# Variability of subgrade reaction modulus on flexible mat foundation

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**Abstract.** The subgrade reaction modulus of a large mat foundation was investigated by using a numerical analysis and a field case study. The emphasis was on quantifying the appropriate method for determining the subgrade reaction modulus for the design of a flexible mat foundation. A series of 3D non-linear FE analyses are conducted with special attention given to the subgrade reaction modulus under various conditions, such as the mat width, mat shape, mat thickness, and soil condition. It is shown that the distribution of the subgrade reaction modulus is non-uniform and that the modulus of subgrade reaction at both the corners and edges should be stiffer than that at the center. Based on the results obtained, a simple modification factor for the subgrade reaction modulus is proposed depending on the relative positions within the foundation in weathered soil and rocks.

**Keywords:** subgrade reaction modulus; mat foundation; numerical analysis; non-uniform; modification factor

## 1. Introduction

A mat foundation is generally used when the ground condition at the foundation level is reliable enough to support the heavy building load within a certain allowable settlement. The analysis and design of mat foundations is conducted using different methods, such as the conventional rigid method, the approximate flexible method, numerical methods such as the finite difference method, finite element method, and finite grid method, and the soil-structure interaction approach (soil spring model).

Soil-structure interaction has been one of the challenging problems in geotechnical engineering. Due to the complexity of soil behavior, the subgrade in soil-structure interaction problems is replaced by a simpler system called a subgrade model. Two methods are mainly used to model the structure-soil interaction. One method is the beam/plate resting on soil springs, and the other is the continuum method that uses finite element analysis (Dutt and Roy 2002, Viktor *et al.* 2009,

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Colasanti 2010, Mohamed and Eric 2011, Suchart et al. 2012).

The continuum method is computationally difficult and requires extensive training due to the three-dimensional and nonlinear nature of the problem. In addition to being time consuming, both in modelling and computation, the continuum method can be exhausting. Conversely, the soil spring method does not give a very realistic representation of the settlement but still gives an indication of what will happen in reality. Despite this growth in technology, there are still many applications in routine practice where analytical software using soil springs is still preferred (Omer and Baki 2010, Dj *et al.* 2013).

The structural part of a mat foundation can be modelled as a flexible or a rigid plate. The conventional rigid method assumes that the mat is a rigid body, which does not consider the mat flexibility and requires a greater thickness. Even very thick mats deflect when loaded by the superstructure loads (Bowles 1996). To overcome this limitation, an improved numerical method for analyzing a mat foundation that can consider mat flexibility and coupled soil springs, YS-MAT, was proposed (Lee *et al.* 2015). Because of the flexibility and deflection of mat foundations, the subgrade reaction modulus of soil is a non-uniform distribution in accordance with its position on the mat foundation.

Plates on soil springs have received considerable attention due to their wide applicability in civil engineering. In preliminary design, engineers prefer to model the soil mass as a series of elastic soil springs, and the elastic constant of the springs are assumed to be the modulus of subgrade reaction  $(k_s)$ . Typically, engineers assume that the spring constant for an edge is half that of the inner mat and twice that of the corner; this concept is based on the assumption that the area that each spring represents has a uniform  $k_s$ . However,  $k_s$  is not uniformly distributed under the mat foundation as assumed by practical engineers.

This paper discusses an appropriate approach for determining the magnitude of the subgrade reaction modulus for a mat foundation design. A series of numerical analyses are performed to take into account various factors influencing the subgrade reaction modulus, i.e., mat width, shape, thickness, and soil condition. Based on the obtained results, a new modification factor is proposed, particularly for large mat foundations.

## 2. Distribution of the subgrade reaction modulus

The analysis of mat foundations is generally performed so that the loads from the columns and a core are supported on the idealized ground model provided by the geotechnical engineers to the structural engineers; this is generally expressed as the modulus of subgrade reaction  $(k_s)$ . In the practical design of mat foundations, the engineers prefer to model the soil as a series of elastic springs, known as a Winkler foundation. The modulus of subgrade reaction can be considered an appropriate interface between the geotechnical and structural engineers.

$$k_s = \frac{q}{\delta} \tag{1}$$

where, q is the soil pressure at a given point and  $\delta$  is the settlement of the mat at the same point.

Since the 1930s, several studies have been performed by many researchers on the subgrade reaction modulus (Biot 1937, Terzaghi 1955, Vlassov 1966, Meyerhof and Baikie 1963, Vesic 1961, Kloppel and Glock 1979, Lee *et al.* 2015, Horvath 1989, Daloglu 2000, Elachachi *et al.* 

2004, Ziaie Moayed and Naeini 2006). Previous authors have each suggested a different but suitable expression. The modulus of subgrade reaction, although simple in its definition, is a difficult parameter to properly evaluate because it is not a unique fundamental property (Becker 2006). Its magnitude depends on several factors, including (1) footing size, (2) footing shape, (3) relative stiffnesses of footing and soil, and (4) type of soil. The plate loading test is usually used by practical engineers to evaluate the values of the subgrade reaction modulus. The plate is rigid, and the mat may be rigid or flexible. These differences between the mat and plate rigidities also produce an error for the values and distribution of  $k_s$ . Directly using a uniform  $k_s$  under a mat foundation from the plate load test without modifications is inappropriate because the size and rigidity of the mat foundation are different than those of the small plate, and these differences will affect the  $k_s$  values. Therefore, the factors (width, shape, and thickness of mat, and soil condition) that influence the modulus of subgrade reaction must be considered.

The magnitude of the modulus of subgrade reaction can vary from one point to another under a footing, mat, or slab, and it significantly affects the behavior of a mat foundation (Farouk and Farouk 2014). It is commonly acknowledged that the assumption of soils acting as elastic springs of uniform stiffness below a footing does not model realistic foundation behavior. The use of a single constant modulus of subgrade reaction can produce misleading results. Therefore, it is very important to accurately determine the distribution of subgrade reaction modulus. The use of an appropriate modulus of subgrade reaction is essential for ensuring the optimal design of a mat foundation.

In this study, three-dimensional finite element analyses (with ABAQUS software) were employed to investigate the non-uniform  $k_s$  values depending upon their position on the large mat foundation. The linear-elastic, perfectly-plastic, Mohr-Coulomb model was adopted to estimate  $k_s$  and to investigate its distribution in the mat foundation. Based on the results obtained, the modification factor of the subgrade reaction modulus was proposed depending on its position on the mat foundation. It is noted that the results may vary depending on the different models used.

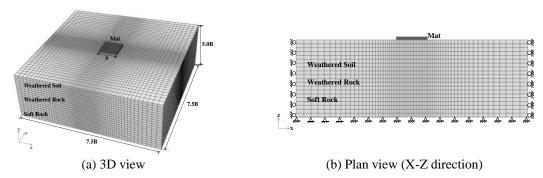


Fig. 1 Typical mesh for FE analysis

Table 1 Summary of material properties (parametric study)

Type	E (MPa)	v	$\gamma (kN/m^3)$	φ (deg.)	c (kPa)	Model
Weathered soil	50	0.32	19	35	15	
Weathered rock	400	0.3	20	38	300	M.C.
Soft rock	1200	0.27	23	40	700	
Mat	28000	0.15	24	-	-	L.E.

# 2.1 FE mesh and boundary conditions

The soil and mat foundation are modeled with finite elements, which allow very rigorous treatment of the soil-structure interaction. The FE package ABAQUS was used. Both the soil and foundation are modeled using solid elements represented by 8-noded brick elements (C3D8R).

Fig. 1 shows a typical idealized 3D FE mesh used in this study. A relatively fine mesh was used near the mat-soil interface and the edge of the mat; the mesh became coarser farther from the mat.

The overall dimensions of the model boundaries comprise a width of 7.5 times the mat width (B) from the mat center and a height equal to 5.0 times the mat width (B). These dimensions were considered adequate to eliminate the influence of boundary effects on the mat foundation performance. A large square and rectangular mat were considered. The bottom boundary was restrained from all movements, and the side boundaries were assumed to be on rollers to allow the downward movement of the soil layers. In numerical analysis, the initial equilibrium state is important. The specified initial stress distributions should match the calculations based on the self-weight of the material. After initial equilibrium, the vertical loading was applied on the top of the mat surface.

### 2.2 Material parameters and interface model

In engineering practice, the mat foundation is bearing on typically 1) weathered soil, 2) weathered rock, and 3) soft rock. Therefore, the 3 types of soils were adopted to analyze variability of subgrade reaction modulus under various ground conditions. The material properties were adopted from some typical values based on the results of a soil investigation in field cases as reported by Seol *et al.* (2008) and Cho *et al.* (2011). An isotropic elastic model was used for the mat foundation, and the material behavior of the soil and rock was modeled with a Mohr-Coulomb model. A mat Young's modulus was applied to a general concrete material parameter. The material properties used in the FE analyses are summarized in Table 1. For the interface model, the contact between the soil (rock) and foundation was described as a slip condition. When contact occurs, the relationship between shear force and normal pressure is governed by a Coulomb's model. A friction coefficient  $\mu$ , where  $\mu$ = $tan(\phi)$ , an elastic stiffness, and a limiting displacement were used to provide convergence. An interface friction coefficient  $\mu$  of 0.7-0.8 was adopted.

#### 2.3 Interpretation of the results

The modulus of subgrade reaction  $(k_s)$  is defined as the ratio of the pressure (q) against the mat to the settlement  $(\delta)$  at a given point. The mat foundation settlement and soil pressure from the 3D FE analyses were used directly. In this study, every increment in soil pressure (q) and settlement  $(\delta)$  of axial load were estimated, and the  $k_s$  values at given points (center, edge, and corner) were calculated as the slope of the q- $\delta$  curve.

## 2.4 Validation of the 3D FEM with field measurements

The validation of the 3D FE model was conducted by a comparison with field measurements for a vertically loaded footing on Korean rock. The footing is  $1.5 \text{ m} \times 1.5 \text{ m}$  with 0.1 m thickness, and it was situated on soft rock. Fig. 2 shows the typical mesh used in this study. A load of 8.5 MN was applied as a uniform load over the footing. The material properties representative

typically results used in this analysis were chosen by the results of field tests and summarized in Table 2.

The comparative results of the 3D FE analysis and field measurements are shown in Fig. 3. Although most settlement values of the 3D FE analysis are smaller than the settlement measured at the center, the predictions by FE analysis are in good agreement with the general trend observed in the field measurements.

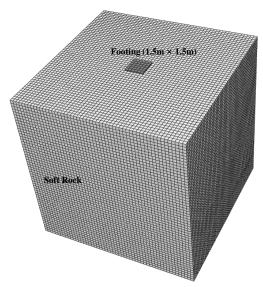


Fig. 2 Typical 3D mesh for validation

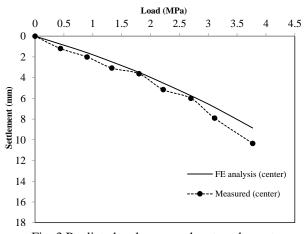


Fig. 3 Predicted and measured mat settlement

Table 2 Summary of material properties (validation case)

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0	E (MPa)	v	$\gamma (kN/m^3)$	φ (deg.)	c (kPa)	Model
Soft rock	900	0.28	27	35	130	M.C
Footing	210000	0.2	75	-	-	L.E.

# 2.5 FE analysis results and discussion

A series of numerical analyses on mat foundation were performed for different mat widths (B), mat lengths (L), mat thicknesses (t) and soil types, as shown in Table 3. A total of 150 cases of rectangular and square mat foundations were considered, as shown in Fig. 4. In this section, only a selection of typical analysis results is presented.

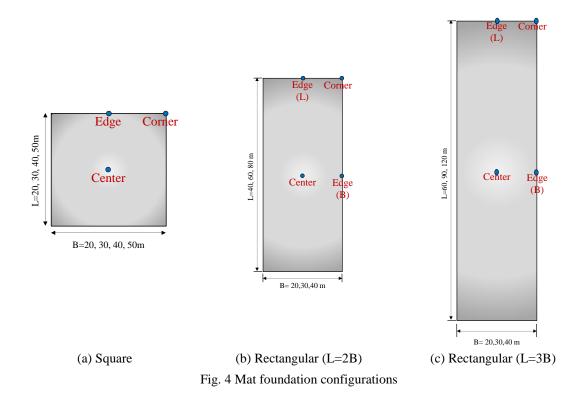
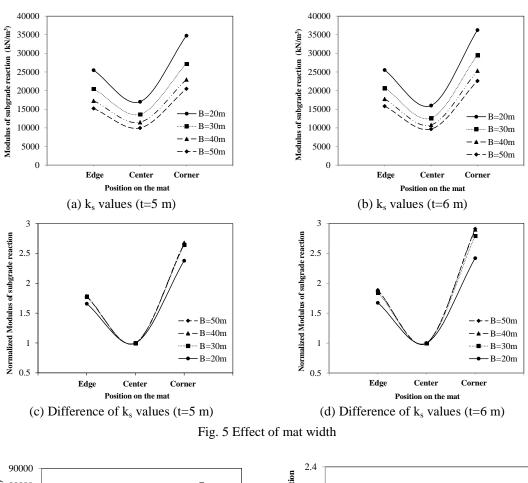


Table 3 Summary of the FE analyses

	Rock type		
B (m)	L (m)	t (m)	
	20		
20	40	2	
	60	<del>-</del>	Weathered soil
	30	3	&
30	60	4	Weathered rock
	90	4	&
	40	5	Soft rock
40	80		
	120	6	
50	50		



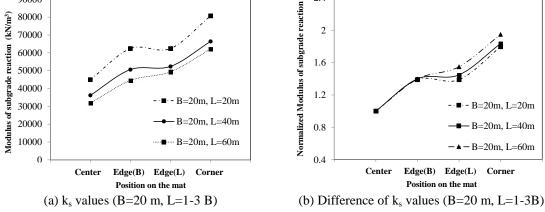


Fig. 6 Effect of mat shape

# 2.5.1 Effect of the mat width

The effect of mat width on the modulus of subgrade reaction  $(k_s)$  was investigated by varying the mat width: B=20 m, 30 m, 40 m, and 50 m. A square mat on weathered rock was considered.

Figs. 5(a)-5(b) present the  $k_s$  values versus the position (center, edge, and corner) on the mat with varying mat width (B) for the same mat thickness (t=5 m and 6 m). The results in all cases of subgrade reaction modulus show a good correlation with mat width. As the mat width increases, the modulus of subgrade reaction decreases almost linearly. Additionally, Figs. 5(c)-5(d) show the  $k_s$  distribution that is normalized by the  $k_s$  value at the center of the mat foundation. With increasing mat width, the difference in  $k_s$  values at each position (center, edge, and corner) increases slightly.

### 2.5.2 Effect of the mat shape

Based on the literature (Bowles 1996), a wide mat will settle more than a narrow mat with the same load, and the stresses below square loaded areas are different from those below long, narrow loaded areas. Therefore, k<sub>s</sub> will differ.

The effect of mat shape on  $k_s$  was studied. Square and rectangular mat foundations on soft rock were considered. The mat length (L) ranged from 1 B to 3 B for a mat width (B)=20 m. Fig. 6 shows the  $k_s$  values versus the position (center, edge (B), edge (L), and corner) on the mat with varying mat shapes under the same mat thickness (t=3 m). The results show that the  $k_s$  values of square and rectangular mats are different. More specially, it is shown that the subgrade reaction modulus decreases as the mat length (L) increase under same width (B). Conversely, normalized  $k_s$  values of square mat at edge and corner were lower than those of rectangular mats.

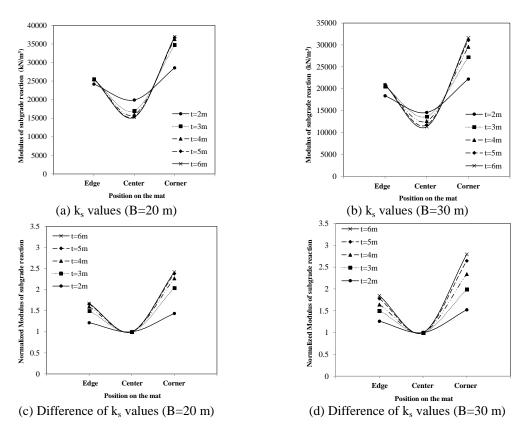


Fig. 7 Effect of mat thickness

#### 2.5.3 Effect of the mat thickness

The effect of the mat thickness (t) was studied. The rigidity of the mat foundation was calculated using Eq. (2).

$$K_{r} = \frac{EI_{b}}{E_{c}B^{3}}$$
 (2)

where E is the modulus of elasticity of the foundation,  $E_s$  is the modulus of elasticity of the soil, B is the foundation width, and  $I_b$  is the moment of inertia of the foundation per unit length perpendicular to B.

ACI Committee 336 (1988) recommends that if the  $K_r$  of a foundation is equal to 0.5 or larger, then the footing can be considered rigid and the variation of soil pressure can be determined based on simple statics. However, if the relative stiffness factor is less than 0.5, then the footing should be designed as a flexible foundation.

Changing the mat thickness affects the mat rigidity, which affects the contact stress distribution under the mat. Therefore,  $k_s$  should be affected by the mat rigidity. Figs. 7(a)-7(b) show the distribution of  $k_s$  values (center, edge, and corner) for different thicknesses (t=2 m, 3 m, 4 m, 5 m, and 6 m) for the same mat foundations on weathered rock. For instance, for a 20 m×20 m square mat, with thicknesses of 3 m, 4 m, 5 m, and 6 m,  $K_r$  is 0.49, 0.93, 1.83, and 3.15, respectively. Therefore, the mat is rigid. Furthermore, in the case of t=2 m, the mat is flexible.

The difference in the contact stress increased and the differential settlement of the mat decreased with increasing mat thickness; the mat thicknesses of 3 m, 4 m, 5 m, and 6 m were relatively rigid compared to the mat thickness of 2 m. For a flexible mat, with a thickness of 2 m, the contact stress is almost similar and settlement is concentrated under the footing center. With increasing the footing rigidity to a rigid mat, with thicknesses of 3 m, 4 m, 5 m, and 6 m, the contact stress near the edge and corner increased and the differential settlement decreased. Therefore,  $k_s$  near the edge and corner increased slightly with slight decreases in the concentration under the footing center. Figs. 7(c)-7(d) show the  $k_s$  values normalized by the  $k_s$  value at the center of the mat foundation. As the mat thickness increases, the difference between  $k_s$  values at each position (center, edge, and corner) increases slightly.

As a result, the distribution of  $k_s$  varied due to the change in footing rigidity. The distribution of  $k_s$  is non-uniform depending on the mat thickness.

# 3. Proposed non-uniform subgrade reaction modulus

The obtained results show that the modulus of subgrade reaction is related to the shape of the contact stress and settlement under the foundation and that the distribution of the modulus of subgrade reaction is non-uniform. The subgrade reaction modulus especially varies when a foundation is subjected to uniform loading because vertical soil pressure near the edges and corners of the foundation are high while the settlements at the same locations are smallest due to the bending of the foundation, producing a high soil spring constant. Conversely, at the center of the foundation, the soil pressure is smaller and the settlement is higher; therefore, the soil spring constant is smaller at that location. With increasing foundation rigidity,  $k_{\rm s}$  is concentrated near the corner and is low under the center of the foundation.

In this section, a correlation between the non-uniform modulus of subgrade reaction and influencing factors is examined based on the results of a broad-based parametric study.

Fig. 8 shows an example of a typical relationship between mat thickness, mat width, and normalized  $k_s$ . As shown in Fig. 8, the  $k_s$  values of the edge and corner increase as the mat width and thickness increase, and these trends of  $k_s$  were confirmed regardless various soil types. Consequently, increasing mat thickness and width increases the  $k_s$  values at both the corner and edge more than that of the center. Additionally, the typical normalized  $k_s$  versus mat shape is shown in Fig. 9. For instance, the rectangular mats on weathered and soft rock are presented. As shown, increasing mat length (L) can result in expanding the normalized  $k_s$  at its respective position. The modulus of subgrade reaction at both the corner and edge is stiffer than that of the center.

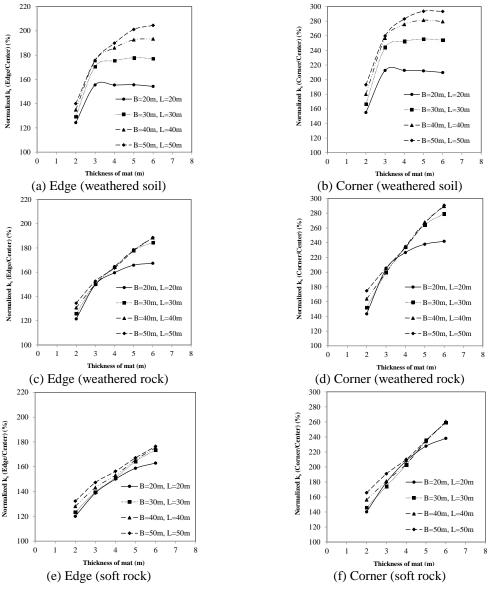


Fig. 8 Effect of mat width, thickness on k<sub>s</sub>

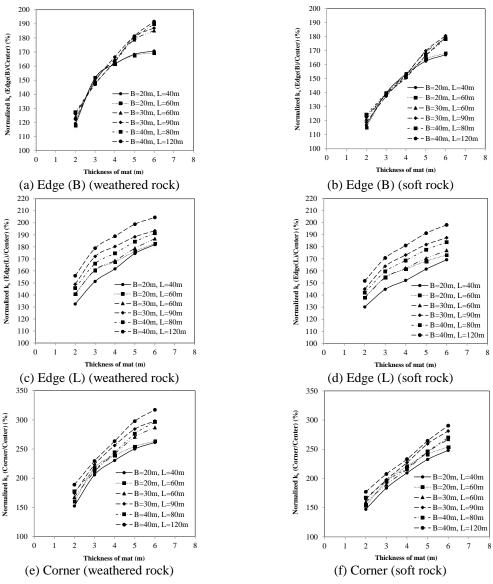


Fig. 9 Effect of mat shape on k<sub>s</sub>

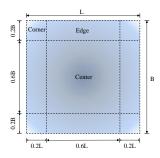


Fig. 10 A typical mat divided into zones

In preliminary design, to model the soil springs appropriately, the pseudo-coupled concept can be used. The idea is to substitute stiffer  $k_s$  values at the corners and edges and smaller  $k_s$  values in the center of the mat foundation. We can implement this technique in the following way: Divide the mat into several zones, such as center, edge, and corner zones, as shown in Fig. 10 (ACI Committee 336 1993). Then, assign a  $k_s$  value to each zone. Based on the conducted results, the recommended modification factor of subgrade reaction modulus at relative positions on the large mat foundation is proposed in Tables 4-6.

Table 4 Recommended modification factor of  $k_{\text{s}}$  in weathered soil condition

	1100011111101			100001 01	115 111		011 001101				
F	3 (m)		20			30			40		50
I	L (m)	20	40	60	30	60	90	40	80	120	50
t (m)	Location		-			-			-		-
_	Edge (B) Edge (L)	1.24	1.24 1.36	1.23 1.46	1.29	1.29 1.45	1.29 1.55	1.35	1.35 1.52	1.35 1.64	1.40
2	Center	1	1	1	1	1	1	1	1	1	1
	Corner	1.55	1.67	1.78	1.67	1.85	1.96	1.80	1.99	2.14	1.93
3	Edge (B) Edge (L)	1.55	1.58 1.68	1.57 1.69	1.71	1.72 1.74	1.74 1.82	1.76	1.77 1.80	1.79 1.92	1.76
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.13	2.30	2.33	2.44	2.55	2.67	2.57	2.67	2.87	2.60
	Edge (B) Edge (L)	1.55	1.59 1.74	1.58 1.77	1.75	1.78 1.85	1.79 1.88	1.86	1.88 1.90	1.89 1.99	1.90
4	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.13	2.36	2.40	2.53	2.70	2.78	2.76	2.88	3.04	2.83
5	Edge (B) Edge (L)	1.56	1.60 1.78	1.50 1.83	1.78	1.83 1.95	1.82 1.97	1.93	1.96 2.02	1.95 2.07	2.01
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.12	2.39	2.47	2.56	2.76	2.86	2.81	3.01	3.12	2.94
	Edge (B) Edge (L)	1.54	1.59 1.78	1.59 1.85	1.77	1.83 1.98	1.83 2.02	1.93	1.99 2.10	1.96 2.12	2.05
6	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.10	2.38	2.47	2.54	2.74	2.85	2.80	3.01	3.09	2.93

Table 5 Recommended modification factor of  $k_{\rm s}$  in weathered rock condition

	B (m)		20			30			40		50
	L (m)	20	40	60	30	60	90	40	80	120	50
t (m)	Location		-			-			-		-
	Edge (B)	1.01	1.19	1.18	1.26	1.24	1.22	1.21	1.27	1.26	1.24
2	Edge (L)	1.21	1.33 1.41	1.26	1.41	1.49	1.31	1.46	1.56	1.34	
2	Center	1	1	1	1	1	1	1	1	1	1
	Corner	1.43	1.53	1.61	1.52	1.68	1.74	1.64	1.77	1.89	1.75

Table 5 Continued

I	B (m)		20			30			40		50
I	L (m)	20	40	60	30	60	90	40	80	120	50
t (m)	Location		-			-			-		-
	Edge (B)	1.50	1.52	1.52	1.50	1.50	1.49	1.50	1.48	1.47	1.52
3	Edge (L)	1.50	1.51	1.61	1.50	1.61	1.72	1.50	1.66	1.79	1.53
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.04	2.06	2.18	2.00	2.11	2.23	1.99	2.14	2.30	2.06
E	Edge (B)	1.60	1.61	1.62	1.64	1.65	1.67	1.65	1.63	1.63	1.63
4	Edge (L)	1.00	1.62	1.68	1.69	1.69	1.80	1.03	1.75	1.89	
4	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.27	2.31	2.39	2.34	2.42	2.56	2.35	2.44	2.64	2.33
	Edge (B)	1.66	1.68	1.68	1.78	1.79	1.81	1.78	1.80	1.81	1.78
5	Edge (L)	1.00	1.75	1.76	1.76	1.79	1.88	1.76	1.84	1.99	
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.38	2.50	2.54	2.64	2.71	2.84	2.68	2.76	2.98	2.67
	Edge (B)	1.67	1.71	1.69	1.85	1.85	1.87	1.88	1.90	1.92	1.89
6	Edge (L)	1.07	1.82	1.83	1.65	1.87	1.93	1.00	1.91	2.05	
6	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.42	2.61	2.64	2.79	2.87	2.98	2.90	2.97	3.17	2.91

Table 6 Recommended modification factor of  $\boldsymbol{k}_{\boldsymbol{s}}$  in soft rock condition

I	3 (m)		20			30			40		50
I	_ (m)	20	40	60	30	60	90	40	80	120	50
t (m)	Location		-			-			-		-
	Edge (B)	1.20	20 1.17	1.15	1.23	1.21	1.19	1.28	1.24	1.23	1.32
2	Edge (L)	1.20	1.30	1.38	38 1.23 1.38 1.45	1.20	1.42	1.52	1.32		
2	Center	1	1	1	1	1	1	1	1	1	1
	Corner	1.40	1.47	1.54	1.46	1.59	1.64	1.56	1.67	1.77	1.66
	Edge (B)	1.39	1.40	1.40	1.40 1.39 1.54	1.39	1.37	1.43	1.40	1.39	1 47
3	Edge (L)	1.39	1.45	1.55		1.64	1.43	1.59	1.71	1.47	
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	1.80	1.83	1.95	1.74	1.90	1.98	1.81	1.95	2.08	1.91
	Edge (B)	1.50	1.53	1.54	1.51	1.53	1.53	1.52	1.51	1.51	1.56
4	Edge (L)	1.50	1.52	1.62	1.51	1.62	1.73	1.53	1.69	1.81	
4	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.08	2.10	2.21	2.03	2.16	2.27	2.04	2.18	2.33	2.10
	Edge (B)	1.50	1.63	1.64	1 61	1.69	1.70	1 65	1.66	1.67	1.67
5	Edge (L)	1.59	1.61	1.68	1.64	1.70	1.82	1.65	1.77	1.91	
3	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.28	2.33	2.42	2.36	2.45	2.59	2.34	2.46	2.64	2.35

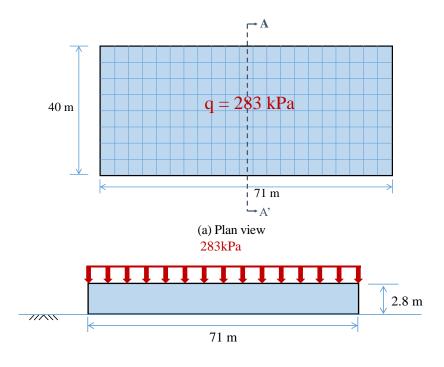
Table 6 Continued

I	B (m)		20			30			40		50
I	L (m)	20	40	60	30	60	90	40	80	120	50
t (m)	Location		-			-			-		-
	Edge (B)	1.62	1.67	1.68	1.74	1.79	1.81	1.75	1.78	1.79	1.76
6	Edge (L)	1.63	1.69	1.73	1./4	1.77	1.87	1./3	1.84	1.98	
6	Center	1	1	1	1	1	1	1	1	1	1
	Corner	2.38	2.48	2.54	2.59	2.67	2.81	2.61	2.70	2.91	2.59

Table 7 Input parameters

Type	E (MPa)	ν	$\gamma (kN/m^3)$	φ (deg.)	c (kPa)
Bouldery soil	80	0.3	19	-	1046
Mat	28,000	0.15	24	-	-
Subgrade reaction	$k_s^{\ a}$	Center	Edge (B) <sup>b</sup>	Edge (L) <sup>b</sup>	Corner <sup>b</sup>
modulus (kN/m³)	27,963	27,963	49,494 (=177% of Center)	50,333 (=180% of Center)	74,661 (=267% of Center)

<sup>&</sup>lt;sup>a</sup>k<sub>s</sub>: The value obtained from literature <sup>b</sup>Based on Table 4



Bouldery soil

(b) Section view

Fig. 11 Schematic diagram of mat foundation

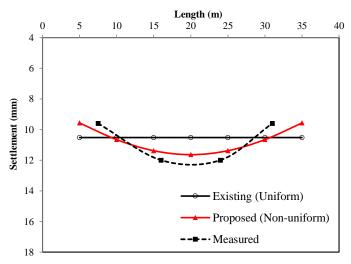


Fig. 12 Mat foundation settlement distribution (A-A')

## 4. Comparison with field measurement

A comparison of numerical analysis based on the results of this study with field measurement was conducted for an application of the proposed subgrade reaction modulus. In this analysis, the subgrade reaction modulus for the weathered soil was adopted under the same conditions like real field case. The non-uniform subgrade reaction modulus at relative positions on the mat were used in the YS-MAT program (Yonsei Mat foundation). As discussed in detail in (Lee *et al.* 2015), YS-MAT can consider the mat flexibility and includes the pseudo-coupled method. This model uses soil springs that have different values of subgrade reaction modulus depending on the relative positions on the mat.

This test site was located near the Central Business District in Singapore. The large mat is  $40\times71$  m with a thickness of 2.8 m and supports the 33-story Savu Tower. An average uniform pressure of 283 kPa was applied over the whole mat area, and the mat was installed in Bouldery soil. A schematic diagram of the mat foundation is shown in Fig. 11.

The settlement behavior of a mat foundation reported by Wong *et al.* (1996) is compared to the predicted values by the proposed method. Each soil spring stiffness at the relative positions in a mat (center, edge, and corner) was used, according to Table 4. The input parameters are summarized in Table 7.

Fig. 12 presents the mat settlements along the centerline predicted by the proposed design method (YS-MAT using the proposed non-uniform subgrade reaction modulus) and YS-MAT using the existing uniform subgrade reaction modulus; this figure compares the design results to the measured settlement. The analysis using uniform subgrade reaction modulus gives a uniform displacement, as reported in Daloglu and Vallabhan (2000). Conversely, the proposed design method gives a dish-shaped settlement of the mat foundation that would be expected for an actual situation and is in good agreement with the measured settlement distribution. As a result, the proposed design method gives a fair approximation of the general trend of the measured settlement compared to the existing uniform subgrade reaction modulus. Therefore, the proposed design method can properly predict the settlement behavior of a mat foundation.

#### 5. Conclusions

- The objective of this study is to discuss an appropriate approach to determine the subgrade reaction modulus for a mat foundation design. A series of 3D FE analyses are conducted to investigate the effect of influencing factors on the modulus of subgrade reaction for a large mat foundation; this effect could not be fully clarified in field and laboratory tests. Numerical modeling techniques are also verified by field measurements. Based on the parametric studies, the conclusions of this study are as follows:
- Based on the obtained results, the modulus of subgrade reaction of a large mat foundation is significantly dependent on the following factors: mat width, mat shape, mat thickness, and soil condition. The results in all of the subgrade reaction modulus  $(k_s)$  cases show a good correlation with the influencing factors.
- With increasing mat size, the  $k_s$  values decrease almost linearly, and the difference in  $k_s$  values at the relative positions increases slightly. Additionally, with increasing mat thickness, the contact stress near the edge and corner increased and the differential settlement decreased. As a result, the  $k_s$  is related to the shape of the contact stress and settlement under the foundation, and the distribution of the modulus of subgrade reaction is non-uniform.
- Most commercial mat analysis programs use the subgrade reaction modulus (Winkler spring) to represent the soil-structure interaction, and these programs can use the pseudo-coupled concept. The  $k_s$  values should be larger at the corner and edge of the mat and smaller at the center of the mat. Consequently, the recommended modification factor of the subgrade reaction modulus at relative positions on the large mat foundation is proposed. Therefore, it is recommended to use an approximately 200% stiffer value of  $k_s$  at the corner and edge of mat compared to the center of the mat foundation.
- The proposed subgrade reaction modulus is validated by comparing it to the field measurements. The proposed subgrade reaction modulus reasonably represents the settlement behavior of a mat foundation. The application of the proposed modification factor at relative positions on a large mat foundation will lead to more practical and appropriate designs for mat foundations.

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