shaped or sigmoid curve functions. One of the most common "S" shape function is logistic function with the following equation

$$f(x) = \frac{1}{1 + me^{-nx}} \tag{13}$$

where x varies in the range of [-∞, +∞], but it is often sufficient to compute the standard logistic function for x over a small range of real numbers such as a range contained in [−5, +5]. Also the value of logistic function varies in the range of [0, 1]. For fitting the logistic function on LDP curves, the radial displacement of tunnel was normalized with the maximum radial displacement and the distance from the face was normalized with the tunnel diameter. Also, m and n are constants which were significantly influenced by geo-mechanical conditions of tunnel.

The logistic function has been used for estimation of LDP in previous studies, similar to the one adopted by Hoek 1999. However, m and n in Hoek's equation are constant for any conditions of tunnel and consequently a single LDP was proposed for all of geo-mechanical conditions. In the present study, the values of m and n were predicted on the basis of rock mass quality and then a more comprehensive equation was proposed for calculation of LDP in different geo-mechanical conditions. Variation of m and n with GSI were depicted in Fig. 4(a) and 4(b), respectively.

As shown in Fig. 4, the constants of m and n significantly change with variation of GSI. However, variation of the constant of m was limited in the range of GSI<20 and variation of the constant of n was limited in the range of GSI>60.

In addition of GSI, tunnel diameter and overburden depth significantly influence on LDP. Fig. 5 and 6 show variation of the both constants of m and n with overburden depth and tunnel diameter, respectively. As shown in Figs. 5 and 6, there are not good correlations between the constants m and n with overburden depth and tunnel diameter. Therefore it is necessary to perform a nonlinear regression analysis to propose a robust equation for calculation of LDP.

4.1 Regression analysis

Regression analysis includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. One of the most appropriate form of mathematical regression analysis is least squares that finds the curve of best fit for a dataset, providing a visual demonstration of the relationship between the data points. Least squares means that the overall solution minimizes the sum of the squares of the residuals made in the results of every single equation

$$err = \sum (d_i)^2 = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + (y_3 - f(x_3))^2 + (y_4 - f(x_4))^2 + \dots \tag{14}$$

The best curve has minimum error between curve and data points, then the derivative of the error with respect to m and n should be set each to zero

$$\frac{\partial err}{\partial m} = 0 \text{ and } \frac{\partial err}{\partial n} = 0 \tag{15}$$

Based on the mentioned equations, the suggested formulation for calculation of LDP is

$$y = \frac{1}{1 + me^{-nX}} \tag{16}$$

where $y = \frac{u_r}{u_r^m}$, $X = \frac{x}{D}$, m and n were found in the different range of GSI as follows

$$m = \begin{cases} 0.74 \times exp\left(\dfrac{23.69 \times h^{0.4}}{D^{1.6} \times GSI^{0.8}}\right) & \text{for GSI} < 20 \\ 1.85 & \text{for GSI} \geq 20 \end{cases} \tag{17}$$

$$n = \begin{cases} 1.73 \times \ln\left(\dfrac{GSI^{0.9}}{h^{0.7}}\right) - 1.36 & \text{for GSI} < 60 \\ 4.3 & \text{for GSI} \geq 60 \end{cases} \tag{18}$$

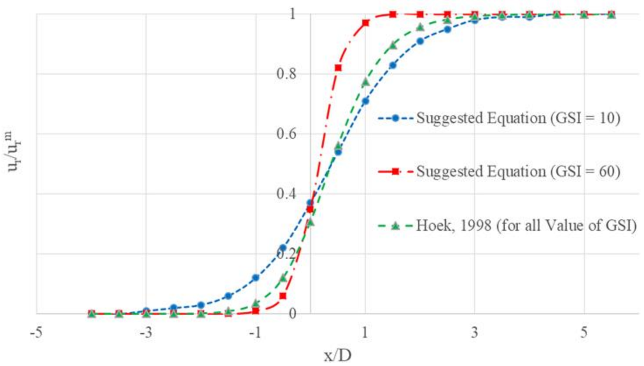


Fig. 7 Comparison of the proposed LDP at different GSI

Indeed, LDP curves of rock masses with high quality (GSI>60) match together because the rock mass deformation varies over an elastic range and is only proportional to the distance from the face. Based on the suggested equation in the present study, the LDP of a circular tunnel with a diameter of 8 m and overburden depth of 100 m at two different value of GSI (GSI = 10 and 60) was depicted in Fig. 7 and compared with the formulation proposed by Hoek, 1999. As shown in this figure, the suggested equation indicated that GSI significantly influences on LDP curve and the Hoek's equation is only valid for the rock masses with rather poor quality.

The amount of tunnel convergence, the rate of deformation and the extent of the plastic zone around the tunnel significantly was influenced by geological and geotechnical conditions, the in situ state of stress relative to rock mass strength, the ground water flow and pore pressure, and the rock mass properties (Ghiasi *et al.* 2012). However, the rock masses with different quality have completely different behaviors. For example the high quality rock masses behave elastically and the tunnel will be stable and deform linearly proportional to the distance from the face. The rocks with poor to moderate quality exhibit nonlinear deformation and the tunnel convergence is not only proportional to the distance from the face (Aydan *et al.* 1993). Alejano *et al.* (2009) evaluated the ground reaction curves defined in three different quality rock masses (good, average and poor) and found that the application of CCM varies according to the quality of the rock mass.

However, LDP is significantly related to the rock mass quality and it is certainly applicable to use GSI, as a well-known rock mass quality index, for calculation of the LDP.

5. Conclusions

In the present study, the available equations for calculation of LDP were evaluated and a comprehensive numerical modeling program was developed to investigate the effect of rock mass quality on the LDP of a circular tunnel. Results of the numerical modelling showed that the previous equations only valid when the tunnel surrounding rock masses deform in the range of linear elastic deformations, because they are only proportional to the distance from the face of tunnel.

The quality of rock mass significantly influences on the LDP and using of a single equation for both strong and weak rocks lead to an inaccurate estimation of LDP.

LDP curves of the tunnel surrounded by high quality rock masses (GSI>60) match together, because the rock mass deformation varies over a linear elastic range.

The suggested equation indicated that GSI significantly influences on LDP curve and the Hoek's equation is only valid for the rock masses with rather poor quality.

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