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Reliability analysis of a mechanically stabilized earth wall using the surface response methodology optimized by a genetic algorithm

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Abstract. A probabilistic study of a reinforced earth wall in a frictional soil using the surface response methodology (RSM) is presented. A deterministic model based on numerical simulations is used (Abdelouhab *et al.* 2011, 2012b) and the serviceability limit state (SLS) is considered in the analysis. The model computes the maximum horizontal displacement of the wall. The response surface methodology is utilized for the assessment of the Hasofer-Lind reliability index and is optimized by the use of a genetic algorithm. The soil friction angle and the unit weight are considered as random variables while studying the SLS. The assumption of non-normal distribution for the random variables has an important effect on the reliability index for the practical range of values of the wall horizontal displacement.

Keywords: reinforced earth walls; reliability analysis; surface Response methodology; limit states; approximate performance function; genetic algorithm

1. Introduction

Traditionally, analysis of the behavior of reinforced earth walls is based on deterministic approaches, where the uncertainties of the geomechanical parameters are addressed through a global safety factor. To account more rigorously for uncertainties, the reliability theory is used. This permits to take into account the hazards of each uncertain parameter via its probability distribution. These approaches have the advantage of giving the system response (the maximum displacement, the safety factor, etc.), not only by a single value representing its mean, but by its mean and variance, or by its reliability index or its probability of failure. Thus, the reliability approaches allow considering the propagation of uncertainties of the input parameters to the response of the system using a mechanical model of the system studied. The design based on reliability was applied to geotechnical engineering by Kulhawy and Phoon (2002), Low (2005, 2014), DV Griffiths *et al.* (2001, 2002), Mollon (2009), Hamrouni *et al.* (2017a, b) and Pan *et al.* (2017a, b, c).

The mechanically stabilized earth (MSE) wall is a composite material formed by the combination of soil and metallic or synthetic strips able to sustain significant tensile loads. The reinforcing strips give to the soil mass an anisotropic cohesion in the direction perpendicular to the reinforcement Schlosser and Elias, (1978). The presence of the strips improves the overall mechanical properties of the soil. The design methods used in these structures are based on the internal and external stability analysis using limit equilibrium methods. For the internal stability, the common method is based on the verification of the strip long-term tensile force and the adherence or bond capacity at the soil/strip interface (AASHTO 2014, NF P 94-270 2009). Although sometimes described as excessively conservative for the synthetic reinforcement (Eliaset *et al.* 2001, Koerner and Soong 2001, Allen *et al.* 2002, Bathurst *et al.* 2005), this straightforward design methodology allows verifying the structure stability (Yoo and Jung 2006, Quang *et al.* 2008) but does not make it possible to determine the deformation state of the structure.

In the numerical studies, two and three-dimensional methods based on finite elements or finite differences (Skinner and Rowe 2005, Hatami and Bathurst 2006, Al Hattamleh and Muhunthan 2006, Bergado and Teerawattanasuk (2008) allow the authors to analyse the deformation and the influence of several parameters in some types of reinforced soil walls. Huang *et al.* (2009) and Ling and Liu (2009) have studied different soil constitutive models and their influence on results (Abdelouhab *et al.* 2010, 2011, 2012a, b).

The retaining walls reliability analysis was proved to be rational due to the soil variability of their geomechanical properties Duncan (2000).

Recent research studies on reinforced retaining walls included consideration of uncertainties in reinforcement properties Chun *et al.* (2004), Sayed *et al.* (2008-2010), Miyata *et al.* (2012), GuhaRay *et al.* (2014), Lin *et al.* (2016), Yu and al (2017), engineering design optimization Yuan *et al.* (2003) and application of Monte Carlo simulation to evaluate the Hasofer-Lind reliability index

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Chalermyanont and Benson (2004). Reliability charts were developed to select geosynthetic materials that will satisfy the minimum requirement of safety against sliding Sia and Dixon (2008). However, the reliability analysis is unable to represent a structure as realistically as possible which is performed by the numerical analyses. Several studies had been conducted on the lateral wall deformation of reinforced retaining wall due to foundation yielding Yoo (2004), Yoo and Song (2006) and due to the influence of the reinforcement creep Kazimierowicz-Frankowska (2005).

This paper presents a reliability study of a mechanically stabilized earth walls reinforced by geosynthetic strips. The granular soil is Hostun R_F sand Gay, (2000); Flavigny and al. (1990). Regarding the deterministic model, a two-dimensionnal numerical model is created using FLAC2D (Abdelouhab *et al.* 2011, 2012b), the serviceability limit state (SLS) is considered in the analysis. The maximum horizontal displacement of the wall facing is considered in this analysis. The uncertain parameters are modeled by random variables. These variables are the physical and mechanical parameters of the soil. The Hasofer-Lind reliability index β_{HL} was adopted. The response surface methodology optimized by a genetic algorithm is used to find the reliability index. After a brief description of the basic concepts of reliability, the probabilistic analysis and the corresponding numerical results are presented and discussed.

2. Ellipsoid approach in reliability theory

The safety of a geotechnical structure can be measured by its reliability index that takes into account the inherent uncertainties of the input parameters. A most widely used reliability index is the Hasofer and Lind (1974) index. Its matrix formulation is Ditlevsen (1981).

$$\beta_{HL} = \min_{G(x)=0} \sqrt{(x-\mu)^T (C)^{-1} (x-\mu)} \quad (1)$$

in which x is a vector representing the n random variables; μ is a vector of their mean values; and C is the covariance matrix. The minimization of Eq. (1) is performed using the constraint $G(x) \leq 0$, where the limit state surface $G(x) = 0$ separates the n-dimensional domain of random variables into two regions: a failure region F represented by $G(x) \leq 0$ and a safe region given by $G(x) > 0$. The conventional method for calculating β_{HL} by Eq. (1) is based on the transformation of the limit state surface initially defined in the space of the physical variables. This surface must be expressed in the space of the normal random variables, centered, reduced and uncorrelated, which is also called standard space. The shortest distance between the origin of the space and the state boundary surface is equal to the β_{HL} reliability index.

An intuitive interpretation of the reliability index was proposed by Low and Tang (2004). The concept of iso-probability ellipsoid leads to a simpler calculation method for the reliability index in the original physical variables (Fig. 1). Low and Tang (2004), Mollon *et al.* (2009), Low (2014), Ji *et al.* (2014) Hamrouni *et al.* (2017a, b) and Hamrouni *et al.* (2018), demonstrated that the reliability index of Hasofer Lind was equal to the ratio between the

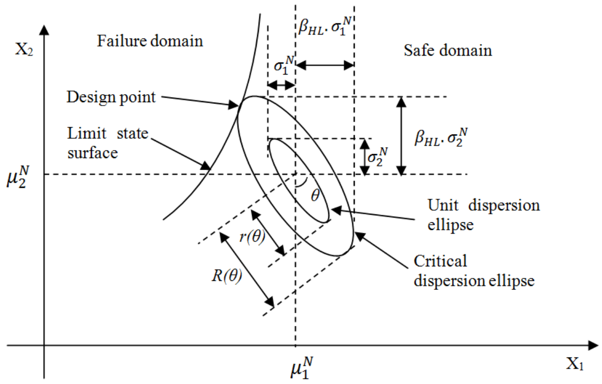


Fig. 1 Design point and equivalent normal dispersion ellipses in the space of two random variables (example of a 2D case)

axes of the critical dispersion ellipsoid (that is to say the smallest ellipsoid dispersion tangent to the boundary surface condition) and the ellipsoid in unit dispersion (the one obtained for $\beta_{HL} = 1$ in Eq. (1), without minimization). They also demonstrated that finding the critical dispersion ellipsoid is equal to find the most probable point of failure, at the point of tangency between the ellipsoid and the limit state surface which is called design point (Fig. 1).

To extend the Hasofer-Lind method to the case of non-normal random variables, Rackwitz and Fiessler (1978) proposed to transform each non-normal random variable into an equivalent normal random variable with a mean μ^N and a standard deviation σ^N . This transformation allows estimating a solution in the reduced space by using the procedure explained in the previous paragraphs. The equivalent parameters evaluated at the design point X_i^* are given by

$$\mu_i^N = -\sigma_i^N \Phi^{-1}\left[F_{X_i}(X_i^*)\right] + X_i^* \quad (2)$$

$$\sigma_i^N = \frac{\phi\{\Phi^{-1}[F_{X_i}(X_i^*)]\}}{f_{x_i}(X_i^*)} \quad (3)$$

where Φ[.] and ϕ[.] are the CDF (Cumulative Density Function) and the PDF (Probabilistic density Function) of the standard variables, respectively, and F_{Xi} (.) and f_{Xi} (.) are the CDF and PDF of the original non-normal random variables. Notice that Eqs. (2) and (3) are derived by equating the cumulative distribution functions and the probability density functions of the actual variables and the equivalent normal variables at the design point on the limit state surface.

In this paper, the method of Low and Tang was used. They set up a tilted ellipsoid and used an optimization algorithm to minimize the dispersion ellipsoid. Eq. (1) may be rewritten as Low and Tang (1997b, 2004).

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x-\mu_x^N}{\sigma_x^N}\right]^T [R]^{-1} \left[\frac{x-\mu_x^N}{\sigma_x^N}\right]} \quad (4)$$

in which $[R]^{-1}$ is the inverse of the correlation matrix. This equation will be used to set up the ellipsoid since the correlation matrix [R] displays the correlation structure more explicitly than the covariance matrix [C].

From the first order reliability method (FORM) and the Hasofer Lind reliability index β_{HL}, one can approximate the failure probability as follows

$$P_f \approx \Phi(-\beta_{HL}) \tag{5}$$

where Φ[.] is the cumulative distribution function of a standard normal variable. In this method, the limit state function is approximated by a hyperplane tangent to the limit state surface at the design point.

3. Response surfaces method optimized by a genetic algorithm

If the objective function has a known analytical form, the reliability index may be easily calculated. When using numerical calculations, it is not possible to obtain an explicit analytical form of the objective function and then the surface response method can be used to approach this function by successive iterations in order to calculate the reliability index and the design point. An algorithm based on the RSM proposed by Tandjiria *et al.* (2000). The basic idea of this method is to approximate the performance function by an explicit function of the random variables and to improve the approximation via iterations. The approximate performance function widely used in literature has a quadratic form. It uses a second order polynomial with squared terms. The expression of this approximation is given by Eq. (6).

$$G(x) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i x_i^2 \tag{6}$$

where x_i is the random variables, n is the number of random variables, (a_i, b_i) are coefficients to be determined.

For more accuracy, a more complex performance function (Eq. (7)) can be used. It contained quadratic and crossed square terms.

$$G(x) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j \tag{7}$$

The set of parameters (a_i, b_{ij}) of the eq (7) are usually determined using an iterative method which is expensive in terms of time computation Youssef Abdel Massih and al. (2008) and Mollon *et al.* (2009). In this paper the set of parameters (a_i, b_{ij}) were determined using a genetic algorithm Bouacha *et al.* (2014), Hamrouni *et al.* (2017a, b 2018).

A genetic algorithm is a search heuristic that mimics the process of natural selection. This type of algorithm can be used to generate useful solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection and crossover.

In the studied problem, the parameters in the optimization problem are the variables a_i and b_{ij}. They are translated into chromosomes with a data string.

An initial population is necessary to begin the genetic algorithm procedure. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions (in our study, a number of 500 was chosen). Traditionally, the population is generated randomly, covering the entire range of possible solutions (the search space, Tang *et al.* 1996).

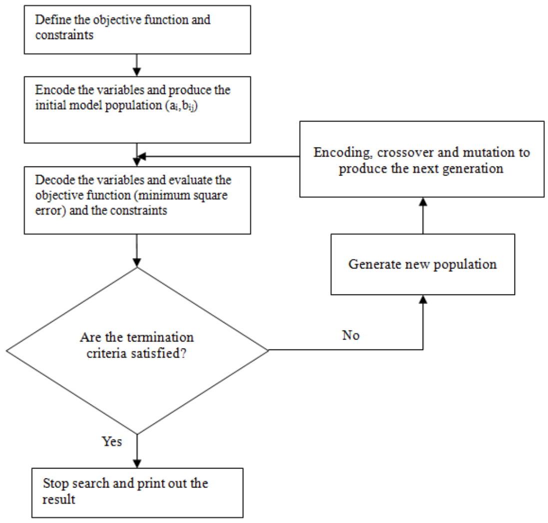


Fig. 2 Principle of optimization with a genetic algorithm

The minimum square error is used as the fitness function in the GA approach to compare the results obtained with Eq. (7) and the numerical results. This permits to determine the values of (a_i, b_{ij}). There are no constraints on the parameters (a_i, b_{ij}).

A range of possible solutions is obtained from the variable space and the fitness of these solutions is compared. If a solution is not obtained, a new population is created from the original (parent) chromosomes. This is achieved using 'crossover' and 'mutation' operations. Crossover involves gene exchange from two random (parent) solutions to form a child (new solution). Mutation involves the random switching of a single variable in a chromosome and is used to maintain population diversity, as the process converges towards a solution.

A flow chart detailing the operation of the GA process is shown in Fig. (2).

The key advantages of GA are:

(a) It is a population-based approach and thus considers a wide range of possible solutions,

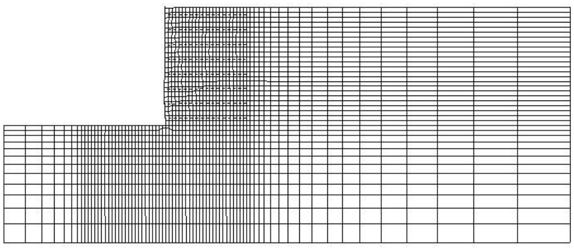
(b) The mutation process restricts the solution falling into local minima that can occur in alternative solution techniques.

4. Presentation of the numerical model

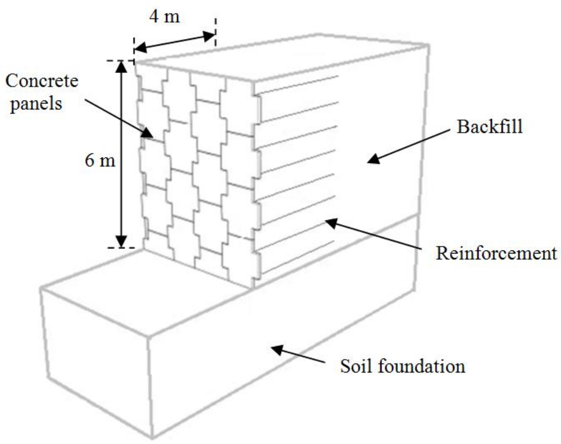
Abdelouhab *et al.* (2011, 2012b) used the Lagrangian explicit finite-difference code FLAC2D to study the behavior of a mechanically reinforced earth wall by geosynthetic strips. The reference study case wall is 6m high and is made of 4 superimposed panels (modeled by beam elements) and reinforced by 8 levels of 4 m long reinforcement layers (modeled by strip elements). The cruciform geometry of the panels (Fig. 3(b)), leads to a complex geometry of the wall. This three-dimensional geometry and the staggered layout are simplified into a two-dimensional model using some simplifications. The panels

Table 1 Mechanical properties of soils, concrete and reinforcement (Abdelouhab *et al.* 2011, 2012b)

Properties (unity)	Fill	Soil foundation	Concrete panels	Reinforcement GS 50
E (Young's modulus) (MPa)	50	50	15000	2500
ν (Poisson ratio)	0.3	0.3	0.2	-
φ (friction angle) (°)	36	-	-	-
Ψ (dilation angle) (°)	6	-	-	-
C (cohesion) (kPa)	0	-	-	-
Unit weight (kN/m^3)	15.6	20	25	-
Width (m)	-	-	-	0.1
Thickness (mm)	-	-	-	3
Strip tensile yield-force limit (kN)	-	-	-	100
Maximum Compressive strength (kPa)	-	-	-	0
Tensile failure strain limit of strip (%)	-	-	-	12



(a) Numerical model developed in FLAC2D



(b) Geometry of the earth reinforced wall.

Fig. 3 Presentation of the studied numerical model

are modeled as rectangular plates of 1.5 m by 1.5 m. The simplification of the geometry makes it possible to use a two-dimensional model with continuous reinforcements.

The model consists of two soils (Fig 3). The embankment soil consists on uniform fine sand, known under the name of Hostun RF sand Gay (2000), Flavigny *et al.* (1990). The constitutive model used for this sand is linear elastic and perfectly plastic with failure criterion of Mohr-Coulomb type and the mechanical properties are obtained after calibration on triaxial tests (Abdelouhab *et al.* 2011, 2012b). For the foundation soil, a linear elastic behavior model is used (Table 1).

For the boundary conditions, the horizontal and vertical displacements are blocked at the bottom model, and only the horizontal displacements are blocked on the lateral sides. In order to reproduce the real building steps, the setting up of the embankment is modeled by 0.375 m layers in several phases:

-Stage 1. Set up of the first concrete panel, the first and the second soil layer and installation of the first strip between the two layers of the reinforced backfill (equilibrium under self weight).

-Stage 2. Placement of the third and the fourth layer, installation of the second strip between the two layers of the reinforced backfill (equilibrium under self-weight).

-Stage 3. Set up of the second beam, the fifth and sixth layer and installation of the third strip between the two layers of the reinforced backfill.

-These phases are repeated up to 6 m height.

In our study, only one type of reinforcement is studied and modeled by the use of structural elements of type "Strip" in Flac2D software. These elements are specially designed to simulate the behavior of reinforcing bands used in Reinforced Earth embankments. Strip elements allow considering tensile strength, compression and but cannot withstand bending moments (Abdelouhab *et al.* 2010, 2012a). The characteristics of these reinforcements are calculated as being the ratio of characteristics for the width of considered ground. In most cases of real walls, (GeoStrap 50) extensible frames are implemented as a pair of 50 mm wide strips (2×50 mm).

5. Reliability analysis of reinforced earth walls

5.1 Influence of soil parameters on the reinforced earth walls

The index of reliability Hasofer-Lind is adopted to calculate the reliability of the mechanically reinforced earth wall. If we consider all the input parameters as random variables, it is going to be a large number of deterministic computations by the numerical model. The probabilistic methodology (RSM) becomes very costly in computation time when the number of random variables increases. To reduce the time calculation, a parametric study has been used to define which input parameter has influence on the wall behaviour. The output parameter considered in this study is the maximum horizontal displacement noted as U_{hmax}.

To test the input parameter effect, the U_{hmax} of the reference case studied was first calculated. A value of U_{hmax} = 7.4 cm was founded. Then a parametric study has been developed, the input range parameter values which have been investigated are presented in Table 2. Illustrative values for the influence of input parameters on U_{hmax} are given in Fig. 4. Note that the effects of the Young modulus, of the Poisson's ratio, of the dilation angle and of the soil / reinforcement friction on the response U_{hmax} are considered as negligible (difference versus the reference case inferior to 5%). The two parameters which have an influence on the ouput parameter are the friction angle φ and the soil unit weigth γ. It is important to note that the soil friction influence is higher than the one of the soil unit weight

Table 2 Influence of soil parameters on the overall behavior

Parameters	Value reference	Variation		$\Delta U/U_{ref}$ (%)	
		Min	Max		
Young modulus (MPa)	50	30	90	1.63	-1.24
Poisson's ratio	0.3	0.2	0.4	0.19	4.21
Friction angle (°)	36	25	40	168.43	-22.97
Dilation angle (°)	6	0	24	0.96	-3.29
Unit weight (kN/m^3)	15.6	13	22	-13.50	33.84
Friction angle at panel/soil interface (°)	24	0	36	1.19	0.08

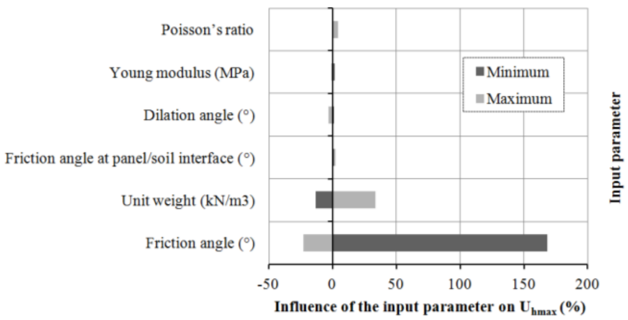


Fig. 4 Influence of input parameters on the maximum displacement of the wall face

Table 3 Probabilistic model

Variable	Mean value (μ)	Coefficient of variation (σ)	Limitations of non-normal variables	Distribution Type	
				Case 1	Case 2
γ (kN/m^3)	15.6	10%	] 0, +∞ [	normal	Log-normal
φ(°)	36	10% - 20%	] 0, 45°[	normal	beta

respectively 170% vs. 34%. These two variables will be considered as random variables on the following work.

The soil Young's modulus has a negligible effect on the facing horizontal displacement. The rigidity of the reinforcing strip, being much larger than the one of the soil, reduces considerably the effect of the Young modulus.

5.2 Performance function for the probabilistic study

The values chosen for the mean and the coefficient of variation of the chosen input parameters are presented in Table 3. For the probability distribution of the random variables, two cases are considered. For the first case, normal distributions are considered for the input parameters. In the second case, the unit weight γ is assumed to follow a log-normal distribution and φ is considered bounded with a Beta distribution, this distribution is recommended due to its flexibility and its lower and upper bounds. It is mostly used to model bounded variables such as friction angle of the soil Fenton and Griffiths (2003). For both cases, non-correlated variables are considered.

The performance function used in this section is given by

$$\boldsymbol{G = U_h - U_{hmax}} \qquad (8)$$

where U_h is the maximum horizontal displacement of the concrete panels given by the numerical calculation. The failure thus occurs when the horizontal maximum displacement of wall face (output random variable) becomes greater than U_{hmax} the ultimate threshold value (considered deterministic). Several values for U_{hmax} are considered in this study from 10 to 25 cm.

The numerical implementation of the surface response algorithm optimized by genetic algorithm for the case of two random variables (γ and φ) is presented and consists on the following steps:

1- Perform numerical deterministic calculations of the maximum horizontal displacements (U_h) with a different set of values of the random variables in each calculation. For each random variable, its value is $\mu \pm m{*}\sigma$. In this analysis, the value of m was chosen equal as 0.5 and 1.0, which gives us five values of each random variable instead of 3 values Low, (2005). It permits to increase the accuracy of the design point values. The total number of samples is equal to 5^n, where n is the number of random variables chosen (25 calculations in our case).

2- Values (a_i) of the following G (γ, φ) performance function (Eq. (9)) using these 25 points were optimized by the genetic algorithm

$$G\,(\gamma,\,\varphi) = a_1 + a_2.\,\varphi + a_3.\,\gamma + a_4.\varphi^2 + a_5.\,\gamma^2 + a_6.\varphi.\,\gamma \qquad (9)$$

3- The Matlab optimization tool (fmincon) is utilized to find the minimum reliability index β_{HL} Eq. (1) and the corresponding design point (γ^*, φ^*) using the condition $G(x) \leq 0$. The obtained design point is then used as an input of the numerical model in order to estimate the accuracy of the result.

A remarkable advantage of the use of a genetic algorithm is the fact that it permits to investigate a given space. The successful use of genetic algorithms depends on how quickly and accurately it converges to the optimal solution, while avoiding local minima. However, in cases where there are a large number of variables, the major drawback of the genetic algorithm is that it necessitates a large amount calculation time before reaching the optimal. In our case study, the number of variables is equal to 6 and the optimization time is less than one minute. For this reason the choice of this optimization method in the studied case remains relevant.

6. Numerical results

The results of the GA optimization approach show that the best combination of parameter values to simultaneously optimize the performance function using the Eq. (9) is: $a1$ = 9.95, $a2$ = -0.6953, $a3$ = 0.3357, $a4$ = 0.0136, $a5$ = 0.0048 and $a6$ = -0.0142.

Fig (5) shows the unit dispersion ellipse, the critical dispersion ellipse and the limit state in the space of the physical variables (γ, φ). The design point for the case U_{hmax} = 15 cm, μ_γ = 15.6 kN/m^3, cov_γ = 10%, μ_φ = 36° and cov_φ = 10% is located in this figure. The reliability index is obtained after convergence and is equal to β_{HL} = 2,353. This β_{HL} value corresponds to a failure probability of 0.93 %, obtained by the FORM approximation. The response surface method optimized by the genetic algorithm thus provides a very satisfying convergence for the reliability

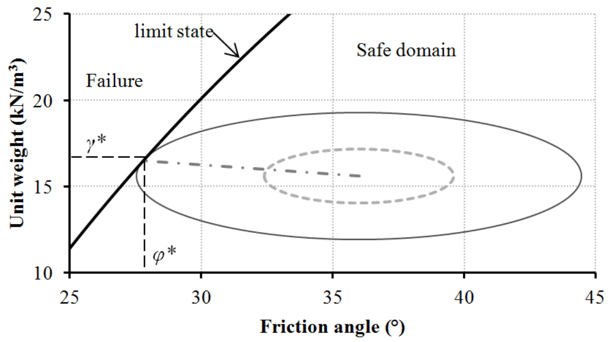


Fig. 5 The limit state surface and ellipsoid

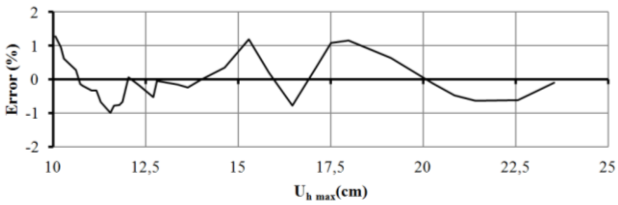


Fig. 6 Error in the performance limit of the model relative to U_{hmax}

study of the serviceability limit state. A way to check the proper convergence of the algorithm is to consider the value provided by the model developed in a deterministic model design based on numerical simulations. The performance function is almost zero at the design point in all cases (Fig. 6). In this article, a quadratic polynom with crossed terms is used as the response surface function. It permits to obtain a good approximation of the performance limit state in previous analyzes.

7. Reliability index point design and partial safety factors

Table 4 provides the values of reliability indices and the coordinates (γ^*, φ^*) of the design points for several displacement limit values (10-25 cm) for normal and non-normal variables. The results in terms of reliability index are also shown in Fig (7). It is observed that increasing the covariance φ leads to a consistently lower index of reliability. It is the same to a smaller extent when considering non-normal variables. For example, to reach a horizontal displacement of 15 cm, the reliability index is on average 25% lower compared with the case of normal variables. This suggests that the simplifying assumption of considering normal variables are safe compared to more complex distribution for the input parameters. It is therefore lead to an uneconomical design of the mechanically reinforced earth wall. The coordinates (γ^*, φ^*) design points obtained can give an idea of partial safety factors of each $F\varphi$ and $F\gamma$ resistance characteristics, expressed as follows

$$F_\varphi = \frac{\tan \mu_\varphi}{\tan \varphi^*} \tag{10}$$

$$F_\gamma = \frac{\gamma^*}{\mu_\gamma} \tag{11}$$

Table 4 Indices of reliability, design points, and partial safety factors

Cov$_\gamma$ =10% Cov$_\varphi$=10%.							Cov$_\gamma$ =10% Cov$_\varphi$=20%.						
Normal variables													
U_{hmax} (cm)	β_{hl}	Pf (%)	φ^* (°)	γ^* (kN/m^3)	F_φ	F_γ	U_{hmax} (cm)	β_{hl}	Pf %	φ^* (°)	γ^* (KN/m^3)	F_φ	F_γ
10	1,135	12.82	32,110	16,140	1,158	1,035	10	0,587	27.86	31,820	15,734	1,171	1,009
15	2,353	0.93	27,690	16,507	1,384	1,058	15	1,204	11.43	27,400	15,835	1,402	1,015
20	3,012	0.13	25,470	16,718	1,525	1,072	20	1,539	6.19	25,000	15,887	1,558	1,018
25	3,523	0.021	23,680	16,905	1,657	1,084	25	1,799	3.6	23,140	15,939	1,700	1,022
Non-normal variables													
10	0,971	16,590	31,920	15,876	1,166	1,018	10	0,525	29,950	31,730	15,610	1,175	1,001
15	1,881	3,000	27,550	16,090	1,393	1,031	15	0,958	16,910	27,300	15,661	1,408	1,004
20	2,379	0,860	25,200	16,240	1,544	1,041	20	1,195	11,600	24,900	15,690	1,565	1,006
25	2,775	0,275	23,390	16,390	1,680	1,051	25	1,385	8,300	23,010	15,700	1,711	1,006

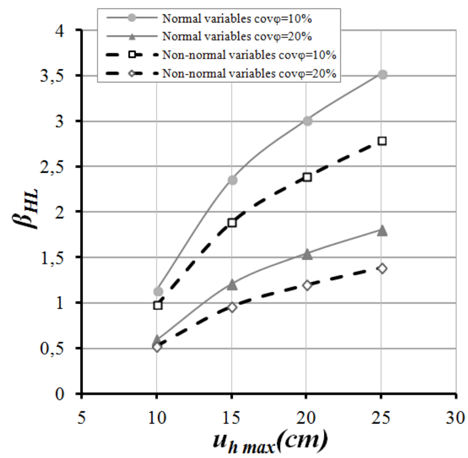


Fig. 7 Reliability index related to the performance limit

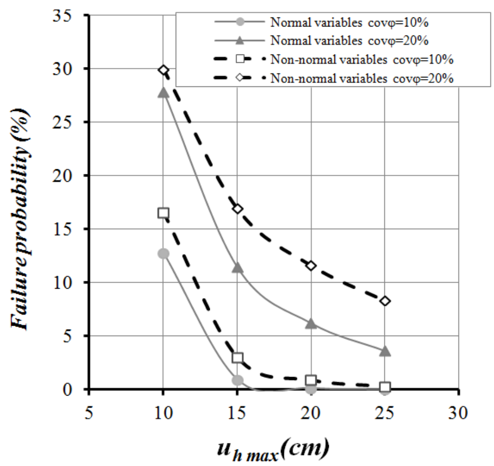


Fig. 8 Failure probability in relation to the performance limit

These factors are also provided in Table 4 for each displacement limit value and for normal or non-normal variables. The partial safety factors are smaller as the displacement limit is reduced and tends to 1 for the limit case equal to the horizontal displacement limits. For the case of the unit weight, however, it is observed that the value of γ^* is slightly greater than the mean value of γ, leading to partial factors $F\gamma$ almost equal to 1. For this reason, a partial safety factor close to 1 for the unit weight does not necessarily indicate the failure of the system, as long as the φ partial safety coefficient is high. This conclusion is similar to that made by Youssef Abdel Massih (2008) and Mollon *et al.* (2009).

From the reliability indices obtained by RSM, the FORM approximation provides directly the failure probability values, which are grouped in Fig (8). The comments already made for reliability indices remain the same. Being confined to use non-normal distributions for the input parameters instead of normal distributions can significantly increase the probability of failure, all things being equal. The assumptions of normal distributions are fairly secure.

8. Conclusions

An analysis based on the reliability of mechanically reinforced earth walls is presented. The state of serviceability limit is used to characterize the behavior of the maximum horizontal displacement of the wall face. A deterministic model based on numerical simulations is used to calculate the face displacement. The index of Hasofer-Lind reliability is adopted here for the reliability assessment of the mechanically reinforced earth wall. The response surface methodology is used to find the reliability indices with an optimization by a genetic algorithm. Only soil parameters are considered as random variables. The main conclusions of this paper can be summarized as follows:

• The main input parameters which influence the movement of the wall are mainly the internal friction angle and the unit weight of the granular soil.

• The use of a genetic algorithm is very effective to optimize the unknown parameters of the performance function (a_i). The genetic algorithm optimization permits to reduce the computation time by eliminating the successive iterative method used by the classical method RSM low (2005).

• The value of the parameter φ at the design point is always smaller than the φ mean values and increase with the decrease of U_{hmax}. The γ^* value slightly exceeds the average value for all the U_{hmax} values. Therefore, a partial safety factor close to 1 for the unit weight does not necessarily indicate a system close to failure, as long as the partial safety factor of φ is high.

• The failure probability is much more sensitive to the uncertainties of the internal friction angle of the soil than of the soil unit weight,

• For a small displacement limit value, the index of reliability is smaller and a high probability of failure indicates the vulnerability of this structure.

• The simplifying assumption of considering normal variables is safe compared to more complex probabilistic models (non-normal variables). It can therefore lead to uneconomical designs of mechanically stabilized earth walls.

The proposed approach is efficient because the problem was essentially based on a specific problem where most of the component properties and geometry are assumed as deterministic.

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