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A new model for T-shaped combined footings part I: Optimal dimensioning

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Abstract. The foundations are classified into shallow and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality, and its constructive systems. The shallow foundations may be of various types according to their function; isolated footings, combined footings, strip footings, and slabs foundation. The isolated footings are of the type rectangular, square and circular. The combined footing may be rectangular, trapezoidal or T-shaped in plan. This paper presents a new model for T-shaped combined footings to obtain the most economical contact surface on the soil (optimal dimensioning) to support an axial load and moment in two directions to each column. The new model considers the soil real pressure, i.e., the pressure varies linearly. The classical model uses the technique of test and error, i.e., a dimension is proposed, and subsequently, the equation of the biaxial bending is used to obtain the stresses acting on each vertex of the T-shaped combined footing, which must meet the conditions following: The minimum stress should be equal or greater than zero, and maximum stress must be equal or less than the allowable capacity that can withstand the soil. To illustrate the validity of the new model, numerical examples are presented to obtain the minimum area of the contact surface on the soil for T-shaped combined footings subjected to an axial load and moments in two directions applied to each column.

Keywords: T-shaped combined footings; optimal dimensioning; contact surface; more economical dimension; minimum area

1. Introduction

The purpose of the foundation is to effectively support the superstructure by transmitting the applied load effects (forces and moments) to the soil below, without exceeding the bearing capacity of the soil, and ensuring that the settlements of the structure are within tolerable limits, and as nearly uniform as possible.

Footings belong to the category of shallow foundations (as opposed to deep foundations such as piles and caissons) and are used when soil has a sufficient strength to a small depth below the ground surface. The shallow foundations (footings) have a large plan area in comparison with the cross-sectional area of the column.

In the design of shallow foundations in terms of the application of loads are: 1) The footings subjected to concentric axial load, 2) The footings subjected to axial load and moment in one direction (uniaxial bending), 3) The footings subjected to axial load and moment in two directions (biaxial bending) (Bowles 2001, Das *et al*. 2006, Calabera 2000, Tomlinson 2008, McCormac and Brown, 2013, González-Cuevas and Robles-Fernandez-Villegas 2005).

Shallow foundations may be of various types according to their function; isolated footing, combined footing, strip footing, or mat foundation.

Some of the different types of footings are presented in Fig. 1: 1) Corner isolated footing, 2) T-shaped combined footing restricted in two opposite end, 3) Rectangular combined footing restricted in two opposite end, 4) Strap footing in two directions for the corners, 5) and 18) Rectangular isolated footing with eccentric load, 6) and 8) Rectangular isolated footing with concentric load, 7) Wall footing with concentric load, 9) Strap footing in a direction, 10) Trapezoidal combined footing restricted in two opposite end, 11) Square isolated footing with concentric load, 12) Trapezoidal combined footing restricted in an end, 13) Wall footing with eccentric load, 14) T-shaped combined footing restricted in an end, 15) Rectangular combined footing restricted in an end, 16) Circular isolated footing with concentric load, 17) Isolated footing with wall to balance the load, 19) Isolated footing with wall in two directions to balance the load.

A combined footing is a long footing supporting two or more columns in (typically two) one row. The combined footing may be rectangular, trapezoidal or T-shaped in plan. The rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted, and the column load of the property line is minor than the other. The trapezoidal footing or T-shaped is provided when the column load of the property line is much more than the other. As a result, both projections of the footing beyond the faces of the columns will be restricted (Kurian 2005, Punmia *et al*. 2007, Varghese 2009).

The combined footings arranged as special footings (two

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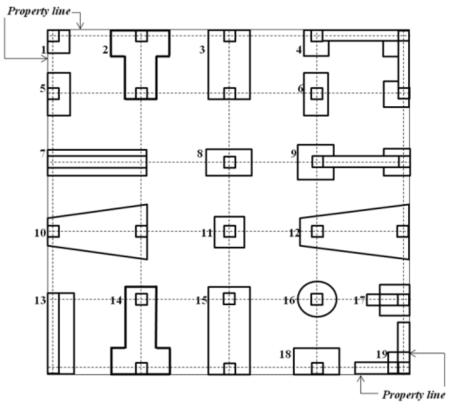

Fig. 1 Types of footings

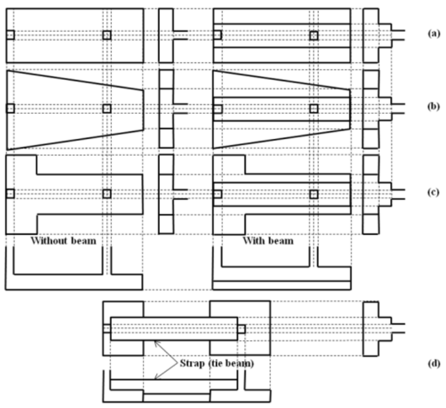

Fig. 2 Kinds of combined footings, (a) Rectangular combined footings, (b) Trapezoidal combined footings, (c) T-shaped combined footings and (d) Strap combined footings

columns, with cantilevered from one end) can be with beam or without beam as shows in Fig. 2 (Uzuner 2016).

Whenever two or more columns in a straight line are carried on a single spread footing, it is called a combined footing. Isolated footings for each column are generally the economical.

Combined footings are provided only when it is absolutely necessary, as

1. When two columns are close together, causing overlap of adjacent isolated footings (for example around elevator shafts and escalators).
2. Where soil bearing capacity is low, causing overlap of the adjacent isolated footings (for example a part of the footing may be occupying the same space of the nearest footing).
3. Proximity of building line or existing building or sewer, adjacent to a building column.

Conventional method for design of combined footings by rigid method assumes that:

1. The footing or mat is infinitely rigid, and therefore, the deflection of the footing or mat does not influence the pressure distribution.

2. The soil pressure is distributed in a straight line or a plane surface such that the centroid of the soil pressure coincides with the line of action of the resultant force of all the loads acting on foundations.

3. The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses.

4. The maximum stress must be equal or less than the allowable capacity that can withstand the soil.

Optimization of building structures is a prime target for designers and has been investigated by many researchers in the past and its papers are: Optimum Design of Unstiffened Built-up Girders (Ha 1993); Shape Optimization of RC Flexural Members (Rath *et al.* 1999); Sensitivity Analysis and Optimum Design Curves for the Minimum Cost Design of Singly and Doubly Reinforced Concrete Beams (Ceranic and Fryer 2000); Optimal Design of a Welded I-Section Frame Using Four Conceptually Different Optimization Algorithms (Jarmai *et al.* 2003); New Approach to Optimization of Reinforced Concrete Beams (Leps and Sejnoha 2003); Cost Optimization of Singly and Doubly Reinforced Concrete Beams with EC2-2001 (Barros *et al.* 2005); Cost Optimization of Reinforced Concrete Flat Slab Buildings (Sahab *et al.* 2005); Multi Objective Optimization for Performance-Based Design of Reinforced Concrete Frames (Zou *et al.* 2007); Design of Optimally Reinforced RC Beam, Column, and Wall Sections (Aschheim *et al.* 2008); Optimum design of reinforced concrete columns subjected to uniaxial flexural compression (Bordignon and Kripka 2012); A hybrid CSS and PSO algorithm for optimal design of structures (Kaveh and Talatahari 2012); Structural Optimization and proposition of pre-sizing parameters for beams in reinforced concrete buildings (Fleith de Medeiros and Kripka 2013); Optimum cost design of RC columns using artificial bee colony algorithm (Ozturk and Durmus 2013); Optimization of a sandwich beam design: analytical and numerical solutions (Awad 2013); Cold-formed steel channel columns Optimization with simulated annealing method (Kripka and Chamberlain Pravia 2013); Cost Optimization of reinforced high strength concrete T-sections in flexure (Tiliouine and Fedghouche 2014); Optimal design of reinforced concrete plane frames using artificial neural networks (Kao and Yeh 2014); Reliability-based design Optimization of structural systems using a hybrid genetic algorithm (Abbasnia *et al.* 2014).

The papers for optimal design of reinforced concrete foundations are: Flexural Strength of Square Spread Footing (Jiang 1983); Closure to “Flexural Strength of Square Spread Footing” by Da Hua Jiang (Jiang 1984); Flexural Limit Design of Column Footing (Hans 1985); Economic Design Optimization of Foundation (Wang and Kulhawy 2008); Reliability-Based Economic Design Optimization of Spread Foundation (Wang 2009); Structural Cost of Optimized Reinforced Concrete Isolated Footing (Al-Ansari 2013); Multi-objective Optimization of foundation using global-local gravitational search algorithm (Khajehzadeh *et al.* 2014).

Some papers show the equations to obtain the more economical dimension of footings, as are: A Mathematical Model for Dimensioning of Footings Rectangular (Luévanos-Rojas 2013); A Mathematical Model for Dimensioning of Footings Square (Luévanos-Rojas 2012a), A Mathematical Model for the Dimensioning of Circular Footings (Luévanos-Rojas 2012b); A New Mathematical Model for Dimensioning of the Boundary Trapezoidal

Combined Footings (Luévanos-Rojas 2015); A Mathematical Model for the Dimensioning of Combined Footings of Rectangular Shape (Luévanos-Rojas 2016); A mathematical model for dimensioning of square isolated footings using optimization techniques: general case (López-Chavarría *et al*. 2017a); Optimal dimensioning for the corner combined footings (López-Chavarría *et al*. 2017b).

This paper presents a new model for T-shaped combined footings to obtain the most economical contact surface on the soil (optimal dimensioning) to support an axial load and moment in two directions to each column. The new model considers the soil real pressure, i.e., the pressure varies linearly. The classical model uses the technique of test and error, i.e., a dimension is proposed, and subsequently, the equation of the biaxial bending is used to obtain the stresses acting on each vertex of the T-shaped combined footing, which must meet the conditions following: The minimum stress should be equal or greater than zero, and maximum stress must be equal or less than the allowable capacity that can withstand the soil. To illustrate the validity of the new model, numerical examples are presented to obtain the minimum area of the contact surface on the soil for T-shaped combined footings subjected to an axial load and moments in two directions applied to each column.

2. Formulation of the new model

General equation for any type of footings subjected to biaxial bending is (Luévanos-Rojas 2012a, b, 2013, 2015, 2016, López-Chavarría *et al*. 2017a, b)

$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{1}$$

where σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the footing, P is the axial load applied at the center of gravity of the footing, M_x is the moment around the axis "X", M_y is the moment around the axis "Y", x is the distance in the direction "X" measured from the axis "Y" to the fiber under study, y is the distance in direction "Y" measured from the axis "X" to the farthest under study, I_y is the moment of inertia around the axis "Y" and I_x is the moment of inertia around the axis "X".

Fig. 3 shows a corner combined footing under axial load and moment in two directions (biaxial bending) in each column, the pressure below the footing vary linearly (Luévanos-Rojas 2012a, b, 2013, 2015, 2016, López-Chavarría *et al*. 2017a, b).

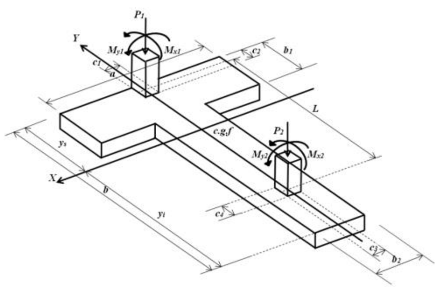

Fig. 3 T-shaped combined footing

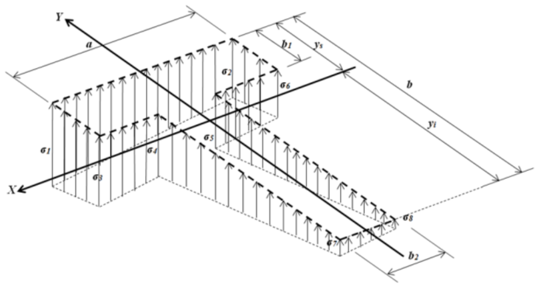

Fig. 4 Diagram of pressure below the footing

Fig. 4 shows the pressure diagram below the T-shaped combined footing, and also the stresses in each vertex are presented.

The stresses in each vertex of the T-shaped combined footing by Eq. (1) are obtained

$$\sigma_1 = \frac{R}{A} + \frac{M_{xT} y_s}{I_x} + \frac{M_{yT} a}{2I_y} \tag{2}$$

$$\sigma_2 = \frac{R}{A} + \frac{M_{xT} y_s}{I_x} - \frac{M_{yT} a}{2I_y} \tag{3}$$

$$\sigma_3 = \frac{R}{A} + \frac{M_{xT}(y_s - b_1)}{I_x} + \frac{M_{yT} a}{2I_y} \tag{4}$$

$$\sigma_4 = \frac{R}{A} + \frac{M_{xT}(y_s - b_1)}{I_x} + \frac{M_{yT} b_2}{2I_y} \tag{5}$$

$$\sigma_5 = \frac{R}{A} + \frac{M_{xT}(y_s - b_1)}{I_x} - \frac{M_{yT} b_2}{2I_y} \tag{6}$$

$$\sigma_6 = \frac{R}{A} + \frac{M_{xT}(y_s - b_1)}{I_x} - \frac{M_{yT} a}{2I_y} \tag{7}$$

$$\sigma_7 = \frac{R}{A} - \frac{M_{xT} y_i}{I_x} + \frac{M_{yT} b_2}{2I_y} \tag{8}$$

$$\sigma_8 = \frac{R}{A} - \frac{M_{xT} y_i}{I_x} - \frac{M_{yT} b_2}{2I_y} \tag{9}$$

where R is the resultant force, M_{xT} is the resultant moment around the axis "X" and M_{yT} is the resultant moment around of the axis "Y" are obtained

$$R = P_1 + P_2 \tag{10}$$

$$M_{xT} = M_{x1} + M_{x2} + P_1\left(y_s - \frac{c_2}{2}\right) - P_2\left(L + \frac{c_2}{2} - y_s\right) \tag{11}$$

$$M_{yT} = M_{y1} + M_{y2} \tag{12}$$

The geometric properties of the T-section are

$$A = (a - b_2)b_1 + bb_2 \tag{13}$$

$$y_s = \frac{(a - b_2){b_1}^2 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{14}$$

$$y_i = \frac{(2b - b_1)(a - b_2)b_1 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{15}$$

$$I_x = \frac{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)(2b^2 - bb_1 + {b_1}^2) + {b_2}^2 (b - b_1)^4}{12[(a - b_2)b_1 + bb_2]} \tag{16}$$

$$I_y = \frac{b_1 a^3 + (b - b_1){b_2}^3}{12} \tag{17}$$

Geometry conditions are

$$b \geq \frac{c_2}{2} + L + \frac{c_4}{2} \tag{18}$$

$$b = y_s + y_i \tag{19}$$

Substituting Eq. (14) into Eq. (11) to obtain of the moment "M_{xT}" in function of "a", "b", "b_1" and "b_2", this is

$$M_{xT} = \frac{R\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{x1} + M_{x2} - \frac{Rc_2}{2} - P_2 L \tag{20}$$

Substituting Eqs. (13) to (17) into Eqs. (2) to (9) to find the stresses in function of "a", "b", "b_1" and "b_2", these are

$$\sigma_1 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{21}$$

$$\sigma_2 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{22}$$

$$\sigma_3 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{23}$$

$$\sigma_4 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{24}$$

$$\sigma_5 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{25}$$

$$\sigma_6 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{26}$$

$$\sigma_7 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{27}$$

$$\sigma_8 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{28}$$

The stresses generated by soil on the contact surface of the combined footing must meet the following conditions: The minimum stress should be equal or greater than zero, and the maximum stress must be equal or less than the soil allowable load capacity "σ_{adm}".

3. Dimensioning for T-shaped combined footings using optimization techniques

Objective function to minimize the contact surface of the total area "A" is

$$A = (a - b_2)b_1 + bb_2 \tag{29}$$

Constraint functions for the dimensioning of T-shaped combined footings are

$$R = P_1 + P_2 \tag{30}$$

$$M_{xT} = \frac{R\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{x1} + M_{x2} - \frac{Rc_2}{2} - P_2 L \tag{31}$$

$$M_{yT} = M_{y1} + M_{y2} \tag{32}$$

$$\sigma_1 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{33}$$

$$\sigma_2 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2){b_1}^2 + b^2 b_2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{34}$$

$$\sigma_3 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{35}$$

$$\sigma_4 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{36}$$

$$\sigma_5 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{37}$$

$$\sigma_6 = \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - a{b_1}^2\right]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}a}{b_1 a^3 + (b - b_1){b_2}^3} \tag{38}$$

$$\sigma_7 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} + \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{39}$$

$$\sigma_8 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 {b_1}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + {b_1}^2\right) + {b_2}^2(b - b_1)^4} - \frac{6M_{yT}b_2}{b_1 a^3 + (b - b_1){b_2}^3} \tag{40}$$

$$0 \leq \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{Bmatrix} \leq \sigma_{adm} \tag{41}$$

$$\frac{c_2}{2} + L + \frac{c_4}{2} \leq b \tag{42}$$

4. Numerical examples

The Tables present four cases for the dimensioning of the T-shaped combined footings, each case varies the moment around of the axis "X", and the moment around of the axis "Y" in the direction, but the value of "R" is the same in all the cases, and in each case shows five types varying in the soil allowable load capacity of "σ_{adm}=250, 225, 200, 175, 150 kN/m^2".

The results presented in the Tables 1, 2, 3 and 4 make the following considerations: 1) The dimensions of the two columns are of 40x40 cm in all cases; 2) The soil allowable load capacity varies for each type; 3) The axial load "P_1" and "P_2" varies for each case, but "R" is equal in all the cases; 4) The distance between columns is L=6.00 m.

Tables 1, 2, 3 and 4 show the results using the optimization techniques; the objective function (minimum area) by Eq. (29) is obtained, and the constraint functions by Eqs. (30)-(42) are found. The optimal areas and dimensions for the T-shaped combined footings are obtained using the MAPLE-15 software.

This problem assumes that the constant parameters are: P_1, M_{x1}, M_{y1}, P_2, M_{x2}, M_{y2}, c_1, c_2, c_3, c_4, L, σ_{adm}, and the decision variables are: R, M_{xT}, M_{yT}, a, b, b_1, b_2, A, σ_1, σ_2, σ_3, σ_4, σ_5, σ_6, σ_7, σ_8. Table 1 takes into account the following considerations: P_1=1250 kN, P_2=250 kN, R=1500 kN, M_{xT} and M_{yT} are not constrained, $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$, A is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$, $0 \leq \sigma_7 \leq \sigma_{adm}$, $0 \leq \sigma_8 \leq \sigma_{adm}$. Table 2 makes the following considerations: P_1=1000 kN, P_2=500 kN, R=1500 kN, M_{xT} and M_{yT} are not constrained, $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$, A is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$, $0 \leq \sigma_7 \leq \sigma_{adm}$, $0 \leq \sigma_8 \leq \sigma_{adm}$. Table 3 considers the following: P_1=750 kN, P_2=750 kN, R=1500 kN, M_{xT} and M_{yT} are not constrained, $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$, A is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$, $0 \leq \sigma_7 \leq \sigma_{adm}$, $0 \leq \sigma_8 \leq \sigma_{adm}$. Table 4 does the following considerations: P_1=500 kN, P_2=1000 kN, R=1500 kN, M_{xT} and M_{yT} are not constrained, $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 = b$, $b_2 = a$ and $6.40 \leq b$, A is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$, $0 \leq \sigma_7 \leq \sigma_{adm}$, $0 \leq \sigma_8 \leq \sigma_{adm}$.

Tables 1, 2, 3 and 4 are presented in Appendix.

5. Results

Table 1 shows the following results: The case 1 and 3 for all the types are the same results, because the stress generated by loads in each vertex is minor than σ_{adm} and the minimum stress is equal to zero. The case 2 and 4 is increased the value of "a", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b", "b_1" and "b_2" are constant.

Table 2 presents the following: The case 1 and 3 is increased the value of "a", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b", "b_1" and "b_2" are constant. The case 2 and 4 is increased the value of "a" and "b", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b_1" and "b_2" are constant.

Table 3 shows the following: The case 1 and 3 is increased the value of "a" and "b_1", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b" and "b_2" are constant. The case 2 and 4 is increased the value of "a" and "b_2", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b" and "b_1" are constant.

Table 4 considers a rectangular combined footing, because the footing with minor load is located in property line and presents the following results: The case 1, 2, 3 and 4 is increased the value of "a" and "b_2", when the soil allowable load capacity decreases, the maximum stress is equal than σ_{adm} and the minimum stress is major to zero, and the value of "b" and "b_1" are constant. The case 1 and 3 has the same dimensions, and the stresses are totally antisymmetric, because the moments around the axis "Y" are equal, but in direction opposite. The case 2 and 4 is presented the same that the case 1 and 3.

If "M_{xT}=0", this means that the resultant force is located in the center of gravity in direction "Y" of the contact area of the footing with soil.

6. Conclusions

The foundation is an essential part of a structure that transmits column or wall loads to the underlying soil below the structure. The mathematical approach suggested in this paper produces results that have a tangible accuracy for all problems, main part of this research to find the more economical dimensions of T-shaped combined footings using the optimization techniques.

A T-shaped combined footing is provided when one column load is much more than the other. As a result, the both projections of footing beyond the faces of the columns will be restricted, where the column more loaded is localized in property line.

The new model presented in this paper for dimensioning of T-shaped combined footings subjected to an axial load and moment in two directions in each column, also it can be applied to others cases: 1) Footings subjected to a concentric axial load in each column, 2) Footings subjected to a axial load and a moment in each column.

The main conclusions are:

1. If the two moments around of the axis "Y" change of direction, the results for the Case 1 and 3 and for the Case 2 and 4 of the Tables 1, 2, 3 and 4 are not altered.

2. The methodology shown in this paper is more accurate and converges more quickly.

3. The classical model will not be practical compared to this methodology, because the classical model is developed proposing the dimensions and then verified to comply with the stresses limits mentioned above.

4. The proposed model can be used for the dimensioning of T-shaped combined footings for two property lines of opposite sides constrained (see Tables 2 and 3).

The model presented in this paper applies only for dimensioning of rectangular and T-shaped combined footings (see Table 4), the structural member is assumed to be rigid and the supporting soil layers elastic, which meet expression of the biaxial bending, i.e., the variation of pressure is linear.

The suggestions for future research are: when another type of soil is presented, by example in totally cohesive soils (clay soils) and totally granular soils (sandy soils), the pressure diagram is not linear and should be treated differently.

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Appendix

Table 1 Results obtained by software for $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 1 (P_1 = 1250 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 250 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	915.82	400	17.10	11.70	6.40	1.00	1.00	131.83	96.89	114.20	98.22	95.23	79.25	2.99	0.00
225	1500	915.82	400	17.10	11.70	6.40	1.00	1.00	131.83	96.89	114.20	98.22	95.23	79.25	2.99	0.00
200	1500	915.82	400	17.10	11.70	6.40	1.00	1.00	131.83	96.89	114.20	98.22	95.23	79.25	2.99	0.00
175	1500	915.82	400	17.10	11.70	6.40	1.00	1.00	131.83	96.89	114.20	98.22	95.23	79.25	2.99	0.00
150	1500	915.82	400	17.10	11.70	6.40	1.00	1.00	131.83	96.89	114.20	98.22	95.23	79.25	2.99	0.00
Case 2 (P_1 = 1250 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 250 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	843.96	400	11.06	5.66	6.40	1.00	1.00	250.00	104.41	229.85	169.92	144.19	84.26	61.12	35.39
225	1500	738.08	400	11.58	6.18	6.40	1.00	1.00	225.00	102.18	207.90	156.42	136.55	85.08	64.06	44.19
200	1500	620.26	400	12.22	6.82	6.40	1.00	1.00	200.00	98.67	186.08	142.84	127.99	84.75	67.67	52.83
175	1500	488.05	400	13.04	7.64	6.40	1.00	1.00	175.00	93.70	164.43	129.10	118.46	83.13	72.00	61.35
150	1500	338.28	400	14.10	8.70	6.40	1.00	1.00	150.00	87.10	142.95	115.11	107.88	80.05	77.03	69.80
Case 3 (P_1 = 1250 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 250 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	915.82	−400	17.10	11.70	6.40	1.00	1.00	96.89	131.83	79.25	95.23	98.22	114.20	0.00	2.99
225	1500	915.82	−400	17.10	11.70	6.40	1.00	1.00	96.89	131.83	79.25	95.23	98.22	114.20	0.00	2.99
200	1500	915.82	−400	17.10	11.70	6.40	1.00	1.00	96.89	131.83	79.25	95.23	98.22	114.20	0.00	2.99
175	1500	915.82	−400	17.10	11.70	6.40	1.00	1.00	96.89	131.83	79.25	95.23	98.22	114.20	0.00	2.99
150	1500	915.82	−400	17.10	11.70	6.40	1.00	1.00	96.89	131.83	79.25	95.23	98.22	114.20	0.00	2.99
Case 4 (P_1 = 1250 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 250 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	843.96	400	11.06	5.66	6.40	1.00	1.00	104.41	250.00	84.26	144.19	169.92	229.85	35.39	61.12
225	1500	738.08	400	11.58	6.18	6.40	1.00	1.00	102.18	225.00	85.08	136.55	156.42	207.90	44.19	64.06
200	1500	620.26	400	12.22	6.82	6.40	1.00	1.00	98.67	200.00	84.75	127.99	142.84	186.08	52.83	67.67
175	1500	488.05	400	13.04	7.64	6.40	1.00	1.00	93.70	175.00	81.13	118.46	129.10	164.43	61.35	72.00
150	1500	338.28	400	14.10	8.70	6.40	1.00	1.00	87.10	150.00	80.05	107.88	115.11	142.95	69.80	77.03

Table 2 Results obtained by software for $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 1 (P_1 = 1000 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 500 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	364.39	400	10.52	5.12	6.40	1.00	1.00	250.00	73.83	240.99	170.12	135.70	64.83	121.48	87.06
225	1500	263.49	400	10.97	5.57	6.40	1.00	1.00	225.00	74.81	218.67	157.07	130.09	68.49	122.91	95.93
200	1500	151.02	400	11.51	6.11	6.40	1.00	1.00	200.00	74.59	196.49	144.04	123.53	71.08	125.07	104.56
175	1500	24.39	400	12.20	6.80	6.40	1.00	1.00	175.00	72.98	174.45	130.94	115.94	72.43	127.98	112.98
150	1500	−138.27	400	13.21	7.81	6.40	1.00	1.00	147.02	69.27	150.00	116.10	106.15	72.24	132.17	122.21
Case 2 (P_1 = 1000 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 500 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	−386.17	400	9.92	4.52	6.40	1.00	1.00	240.02	17.74	250.00	165.47	114.25	27.72	217.37	168.16
225	1500	−492.05	400	10.34	4.94	6.40	1.00	1.00	212.68	24.07	225.00	149.81	111.59	36.39	216.34	178.12
200	1500	−508.82	400	11.05	5.47	6.58	1.00	1.00	188.65	33.72	200.00	136.68	108.38	45.06	200.00	171.69
175	1500	−467.33	400	12.09	6.18	6.91	1.00	1.00	166.31	43.77	175.00	123.64	103.82	52.46	175.00	155.18
150	1500	−419.44	400	13.46	7.14	7.32	1.00	1.00	143.71	51.16	150.00	110.21	97.24	57.45	150.00	137.04
Case 3 (P_1 = 1000 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 500 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	364.39	−400	10.52	5.12	6.40	1.00	1.00	73.83	250.00	64.83	135.70	170.12	240.99	87.06	121.48
225	1500	263.49	−400	10.97	5.57	6.40	1.00	1.00	74.81	225.00	68.49	130.09	157.07	218.67	95.93	122.91

Table 2 Continued

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 3 (P_1 = 1000 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 500 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
200	1500	151.02	−400	11.51	6.11	6.40	1.00	1.00	74.59	200.00	71.08	123.53	144.04	196.49	104.56	125.07
175	1500	24.39	−400	12.20	6.80	6.40	1.00	1.00	72.98	175.00	72.43	115.94	130.94	174.45	112.98	127.98
150	1500	−138.27	−400	13.21	7.81	6.40	1.00	1.00	69.27	147.02	72.24	106.15	116.10	150.00	122.21	132.17
Case 4 (P_1 = 1000 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 500 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	−386.17	−400	9.92	4.52	6.40	1.00	1.00	17.74	240.02	27.72	114.25	163.47	250.00	168.16	217.37
225	1500	−492.05	−400	10.34	4.94	6.40	1.00	1.00	24.07	212.68	36.39	111.59	149.81	225.00	178.12	216.34
200	1500	−508.82	−400	11.05	5.47	6.58	1.00	1.00	33.72	188.65	45.06	108.38	136.68	200.00	171.69	200.00
175	1500	−467.33	−400	12.09	6.18	6.91	1.00	1.00	43.77	166.31	52.46	103.82	123.64	175.00	155.18	175.00
150	1500	−419.44	−400	13.46	7.14	7.32	1.00	1.00	51.16	143.71	57.45	97.24	110.21	150.00	137.04	150.00

Table 3 Results obtained by software for $b_1 \geq 1.00$, $b_2 \geq 1.00$, $b_1 \leq b$, $b_2 \leq a$ and $6.40 \leq b$

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 1 (P_1 = 750 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 750 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	0.00	400	11.33	1.98	6.40	5.02	1.00	250.00	14.83	250.00	191.75	73.07	14.83	191.75	73.07
225	1500	0.00	400	12.14	2.12	6.40	5.13	1.00	225.00	22.03	225.00	171.40	75.63	22.03	171.40	75.63
200	1500	0.00	400	13.15	2.29	6.40	5.23	1.00	200.00	28.19	200.00	151.61	76.58	28.19	151.61	76.58
175	1500	0.00	400	14.41	2.51	6.40	5.32	1.00	175.00	33.16	175.00	132.38	75.78	33.16	132.38	75.78
150	1500	0.00	400	16.07	2.79	6.40	5.40	1.00	150.00	36.74	150.00	113.68	73.07	36.74	113.68	73.07
Case 2 (P_1 = 750 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 750 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	0.00	400	11.73	1.68	7.00	7.00	1.68	250.00	5.77	250.00	250.00	5.77	5.77	250.00	5.77
225	1500	0.00	400	12.59	1.80	7.00	7.00	1.80	225.00	13.19	225.00	250.00	13.19	13.19	250.00	13.19
200	1500	0.00	400	13.65	1.95	7.00	7.00	1.95	200.00	19.74	200.00	200.00	19.74	19.74	200.00	19.74
175	1500	0.00	400	14.98	2.14	7.00	7.00	2.14	175.00	25.27	175.00	175.00	25.27	25.27	175.00	25.27
150	1500	0.00	400	16.70	2.39	7.00	7.00	2.39	150.00	29.59	150.00	150.00	29.59	29.59	150.00	29.59
Case 3 (P_1 = 750 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 750 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	0.00	−400	11.33	1.98	6.40	5.02	1.00	14.83	250.00	14.83	73.07	191.75	250.00	73.07	191.75
225	1500	0.00	−400	12.14	2.12	6.40	5.13	1.00	22.03	225.00	22.03	75.63	171.40	225.00	75.63	171.40
200	1500	0.00	−400	13.15	2.29	6.40	5.23	1.00	28.19	200.00	28.19	76.58	151.61	200.00	76.58	151.61
175	1500	0.00	−400	14.41	2.51	6.40	5.32	1.00	33.16	175.00	33.16	75.78	132.38	175.00	75.78	132.38
150	1500	0.00	−400	16.07	2.79	6.40	5.40	1.00	36.74	150.00	36.74	73.07	113.68	150.00	73.07	113.68
Case 4 (P_1 = 750 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 750 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	0.00	−400	11.73	1.68	7.00	7.00	1.68	5.77	250.00	5.77	5.77	250.00	250.00	5.77	250.00
225	1500	0.00	−400	12.59	1.80	7.00	7.00	1.80	13.19	225.00	13.19	13.19	225.00	225.00	13.19	225.00
200	1500	0.00	−400	13.65	1.95	7.00	7.00	1.95	19.74	200.00	19.74	19.74	200.00	200.00	19.74	200.00
175	1500	0.00	−400	14.98	2.14	7.00	7.00	2.14	25.27	175.00	25.27	25.27	175.00	175.00	25.27	175.00
150	1500	0.00	−400	16.70	2.39	7.00	7.00	2.39	29.59	150.00	29.59	29.59	150.00	150.00	29.59	150.00

Table 4 Results obtained by software for $b_1 \geq 1.00$, $b_2 \geq 1.00$, $a = b_2$, $b = b_1$ and $6.40 \leq b$

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 1 (P_1 = 500 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 1000 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	0.00	400	12.48	1.60	7.80	7.80	1.60	240.38	0.00	240.38	240.38	0.00	0.00	240.38	0.00
225	1500	0.00	400	13.04	1.67	7.80	7.80	1.67	225.00	4.98	225.00	225.00	4.98	4.98	225.00	4.98

Table 4 Continued

Soil allowable load capacity	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex							
σ_{adm} kN/m^2	R kN	M_{xT} $kN\text{-}m$	M_{yT} $kN\text{-}m$	A m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2	σ_7 kN/m^2	σ_8 kN/m^2
Case 1 (P_1 = 500 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 1000 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
200	1500	0.00	400	14.13	1.81	7.80	7.80	1.81	200.00	12.37	200.00	200.00	12.37	12.37	200.00	12.37
175	1500	0.00	400	15.48	1.98	7.80	7.80	1.98	175.00	18.78	175.00	175.00	18.78	18.78	175.00	18.78
150	1500	0.00	400	17.24	2.21	7.80	7.80	2.21	150.00	24.02	150.00	150.00	24.02	24.02	150.00	24.02
Case 2 (P_1 = 500 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = 200 $kN\text{-}m$; P_2 = 1000 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = 200 $kN\text{-}m$)																
250	1500	0.00	400	14.40	1.60	9.00	9.00	1.60	208.33	0.00	208.33	208.33	0.00	0.00	208.33	0.00
225	1500	0.00	400	14.40	1.60	9.00	9.00	1.60	208.33	0.00	208.33	208.33	0.00	0.00	208.33	0.00
200	1500	0.00	400	14.80	1.64	9.00	9.00	1.64	200.00	2.73	200.00	200.00	2.73	2.73	200.00	2.73
175	1500	0.00	400	16.19	1.80	9.00	9.00	1.80	175.00	10.26	175.00	175.00	10.26	10.26	175.00	10.26
150	1500	0.00	400	18.00	2.00	9.00	9.00	2.00	150.00	16.67	150.00	150.00	16.67	16.67	150.00	16.67
Case 3 (P_1 = 500 kN; M_{x1} = 300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 1000 kN; M_{x2} = 150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	0.00	−400	12.48	1.60	7.80	7.80	1.60	0.00	240.38	0.00	0.00	240.38	240.38	0.00	240.38
225	1500	0.00	−400	13.04	1.67	7.80	7.80	1.67	4.98	225.00	4.98	4.98	225.00	225.00	4.98	225.00
200	1500	0.00	−400	14.13	1.81	7.80	7.80	1.81	12.37	200.00	12.37	12.37	200.00	200.00	12.37	200.00
175	1500	0.00	−400	15.48	1.98	7.80	7.80	1.98	18.78	175.00	18.78	18.78	175.00	175.00	18.78	175.00
150	1500	0.00	−400	17.24	2.21	7.80	7.80	2.21	24.02	150.00	24.02	24.02	150.00	150.00	24.02	150.00
Case 4 (P_1 = 500 kN; M_{x1} = −300 $kN\text{-}m$; M_{y1} = − 200 $kN\text{-}m$; P_2 = 1000 kN; M_{x2} = −150 $kN\text{-}m$; M_{y2} = − 200 $kN\text{-}m$)																
250	1500	0.00	−400	14.40	1.60	9.00	9.00	1.60	0.00	208.33	0.00	0.00	208.33	208.33	0.00	208.33
225	1500	0.00	−400	14.40	1.60	9.00	9.00	1.60	0.00	208.33	0.00	0.00	208.33	208.33	0.00	208.33
200	1500	0.00	−400	14.80	1.64	9.00	9.00	1.64	2.73	200.00	2.73	2.73	200.00	200.00	2.73	200.00
175	1500	0.00	−400	16.19	1.80	9.00	9.00	1.80	10.26	175.00	10.26	10.26	175.00	175.00	10.26	175.00
150	1500	0.00	−400	18.00	2.00	9.00	9.00	2.00	16.67	150.00	16.67	16.67	150.00	150.00	16.67	150.00