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Dynamic impedance of a floating pile embedded in poro-visco-elastic soils subjected to vertical harmonic loads

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Abstract. Based on the theory of porous media, an interaction system of a floating pile and a saturated soil in cylindrical coordinates subjected to vertical harmonic load is presented in this paper. The surrounding soil is separated into two distinct layers. The upper soil layer above the level of pile base is described as a saturated viscoelastic medium and the lower soil layer is idealized as equivalent spring-dashpot elements with complex stiffness. Considering the cylindrically symmetry and the pile-soil compatibility condition of the interaction system, a frequency-domain analytical solution for dynamic impedance of the floating pile embedded in saturated viscoelastic soil is also derived, and reduced to verify it with existing solutions. An extensive parametric analysis has been conducted to reveal the effects of the impedance of the lower soil base, the interaction coefficient and the damping coefficient of the saturated viscoelastic soil layer on the vertical vibration of the pile-soil interaction system. It is shown that the vertical dynamic impedance of the floating pile significantly depends on the real stiffness of the impedance of the lower soil base, but is less sensitive to its dynamic damping variation; the behavior of the pile in poro-visco-elastic soils is totally different with that in single-phase elastic soils due to the existence of pore liquid; the effect of the interaction coefficient of solid and liquid on the pile-soil system is limited.

Keywords: analytical solution; dynamic impedance; pile-soil interaction; vertical vibration; porous medium; viscoelastic soil

1. Introduction

Dynamic impedance of piles in soil media to time-harmonic loads is of important theoretical significance in the field of geotechnical engineering and structural engineering (Bose and Haldar 1985, Dobry and Gazetas 1988, Amin *et al.* 2015). This introduction provides an overview of the key literature relevant to the development of theoretical models to obtain the dynamic impedance of piles embedded in soils, with various mathematical models having been developed by researchers. The Winkler model is extensively employed due to its simplicity in which soil layers are represented by equivalent spring-dashpot elements. However, the Winker model has limitations when describing the mechanism of wave propagation within the pile-soil system (Anoyatis and Mylonakis 2012, Wu *et al.* 2014, Ding *et al.* 2014). Novak *et al.* (1978) presented a plane-strain model for the pile-soil interaction system and considered the soil as a linear viscoelastic layer with hysteretic-type damping. Manna and Baidya (2009) investigated the possible factors for the unsatisfactory performance of the Novak's model and showed these to be the effective pile length for significantly under-loaded piles and the real embedment effect. Furthermore, the three dimensional wave effect of an end bearing pile is considered within the pile-soil interaction system by modelling the soil as a three-dimensional axisymmetric continuum in which both its radial and vertical displacements are taken into account (Yang *et al.* 2009, Wu *et al.* 2013).

The surrounding soil of the pile in the above studies of the pile-soil system is assumed as a single-phase medium. However, soil is generally a multiphase medium that can be modelled as a liquid-filled porous medium. In recent decades, pile-soil dynamic interaction considering the effect of liquid-saturated media has become one of the key topics of pile-soil interaction (Liu *et al.* 2014). In much of the research to date, Biot's model has been employed to describe the macro-mechanical behaviour of a saturated soil in pile-soil interaction systems. Rajapakse and Senjuntichai (1995) explicitly derived rigorous analytical solutions of the vertical vibration that describes the relationship between the generalized displacement and the force of multilayered porous media in the Fourier-frequency space. Zhou *et al.* (2009) investigated the dynamic response of a pile embedded in a saturated half space subjected to transient vertical loading by adopting Biot's porous elastodynamic equations. Cai and Hu (2010) used Biot's elastodynamic theory as the basis to present the analytical solution for the vertical vibration of a rigid foundation embedded in a poroelastic half-space. In addition, Zheng *et al.* (2015) presented an analytical method to study the vertical vibration of a floating pile embedded in poroelastic soil using the Biot's theory.

The framework of Biot's model is essentially based on a phenomenological methodology and an engineering

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description. Bowen (1980) proposed the theory of porous media (TPM) by integrating the continuum theory of mixtures with the concept of volume fractions. In contrast to Biot's theory, the theory of porous media has also been proven to provide a comprehensive and extensive modelling framework (Edelman and Wilmanski 2002, Heider *et al.* 2012). Substantial developments with respect to the theory of porous media have been extended to geomechanical problems and were contributed to by De Boer's pioneering work (De Boer *et al.* 1990, 1994, 1996a, b, c). In addition to De Boer's pioneering work, Liu *et al.* (1999) investigated inhomogeneous wave propagation in saturated porous soils by using the theory of porous media (TPM). The general solutions of plane longitudinal and transverse waves in saturated porous media were obtained, and simultaneously, the explicit expressions for the mean energy flux vectors and the mean energy dissipation rate were also presented by Zheng *et al.* (2005). Kumar and Hundal (2005) derived characteristic equations for discontinuities across the wave fronts in a fluid-saturated incompressible porous medium, and took the Heaviside step input function for the numerical investigation of the symmetric wave propagation.

As for studies on the dynamic behaviour of piles in saturated soil based on the theory of porous media (TPM), some substantial developments have been made by some investigators. For example, vertical vibrations of an end-bearing pile and complex stiffness at the pile head were investigated by Liu and Yang (2009). In addition, the axisymmetrical analytical solutions for vertical vibrations in an end-bearing pile in a saturated viscoelastic soil layer were obtained by Yang and Pan (2010). The effects of the saturated soil parameters, modulus ratio of the pile to soil, slenderness ratio of pile and pile's Poisson ratio on the stiffness factor and damping were also examined by Yang and Pan (2010). Subsequently, Cui *et al.* (2016) deduced an axisymmetrical analytical solution for the vertical time-harmonic vibration of a pile in a saturated viscoelastic soil layer overlaying bedrock using the method of differential operators, and investigated the effect of relative bedrock depth to the pile on the dynamic response of pile-soil system.

The aforementioned studies are devoted to the end-bearing pile case. However, based on an extensive review of the literature, for the moment no study has been reported to the dynamic behaviour of floating piles in a viscoelastic saturated soil using the theory of porous media (TPM) by integrating the continuum theory of mixtures with the concept of volume fractions. Consequently, the vertical vibration of a single floating pile in a poro-visco-elastic soil layer is investigated in this study. The saturated soil layer is modelled as a two-phase medium, of which the governing equations are described in the mentioned porous media theory, while the pile is treated as an one-dimensional rod and described by the theory of beam vibration (Rayleigh 1945). Firstly, based on the theory of porous media (TPM), the axisymmetrical fundamental solution of the soil reaction around the floating pile is obtained in a cylindrical coordinate system using the differential operator theory and the variable separation method. Secondly, the partial differential equations for the vertical vibration of a floating pile are established on the basis of the fundamental solution presented for the soil reaction around the pile. An analytical solution for the vertical displacement at the head of a single floating pile is derived by considering the pile-soil comparability condition. Finally, the vertical dynamic impedance solution of a single floating pile in a saturated soil layer is obtained and compared with the existing solutions. The effects of the various parameters associated with the saturated soil on the vertical impedance solution of the pile are also revealed.

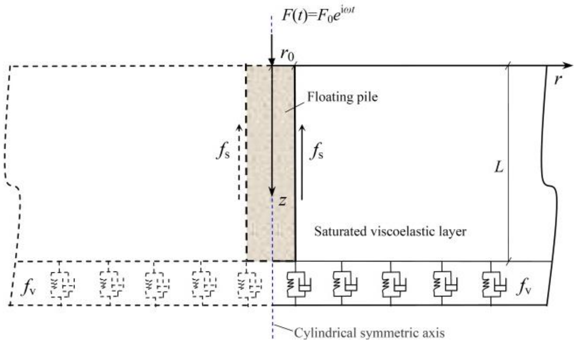


Fig. 1 Dynamic interaction model of a floating pile embedded in saturated soil subjected to harmonic axial load

2. Conceptual model and formulation of governing equations

The mechanical model for the problem under consideration and the interaction system of a floating pile and a saturated soil in cylindrical coordinates subjected to a vertical harmonic exciting force are shown in Fig. 1. In this model, due to the difficulty of establishing a coupled continuum model for the pile-soil interaction in a strictly mathematical and physical manner, as suggested by Baranov (1967), Novak and Beredugo (1972), Hu (2003), and Cai and Hu (2010), the upper soil layer above the level of the pile base is considered as a continuum, while the lower soil layer is idealized as equivalent spring-dashpot elements with a complex stiffness fv. Furthermore, it assumes that the vibration of the pile-soil system is infinitesimal, and the displacements and stresses at the interface between pile and soil are continuous. As the cylindrical pile is in a harmonic vertical vibration, the motion of the saturated viscoelastic soil layers will also be cylindrically symmetric and time-harmonic.

2.1 Governing equations for saturated soil layer

On the basis of the theory of porous media (TPM) by integrating the continuum theory of mixtures with the concept of volume fractions (De Boer and Liu 1996), the three dimensional dynamic governing equations for the saturated viscoelastic soil layer can be described using Eqs. (1a)-(1c):

Momentum balance equation for solid skeleton

$$(\lambda^{S}+\mu^{S})\,\mathrm{grad\,div}\,\mathbf{U}_{S}+\mu^{S}\,\mathrm{div\,grad}\,\mathbf{U}_{S}-n^{S}\,\mathrm{grad}\,p-\rho^{S}\ddot{\mathbf{U}}_{S}+S_{v}(\dot{\mathbf{U}}_{L}-\dot{\mathbf{U}}_{S})=0\,,\qquad(1a)$$

Momentum balance equation for pore liquid,

$$-n^{L}\,\mathrm{grad}\,p-\rho^{L}\ddot{\mathbf{U}}_{L}-S_{v}(\dot{\mathbf{U}}_{L}-\dot{\mathbf{U}}_{S})=0\,,\qquad(1b)$$

Mass balance equation for solid-liquid aggregate,

$$\mathrm{div}(n^{\mathrm{S}}\dot{\mathbf{U}}_{\mathrm{S}}+n^{\mathrm{L}}\dot{\mathbf{U}}_{\mathrm{L}})=0, \tag{1c}$$

where, λ^{S} and μ^{S} are complex Lamé constants, and $\mathbf{U}_{\mathrm{S}}$ and $\mathbf{U}_{\mathrm{L}}$ are the displacement vectors for the soil skeleton and pore water, respectively. p is the pore pressure of the incompressible pore fluid, and ρ^{S} and ρ^{L} denote the densities of the solid and fluid phases, respectively, n^{S} and n^{L} are the volume fractions satisfying $n^{\mathrm{S}}+n^{\mathrm{L}}=1$. In Eq. (1a), $\mu^{\mathrm{S}}=G(1+\mathrm{i}\xi)$, $\lambda^{\mathrm{S}}=\frac{2v}{1-2v}\mu^{\mathrm{S}}$, where G is the shear modulus of soil, ξ is the damping coefficient, and v is the Poisson's ratio. $S_v=\frac{(n^{\mathrm{L}})^2\gamma^{\mathrm{LR}}}{k^{\mathrm{L}}}$, in Eqs. (1a) and (1b), denotes the coupled interaction between the soil skeleton and the pore water, in which γ^{LR} is the effective specific weight of the liquid and k^L is the Darcy's permeability coefficient of the porous medium. It is noted that the last term in Eq. (1a) represents the linear drag force of the pore liquid exerting on the solid skeleton, and then S_v can be also considered as an internal friction coefficient. For totally permeable or totally impermeable system, these equations are still valid.

For an interaction system of a pile and saturated soil layers subjected to a harmonic axial load $F(t)=F_0e^{\mathrm{i}\omega t}$ ($\mathrm{i}^2=-1$), all field variables are time-harmonic with the term $e^{\mathrm{i}\omega t}$, i.e.,

$$u_{\mathrm{S}}=U_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ u_{\mathrm{L}}=U_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t},\ w_{\mathrm{S}}=W_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ w_{\mathrm{L}}=W_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t},\ w_{\mathrm{P}}=W_{\mathrm{P}}\mathrm{e}^{\mathrm{i}\omega t},\ p=P\mathrm{e}^{\mathrm{i}\omega t}. \tag{2a}$$

Then one can get their derivatives with respect to time as

$$\frac{\partial u_{\mathrm{S}}}{\partial t}=\mathrm{i}\omega U_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial^2 u_{\mathrm{S}}}{\partial t^2}=-\omega^2 U_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial u_{\mathrm{L}}}{\partial t}=\mathrm{i}\omega U_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial^2 u_{\mathrm{L}}}{\partial t^2}=-\omega^2 U_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t}, \tag{2b}$$

$$\frac{\partial w_{\mathrm{S}}}{\partial t}=\mathrm{i}\omega W_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial^2 w_{\mathrm{S}}}{\partial t^2}=-\omega^2 W_{\mathrm{S}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial w_{\mathrm{L}}}{\partial t}=\mathrm{i}\omega W_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial^2 w_{\mathrm{L}}}{\partial t^2}=-\omega^2 W_{\mathrm{L}}\mathrm{e}^{\mathrm{i}\omega t}, \tag{2c}$$

$$\frac{\partial w_{\mathrm{P}}}{\partial t}=\mathrm{i}\omega W_{\mathrm{P}}\mathrm{e}^{\mathrm{i}\omega t},\ \frac{\partial^2 w_{\mathrm{P}}}{\partial t^2}=-\omega^2 W_{\mathrm{P}}\mathrm{e}^{\mathrm{i}\omega t}, \tag{2d}$$

where u_{S} and u_{L} represent the radial displacements of the soil skeleton and pore water at r direction, w_{S} and w_{L} represent the vertical displacements of the soil skeleton and pore water at z direction, and w_{P} represents the vertical displacement of the pile.

Furthermore, considering the cylindrically symmetric conditions of the interaction system of a pile and saturated soil layers under harmonic axial load, Eqs. (1a) to (1c) can be expressed in the frequency domain by Eqs. (3a) to (3e)

$$\begin{aligned}&(\lambda^{\mathrm{S}}+\mu^{\mathrm{S}})\frac{\partial\Theta}{\partial r}+\mu^{\mathrm{S}}(\nabla^2-\frac{1}{r^2})U_{\mathrm{S}}-\frac{\partial P}{\partial r}\\&+\rho^{\mathrm{S}}\omega^2U_{\mathrm{S}}+\rho^{\mathrm{L}}\omega^2U_{\mathrm{L}}=0\end{aligned} \tag{3a}$$

$$\begin{aligned}&(\lambda^{\mathrm{S}}+\mu^{\mathrm{S}})\frac{\partial\Theta}{\partial z}+\mu^{\mathrm{S}}\nabla^2W_{\mathrm{S}}-\frac{\partial P}{\partial z}\\&+\rho^{\mathrm{S}}\omega^2W_{\mathrm{S}}+\rho^{\mathrm{L}}\omega^2W_{\mathrm{L}}=0\end{aligned} \tag{3b}$$

$$n^{\mathrm{L}}\frac{\partial P}{\partial r}+\rho^{\mathrm{L}}(-\omega^2U_{\mathrm{L}})+S_v\cdot\mathrm{i}\omega(U_{\mathrm{L}}-U_{\mathrm{S}})=0 \tag{3c}$$

$$n^{\mathrm{L}}\frac{\partial P}{\partial z}+\rho^{\mathrm{L}}(-\omega^2W_{\mathrm{L}})+S_v\cdot\mathrm{i}\omega(W_{\mathrm{L}}-W_{\mathrm{S}})=0 \tag{3d}$$

$$\begin{aligned}&n^{\mathrm{S}}\frac{\partial U_{\mathrm{S}}}{\partial r}+n^{\mathrm{L}}\frac{\partial U_{\mathrm{L}}}{\partial r}+\frac{1}{r}(n^{\mathrm{S}}U_{\mathrm{S}}+n^{\mathrm{L}}U_{\mathrm{L}})\\&+n^{\mathrm{S}}\frac{\partial W_{\mathrm{S}}}{\partial z}+n^{\mathrm{L}}\frac{\partial W_{\mathrm{L}}}{\partial z}=0\end{aligned} \tag{3e}$$

where $\Theta=\frac{\partial U_{\mathrm{S}}}{\partial r}+\frac{U_{\mathrm{S}}}{r}+\frac{\partial W_{\mathrm{S}}}{\partial z}$ is the volume strain of the soil skeleton, and $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$ is Laplacian operator.

Henceforth, the non-dimensional quantities and variables are introduced by Eqs. (4a) to (4c)

$$\bar{\lambda}^{\mathrm{S}}=\frac{\lambda^{\mathrm{S}}}{G},\ \bar{\mu}^{\mathrm{S}}=\frac{\mu^{\mathrm{S}}}{G},\ \bar{E}=\frac{E}{G},\ \bar{\rho}^{\mathrm{S}}=\frac{\rho^{\mathrm{S}}}{\rho},\ \bar{\rho}^{\mathrm{L}}=\frac{\rho^{\mathrm{L}}}{\rho},\ \bar{S}_v=\frac{r_0S_v}{\sqrt{\rho G}},\ \bar{f}_v=\frac{f_v}{Gr_0},\ \bar{r}_0=\frac{r_0}{1\mathrm{m}} \tag{4a}$$

$$\bar{r}=\frac{r}{r_0},\ \bar{z}=\frac{z}{r_0},\ \bar{U}_{\mathrm{S}}=\frac{U_{\mathrm{S}}}{r_0},\ \bar{U}_{\mathrm{L}}=\frac{U_{\mathrm{L}}}{r_0},\ \bar{W}_{\mathrm{S}}=\frac{W_{\mathrm{S}}}{r_0},\ \bar{W}_{\mathrm{L}}=\frac{W_{\mathrm{L}}}{r_0},\ \bar{W}_{\mathrm{P}}=\frac{W_{\mathrm{P}}}{r_0} \tag{4b}$$

$$\bar{P}=\frac{P}{G},\ \bar{\sigma}_z=\frac{\sigma_z}{G},\ \bar{\tau}_{rz}=\frac{\tau_{rz}}{G},\ \bar{F}_s=\frac{F_s}{Gr_0},\ a_0=\sqrt{\frac{\rho}{G}}r_0\omega, \tag{4c}$$

where E is the elastic modulus of the soil skeleton; $\rho=n^{\mathrm{S}}\rho^{\mathrm{S}}+n^{\mathrm{L}}\rho^{\mathrm{L}}$ is the total density of liquid-solid mixture; $\Theta=\frac{\partial\bar{U}_{\mathrm{S}}}{\partial\bar{r}}+\frac{\bar{U}_{\mathrm{S}}}{\bar{r}}+\frac{\partial\bar{W}_{\mathrm{S}}}{\partial\bar{z}}=\frac{\partial U_{\mathrm{S}}}{\partial r}+\frac{U_{\mathrm{S}}}{r}+\frac{\partial W_{\mathrm{S}}}{\partial z}$; $\nabla^2=\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}+\frac{\partial^2}{\partial\bar{z}^2}$.

Inserting Eqs. (4a) to (4c) into Eqs. (3a) to (3e), Eqs. (5a) to (5e) are obtained

$$\begin{aligned}&(\bar{\lambda}^{\mathrm{S}}+\bar{\mu}^{\mathrm{S}})\frac{\partial\Theta}{\partial\bar{r}}+\bar{\mu}^{\mathrm{S}}(\nabla^2-\frac{1}{\bar{r}^2})\bar{U}_{\mathrm{S}}\\&-\frac{\partial\bar{P}}{\partial\bar{r}}+\bar{\rho}^{\mathrm{S}}a_0^2\bar{U}_{\mathrm{S}}+\bar{\rho}^{\mathrm{L}}a_0^2\bar{U}_{\mathrm{L}}=0\end{aligned} \tag{5a}$$

$$\begin{aligned}&(\lambda^{\mathrm{S}}+\mu^{\mathrm{S}})\frac{\partial\Theta}{\partial z}+\mu^{\mathrm{S}}\nabla^2W_{\mathrm{S}}-\frac{\partial P}{\partial z}\\&+\rho^{\mathrm{S}}\omega^2W_{\mathrm{S}}+\rho^{\mathrm{L}}\omega^2W_{\mathrm{L}}=0\end{aligned} \tag{5b}$$

$$n^{\mathrm{L}}\frac{\partial P}{\partial r}+\rho^{\mathrm{L}}(-\omega^2U_{\mathrm{L}})+S_v\cdot\mathrm{i}\omega(U_{\mathrm{L}}-U_{\mathrm{S}})=0 \tag{5c}$$

$$n^{\mathrm{L}}\frac{\partial P}{\partial z}+\rho^{\mathrm{L}}(-\omega^2W_{\mathrm{L}})+S_v\cdot\mathrm{i}\omega(W_{\mathrm{L}}-W_{\mathrm{S}})=0 \tag{5d}$$

$$\begin{aligned}&n^{\mathrm{S}}\frac{\partial U_{\mathrm{S}}}{\partial r}+n^{\mathrm{L}}\frac{\partial U_{\mathrm{L}}}{\partial r}+\frac{1}{r}(n^{\mathrm{S}}U_{\mathrm{S}}+n^{\mathrm{L}}U_{\mathrm{L}})\\&+n^{\mathrm{S}}\frac{\partial W_{\mathrm{S}}}{\partial z}+n^{\mathrm{L}}\frac{\partial W_{\mathrm{L}}}{\partial z}=0\end{aligned} \tag{5e}$$

From the model in Fig. 1, the boundary conditions of the soil-pile system is specified in non-dimensional form as follows:

(i) The displacements and stresses are zero at infinity, i.e.,

$$\bar{U}_{\mathrm{S}}(\infty,\bar{z})=0,\ \bar{\tau}_{rz}(\infty,\bar{z})=0,\ \text{etc.}\qquad(\bar{r}\to\infty) \tag{6a}$$

(ii) The surface of the soil layer is free-traction and permeable, i.e.,

$$\bar{\sigma}_z(\bar{r},0)=0,\ \bar{\tau}_{rz}(\bar{r},0)=0,\ \bar{P}=0,\quad(\bar{z}=0) \tag{6b}$$

(iii) The stresses and displacements at the interface between the saturated soil layer and the lower layer are assumed to be continuous, i.e.,

$$\bar{E}\frac{\partial \overline{W}_{\mathrm{S}}}{\partial \bar{z}} = \bar{f}_v \overline{W}_{\mathrm{S}}(\bar{r}, \frac{L}{r_0})\bar{r}_0^2 \qquad (\bar{z} = \frac{L}{r_0}) \tag{6c}$$

(iv) The saturated soil and floating pile are bonded on their interfaces, and the pile is assumed to be impermeable and an one-dimensional Euler-Bernoulli rod. Thus, the vertical displacements at the interface are identical, and the radial displacements of the liquid and the soil skeleton at the interface are zero, i.e.,

$$\overline{W}_{\mathrm{S}}(1,\bar{z}) = \overline{W}_{\mathrm{P}}(\bar{z}),\ \overline{U}_{\mathrm{L}}(1,\bar{z}) = 0,\ \overline{U}_{\mathrm{S}}(1,\bar{z}) = 0,\ (\bar{r}=1) \tag{6d}$$

2.2 Fundamental solution for a saturated soil around a pile

After rearranging the terms of Eq. (5c) and Eq. (5d), then Eqs. (7a) and (7b) are obtained

$$\overline{U}_{\mathrm{L}} = D_1(\overline{U}_{\mathrm{S}} - \frac{n^{\mathrm{L}}}{\mathrm{i}\, a_0 \bar{S}_v}\frac{\partial \overline{P}}{\partial \bar{r}}) \tag{7a}$$

$$\overline{W}_{\mathrm{L}} = D_1(\overline{W}_{\mathrm{S}} - \frac{n^{\mathrm{L}}}{\mathrm{i}\, a_0 \bar{S}_v}\frac{\partial \overline{P}}{\partial \bar{z}}) \tag{7b}$$

where $D_1 = \dfrac{\mathrm{i}\, a_0 \bar{S}_v}{\mathrm{i}\, a_0 \bar{S}_v - a_0^2 \bar{\rho}^{\mathrm{L}}}$.

Combining Eq. (5a) and Eq. (5b) according to the operation $\frac{\partial}{\partial \bar{r}}(5\mathrm{a}) + \frac{1}{\bar{r}}(5\mathrm{a}) + \frac{\partial}{\partial \bar{z}}(5\mathrm{b})$ with the substitution of Eq. (7a) and Eq. (7b), then one obtains Eq. (8a)

$$k_1\nabla^2\Theta - k_2\nabla^2\overline{P} + k_3\Theta = 0 \tag{8a}$$

where $k_1 = \bar{\lambda}^{\mathrm{S}} + 2\bar{\mu}^{\mathrm{S}}$, $k_2 = 1 + \dfrac{D_1\bar{\rho}^{\mathrm{L}} a_0 n^{\mathrm{L}}}{\mathrm{i}\bar{S}_v}$, $k_3 = \bar{\rho}^{\mathrm{S}} a_0^2 + \bar{\rho}^{\mathrm{L}} a_0^2 D_1$.

Similarly, combining Eq. (7a) and Eq. (7b) according to the operation $\frac{\partial}{\partial \bar{r}}(7\mathrm{a}) + \frac{1}{\bar{r}}(7\mathrm{a}) + \frac{\partial}{\partial \bar{z}}(7\mathrm{b})$ and inserting the resulting expression into Eq. (5e) yields Eq. (8b)

$$\nabla^2\overline{P} = D_2\Theta \tag{8b}$$

where $D_2 = \dfrac{(n^{\mathrm{S}} + n^{\mathrm{L}} D_1)\mathrm{i}\, a_0 \bar{S}_v}{(n^{\mathrm{L}})^2 D_1}$.

Furthermore, Eq. (8a) and Eq. (8b) can be united and rewritten in the form

$$\begin{bmatrix} k_1\nabla^2 + k_3 & -k_2\nabla^2 \\ D_2 & -\nabla^2 \end{bmatrix}\begin{bmatrix} \Theta \\ \overline{P} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

On account of the non-trivial solution for Eq. (9), the determinant of the coefficient matrix for Eq. (9) should be zero, i.e.,

$$\begin{vmatrix} k_1\nabla^2 + k_3 & -k_2\nabla^2 \\ D_2 & -\nabla^2 \end{vmatrix} = \nabla^4 + \frac{k_3 - k_2 D_2}{k_1}\nabla^2 = 0 \tag{10}$$

If $\beta_1^2 = \dfrac{k_2 D_2 - k_3}{k_1}$, then Eq. 10 can be rewritten as

$$\nabla^2(\nabla^2 - \beta_1^2) = 0 \tag{11}$$

According to differential operator theory (Senjuntichai and Rajapakse 1993), suppose that $\Theta = \Theta_1 + \Theta_2$ satisfies

$$(\nabla^2 - \beta_1^2)\Theta_1 = 0 \tag{12a}$$

$$\nabla^2\Theta_2 = 0 \tag{12b}$$

Using the variable separation method, and substituting $\Theta_1 = R(\bar{r})S(\bar{z})$ into Eq. (12a) produces

$$\frac{1}{R(\bar{r})}\frac{\mathrm{d}^2R(\bar{r})}{\mathrm{d}\bar{r}^2} + \frac{1}{R(\bar{r})}\frac{1}{\bar{r}}\frac{\mathrm{d}R(\bar{r})}{\mathrm{d}\bar{r}} + \frac{1}{S(\bar{z})}\frac{\mathrm{d}^2S(\bar{z})}{\mathrm{d}\bar{z}^2} - \beta_1^2 = 0 \tag{13}$$

The solutions of Eq. (13) are given by

$$R(\bar{r}) = C_1K_0(g_2\bar{r}) + C_2I_0(g_2\bar{r}) \tag{14a}$$

$$S(\bar{z}) = A_1\mathrm{e}^{g_1\bar{z}} + A_2\mathrm{e}^{-g_1\bar{z}} \tag{14b}$$

of which the corresponding derivation is expressed in the Appendix.

Thus, Θ_1 can be determined from

$$\Theta_1 = (A_1\mathrm{e}^{g_1\bar{z}} + A_2\mathrm{e}^{-g_1\bar{z}}) \times \left[C_1K_0(g_2\bar{r}) + C_2I_0(g_2\bar{r})\right] \tag{15a}$$

Similarly, the corresponding expression of Θ_2 can be expressed from Eq. (12b) in the following form

$$\Theta_2 = (A_3\mathrm{e}^{g_3\bar{z}} + A_4\mathrm{e}^{-g_3\bar{z}}) \times \left[C_3K_0(g_4\bar{r}) + C_4I_0(g_4\bar{r})\right] \tag{15b}$$

in which $I_0(g_2\bar{r})$ and $I_0(g_4\bar{r})$ are the modified zero-order Bessel functions of the first kind, and $K_0(g_4\bar{r})$ is the modified zero-order Bessel functions of the second kind, respectively. A_1, A_2, A_3, A_4, C_1, C_2, C_3 and C_4 are undetermined coefficients. Moreover, g_3 and g_4 satisfy the following conditions, $g_3^2 + g_4^2 = 0$, $Re(g_3) > 0$, $Re(g_4) > 0$.

From Eqs. (15a) and (15b), it can be written as

$$\begin{aligned}\Theta &= (A_1\mathrm{e}^{g_1\bar{z}} + A_2\mathrm{e}^{-g_1\bar{z}})\left[C_1K_0(g_2\bar{r}) + C_2I_0(g_2\bar{r})\right] \\ &+ (A_3\mathrm{e}^{g_3\bar{z}} + A_4\mathrm{e}^{-g_3\bar{z}})\left[C_3K_0(g_4\bar{r}) + C_4I_0(g_4\bar{r})\right]\end{aligned} \tag{16a}$$

In the same way, it can be written as

$$\begin{aligned}\overline{P} &= (A_5\mathrm{e}^{g_1\bar{z}} + A_6\mathrm{e}^{-g_1\bar{z}})\left[C_5K_0(g_2\bar{r}) + C_6I_0(g_2\bar{r})\right] \\ &+ (A_7\mathrm{e}^{g_3\bar{z}} + A_8\mathrm{e}^{-g_3\bar{z}})\left[C_7K_0(g_4\bar{r}) + C_8I_0(g_4\bar{r})\right]\end{aligned} \tag{16b}$$

where A_5, A_6, A_7, A_8, C_5, C_6, C_7 and C_8 are undetermined coefficients.

Taking into account the boundary conditions expressed in Eq. (6a),

$$C_2 = C_4 = C_6 = C_8 = 0 \tag{17a}$$

$$A_5 + A_6 = 0 \tag{17b}$$

$$A_7 + A_8 = 0 \tag{17c}$$

Using Eqs. (17a) to (17c), Eqs. (16a) and (16b) can be reduced to

$$\begin{aligned}\Theta &= (B_1 e^{g_1\bar{z}} + B_2 e^{-g_1\bar{z}})K_0(g_2\bar{r}) \\ &+(B_3 e^{g_3\bar{z}} + B_4 e^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{18a}$$

$$\begin{aligned}\bar{P} &= B_5(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_0(g_2\bar{r}) \\ &+B_6(e^{g_3\bar{z}} - e^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{18b}$$

where $C_1A_1 = B_1$, $C_1A_2 = B_2$, $C_3A_3 = B_3$, $C_3A_4 = B_4$, $C_5A_6 = B_5$, and $C_7A_7 = B_6$.

Substituting Eqs. (18a) and (18b) into Eqs. (6c) and (6d) gives

$$B_3 = B_4 = 0\ ,\ B_1 = -B_2 = \frac{\beta_1^2}{D_2}B_5 . \tag{19}$$

Inserting Eq. (19) into Eqs. (18a) and (18b), Θ and $\bar{P}$ become

$$\Theta = \frac{\beta_1^2}{D_2}B_5(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_0(g_2\bar{r}) \tag{20a}$$

$$\begin{aligned}\bar{P} &= B_5(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_0(g_2\bar{r}) \\ &+ B_6(e^{g_3\bar{z}} - e^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{20b}$$

Therefore Eqs. (5a) and (5b) can be reduced to the following form

$$\begin{aligned}&\bar{\mu}^S\nabla^2\bar{U}_S + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{U}_S - \frac{\bar{\mu}^S}{\bar{r}^2}\bar{U}_S \\ &-(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})\frac{\partial\bar{P}}{\partial\bar{r}} + (\bar{\lambda}^S + \bar{\mu}^S)\frac{\partial\Theta}{\partial\bar{r}} = 0\end{aligned} \tag{21a}$$

$$\begin{aligned}&\bar{\mu}^S\nabla^2\bar{W}_S + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{W}_S \\ &-(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})\frac{\partial\bar{P}}{\partial\bar{z}} + (\bar{\lambda}^S + \bar{\mu}^S)\frac{\partial\Theta}{\partial\bar{z}} = 0\end{aligned} \tag{21b}$$

It is easy to see that Eqs. (21a) and (21b) are inhomogeneous equations with respect to $\bar{U}_S$ and $\bar{W}_S$, respectively. The corresponding homogeneous equations to Eqs. (21a) and (21b) are given by

$$\bar{\mu}^S\nabla^2\bar{U}_S^1 + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{U}_S^1 - \frac{\bar{\mu}^S}{\bar{r}^2}\bar{U}_S^1 = 0 \tag{22a}$$

$$\bar{\mu}^S\nabla^2\bar{W}_S^1 + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{W}_S^1 = 0 \tag{22b}$$

Setting $\beta_2^2 = -\frac{\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1}{\bar{\mu}^S}$, Eqs. (22a) and (22b) can be simplified to

$$\nabla^2\bar{U}_S^1 - (\beta_2^2 + \frac{1}{\bar{r}^2})\bar{U}_S^1 = 0 \tag{23a}$$

$$\nabla^2\bar{W}_S^1 - \beta_2^2\bar{W}_S^1 = 0 \tag{23b}$$

The solutions to Eqs. (23a) and (23b) are given by

$$\bar{U}_S^1 = (d_1 e^{g_5\bar{z}} + d_2 e^{-g_5\bar{z}})[d_3K_1(g_6\bar{r}) + d_4I_1(g_6\bar{r})] \tag{24a}$$

$$\bar{W}_S^1 = (f_1 e^{g_7\bar{z}} + f_2 e^{-g_7\bar{z}})[f_3K_0(g_8\bar{r}) + f_4I_0(g_8\bar{r})] \tag{24b}$$

where $g_5^2 + g_6^2 = \beta_2^2$, $g_7^2 + g_8^2 = \beta_2^2$, $Re(g_5) > 0$, $Re(g_6) > 0$, $Re(g_7) > 0$, $Re(g_8) > 0$.

From the boundary conditions expressed in Eq. (6a),

$$d_4 = 0,\quad f_4 = 0 \tag{25}$$

and

$$\bar{U}_S^1 = (d_5 e^{g_5\bar{z}} + d_6 e^{-g_5\bar{z}})K_1(g_6\bar{r}) \tag{26a}$$

$$\bar{W}_S^1 = (f_5 e^{g_7\bar{z}} + f_6 e^{-g_7\bar{z}})K_0(g_8\bar{r}) \tag{26b}$$

where $d_5 = d_1d_3$, $d_6 = d_2d_3$, $f_5 = f_1f_3$, $f_6 = f_2f_3$.

Substituting Eqs. (20a) and (20b) into Eqs. (21a) and (21b), respectively, and with rearrangement produces

$$\begin{aligned}&\bar{\mu}^S\nabla^2\bar{U}_S + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{U}_S - \frac{\bar{\mu}^S}{\bar{r}^2}\bar{U}_S \\ &= -(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})[B_5 g_2(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_1(g_2\bar{r})] \\ &-(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})[B_6 g_4(e^{g_3\bar{z}} - e^{-g_3\bar{z}})K_1(g_4\bar{r})] \\ &+(\bar{\lambda}^S + \bar{\mu}^S)\frac{\beta_1^2}{D_2}B_5 g_2(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_1(g_2\bar{r})\end{aligned} \tag{27a}$$

$$\begin{aligned}&\bar{\mu}^S\nabla^2\bar{W}_S + (\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1)\bar{W}_S \\ &= (1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})[B_5 g_1(e^{g_1\bar{z}} + e^{-g_1\bar{z}})K_0(g_2\bar{r})] \\ &+(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})[B_6 g_3(e^{g_3\bar{z}} + e^{-g_3\bar{z}})K_0(g_4\bar{r})] \\ &-(\bar{\lambda}^S + \bar{\mu}^S)\frac{\beta_1^2}{D_2}B_5 g_1(e^{g_1\bar{z}} + e^{-g_1\bar{z}})K_0(g_2\bar{r})\end{aligned} \tag{27b}$$

Since $K_1(g_2\bar{r})$ and $K_1(g_4\bar{r})$ are linearly independent, then the particular solution to Eq. (27a) can be given by

$$\begin{aligned}\bar{U}_S^2 &= d_7(e^{g_1\bar{z}} - e^{-g_1\bar{z}})K_1(g_2\bar{r}) \\ &+d_8(e^{g_3\bar{z}} - e^{-g_3\bar{z}})K_1(g_4\bar{r})\end{aligned} \tag{28a}$$

Similarly, the particular solution to Eq. (27b) can be given by

$$\begin{aligned}\bar{W}_S^2 &= f_7(e^{g_1\bar{z}} + e^{-g_1\bar{z}})K_0(g_2\bar{r}) \\ &+ f_8(e^{g_3\bar{z}} + e^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{28b}$$

Inserting Eq. (28a) into Eq.(27a) gives

$$d_7 = \frac{\left[(\bar{\lambda}^S + \bar{\mu}^S)\frac{\beta_1^2}{D_2} - (1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})\right]g_2B_5}{\bar{\mu}^S\beta_1^2 + \bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1} \tag{29a}$$

$$d_8 = -\frac{(1+\frac{\bar{\rho}^L a_0 D_1 n^L}{i\bar{S}_v})g_4B_6}{\bar{\rho}^S a_0^2 + \bar{\rho}^L a_0^2 D_1} \tag{29b}$$

Following a similar procedure as above, we have

$$f_7=\frac{\left[(1+\frac{\bar{\rho}^{\mathrm{L}}a_0D_1n^{\mathrm{L}}}{\mathrm{i}\bar{S}_v})-(\bar{\lambda}^{\mathrm{S}}+\bar{\mu}^{\mathrm{S}})\frac{\beta_1^2}{D_2}\right]g_1B_5}{\bar{\mu}^{\mathrm{S}}\beta_1^2+\bar{\rho}^{\mathrm{S}}a_0^2+\bar{\rho}^{\mathrm{L}}a_0^2D_1} \tag{29c}$$

$$f_8=\frac{(1+\frac{\bar{\rho}^{\mathrm{L}}a_0D_1n^{\mathrm{L}}}{i\bar{S}_v})g_3B_6}{\bar{\rho}^{\mathrm{S}}a_0^2+\bar{\rho}^{\mathrm{L}}a_0^2D_1} \tag{29d}$$

This means the solutions to Eqs. (21a) and (21b) can be given by

$$\begin{aligned}\bar{U}_{\mathrm{S}}&=\bar{U}_{\mathrm{S}}^1+\bar{U}_{\mathrm{S}}^2=(d_5\mathrm{e}^{g_5\bar{z}}+d_6\mathrm{e}^{-g_5\bar{z}})K_1(g_6\bar{r})+\\&d_7(\mathrm{e}^{g_1\bar{z}}-\mathrm{e}^{-g_1\bar{z}})K_1(g_2\bar{r})+d_8(\mathrm{e}^{g_3\bar{z}}-\mathrm{e}^{-g_3\bar{z}})K_1(g_4\bar{r})\end{aligned} \tag{30a}$$

$$\begin{aligned}\bar{W}_{\mathrm{S}}&=\bar{W}_{\mathrm{S}}^1+\bar{W}_{\mathrm{S}}^2=(f_5\mathrm{e}^{g_7\bar{z}}+f_6\mathrm{e}^{-g_7\bar{z}})K_0(g_8\bar{r})+f_7\\&(\mathrm{e}^{g_1\bar{z}}+\mathrm{e}^{-g_1\bar{z}})\times K_0(g_2\bar{r})+f_8(\mathrm{e}^{g_3\bar{z}}+\mathrm{e}^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{30b}$$

By substituting Eqs. (30a) and (30b) into $\Theta=\frac{\partial\bar{U}_{\mathrm{S}}}{\partial\bar{r}}+\frac{\bar{U}_{\mathrm{S}}}{\bar{r}}+\frac{\partial\bar{W}_{\mathrm{S}}}{\partial\bar{z}}$ gives

$$g_6=g_8,\ g_5=g_7,\quad g_7f_5=g_6d_5,\quad g_6d_6=-g_7f_6,\quad g_3f_8=g_4d_8,\quad g_1f_7-g_2d_7=\frac{\beta_1^2}{D_2}B_5. \tag{31}$$

Considering the boundary conditions in Eq. (6b), further expressions can be written as

$$d_5=-d_6,\quad f_5=f_6. \tag{32}$$

Eqs. (30a) and (30b) can then be reduced to

$$\begin{aligned}\bar{U}_{\mathrm{S}}&=d_5(\mathrm{e}^{g_5\bar{z}}-\mathrm{e}^{-g_5\bar{z}})K_1(g_6\bar{r})+d_7(\mathrm{e}^{g_1\bar{z}}-\mathrm{e}^{-g_1\bar{z}})\times\\&K_1(g_2\bar{r})+d_8(\mathrm{e}^{g_3\bar{z}}-\mathrm{e}^{-g_3\bar{z}})K_1(g_4\bar{r})\end{aligned} \tag{33a}$$

$$\begin{aligned}\bar{W}_{\mathrm{S}}&=f_5(\mathrm{e}^{g_5\bar{z}}+\mathrm{e}^{-g_5\bar{z}})K_0(g_6\bar{r})+f_7(\mathrm{e}^{g_1\bar{z}}+\mathrm{e}^{-g_1\bar{z}})\times\\&K_0(g_2\bar{r})+f_8(\mathrm{e}^{g_3\bar{z}}+\mathrm{e}^{-g_3\bar{z}})K_0(g_4\bar{r})\end{aligned} \tag{33b}$$

Inserting Eqs. (20b), (33a) and (33b) into Eqs. (7a) and (7b), respectively, the following can be obtained

$$\begin{aligned}\bar{U}_{\mathrm{L}}&=D_1[d_5(\mathrm{e}^{g_5\bar{z}}-\mathrm{e}^{-g_5\bar{z}})K_1(g_6\bar{r})\\&+(\frac{n^{\mathrm{L}}g_2B_5}{\mathrm{i}a_0\bar{S}_v}+d_7)(\mathrm{e}^{g_1\bar{z}}-\mathrm{e}^{-g_1\bar{z}})K_1(g_2\bar{r})\\&+(\frac{n^{\mathrm{L}}g_4B_6}{\mathrm{i}a_0\bar{S}_v}+d_8)(\mathrm{e}^{g_3\bar{z}}-\mathrm{e}^{-g_3\bar{z}})K_1(g_4\bar{r})]\end{aligned} \tag{34a}$$

$$\begin{aligned}\bar{W}_{\mathrm{L}}&=D_1[\ b_5(\mathrm{e}^{g_5\bar{z}}+\mathrm{e}^{-g_5\bar{z}})K_0(g_6\bar{r})\\&+(b_7-\frac{n^{\mathrm{L}}g_1B_5}{\mathrm{i}a_0\bar{S}_v})(\mathrm{e}^{g_1\bar{z}}+\mathrm{e}^{-g_1\bar{z}})K_0(g_2\bar{r})\\&+(b_8-\frac{n^{\mathrm{L}}g_3B_6}{\mathrm{i}a_0\bar{S}_v})(\mathrm{e}^{g_3\bar{z}}+\mathrm{e}^{-g_3\bar{z}})K_0(g_4\bar{r})]\end{aligned} \tag{34b}$$

Substituting Eq. (33b) into Eq. (6c) yields

$$\begin{cases}g_1(e^{\frac{g_1L}{r_0}}-e^{-\frac{g_1L}{r_0}})=\frac{\bar{f}_v\bar{r}_0^2}{\bar{E}}(e^{\frac{g_1L}{r_0}}+e^{-\frac{g_1L}{r_0}})\\g_3(e^{\frac{g_3L}{r_0}}-e^{-\frac{g_3L}{r_0}})=\frac{\bar{f}_v\bar{r}_0^2}{\bar{E}}(e^{\frac{g_3L}{r_0}}+e^{-\frac{g_3L}{r_0}})\\g_5(e^{\frac{g_5L}{r_0}}-e^{-\frac{g_5L}{r_0}})=\frac{\bar{f}_v\bar{r}_0^2}{\bar{E}}(e^{\frac{g_5L}{r_0}}+e^{-\frac{g_5L}{r_0}})\end{cases} \tag{35}$$

Furthermore, inserting Eqs. (33a) and (34a) into Eq. (6d), leads to

$$d_5K_1(g_6)+d_7K_1(g_2)+d_8K_1(g_4)=0 \tag{36a}$$

$$\begin{aligned}&d_5K_1(g_6)+(d_7+\frac{n^{\mathrm{L}}g_2B_5}{\mathrm{i}a_0\bar{S}_v})K_1(g_2)\\&+(d_8+\frac{n^{\mathrm{L}}g_4B_6}{\mathrm{i}a_0\bar{S}_v})K_1(g_4)=0\end{aligned} \tag{36b}$$

By solving the simultaneous Eqs. (36a) and (36b), the following expressions are obtained

$$B_6=-\frac{g_2K_1(g_2)B_5}{g_4K_1(g_4)} \tag{37a}$$

$$d_5=-\left[\frac{(\bar{\lambda}^{\mathrm{S}}+\bar{\mu}^{\mathrm{S}})\frac{\beta_1^2}{D_2}-k_2}{\bar{\mu}^{\mathrm{S}}\beta_1^2+k_3}+\frac{k_2}{k_3}\right]\frac{g_2K_1(g_2)B_5}{K_1(g_6)} \tag{37b}$$

Thus, the shear stresses at the pile-soil interface in the saturated soil layer can be expressed as

$$\begin{aligned}\bar{\tau}_{zr}\big|_{\bar{r}=1}&=\left[\bar{\mu}^{\mathrm{S}}(\frac{\partial\bar{U}_{\mathrm{S}}}{\partial\bar{z}}+\frac{\partial\bar{W}_{\mathrm{S}}}{\partial\bar{r}})\big|_{\bar{r}=1}\right]\times\mathrm{e}^{\mathrm{i}\omega t}\\&=\bar{\mu}^{\mathrm{S}}[(d_5g_5-f_5g_6)K_1(g_6)(\mathrm{e}^{g_5\bar{z}}+\mathrm{e}^{-g_5\bar{z}})\\&+(d_7g_1-f_7g_2)K_1(g_2)(\mathrm{e}^{g_1\bar{z}}+\mathrm{e}^{-g_1\bar{z}})\\&+(d_8g_3-f_8g_4)K_1(g_4)(\mathrm{e}^{g_3\bar{z}}+\mathrm{e}^{-g_3\bar{z}})]\times\mathrm{e}^{\mathrm{i}\omega t}\end{aligned} \tag{38}$$

Integrating $\bar{\tau}_{rz}\big|_{\bar{r}=1}$ along the cylindrical interface between the floating pile and the saturated soil, the corresponding fundamental solution of soil reaction against pile can be given by

$$f_s=F_s\mathrm{e}^{\mathrm{i}\omega t}=2\pi r_0\tau_{rz}\big|_{r=r_0} \tag{39a}$$

Furthermore, introducing non-dimension quantities into Eq. (39a) yields

$$\frac{f_s}{Gr_0}=\bar{F}_s\mathrm{e}^{\mathrm{i}\omega t}=2\pi\bar{\tau}_{rz}\big|_{\bar{r}=1} \tag{39b}$$

2.3 Dynamic pile impedance

Based on the previous fundamental solution of the soil reaction against a pile in Eq. (39b), the governing equations for the vertical vibration of a pile in a saturated viscoelastic soil layer can be expressed by

$$E_{\mathrm{P}}\pi r_0^2\frac{\partial^2w_{\mathrm{P}}(t)}{\partial z^2}+f_s=\rho_{\mathrm{P}}\pi r_0^2\frac{\partial^2w_{\mathrm{P}}(t)}{\partial t^2} \tag{40}$$

where $w_{\mathrm{P}}(t)=W_{\mathrm{P}}\mathrm{e}^{i\omega t}$, $f_s=F_s\mathrm{e}^{i\omega t}$; E_P and ρ_P denote the elastic modulus and density of the pile, respectively.

$$\bar{W}_{\mathrm{P}}=\frac{W_{\mathrm{P}}}{r_0},\quad \bar{E}_{\mathrm{P}}=\frac{E_{\mathrm{P}}}{G},\quad \bar{\rho}_{\mathrm{P}}=\frac{\rho_{\mathrm{P}}}{\rho},\quad a_0=\sqrt{\frac{\rho}{G}}r_0\omega,\quad \bar{F}_0=\frac{F_0}{Gr_0^2}. \tag{41}$$

After substituting Eq. (41) into Eq. (40) and considering the boundary conditions of the floating pile in a saturated viscoelastic soil, we can obtain:

Dimensionless equation of motion of the pile,

$$\frac{\partial^2 \overline{W}_{\mathrm{P}}}{\partial \bar{z}^2} + \frac{\bar{\rho}_{\mathrm{P}}}{\bar{E}_{\mathrm{P}}} a_0^2 \overline{W}_{\mathrm{P}} = -\frac{2}{\bar{E}_{\mathrm{P}}} \frac{\bar{\tau}_{rz}\big|_{\bar{r}=1}}{\mathrm{e}^{\mathrm{i}\omega t}} \tag{42a}$$

Dimensionless stress continuity (including displacement continuity) at the pile bottom,

$$\bar{E}_{\mathrm{P}} \frac{\mathrm{d}\overline{W}_{\mathrm{P}}}{\mathrm{d}\bar{z}}\bigg|_{\bar{z}=\frac{L}{r_0}} = \bar{f}_v \overline{W}_{\mathrm{P}}\left(\frac{L}{r_0}\right)\bar{r}_0^2 \tag{42b}$$

Dimensionless stress continuity at the pile head,

$$\frac{\mathrm{d}\overline{W}_{\mathrm{P}}}{\mathrm{d}\bar{z}}\Big|_{\bar{z}=0} = \frac{\bar{F}_0}{\bar{E}_{\mathrm{P}}\pi} \tag{42c}$$

The general solution for the homogeneous equation corresponding to Eq. (42a) can be given by

$$\overline{W}_{\mathrm{P}}^{1} = a_1 \cos(\lambda \bar{z}) + a_2 \sin(\lambda \bar{z}) \tag{43a}$$

and the particular solution for Eq. (42a) can be given by

$$\overline{W}_{\mathrm{P}}^{2} = Q \frac{\bar{\tau}_{rz}\big|_{\bar{r}=1}}{\mathrm{e}^{\mathrm{i}\omega t}} \tag{43b}$$

where $\lambda = \sqrt{\frac{\bar{\rho}_{\mathrm{P}}}{\bar{E}_{\mathrm{P}}}} a_0$, a_1, a_2 and Q are undetermined coefficients.

It is found that Eq. (35) is substantially a characteristic equation with multi-solution g_n which can be solved numerically. It is obvious that the linear combination of g_n also satisfy Eq. (35). Then, $\frac{\bar{\tau}_{rz}|_{\bar{r}=1}}{\mathrm{e}^{\mathrm{i}\omega t}}$ can be written as the following form by the principle of superposition

$$\frac{\bar{\tau}_{rz}\big|_{\bar{r}=1}}{\mathrm{e}^{\mathrm{i}\omega t}} = \sum_{n=1}^{\infty} Y_{1n} B_{5n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \tag{44}$$

where g_n is the solution of Eq. (35), and n=1,2,3,∞.

g_{2n}=g_2, g_{6n}=g_6, $k_4 = \frac{(\bar{\lambda}^{\mathrm{S}} + \bar{\mu}^{\mathrm{S}})\beta_1^2 / D_2 - k_2}{\bar{\mu}^{\mathrm{S}}\beta_1^2 + k_3}$, $k_5 = -(k_4 + k_2/k_3)$

$$Y_{1n} = \bar{\mu}^{\mathrm{S}} \left[\frac{g_n^2 - g_{6n}^2}{g_n} k_5 + (2k_4 + \frac{2k_2}{k_3}) g_n \right] g_{2n} K_1(g_{2n}) .$$

Thus, Eq. (43b) can be rewritten as

$$\overline{W}_{\mathrm{P}}^{2} = \sum_{n=1}^{\infty} Q_n (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \tag{45}$$

$$Q_n = \frac{-2Y_{1n} B_{5n}}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2} \tag{46}$$

By inserting Eq. (45) into Eq. (42a), we have

$$\overline{W}_{\mathrm{P}} = \overline{W}_{\mathrm{P}}^{1} + \overline{W}_{\mathrm{P}}^{2} = a_1 \cos(\lambda \bar{z}) + a_2 \sin(\lambda \bar{z}) + \sum_{n=1}^{\infty} \frac{-2Y_{1n} B_{5n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2} \tag{47}$$

Similarly, expanding $\overline{W}_{\mathrm{S}}(1, \bar{z})$ into series form yields

$$\overline{W}_{\mathrm{S}}(1, \bar{z}) = \sum_{n=1}^{\infty} Y_{2n} B_{5n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \tag{48}$$

where

$$Y_{2n} = \frac{g_{2n} g_{6n} k_5 K_1(g_{2n}) K_0(g_{6n})}{g_n K_1(g_{6n})} - k_4 g_n K_0(g_{2n}) - \frac{k_2 g_n g_{2n} K_1(g_{2n}) K_0(g_{4n})}{k_3 g_{4n} K_1(g_{4n})}, \quad g_{4n} = g_4 .$$

Substituting Eqs. (47) and (48) into Eq. (6d) and rearranging produces

$$a_1 \cos(\lambda \bar{z}) + a_2 \sin(\lambda \bar{z}) + \sum_{n=1}^{\infty} \frac{-2Y_{1n} B_{5n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2} = \sum_{n=1}^{\infty} Y_{2n} B_{5n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \tag{49}$$

It is found that the function series $(\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})$ has the orthogonality which is given by,

$$\int_0^{\frac{L}{r_0}} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})(\mathrm{e}^{g_m \bar{z}} + \mathrm{e}^{-g_m \bar{z}}) d\bar{z} = \begin{cases} \left(\frac{2L}{r_0} + \frac{\mathrm{e}^{\frac{2g_n L}{r_0}} - \mathrm{e}^{-\frac{2g_n L}{r_0}}}{2g_n}\right), & n = m \\ 0, & n \neq m \end{cases}$$

Thus, multiplying both sides of Eq. (50) by $(\mathrm{e}^{g_m \bar{z}} + \mathrm{e}^{-g_m \bar{z}})$ and integrating it between the limits $[0, L/r_0]$ leads to

$$B_{5n} = X_{1n} a_1 + X_{2n} a_2 \tag{50}$$

where

$$X_{1n} = \frac{\int_0^{\frac{L}{r_0}} \cos(\lambda \bar{z})(\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \mathrm{d}\bar{z}}{\left(Y_{2n} + \frac{2Y_{1n}}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2}\right)\left(\frac{2L}{r_0} + \frac{\mathrm{e}^{\frac{2g_n L}{r_0}} - \mathrm{e}^{-\frac{2g_n L}{r_0}}}{2g_n}\right)},$$

$$X_{2n} = \frac{\int_0^{\frac{L}{r_0}} \sin(\lambda \bar{z})(\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}}) \mathrm{d}\bar{z}}{\left(Y_{2n} + \frac{2Y_{1n}}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2}\right)\left(\frac{2L}{r_0} + \frac{\mathrm{e}^{\frac{2g_n L}{r_0}} - \mathrm{e}^{-\frac{2g_n L}{r_0}}}{2g_n}\right)}.$$

Therefore, Eq. (47) can be rewritten as (i.e., the fundamental solution for the vertical vibration of a pile)

$$\overline{W}_{\mathrm{P}} = a_1 \left[\cos(\lambda \bar{z}) + \sum_{n=1}^{\infty} \frac{-2Y_{1n} X_{1n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2} \right] + a_2 \left[\sin(\lambda \bar{z}) + \sum_{n=1}^{\infty} \frac{-2Y_{1n} X_{2n} (\mathrm{e}^{g_n \bar{z}} + \mathrm{e}^{-g_n \bar{z}})}{\bar{E}_{\mathrm{P}} g_n^2 + \bar{\rho}_{\mathrm{P}} a_0^2} \right] \tag{51}$$

By inserting Eq. (51) into Eqs. (42b) and (42c), respectively, leads to

$$a_1 = \frac{a_2\left(\frac{\bar{f}_v \bar{r}_0^2 X_6}{\bar{E}_{\mathrm{P}}} - X_4\right)}{X_3 - \frac{\bar{f}_v \bar{r}_0^2 X_5}{\bar{E}_{\mathrm{P}}}}, \quad a_2 = \frac{\bar{P}_0}{\lambda \pi \bar{E}_{\mathrm{P}}} \tag{52}$$

where

$$X_3 = -\lambda\sin(\frac{\lambda L}{r_0}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{1n}g_n(\mathrm{e}^{\frac{g_nL}{r_0}} - \mathrm{e}^{-\frac{g_nL}{r_0}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2},$$

$$X_4 = \lambda\cos(\frac{\lambda L}{r_0}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{2n}g_n(\mathrm{e}^{\frac{g_nL}{r_0}} - \mathrm{e}^{-\frac{g_nL}{r_0}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}$$

$$X_5 = \cos(\frac{\lambda L}{r_0}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{1n}(\mathrm{e}^{\frac{g_nL}{r_0}} + \mathrm{e}^{-\frac{g_nL}{r_0}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2},$$

$$X_6 = \sin(\frac{\lambda L}{r_0}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{2n}(\mathrm{e}^{\frac{g_nL}{r_0}} + \mathrm{e}^{-\frac{g_nL}{r_0}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}.$$

From Eq. (51), the normal stress component in the z direction of the pile can be expressed by

$$\begin{aligned} N(z) &= E_{\mathrm{P}}\frac{\mathrm{d}W_{\mathrm{P}}}{\mathrm{d}z} = E_{\mathrm{P}}\frac{\mathrm{d}\bar{W}_{\mathrm{P}}}{\mathrm{d}\bar{z}} \\ &= E_{\mathrm{P}}a_1\left[-\lambda\sin(\lambda\bar{z}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{1n}g_n(\mathrm{e}^{g_n\bar{z}} - \mathrm{e}^{-g_n\bar{z}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}\right] \\ &+ E_{\mathrm{P}}a_2\left[\lambda\cos(\lambda\bar{z}) + \sum_{n=1}^{\infty}\frac{-2Y_{1n}X_{2n}g_n(\mathrm{e}^{g_n\bar{z}} - \mathrm{e}^{-g_n\bar{z}})}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}\right] \end{aligned} \tag{53}$$

Therefore, the dynamic impedance of the pile can be defined as

$$K_d(a_0) = \frac{\pi r_0^2 N(0)}{W_{\mathrm{P}}(0)} = \frac{\pi r_0^2 N(0)}{r_0\bar{W}_{\mathrm{P}}(0)} = \frac{\pi r_0 E_{\mathrm{P}}\lambda a_2}{a_1X_7 + a_2X_8} \tag{54}$$

where $X_7 = 1 + \sum_{n=1}^{\infty}\frac{-4Y_{1n}X_{1n}}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}$, $X_8 = \sum_{n=1}^{\infty}\frac{-4Y_{1n}X_{2n}}{\bar{E}_{\mathrm{P}}g_n^2 + \bar{\rho}_{\mathrm{p}}a_0^2}$.

Furthermore, Eq. (54) can also be rewritten in the following non-dimensional form

$$\bar{K}_d(a_0) = \frac{K_d(a_0)}{Gr_0} = \frac{\pi\bar{E}_{\mathrm{P}}\lambda a_2}{a_1X_7 + a_2X_8} \tag{55}$$

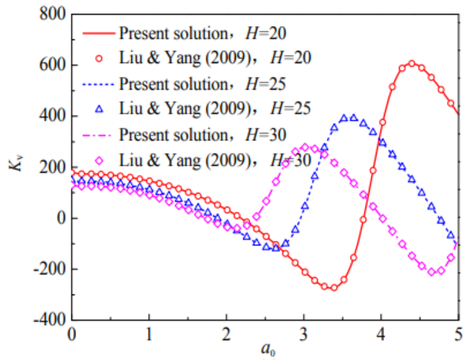
If we define the dimensionless dynamic impedance of the pile as

$$\bar{K}_d = K_v + \mathrm{i}C_v \tag{56}$$

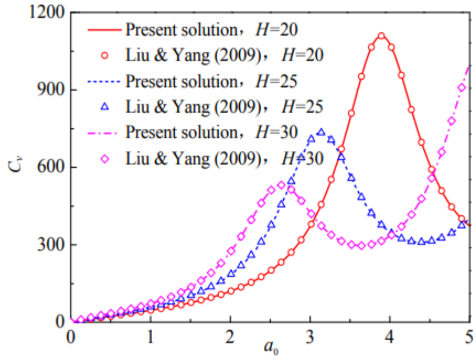
then the real part $K_v = \mathrm{Re}(\bar{K}_d)$ and the imaginary part $C_v = \mathrm{Im}(\bar{K}_d)$ describe the true stiffness and equivalent damping of the pile head, respectively.

3. Results and discussions

In this section, numerical results are presented to demonstrate the validity of the obtained analytical solutions and to investigate the vertical vibration characteristics of the floating pile embedded in the saturated porous viscoelastic soil. Unless otherwise specified, the following parameter values are used, G=20 MPa, $\upsilon = 0.2$, $n^L = 0.4$, ρ^S=1800 kg/m^3, ρ^L=1000 kg/m^3, E_p=20 GPa, ρ_p=2500 kg/m^3, k^L=1×10^{-6} m/s, and the pile slenderness ratio $H = L/r_0 = 15$ (r_0=0.25 m).

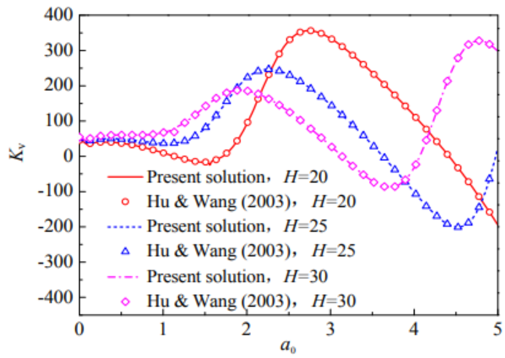


(a) Real part of the dimensionless impedance of the pile

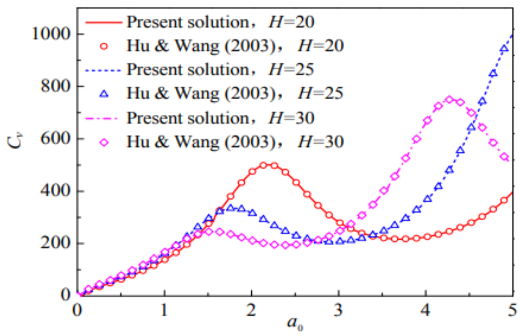


(b) Imaginary part of the dimensionless impedance of the pile

Fig. 2 Comparison of the impedance solution in reduced form (f_v→∞) with the end-bearing pile solution of Liu and Yang (2009)

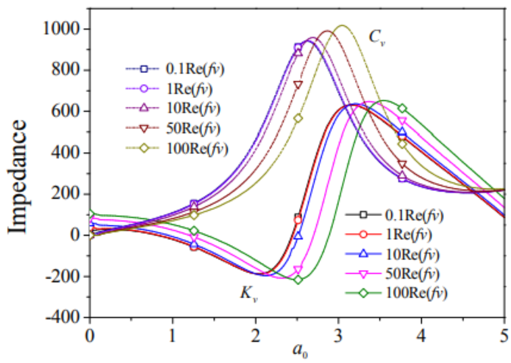


(a) Real part of the dimensionless impedance of the pile

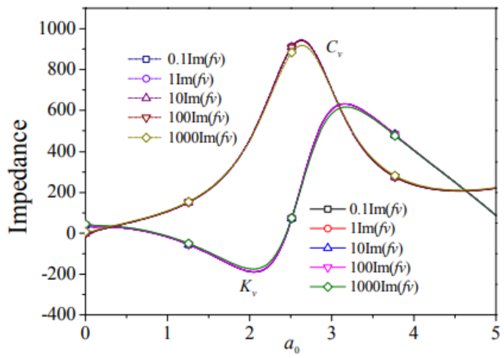


(b) Imaginary part of the dimensionless impedance of the pile

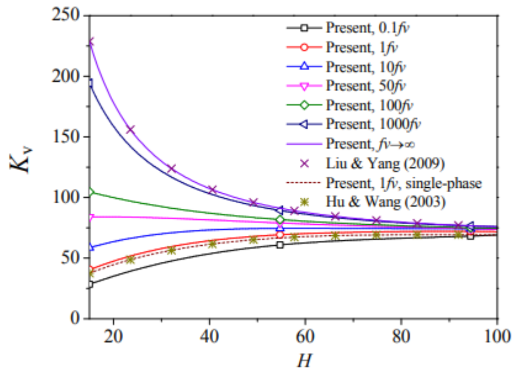
Fig. 3 Comparison of the impedance solution in reduced form ($S_v \to 0, \rho^L \to 0$) with the single-phase soil solution of Hu and Wang (2003)



(a) Only varying the real part of the f_v



(b) Only varying the imaginary part of the f_v



(c) The static impedance of the pile with different pile slenderness ratios

Fig. 4 The dimensionless dynamic impedance vs. the dimensionless frequency a_0 with different values of the impedance f_v of the lower soil base

3.1 Verification of the present solution

In this section, two examples are presented to verify the present solution. First, the impedance solution expressed in Eq. (56) for a floating pile can be reduced to describe the vertical vibration of an end bearing pile by setting the complex stiffness of the equivalent spring-dashpot elements beneath the pile base $f_v\to\infty$. Therefore, based on the same parameters, the solution of $\overline{K}_d$ can be verified by comparing it with the existing solutions for an end-bearing pile embedded in the saturated viscoelastic soil. Fig. 2 shows the comparison of the complex impedance in reduced form ($f_v\to\infty$) evaluated using Eqs. (54) to (56) with the solution of Liu and Yang (2009). It can be seen that the present solution of dynamic impedance with different values of the pile slenderness ratio $H = L/r_0$ is in very good agreement with that proposed by Liu and Yang (2009).

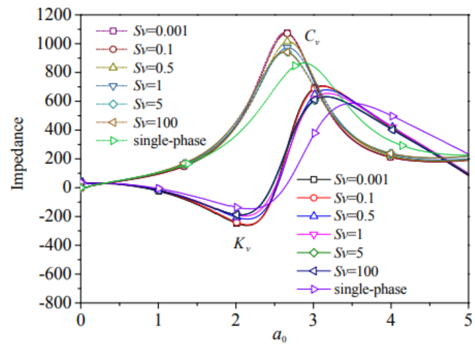


Fig. 5 The dimensionless dynamic impedance vs. the dimensionless frequency a_0 with different interaction coefficient S_v between the soil skeleton and the pore water

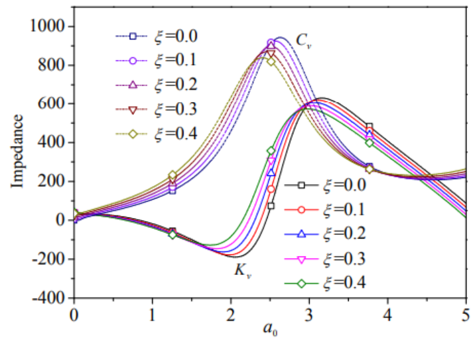


Fig. 6 The dimensionless dynamic impedance vs. the dimensionless frequency a_0 with different damping coefficient ξ of the porous viscoelastic soil

Secondly, by setting $S_v\to 0$ and $\rho^{\mathrm{L}}\to 0$, the present model can be reduced to describe the vertical vibration of a floating pile in the single-phase soil. Fig. 3 shows the comparison of the complex impedance in reduced form ($S_v\to 0$ and $\rho^{\mathrm{L}}\to 0$) with Hu's solution (2003) for the case of single-phase soil. It is clear that the present solution agrees well with Hu's solution. It is noted that in this example of comparison, for keeping consistent with Hu's choice of impedance function of the lower soil base, here f_v follows the simplified form proposed by Lysmer and Richart (1966) for the case of single-phase soil which is given by

$$f_v = \frac{4\mu^s r_0}{1-\upsilon} + \mathrm{i}\frac{3.4(r_0)^2\rho\sqrt{\mu^s/\rho^s}}{1-\upsilon} \tag{57}$$

where the real and imaginary parts denote the true stiffness and equivalent damping of the subsoil beneath the pile base, respectively.

In addition, the comparisons for the static impedance at the pile head versus pile slenderness ratio are also shown in Fig. 4(c). It is observed that the results have a good agreement. And with the increase of the pile slenderness ratio, the stiffness of the end-bearing pile decreases while the stiffness of the floating pile increases. With the increase of the pile slenderness ratio, the stiffness finally approaches a steady value. These phenomenons are consistent with the reported knowledge for the mechanism of end-bearing piles and floating piles in practice engineering. Therefore, the validity of the present solution is confirmed with these independent comparisons.

3.2 Parametric analyses of the present solution

Fig. 4 shows the effect of the impedance f_v of the lower soil base on the dynamic impedance of the floating pile. Here the saturated soil case is considered, and the impedance function proposed by Cai and Hu (2010) for poroelastic half-spaces is taken to represent f_v. It can be seen that the stiffness of the base soil has a significant effect on the dynamic impedance of the pile head. As $\mathrm{Re}(f_v)$ increases, the resonant frequency of the pile-soil system increases, and the corresponding resonance amplitude and static impedance of the pile head increase as well. Furthermore, one can see the transformation process of the mechanical characteristic from floating piles to end-bearing piles with the increase of $\mathrm{Re}(f_v)$. In contrast, the dynamic impedance of the pile head is less sensitive to the damping change of the base soil.

Fig. 5 shows the effect of the interaction coefficient S_v between the soil skeleton and the pore water on the dynamic impedance of the pile. Compared with the traditional elastodynamic solution for single-phase case obtained by Hu and Wang (2003), the proposed poroelastic pile-soil system has lower resonant frequencies and higher peak values of the dynamic impedance. And the effects of S_v on the dynamic impedance are limited. When S_v is rather small or very large, it has negligible effects on the pile-soil system, which are corresponding to a nice dissipation condition and an undrained condition, respectively. Furthermore, for the increase of S_v generally means the decrease of the permeability coefficient k^L, the pore liquid becomes difficult to dissipate from the soil skeleton pore. Thus, within the effective range of S_v, the peak values of real part of the dynamic impedance of the pile head increase, and the peak values of the imaginary part decrease, with the increase of S_v respectively. In contrast, S_v has little effects on the resonant frequency of the pile-soil system.

The effects of the damping coefficient ξ of the porous viscoelastic soil on the dynamic impedance of the pile are shown in Fig. 6. This shows that the damping coefficient ξ of the porous viscoelastic soil has significant effect on the vertical dynamic impedance of the floating pile. The resonance frequencies and the oscillation amplitudes at the resonance frequencies of both the real and imaginary parts decrease as the damping coefficient ξ of the porous viscoelastic soil increases. Obviously, the soil surrounding the floating pile can be reduced to a saturated porous elastic soil if ξ=0. It is indicated that the resonance frequencies and the oscillation amplitudes of the dynamic impedance can be overestimated when the viscosity of the saturated soil is ignored.

4. Conclusions

Based on the theory of porous media (TPM), a new mathematical model for the dynamic response of a floating pile embedded in a saturated viscoelastic soil layer subjected to a vertical harmonic load is proposed. And a corresponding frequency-domain solution for the dynamic impedance of a floating pile embedded in a saturated viscoelastic soil is also derived and subsequently verified by comparing it with the existing solutions.

- The parametric analyses show that the dynamic impedance of floating piles significantly depends on the real stiffness of the impedance function of the soil base, but is less sensitive to its damping variation; due to the existence of pore liquid, the mechanical behavior of piles in poro-visco-elastic soil is obviously different with the single-phase elastic soil case, and the effect of interaction coefficient between soil skeleton and pore liquid on the dynamic impedance of the pile head is limited; if the viscosity of the saturated porous soil layer is ignored, the resonance frequencies and the peak values of dynamic impedance of the pile-soil system will be apparently overestimated.
- The proposed model and obtained solution provide an extensive scope of application, compared with the related existing solutions. The present solution can be reduced to analyze the vertical vibration problem of end-bearing piles in saturated soil and piles embedded in single-phase soil described in related previous studies. Furthermore, by the combination with different impedance functions of the lower soil base f_v, the obtained solution can be conveniently further extended to investigate the vertical vibration problem of floating piles embedded in poro-visco-elastic half-spaces, and finite soil layers.

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