

## A study on surface wave dispersion due to the effect of soft layer in layered media

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**Abstract.** Surface wave techniques are widely used as non-invasive method for geotechnical site characterization. Field surface wave data are collected and analyzed using different processing techniques to generate the dispersion curves, which are further used to extract the shear wave velocity profile by inverse problem solution. Characteristics of a dispersion curve depend on the subsurface layering information of a vertically heterogeneous medium. Sometimes soft layer can be found between two stiff layers in the vertically heterogeneous media, and it can affect the wave propagation dramatically. Now most of the surface wave techniques use the fundamental mode Rayleigh wave propagation during the inversion, but this may not be the actual scenario when a soft layer is present in a vertically layered medium. This paper presents a detailed and comprehensive study using finite element method to examine the effect of soft layers which sometimes get trapped between two high velocity layers. Determination of the presence of a soft layer is quite important for proper mechanical characterization of a soil deposit. Present analysis shows that the thickness and position of the trapped soft layer highly influence the dispersion of Rayleigh waves while the higher modes also contribute in the resulting wave propagation.

**Keywords:** dispersion; surface wave; finite element; frequency-wave number analysis; soft layer; Rayleigh wave

### 1. Introduction

Surface wave methods which utilize the dispersive characteristics of Rayleigh waves are being widely used for geotechnical site characterization. These dispersion properties of Rayleigh waves are utilized in different surface wave techniques by the researchers to investigate the shear wave velocity profile at shallow depths (e.g., Nazarian and Stokoe 1983, Xia *et al.* 1999, Park *et al.* 1999, Louie 2001) as well as at deeper depths. Surface wave techniques initially started with Steady State Rayleigh wave method (Jones 1958) but its popularity came with the introduction of Spectral Analysis of Surface Wave (SASW). Stokoe and Nazarian presented this method in 1983 and analyzed the dispersion curve of a ground roll to produce near-surface shear wave velocity profiles.

The SASW method is quite tedious to carry out as it requires repeating the test for several spacing configurations to get the information up to a reasonable frequency band. Park *et al.* (1999)

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introduced the Multichannel Analysis of Surface Wave (MASW) which uses multiple receivers array to explore the subsurface information up to a frequency band of engineering interest in a single shot. Since then, MASW is being progressively used in geotechnical engineering to characterize a shallow site. Site characterization using surface wave technique involves three steps. First, surface wave data are collected by conducting the field tests. Secondly, these data are analyzed to develop the dispersion curve by using different signal processing techniques (Socco and Strobbia 2004, Strobbia and Foti 2006). Finally, the shear-wave velocity profiles are extracted by inverse problem solution of those dispersion curves. Inversion which is a critical step of surface wave technique is required a skillful way to tackle the problem. There are several methods already being used for inversion like Genetic Algorithms (Lomax and Snieder 1994), Simulated Annealing (Sen and Stoffa 1991), and Neighborhood Algorithm (Sambridge 1999).

Although surface wave methods are widely used for characterization of soil media, it may suffer from several uncertainties like data measurement uncertainty (Jakka *et al.* 2014), which originates when surrounding noises get mixed up while conducting the field tests. Inversion problem is non-unique and it may lead to several equivalent velocity profiles with good fit with the experimental dispersion curve (Foti *et al.* 2009, Roy *et al.* 2013, Boaga *et al.* 2011). The final best fitting velocity profile largely depends on the initial choice of velocity model used in the inversion. So, it is required to have some prior information regarding the soil stratification before starting the inversion. Near field effects generally result into the underestimation or sometimes overestimation of Rayleigh wave phase velocity near to the source (Roberts and Asten 2008, Yoon and Rix 2009). Lateral heterogeneity is another source of uncertainty which arises when the irregularities present in the soil layer and violates the assumption of horizontally layered media of surface wave testing (Vignoli and Cassiani 2010, Boaga *et al.* 2012, Boiero and Socco 2011).

Dispersion curve is generated after analyzing the recorded signals, and it largely depends on the soil medium through which the Rayleigh waves propagate. There are mainly three types of dispersion which are possible. When the shear-wave velocity increases with depth, the medium is known as normally dispersive. If there is a soft or hard layer trapped in-between two layers and the gradual increase of velocity is hindered, then the medium is said to be as inversely dispersive medium. For normally dispersive profile, fundamental mode is generally dominant in the wave propagation but for inversely dispersive cases, the complicity increases as participation of higher modes are sometimes observed (Gucunski and Woods 1992, Tokimatsu *et al.* 1992). Few researches have been carried out to model the surface wave propagation in vertically heterogeneous or homogeneous medium by solving the wave equation. Kausel (1981) developed the explicit, closed-form expressions for the Green's functions when a dynamic load is applied in a layered media. Several researchers used this method to model the surface wave propagation as an axisymmetric problem (Gucunski and Woods 1992, Ganji *et al.* 1998, Yoon and Rix 2009). Finite element (FE) method can also be used as an alternative to model a surface wave test (Gucunski *et al.* 1996, Aung and Leong 2010, 2011, Evangelista and Santucci 2015). When there exists inhomogeneity and nonlinearity in the medium, FE serves as the most efficient method to model the surface wave (Estorff *et al.* 1990). Presently, surface wave tests are widely used to detect the subsurface inhomogeneities which can also be modeled using FE method. In this paper, a detailed numerical study is carried out to model the surface wave tests using finite element method. Generated velocity time histories are then analyzed to extract the dispersion curve using frequency-wavenumber ( $f-k$ ) method. The effect of thickness and the position of soft layer on surface wave dispersion in a vertically layered medium are studied considering different 1D soil models for developing a better understanding and to get a clear insight of the problem.

## 2. Numerical modeling

Numerical simulation is performed with the help of a 2D axisymmetric model using finite element method implemented in PLAXIS2D (Brinkgreve 2002) computer program package. Vertical particle motion displacements have been computed at several receiver positions from the source using an impulse loading. PLAXIS2D solves the wave equation under an impact loading in vertically layered medium. The equation of motion of a N degrees of freedom system subjected to an impact loading can be written as follows

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t) \quad (1)$$

where  $M$ ,  $C$  and  $K$  are the NxN mass, damping and stiffness matrixes, respectively.  $\ddot{u}(t)$ ,  $\dot{u}(t)$  and  $u(t)$  are the acceleration, velocity and displacement vectors, respectively, and  $F(t)$  represents the applied impulsive force. By using the implicit time integration scheme of Newmark, the displacement and the velocity at the point in time  $(t+\Delta t)$  can be expressed as

$$u^{t+\Delta t} = u^t + \dot{u}^t \Delta t + \left( \left( \frac{1}{2} - \alpha \right) \ddot{u}^t + \alpha \ddot{u}^{t+\Delta t} \right) \Delta t^2 \quad (2)$$

$$\dot{u}^{t+\Delta t} = \dot{u}^t + \left( (1 - \beta) \ddot{u}^t + \beta \ddot{u}^{t+\Delta t} \right) \Delta t \quad (3)$$

where  $\Delta t$  is the time step,  $\alpha$  and  $\beta$  are the Newmark's parameters which determine the accuracy of the numerical time integration. For a stable solution, the following condition should be satisfied.

$$\beta \geq 0.5 \text{ and } \alpha \geq 0.25(0.5 + \beta)^2 \quad (4)$$

So, the dynamic problem is solved with the help of a 2D axisymmetric model using finite element method implemented in PLAXIS computer program package. Special absorbent boundaries are introduced at the boundaries to avoid the spurious reflections of the waves from the model boundaries. These absorbant boundaries are provided based on the Lysmer and Kohlmeyer (1969) model. According to this model, the normal and shear stress components are absorbed by providing a damper, which can be determined as follows

$$\sigma_n = -C_1 \rho V_p \dot{u}_x \quad (5)$$

$$\tau = -C_2 \rho V_s \dot{u}_y \quad (6)$$

where  $\rho$  is the mass density,  $V_s$  is the shear wave velocity,  $V_p$  is the longitudinal wave velocity,  $\dot{u}_x$  and  $\dot{u}_y$  are the velocities of particle motion in the direction of  $x$  and  $y$ , respectively, and  $C_1$  and  $C_2$  are relaxation coefficients used to improve the wave absorption at the boundaries.  $C_1$  corrects the dissipation in the direction normal to the boundary and  $C_2$  in the tangential direction. A value of  $C_1=1$  and  $C_2=0.25$  results in better absorption of waves. Impulse loading is simulated using triangular pulse acting for short duration. The load is acting for a short duration of 0.005s. Total time duration of the simulation is 2s with a time step of 0.002s. The time step is selected maintaining the resolution at higher frequencies. Here the sampling frequency is 500 Hz and the Nyquist frequency 250 Hz, which is well enough to provide better resolution at higher frequencies which have been simulated here.

### 3. Dispersion calculation

The simulated velocity time histories at different locations from the source are taken out and analyzed using the frequency-wave number ( $f$ - $k$ ) method to extract the Rayleigh wave phase velocity. The  $f$ - $k$  method is widely used for the dispersion analyses of surface wave array data (Zywicki 1999, Hebler 2001, Yoon and Rix 2009). For this purpose, we used geopsy program (Wathelet *et al.* 2008), which uses the  $f$ - $k$  method to develop the dispersion curve from an array of surface wave data. This method is based on the identification of maximum energy peak which develops in the  $f$ - $k$  spectrum for a particular wave number. Once the peak occurs in the  $f$ - $k$  spectrum, we can calculate the wave numbers of dominant modes of propagating Rayleigh wave. Phase velocity can be calculated using the following equation

$$V_R = \frac{2\pi f}{|k_{peak}|} \quad (7)$$

where  $V_R$  is the Rayleigh wave phase velocity,  $f$  is the frequency of the propagating wave and  $k_{peak}$  is the wave-number associated with the maximum energy peak.

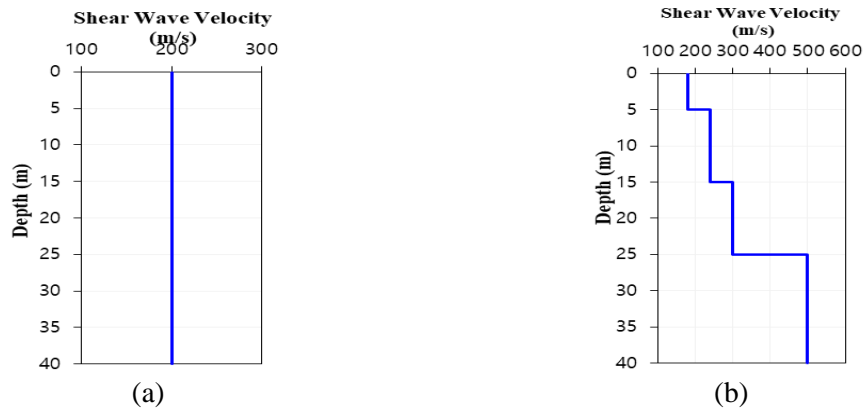


Fig. 1 Considered velocity profiles used for the validation, (a) Homogeneous case and (b) Layered profile case

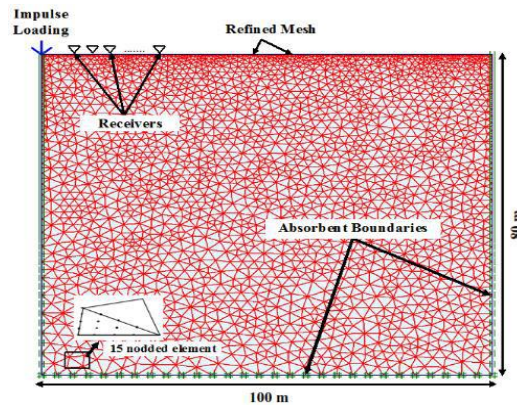


Fig. 2 Developed finite element model in PLAXIS2D for homogeneous soil case

#### 4. Validation of model

Before starting the simulation for different cases of the considered soil profiles to study the effect of soft layer, it is necessary to validate the developed soil model. For this purpose, two simple axisymmetric models, homogeneous and layered soil model, have been prepared in finite element using PLAXIS2D. In wave propagation problem using finite element, element dimensions are chosen with respect to the highest frequency ( $f_{max}$ ) for the lowest Rayleigh wave phase velocity ( $V_R$ ). If the element dimensions are kept too large, it will filter out high frequencies, whereas very small element dimensions can introduce numerical instability as well as require considerable computational resources. The minimum wavelength ( $\lambda_{min}$ ) is calculated as follows

$$\lambda_{min} = \frac{V_R}{f_{max}} \quad (8)$$

The mesh size near the surface is always maintained less than 0.5 times of  $\lambda_{min}$  and increased gradually with the depth. Homogeneous soil model possesses a velocity of 200 m/s (Fig. 1(a)) and layered profile is having three layers plus half-space with velocities 180, 240, 300 and 500 m/s (Fig. 1(b)) and thicknesses 5 m, 7 m and 12 m, respectively. Poisson's ratio and density are assumed as 0.3 and 18 kN/m<sup>3</sup>, respectively for all the layers, and these are the typical values of shallow geologic materials. Fig. 2 shows the developed axisymmetric finite element model after generating mesh for homogeneous soil case. The size of the model is decided on the basis of criteria given by Aung and Leong (2011) based on maximum available wavelength of the generated waves. They proposed that the model size should be twice the longest wavelength available to avoid the errors due to the wave reflected from the boundaries. If we see Fig. 2, the shear wave velocity of the half-space is 200 m/s, and the minimum frequency ( $f_{min}$ ) that we extracted is 5 Hz so the maximum available wavelength ( $\lambda_{max}$ ) is 40 m. Based on the criterion the size of the model should be 80 m. So, we considered the height of the model 80 m and length of the model 100 m fulfilling the criteria.

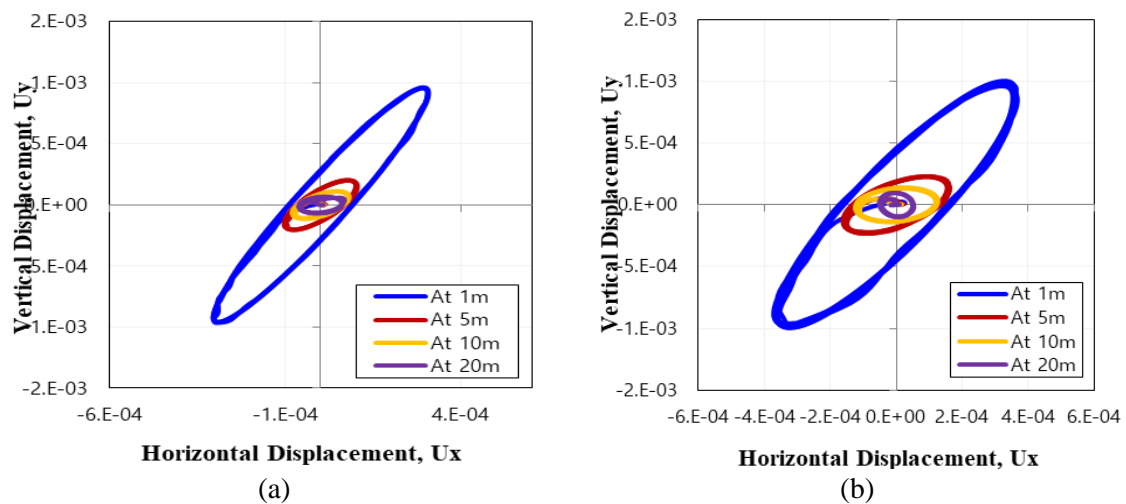


Fig. 3 Particle displacements at different locations from source for two different harmonic motions, (a) 10 Hz and (b) 20Hz

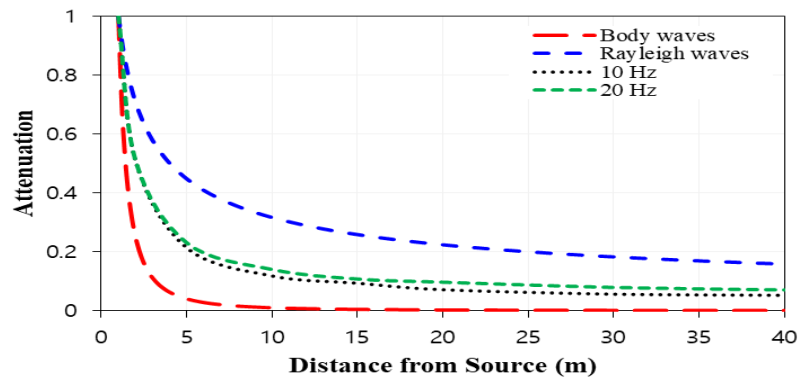


Fig. 4 Attenuation of particle motion at different distances from the source (for geometric spreading)

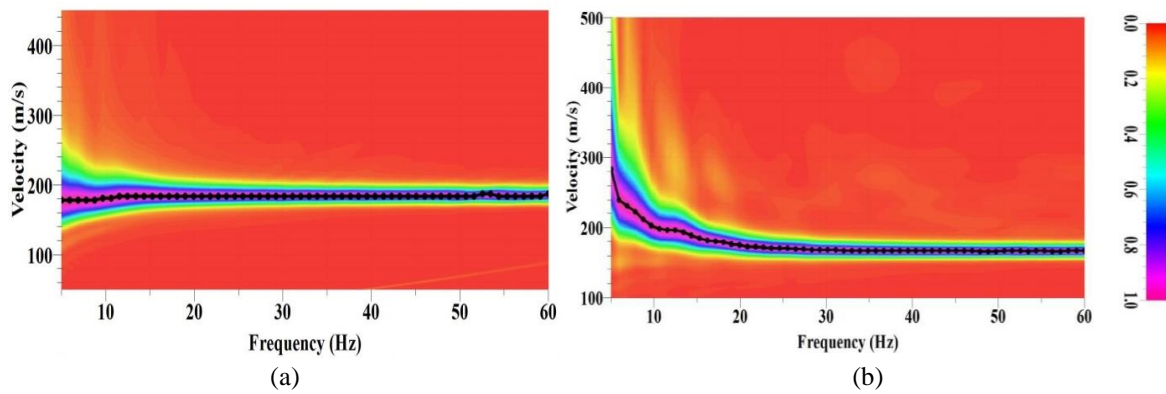


Fig. 5 Dispersion spectra after frequency-wavenumber ( $f-k$ ) analysis, (a) Homogeneous soil case and (b) Layered soil case

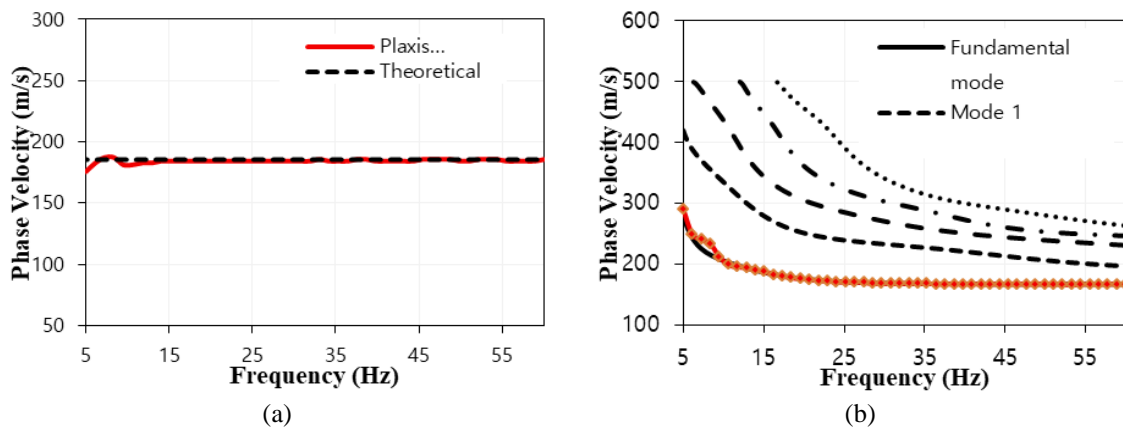


Fig. 6 Comparison of dispersion curves from PLAXIS and modal dispersion curves, (a) Homogeneous Case and (b) Layered soil case

Before applying the pulse load the model is validated using two different harmonic loads of frequencies 10 Hz and 20 Hz with an amplitude multiplier of 100. Fig. 3 shows the plot of vertical

and horizontal displacements for 10 Hz (Fig. 3(a)) and 20 Hz (Fig. 3(b)) at different locations from the source. Here, it is quite evident that the generated surface waves are having a backward type polarization. The positive ratio of  $U_x/U_y$  signifies the prograde or direct orbit of particle motion. Here, it can also be observed that the major axis of ellipse is having some inclination with the vertical axis, which basically signify that the wavefield is not purely of plane Rayleigh wave and P and S waves are also present in the generated wavefield (Evangelista and Santucci de Magistris, 2015). To verify this phenomena a plot is prepared (Fig. 4) with the theoretical attenuation of body and surface wave and attenuation of the maximum displacements of the harmonic signals of 10 Hz and 20 Hz frequency, moving away from the source. Attenuation of harmonic signals is very quick near the source because of the simultaneous presence P-and S-waves together with Rayleigh waves, but the attenuation becomes smooth and negligible away from the source due to the separation of P- and S- wave components. This aspect is later confirmed during the dispersion analysis. The Rayleigh wave phase velocity is found to be underestimated at lower frequencies.

Once the model is validated with harmonic motion, now a triangular pulse load is applied at the point of symmetry and the response is simulated and sampled at different receiver locations. The time history data are analyzed using the  $f-k$  method to generate the dispersion curve. Absorbent boundaries are introduced at the bottom and right hand side of the models to absorb the reflection of waves from the boundaries (Fig. 2). The elements in the model near to the surface are refined to accurately estimate the displacements of higher frequencies at the surface level due to an active source. We spatially sampled the displacements generated by the impulse source at different locations from the source.

Particle velocity time histories are generated at 96 receiver locations with 1m spacing and a source-to-receiver distance of 1 m. Using the array of velocity variation data, dispersion curves are generated using the Geopsy program which uses the  $f-k$  method for the data analysis. Fig. 5 shows the dispersion spectra for homogeneous (Fig. 5(a)) and layered (Fig. 5(b)) soil model. Dispersion curves are generated from the maximum concentrated energy of the developed dispersion spectra and superimposed with the modal theoretical dispersion curves. Fig. 6 represents the superimposed picked dispersion curves with modal dispersion curves for homogeneous soil profile (Fig. 6(a)) and layered soil profile (Fig. 6(b)). Modal dispersion curves (fundamental and four higher modes) have been generated using Thomson (1950) and Haskell (1953) algorithm later modified by Dunkin (1965) and Knopoff (1964). The comparison shows an excellent agreement between theoretical and numerical dispersion curves. Now once we validate our model and the efficiency of PLAXIS finite element program to model the surface wave propagation, a detailed numerical study is performed to study the effect of soft layer thickness and its position in a vertically layered media.

## 5. Model parameterization

In this study, numerical simulations are performed to study the effect of thickness and position of soft layer trapped between two stiff layers in a vertically heterogeneous medium. First, we considered a normally dispersive profile with a gradual increase of velocity, which is shown in Fig. 7(a), and a velocity drop is introduced at third layer to form an inversely dispersive soil profile (Fig. 6(b)). Poisson's ratio (0.3) and mass density ( $19 \text{ kN/m}^3$ ) have been kept constant for all the layers as these parameters are having a minimal effect on the surface wave dispersion. To come up with an optimal number of receivers, we generated the dispersion curves for 10, 20, 30 and 40

receivers with a source-to-receiver distance of 1 m and an inter-receiver spacing of 1 m for normally dispersive case profile which are presented in Fig. 8. As it is very much evident from the dispersion images that with the increase of number of receivers, the resolution increases dramatically, and we selected a 40 receiver array for the rest of our analysis to get a better resolution in the dispersion spectra.

For normally and inversely dispersive profiles, we calculated the velocity time histories at 40 receiver locations with a source-to-first receiver and inter-receiver spacing of 1 m. Poisson's ratio and density are kept same as normally dispersive case. Dispersion spectra are generated using  $f-k$  analysis and dispersion curves are picked from the maximum concentrated energy of the dispersion spectra. Fig. 9 plots the dispersion curves of normally dispersive soil profile and inversely dispersive profile. Both the dispersion curves are superimposed with the respective modal dispersion curves like earlier. Fig. 9(a) shows the generated dispersion spectra after  $f-k$  analysis and Fig. 9(b) presents the superimposed curve of the picked dispersion curve (red color) with theoretical modal dispersion curves (black colors). Normally dispersive profile reflects that the fundamental mode is dominant as the maximum energy is concentrated on the fundamental mode only, which is clearly observed from modal dispersion curves (Fig. 9(b)), where fundamental mode curve (black continuous line) exactly matches with our numerical dispersion curve. Whereas, in inversely dispersive soil profile, higher mode also contribute in the propagation, which can be identified from Fig. 9(c) as there is a distinct jump in the energy spectra. Fig. 9(d), which plots the modal curves with picked dispersion curve (red color), confirms the modal jump of the dispersion spectra. In inversely dispersive case, lower frequency waves travel at fundamental mode, and a jump occurs towards the first higher mode (black dashed line) at  $\sim 15$  Hz frequency and after that higher mode dominates the propagation of all the higher frequencies (Fig. 9(d)). So, the presence of soft layer can alter the wave propagation and it makes the phenomena complicated. Now a detailed parametric study is conducted to ascertain the influence of soft layer with varying thicknesses and at different positions in the following sections.

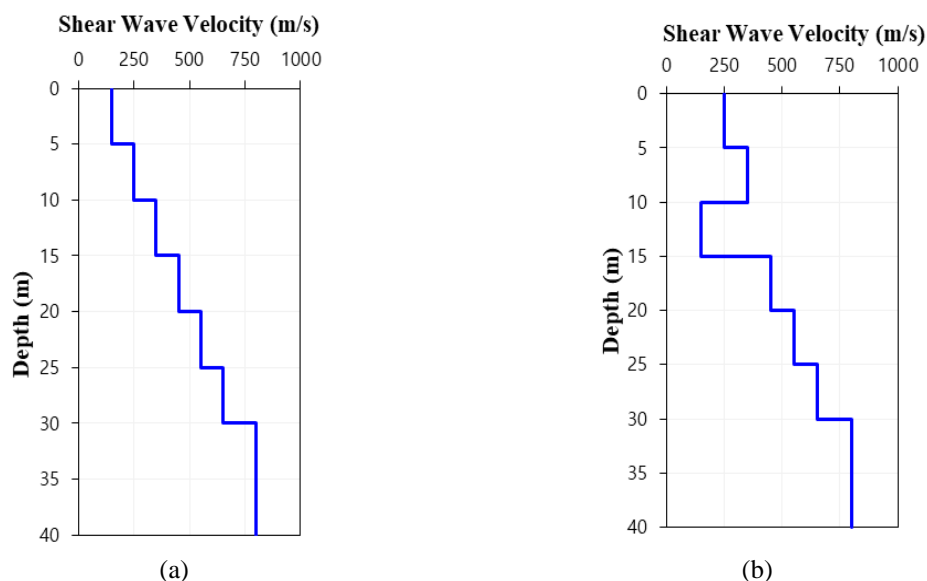


Fig. 7 Shear wave velocity profile for (a) Normally dispersive and (b) Inversely dispersive soil profile



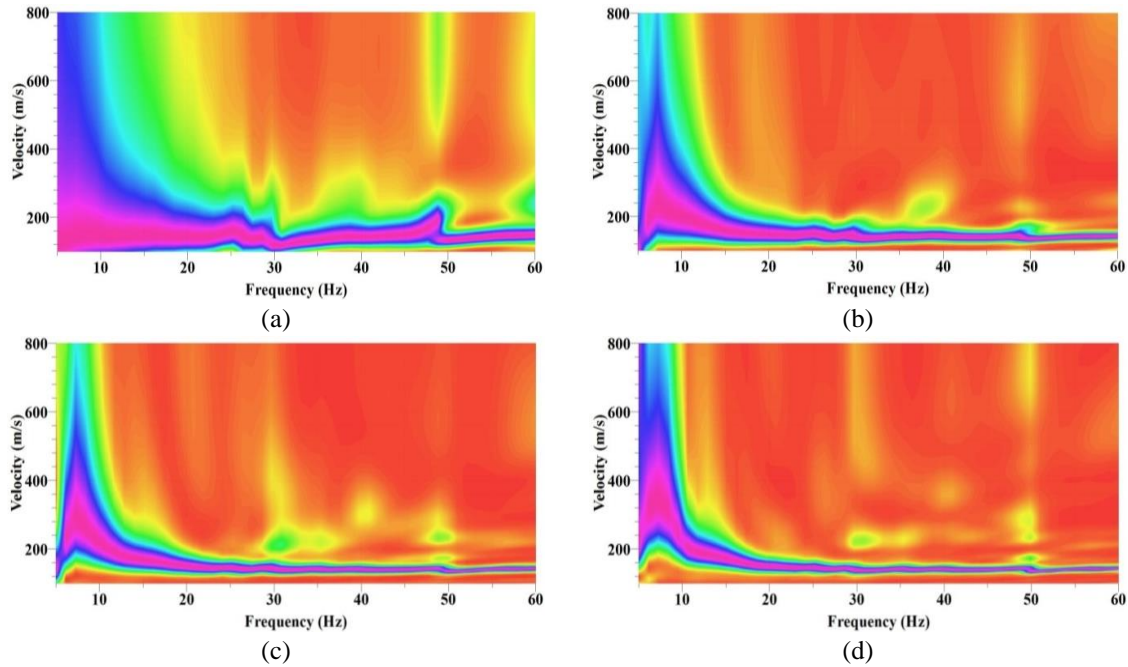


Fig. 8 Effect of number of receivers in Rayleigh wave dispersion, (a) 10 receivers array, (b) 20 receivers array, (c) 30 receivers array and (d) 40 receivers array

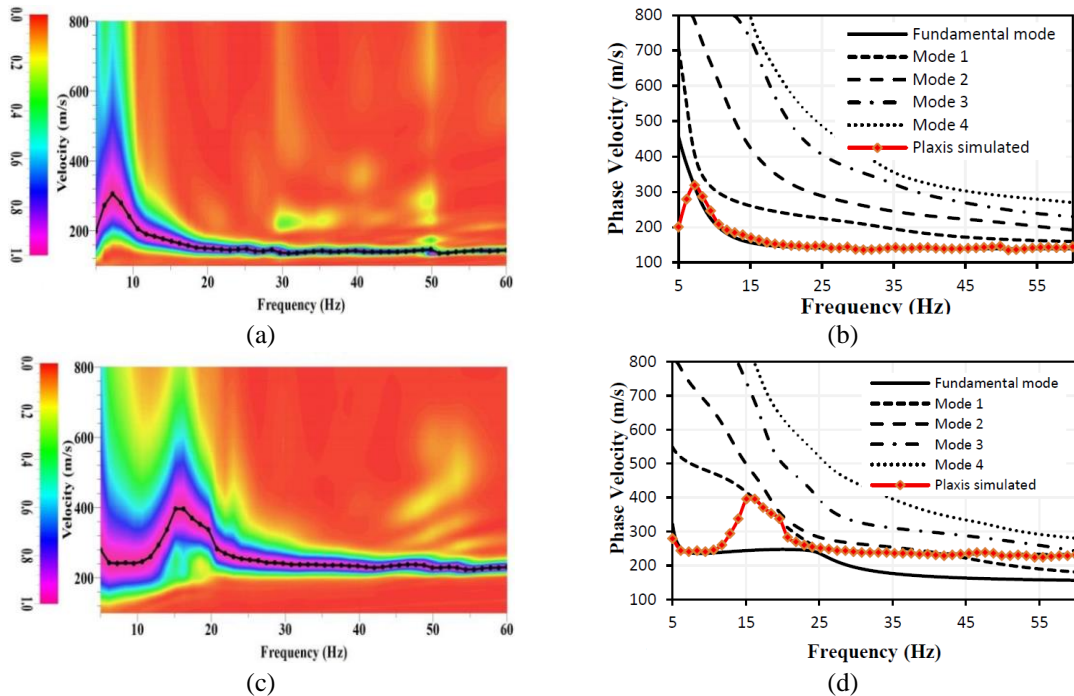


Fig. 9 Normally dispersive soil profile; (a) Dispersion image, and (b) Superimposed curves of the picked dispersion curve with modal curves, Inversely dispersive soil profile, (c) Dispersion image and (d) Superimposed curves of the picked dispersion curve with modal curves

### 5.1 Effect of soft layer thickness

The presence of soft layer between two stiff layers will suddenly change the shear wave velocity of soil profile and hence the analyses of effect of its presence at different depths and of varying thicknesses are a necessary aspect in the field of geotechnical design and engineering. As we had already seen the presence of soft layer increases the complicity while the higher modes also participate in the propagation. To study the effect of thickness of soft layer in the surface wave dispersion, we considered five different cases of soil layer thickness: 1 m (Case 1), 3 m (Case 2), 5 m (Case 3), 7 m (Case 4) and 10 m (Case 5) of the soft layer of the earlier considered inversely dispersive soil profile. The profiles with varying thickness of five different cases are shown in Fig. 10. For each case, velocity time histories are generated using PLAXIS2D finite element program under the action of an impulse load at 40 receiver locations with the similar source-to-first receiver and inter-receiver spacing like before. The data are generated at a time step of 0.002 to ensure the resolution at higher frequencies, which provide the information of shallow layers.

The dispersion analysis is performed separately and dispersion spectra for each case are generated. Figs. 11(a)-11(j) show the dispersion image and corresponding superimposed curve for 1 m, 3 m, 5 m, 7 m and 10 m thick soft layer case, respectively. The dispersion spectra show that with the increase of thickness of soft layer, the jump in the energy spectrum occurs towards the higher modes and it is quite significant. For case 1 (Figs. 11(a) and 11(b)), i.e., 1m thick soft layer, there is no such jump in the energy of propagating wave is observed, and its propagation is similar like the normally dispersive case and the picked dispersion curve follow the fundamental mode of propagation. As the thickness of the soft layer increases, the jumps occur are quite distinct and complicity in the dispersion curve is also found to increase (Figs. 11(g)-11(j)). At Case 4, i.e., 10 m thick soft layer case, the dispersion curve jumps towards the 3<sup>rd</sup> higher mode (Figs. 11(i) and 11(j)) and complicity increases in the dispersion spectra which is quite very clear even with less number of receivers. So, it is quite evident that more the thickness of soft soil layer, multiple jumps are observed in the dispersion spectra and more number of higher modes plays a significant role in the resulting wave propagation phenomena, making the wave propagation phenomena more complicated.

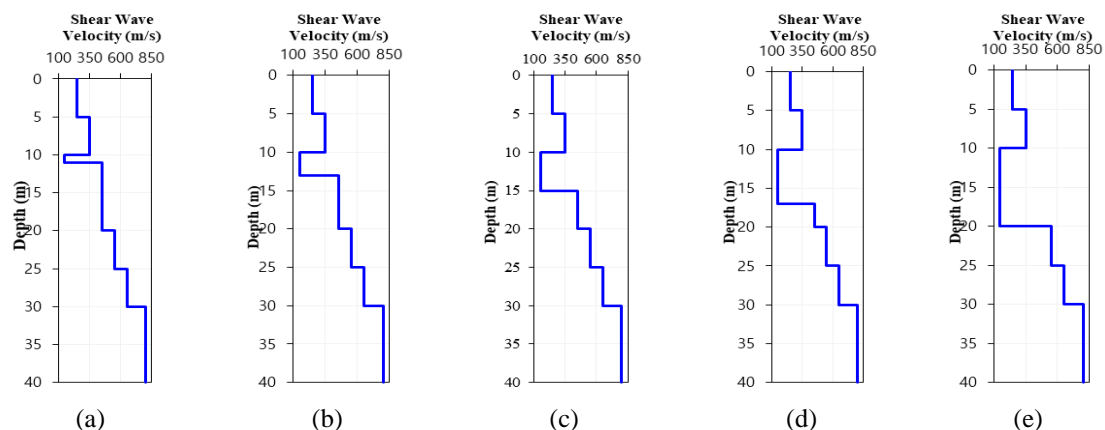


Fig. 10 Shear wave velocity profiles of varying thicknesses of the trapped soft layer, (a) 1 m (Case 1), (b) 3 m (Case 2), (c) 5 m (Case 3), (d) 7 m (Case 4) and (e) 10 m (Case 5)

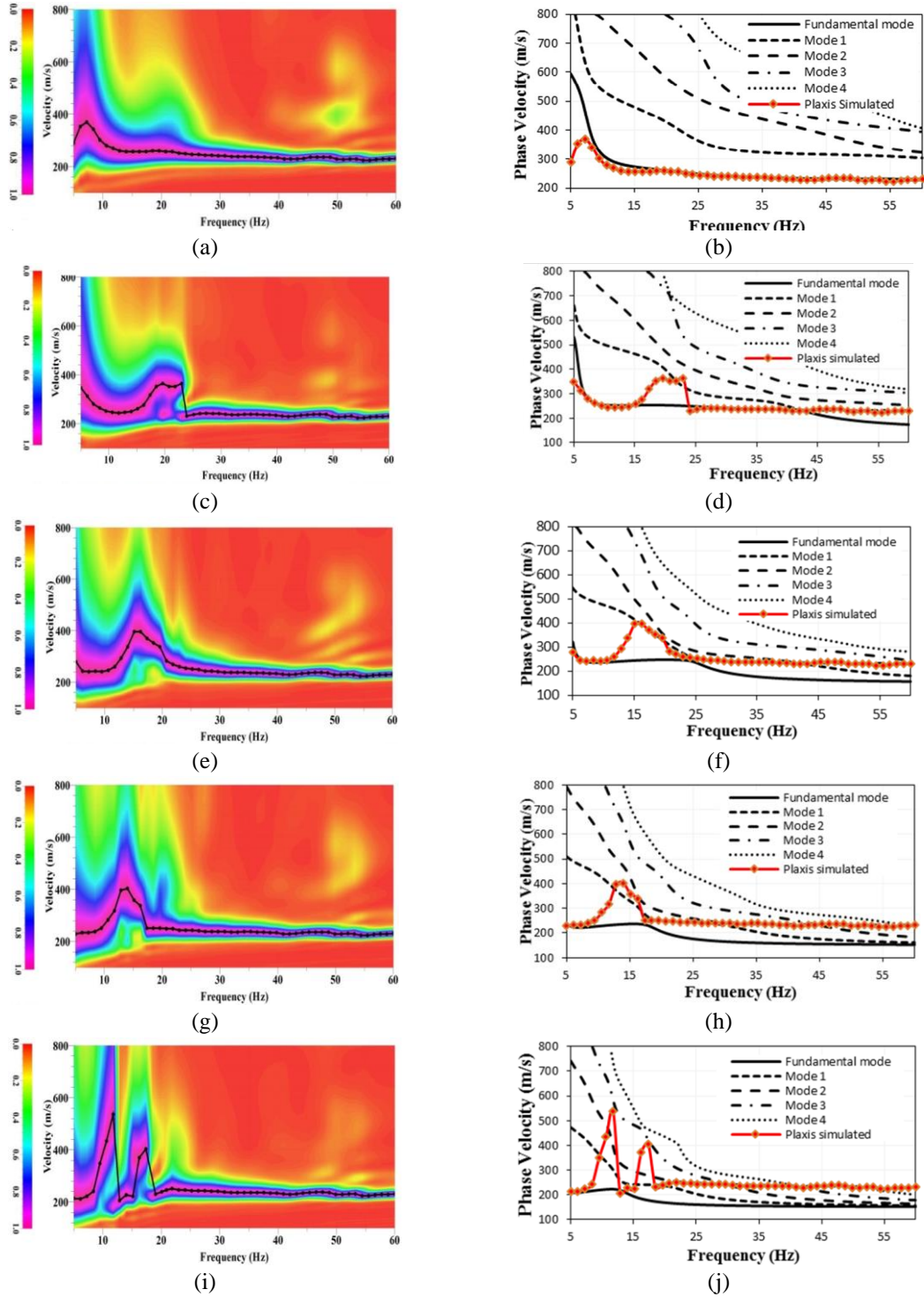


Fig. 11 Dispersion image and superimposed curve of picked dispersion curve with theoretical modal dispersion curves for Case 1 ((a) and (b)), Case 2 ((c) and (d)), Case 3 ((e) and (f)), Case 4 ((g) and (h)) and Case 5 ((i) and (j)), respectively

To get a better understanding on the problem, all the extracted dispersion curves are plotted together in Fig. 12. Now as the thickness of the soft layer increases, the shifting of the jumps is observed towards the lower frequencies. If we see 3 m thick soft layer, i.e., Case 2 (Fig. 10(b)), the jump is occurring at near about  $\sim 14$  Hz frequency, but for 7 m thick soft layer, i.e., Case 4 (Fig. 10(d)), the jump is shifted at  $\sim 8.5$  Hz frequency. For all cases, at higher frequencies the dispersion curves attain the constant same velocity of the first layer, which is clearly evident from Fig. 12. So, the present analysis gives us a clear insight and better understanding on how the thickness of the trapped soft layer is affecting the generated dispersion curves.

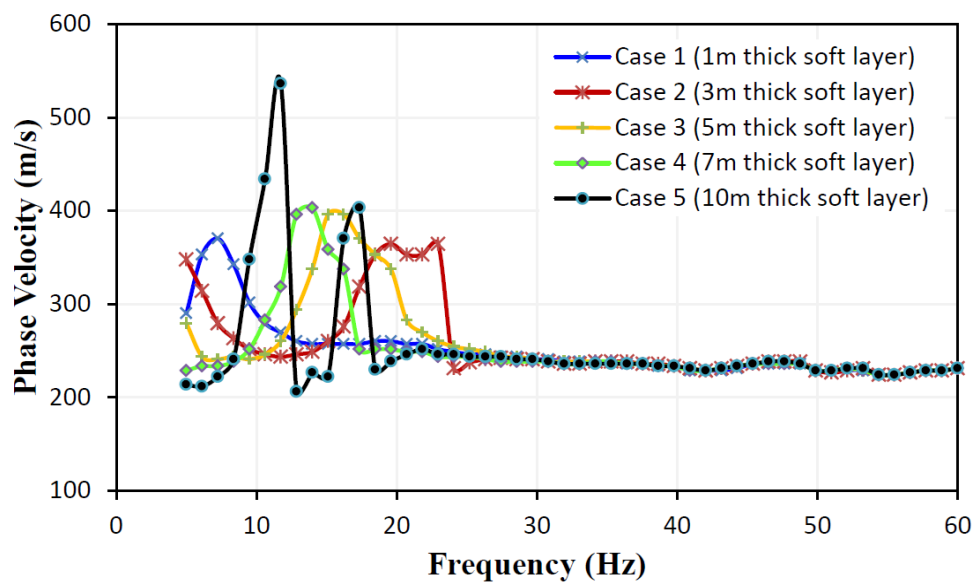


Fig. 12 Comparison of the extracted dispersion curves for all the considered cases

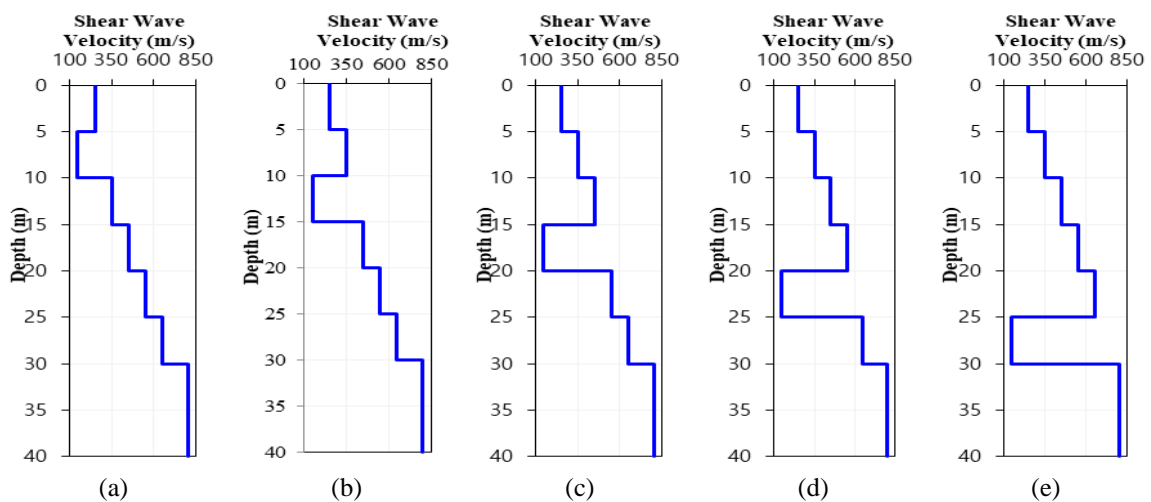


Fig. 13 Shear wave velocity profiles for different positions of trapped soft layer: (a) 5 m, (b) 10 m, (c) 15 m, (d) 20 m and (e) 25 m



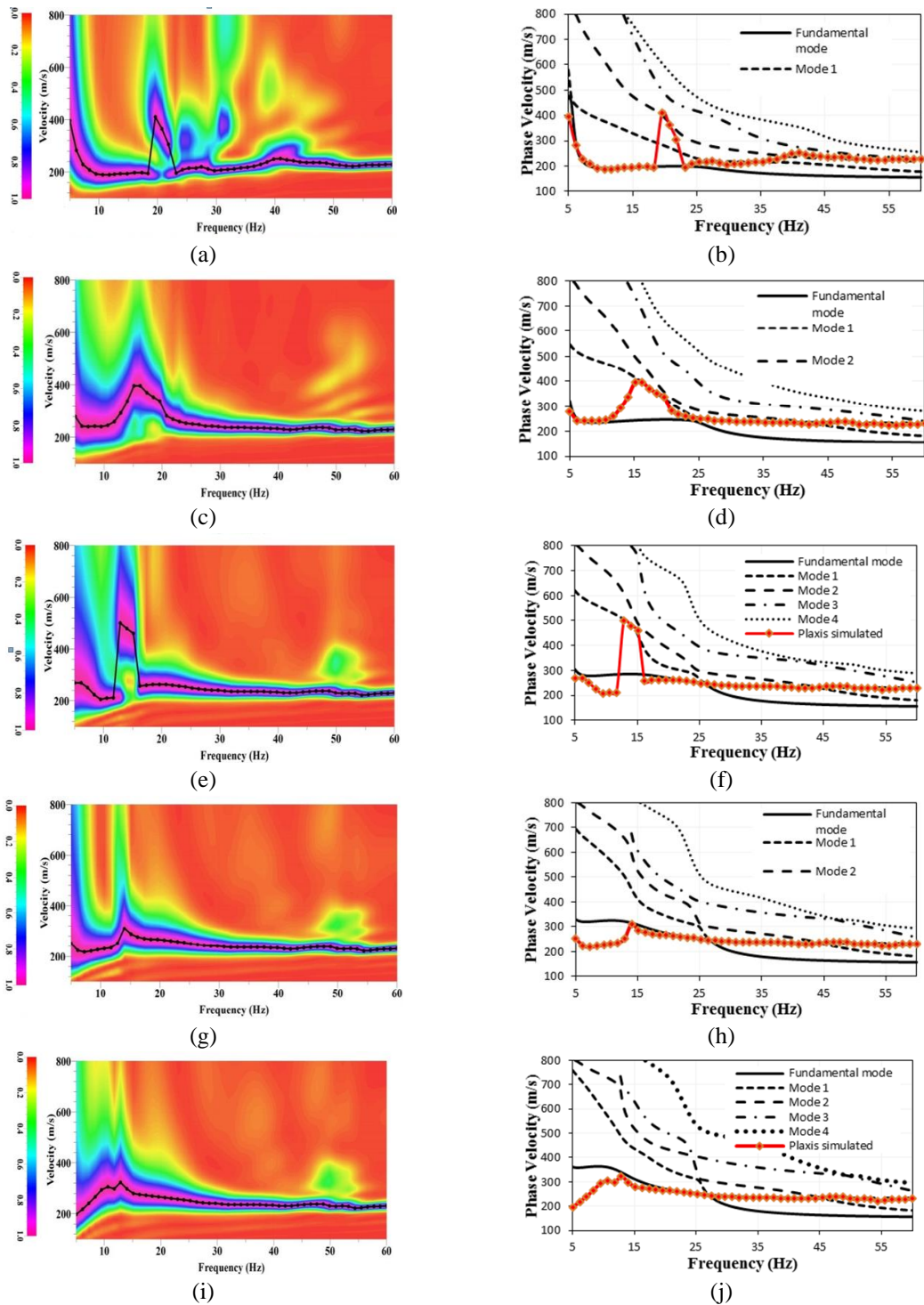


Fig. 14 Dispersion image and superimposed curve of picked dispersion curve with theoretical modal dispersion curves for Case 1 ((a) and (b)), Case 2 ((c) and (d)), Case 3 ((e) and (f)), Case 4 ((g) and (h)) and Case 5 ((i) and (j)), respectively

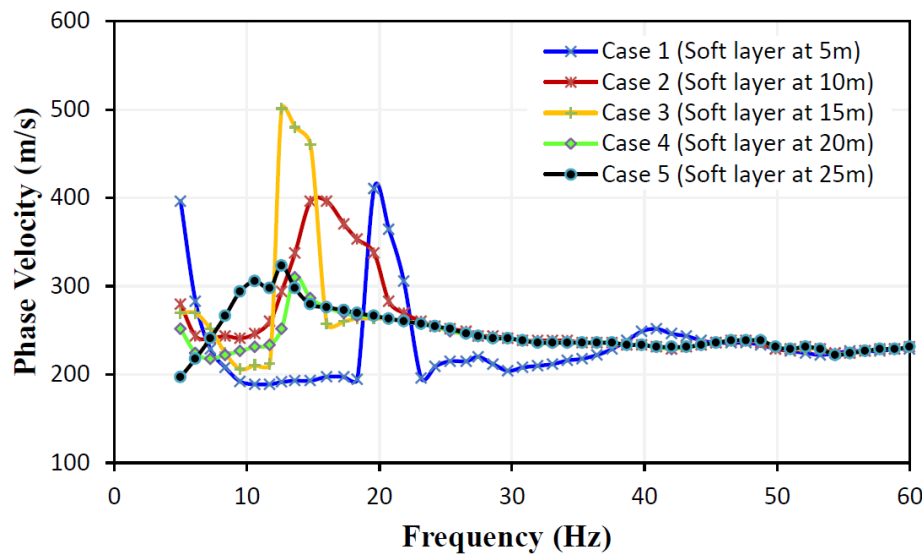


Fig. 15 Comparison of the extracted dispersion curves for all the considered cases of different position of soft layer

### 5.2 Effect of position of soft layer

To study the effect of position of soft layer, we conducted another parametric study placing the 5 m thick soft layer at different depths: 5 m (case 1), 10 m (case 2), 15 m (case 3), 20 m (case 4) and 25 m (case 5) from the surface. Fig. 13 represents the soil profiles of considered five different cases of soft layer position used for the simulation. The dispersion images for all the considered cases are plotted in Fig. 14. Figs 14(a)-14(j) present the dispersion image and superimposed curves of picked dispersion curve and modal curves, for 5 m (case 1), 10 m (case 2), 15 m (case 3), 20 m (case 4) and 25 m (case 5) thick soft layer cases, respectively.

It is very much evident from the plots that when the soft layer is placed at 5 m depth (case 1) (Figs. 14(a) and (b)), high irregularities in the dispersion spectra are observed, and superimposed curves show that the picked dispersion curve jumps towards the second higher mode at  $\sim 18.5$  Hz frequency and it comes back to first higher mode at  $\sim 27.5$  Hz. Another jump occurs at  $\sim 38.5$  Hz to the second higher modes and attains the asymptotic value of first soil layer. As the depth of the soft layer increases, the jump is confined to the first higher mode (Figs. 14(d) and 14(f)) at lower frequencies, and for the last two cases, i.e., soft layer position at 20 and 25 m, there is no such jumps are observed (Figs. 14(h) and 14(j)), dispersion curves follow the fundamental mode at lower frequencies. For all the cases, we can observe that the higher frequencies always following the higher modes to reach the first layer velocity. And to reach to this asymptotic value of first soil layer, in all cases we find a jump from first higher mode to the second higher mode at  $\sim 38$ - $40$  Hz frequency. And at lower frequency region, if the soft layer is present at higher depth, it does not affect the dispersion spectra and extracted dispersion coincides with the fundamental mode of propagation. The frequencies which are reaching up to the depth of soft layer are getting affected by the velocity drop. Frequencies which are confined above the soft layer, exhibits the normally dispersive behavior in the dispersion spectra.

So, to have a better understanding by comparing all the dispersion curves together, all the

picked dispersion curves are plotted in the same plot and shown in Fig. 15. Case 1, i.e., soft layer at 5 m shows high irregularities in the dispersion curve (Fig. 15). Cases 4 and 5, i.e., soft layer at 20 and 25 m, exhibit subtle jump in the dispersion curve but not enough to reach to the 1<sup>st</sup> higher mode. Here also a little frequency shifting of the jump towards the lower frequencies is observed.

So, the presence of soft layer can significantly affect the surface wave dispersion. This parametric study clearly identifies and depicts the consequences of the presence of a soft layer. Sometimes, prior to inversion, some initial information about the soil stratification is required to extract the actual shear wave velocity profile. It is quite difficult from the generated dispersion spectra to predict the accurate layering information, and it may result in different velocity profiles, which are having good fit with the target dispersion curve. This study shows that if there is a presence of any soft layer, it is possible to identify it from the dispersion image and this information can be added in the inversion analysis to get more realistic soil profile when the layered media is inversely dispersive.

## **6. Conclusions**

The surface wave techniques are becoming popular among geotechnical engineers for mechanical characterization of near surface materials. So, the generated dispersion curve can play an important role on the selection of initial model during inversion if the bore-log information is not available. Most of the inversion techniques assume the fundamental mode of propagation, which is true for normally dispersive profile, but this may not be true for inversely dispersive profile. So, it is necessary to accurately extract the proper information from a generated dispersion curves. This paper analyzes how a trapped soft layer in layered media affects the surface wave dispersion. A detailed numerical simulation is performed using finite element method to study the effect of thickness of soft layer and its position in a layered media. The critical observations from the above analysis are as follows:

(i) Normally dispersive soil profiles follow the fundamental mode of propagation, but for inversely dispersive profiles (i.e., when a soft layer is trapped) higher modes also play an important role in the wave propagation. When a soft layer is trapped between two stiff layers in a vertically heterogeneous media, the resulting dispersion spectra is formed by the superposition of several Rayleigh wave modes.

(ii) The presence of soft layer results in mode jump of the generated dispersion spectra towards higher mode and when the thickness of the soft layer increases, there is a shift of frequency of the jump towards the lower frequency is observed. When the thickness of the soft layer is very thin, it is difficult to identify its presence from the dispersion curve, as it behaves as a normally dispersive profile.

(iii) With the increase in thickness of the soft layer, jump towards the higher modes is also quite significant. Multiple mode jumps can be observed towards the higher modes in the dispersion spectra and it makes the wave propagation more complicated while thickness of soft layer increases.

(iv) The position of the soft layer leads to irregularities and complicity in the dispersion curve when it is placed at lower depth. When the soft layer is present at higher depth, the dispersion spectra becomes quite smoother and follows the fundamental mode of propagation at lower frequencies, but at higher frequency, the mode jumps continue till it reach to the velocity of the first soil layer.

(v) The frequencies which are only reaching up to the depth of low velocity layer get affected by the velocity drop. Frequencies which are confined above the soft layer, exhibit only the normally dispersive behavior in the dispersion spectra.

This study exhibits the problems associated with surface wave methods in the presence of soft layer in vertically layered medium. Detection of existence of soft layer is very important in proper mechanical characterization of shallow sub-soil. Use of surface wave methods are highly challenging in this type of soil stratification. Usual practice of considering fundamental mode only is not sufficient for these sites. Higher modes significantly contribute in the dispersion and are required to be considered in the inversion analysis to get more accurate shear wave velocity profiles.

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