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Collapse analysis of shallow tunnel subjected to seepage in layered soils considering joined effects of settlement and dilation

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Abstract. The stability prediction of shallow buried tunnels is one of the most difficult tasks in civil engineering. The aim of this work is to predict the state of collapse in shallow tunnel in layered soils by employing non-associated flow rule and nonlinear failure criterion within the framework of upper bound theorem. Particular emphasis is first given to consider the effects of dilation on the collapse mechanism of shallow tunnel. Furthermore, the seepage forces and surface settlement are considered to analyze the influence of different dilation coefficients on the collapse shape. Two different curve functions which describe two different soil layers are obtained by virtual work equations under the variational principle. The distinct characteristics of falling blocks up and down the water level are discussed in the present work. According to the numerical results, the potential collapse range decreases with the increase of the dilation coefficient. In layered soils, both of the single layer's dilation coefficient and two layers' dilation coefficients increase, the range of the potential collapse block reduces.

Keywords: collapse; layered soils; non-associated flow rule; seepage force; surface settlement

1. Introduction

A massive number of mountainous tunnels have been excavated in practical engineering projects, and a large number of collapses often occur to the shallow tunnels. The collapse of the tunnel exerts great threats to both the life and property of the constructors. In order to avoid the loss of the collapse of the shallow tunnels, different sorts of theoretical calculation methods for predicting the stability of shallow tunnels are used now.

Among the large number of methods which are suitable for solving the stability problem of shallow tunnels, limit equilibrium method, finite element theory and the limit analysis method are widely used. Due to the limitations of the limit equilibrium method and finite element theory, the limit analysis method becomes increasingly popular to evaluate the stability problems. Since 1970, the upper bound theorem was widely used in engineering (Chen 1975). Then this theorem had great importance in the field of tunnel engineering because of its great validity in dealing with the stability problems in underground structures. By establishing three dimensional failure

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mechanisms, Leca and Dormieux (1990) used the limit analysis method and model test to prove that the mode is rational. On the basis of the previous work and reliability theory, Yang and Li (2017) examined the roof stability of the shallow tunnel with surface settlement, and the results had a great improvement. The earth pressure of tunnel face was estimated with nonlinear failure criterion and reliability theory (Yang *et al*. 2016). The three dimensional stability of the shallow circular tunnel were analyzed by Mollon *et al*. (2009, 2011), with the help of the combination of the reliability analysis and limit analysis. Saada *et al*. (2012) investigated the failure mechanism of rock slope considering the seepage effects.

By considering the nonlinear mechanical characteristics of material in tunnel project, the linear criterion which is used to evaluate the stability of the tunnels has been replaced by the nonlinear criterion (Fraldi and Guarracino 2009, 2010, 2011, Hoek and Brown 1997, Mohammadi and Tavakoli 2015). During the process of applying nonlinear failure criterion, a generalized tangential methodology was suggested to calculate the energy dissipation rate and the external work rate accurately (Li and Yang 2016). The Hoek-Brown failure criterion has been widely used among many kinds of nonlinear criterion. Especially on the basis of Hoek-Brown failure criterion, Fraldi and Guarracino (2009, 2010, 2011) characterized the collapse shape. The curved failure mechanism was constructed according to the theory that the energy dissipation rate and external work rate were calculated along the velocity discontinuity (Subrin and Wong 2012).

In terms of shallow tunnels, the ground water plays a significant role in the stability analysis of underground structures. However, the effect of water pressures and seepage forces were ignored in the stability analysis in some scholars' researches by considering the fact that piles of underground structures were located in the saturated stratum. It is found that there exists an optimum size for grouting zone to supporting pressure. Huang *et al*. (2013) used the approach of conformal mapping to obtain the analytical solution to steady ground water flowing into a horizontal tunnel. Feng *et al*. (2012) put forward that the crown and bottom of underwater shield tunnel with large profile are liable to collapse by model test. Based on what mentioned above, the seepage forces and surface settlement would be considered in this work.

At the beginning, using limit analysis to study the stability problems of geotechnical engineering were mainly based on associated flow rule and homogeneous materials (Lee 2016). Later, some scholars began to discuss the nonhomogeneous and layered materials in the limit analysis so as to obtain more accurate failure loads (Yang and Du 2016, Yang and Li 2016, Yang and Xu 2017, Yang and Yao 2017). Drescher and Detourany (1993) have a substantial contribution to determine the limit load in a translational failure mechanism for non-associative materials with a coaxiality of the principal directions stresses and deformation rates. Kumar (2004) calculated the stability coefficient of soil slope based on associated flow rule and non-associated flow rule respectively, and analyzed the influence of dilation angle on the slope stability coefficient with coaxial non-associated flow rule and non-coaxial non-associated flow rule. Combing nonlinear failure criterion, Yang and Xiao (2016) analyzed the surrounding rock stability problem of shallow tunnel in karst region.

However, there are few studies about the influence of non-associated flow rule on the collapse mechanism of shallow tunnels subjected to seepage with help of vatiation principle. In this paper collapse mechanisms in circular tunnels which are excavated in single soil layer with different water levels and layered soils are discussed through combining nonlinear failure criterion with non-associated flow rule within the framework of variation theory. The failure mechanism analysis provide theoretical basis and reference for the stability analysis of shallow tunnels and the optimization design of supporting system on tunnels in future.

2. Associative and non-associative flow rules

For ideal plastic materials (Chen 1975), the yield function f is only related to the stress component σ_{ij} and has nothing to do with the strain component ε_{ij}, which can be expressed as

$$f\left(\sigma_{ij}\right)=0 \tag{1}$$

In stress space: (1) If the stress point is located in the curved surface, then $f(\sigma_{ij}) < 0$, which means that it is under elastic state; (2) if the stress point is located above the surface, then $f(\sigma_{ij}) = 0$, which means that it is under plastic state, and the plastic strain increment takes the following form

$$\mathrm{d}\varepsilon_{ij}^{P}=\mathrm{d}\lambda\frac{\partial f}{\partial\sigma_{ij}} \tag{2}$$

where $\mathrm{d}\varepsilon_{ij}^{P}$ is plastic strain increment. λ is the proportionality coefficient, $\lambda > 0$.

With the yield function f, the plastic potential function g is also a function subjected to stress component σ_{ij}, which can be expressed as

$$g\left(\sigma_{ij}\right)=0 \tag{3}$$

In the plastic theory, the direction of plastic strain increment is determined by flow rule, and at any stress points, it must be perpendicular to its plastic potential surface, as shown in Fig. 1. Therefore, the flow rule is also known as orthogonal law, and the plastic strain increment can be expressed as

$$\mathrm{d}\varepsilon_{ij}^{P}=\mathrm{d}\lambda\frac{\partial g}{\partial\sigma_{ij}} \tag{4}$$

Based on the postulate, stabilizing materials satisfy the following equation

$$\mathrm{d}\sigma_{ij}\mathrm{d}\varepsilon_{ij}^{P}\geq 0 \tag{5}$$

To satisfy the above formula, the plastic strain increment and the yield surface must be

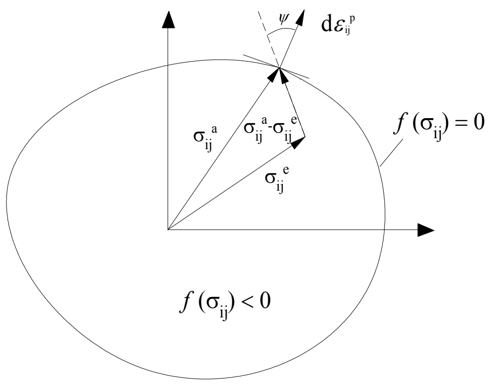


Fig. 1 Illustration of yield surface and flow rule

orthogonal and the yield surface should be convex. The plastic potential surface g and the yield surface f must be coincident, which is the so-called associated flow rule, namely

$$f = g \tag{6}$$

According to assumptions of upper bound theorem of limit analysis, geotechnical materials obey the associated flow rule when plastic deformation occurs. However, a large number of results prove that, based on the hypothesis, the dilation of geotechnical materials usually satisfy non-associated flow rule in general, and the associated flow rule is only a special case. Making reference to the associated flow rule, the dilation angle of geotechnical materials is equal to the internal frictional angle. While according to the non-associated flow rule, the dilation angle is less than the internal frictional angle. In common limit analysis the relationship between inter-block forces and displacements is governed by the associative flow rule, which means that sliding along a joint will be accompanied by separation, or dilation (Drucker 1954). Thus a tangential displacement δ_t along a joint will be accompanied by a normal displacement δ_n. According to the following equation (Babiker *et al*. 2014)

$$\delta_n = \delta_t \tan\psi \tag{7}$$

where the angle ψ is the dilation angle, and $\psi = 0$ means incompressible plastic flow. The angle ψ may vary within the range $\phi > \psi \geq 0$ on the basis of the work of Drescher and Detourany (1993). And it will equal the friction angle ϕ when using a conventional limit analysis approach due to the normality principle, leading to what is termed 'associative friction'.

For non-associative materials with a coaxial flow rule and a linear MC yield condition, the components of tractions and velocity jumps at the velocity discontinuity lines satisfy

$$\tau_n = c^* + \sigma_n \tan\phi^* \tag{8}$$

$$[v]_n = [v]_t \tan\psi \tag{9}$$

where $\tan\phi^* = \eta \tan\phi$, $c^* = \eta c$ and $\eta = \dfrac{\cos\psi\cos\phi}{1-\sin\psi\sin\phi}$

For non-associative materials with a coaxial flow rule and a nonlinear Power-Law yield condition, the parameters can be modified on the basis of the above work.

$$c^* = \eta c = \eta c_0 \tag{10}$$

$$\tan\phi^* = \eta\tan\phi = \eta c_0 / \sigma_t \tag{11}$$

The nonlinear Power-Law failure criterion is expressed as

$$\tau_n = c_0(1+\sigma_n/\sigma_t)^{1/m} \tag{12}$$

Substituting Eqs. (10) and (11) into Eq.(12), the expression based on nonlinear failure criterion and non-associated flow rule is revised

$$\tau_n = \eta c_0 \left(1 + \sigma_n \cdot \left(\eta \frac{c_0}{\sigma_t}\right) \Big/ \eta c_0\right)^{\frac{1}{m}} = \eta c_0 (1 + \frac{\sigma_n}{\sigma_t})^{1/m} \tag{13}$$

When $\eta = 1$, the expression is turned into nonlinear failure criterion relating to the flow rule.

3. Limit analysis with nonlinear failure criterion

3.1 Power-law criterion

Various strength functions (Mohr envelopes), have been proposed to represent nonlinear strength functions for soils. Many publications utilized a simple power law relation of the form. According to the non-linear strength criterion, the normal component of stress can be written as

$$\sigma_n = \sigma_t - \sigma_t \left(\frac{m\sigma_t}{\eta c_0}\right)^{\frac{m}{1-m}} f'(x)^{\frac{m}{m-1}} \tag{14}$$

$$\tau_n = -\eta c_0 \left(\frac{m\sigma_t}{\eta c_0}\right)^{\frac{1}{1-m}} f'(x)^{\frac{1}{m-1}} \tag{15}$$

So that, by virtue of the Greenberg minimum principle, the effective collapse mechanism can be found by minimizing the total dissipation, the dissipation density of the internal forces on the detaching surface, $\dot{D}_i$, results

$$\dot{D}_i = \sigma_n \dot{\varepsilon}_n + \tau_n \dot{\gamma}_n = \left\{\sigma_t - \left(\frac{1-m}{m}\right)(\eta c_0)^{\frac{m}{m-1}} (m\sigma_t)^{\frac{1}{1-m}} f'(x)^{\frac{m}{m-1}} \Big/ \left[w\sqrt{1+f'(x)^2}\right]\right\} v \tag{16}$$

where $\dot{\varepsilon}_n$ is normal plastic strain, $\dot{\gamma}_n$ is shear plastic strain rates, w is the thickness of the plastic detaching zone, and $\dot{u}$ is the velocity of the collapse block.

3.2 Upper bound theorem

zThe theory of the upper bound has been widely used to the predictions of the stability of the tunnels. The upper bound theorem of limit analysis can be depicted as: when the velocity boundary conditions and consistency conditions for strain and velocity are satisfied by the maneuvering-allowable velocity field which is built, the actual loads should be no more than the values of the calculated loads which is derived from the equation constituted by equating the external rate of work and the rates of the internal energy dissipation. According to Chen (1975), the upper bound theorem with seepage forces effect can be written as follows

$$\int_V \sigma_{ij} \cdot \dot{\varepsilon}_{ij} dV \geq \int_S T_i \cdot v_i dS + \int_V X_i \cdot v_i dV + \int_V -grad\, u \cdot v_i dV \tag{17}$$

where σ_{ij} is the stress tensor, $\dot{\varepsilon}_{ij}$ is the strain rate in the kinematically admissible velocity field, respectively. T_i is the limit load exerted on the boundary surface. S is the length of velocity discontinuity, X_i is the body strength which is caused by weight, V is the volume of the plastic zone, v_i is the velocity along the velocity discontinuity surface.

4. Collapse mechanism for shallow tunnel with varying water table

Many scholars have studied the failure modes of the deep tunnel roof with arbitrary cross sections in layered rocks. By considering the fact that a large number of shallow-buried tunnels are also constructed in the layered stratums, this work investigates the failure mechanism of shallow tunnels under the condition of varying water table considering the surface settlement. The upper bound solutions derived from this work have general applicability and can be used more widely. Due to the deformation continuity of the failure shape of the collapse mass, the boundary conditions along the detaching lines is satisfied to make sure the geometry continuity. In order to get the upper bound solutions to describe the shape of the collapse block, the first work is to calculate the internal energy dissipation rate produced by the shear stress and normal stress along the two different detaching lines. Furthermore the objective function consisting of the internal and external work should be constructed. Lastly two failure shape curves $y = f_1(x)$ and $y = f_2(x)$ can be obtained by the variational approach to reflect the distinct characteristics of falling blocks up and down the water level, as illustrated in Fig. 2.

The failure shape of the collapse block should be obtained with conditions. Given the behavior of the layered soils which are under the condition of varying water table should be ideally plastic and follow an associated flow rule, that is, the stress points of the ideally plastic soils should not exceed the yield surface. The width of collapse block at the ground level, $5i$, is only determined by the depth from the center of tunnel to ground surface. Moreover the relationship of the i and H can be described as $i = k \cdot H$, where i stands for the distance between the tunnel centerline and the point of the trough inflexion, k is a coefficient. By considering the shape of the collapse block to be symmetrical with respect to the y-axis, the profile of the detaching curves should be smooth and continuous. The collapse block can be seen as a rigid block without considering the arch effect of shallow circle tunnels.

4.1 Computation of internal energy dissipation

From the assumptions above, without considering the geometric difference in different soil

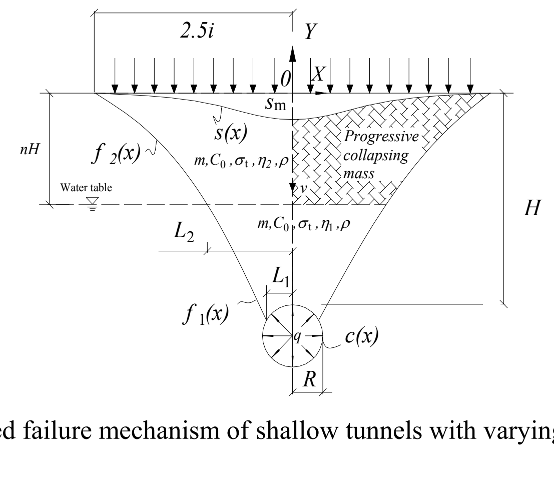


Fig. 2 Curved failure mechanism of shallow tunnels with varying water level

layers, the parameters of the Power-law criterion are assumed to be the same. Due to the presence of velocity detaching line existed in the soil layer of tunnel roofs, the impending failure would slide in a limit state along with the velocity discontinuous surfaces. During the process of the impending collapse, the dissipation densities of the internal forces on the detaching surface is

$$\begin{aligned} W_D &= \int_{L_1}^{L_2} \dot{D}_{i1} \cdot w\sqrt{1+f_1'(x)^2}\,dx + \int_{L_2}^{2.5i} \dot{D}_{i2} \cdot w\sqrt{1+f_2'(x)^2}\,dx \\ &= \int_{L_1}^{L_2} \left[\sigma_t - \left(\frac{1-m}{m}\right) (\eta_1 c_0)^{\frac{m}{m-1}} (m\sigma_t)^{\frac{1}{1-m}} f_1'(x)^{\frac{m}{m-1}} \right] v dx \\ &+ \int_{L_2}^{2.5i} \left[\sigma_t - \left(\frac{1-m}{m}\right) (\eta_2 c_0)^{\frac{m}{m-1}} (m\sigma_t)^{\frac{1}{1-m}} f_2'(x)^{\frac{m}{m-1}} \right] v dx \end{aligned} \tag{18}$$

where L_1 and L_2 characterize the collapse width of upper and lower blocks illustrated in Fig. 2. $y = f_1'(x)$ and $y = f_2'(x)$ are the first derivative of $y = f_1(x)$ and $y = f_2(x)$, respectively.

4.2 Calculation of external work

The work rate of failure block produced by weight can be calculated by integral process

$$\begin{aligned} W_e &= \rho' \int_0^{L_1} c(x) \cdot v dx + \rho' \int_{L_1}^{L_2} f_1(x) \cdot v dx \\ &+ \rho \int_{L_2}^{2.5i} f_2(x) \cdot v dx - \rho \int_0^{2.5i} s(x) \cdot v dx + v \cdot \rho_w L_2 f_1(L_2) \end{aligned} \tag{19}$$

where ρ' is the buoyant weight per unit volume of the rocks. $\rho' = \rho - \rho_w$, in which ρ is the weight per unit volume of the rock, and ρ_w is the unit weight of water. The function $c(x)$ is the function describing the circular tunnel profile. $s(x)$ characterizes the shape of surface settlement.

$$c(x) = -H - R + \sqrt{R^2 - x^2} \tag{20}$$

Based on the research of Osman (2010) and Fahimifar *et al*. (2015), the surface settlement is defined by a Gaussian distribution curve. The area of surface settlement trough can be obtained

$$\int_{-2.5i}^{2.5i} s(x)dx = \sqrt{2\pi} i s_m \tag{21}$$

The distribution of excess pore pressure which is derived from the study of Saada *et al*. (2012) can be written as

$$u = p - p_w = p - \rho_w h \tag{22}$$

where p is the pore water pressure at the considered point which can be obtained by a suitable method $p = r_u \rho h$, r_u stands for pore pressure coefficient, and $p_w = \rho_w h$ is the hydrostatic distribution for pore pressure, h is the vertical distance between the roof of the tunnel and the top of the failure block. So $-grad\ u$ can be defined as

$$-grad\ u = \rho_w - r_u \rho \tag{23}$$

Therefore the work rate produced by seepage forces along the velocity discontinuity surface is

$$W_u = v\int_0^{L_1}(\rho_w - r_u\rho)\left[c(x) - f_1\left(L_2\right)\right]dx + v\int_{L_1}^{L_2}(\rho_w - r_u\rho)\left[f_1(x) - f_1\left(L_2\right)\right]dx \tag{24}$$

Due to the fact that the tunnel is buried in the shallow strata, the supporting structure is unavoidable for the requirement of safety and stability. Therefore, the work rate of supporting pressure in the shallow circular tunnel is

$$W_q = Rqv\arcsin(L_1 / R) \tag{25}$$

where q is the supporting pressure exerting on the circumference of tunnel lining. Meanwhile, because of the fact that the external force always exerts on the underground structure, the work rate of extra force which puts on the ground surface cannot be ignored. The expressions can be written as

$$W_S = -2.5i\sigma_S v \tag{26}$$

where σ_s stands for the surcharge load put on the ground surface.

4.3 Analytical solution for characterizing collapse shape

In order to describe the shape and extension of the failure collapse block, it is of essential to obtain the explicit expressions of $y = f_1(x)$ and $y = f_2(x)$ by constructing an objective function consisting of the external rate of work and the rate of the internal energy dissipation

$$\Lambda = W_D - W_e - W_q - W_u - W_S \tag{27}$$

So that, the effective collapse mechanism can be obtained by minimizing the objective function Λ according to the kinematic theorem of limit analysis.

Then substituting Eqs. (18), (19), (24), (25) and (26) into Eq. (27), the expression of objective function is given

$$\begin{aligned}\Lambda = &\int_{L_1}^{L_2}\psi_1[f_1(x), f_1'(x), x]vdx + \int_{L_2}^{2.5i}\psi_2[f_2(x), f_2'(x), x]vdx - \rho'\int_0^{L_1}c(x)vdx \\ &-\left(\rho_w - r_u\rho\right)\int_0^{L_1}\left(c(x) - f_1\left(L_2\right)\right)\cdot vdx + \rho\int_0^{2.5i}s(x)vdx + 2.5i\sigma_s v - Rqv\arcsin\frac{L_1}{R}\end{aligned} \tag{28}$$

in which

$$\psi_1[f_1(x), f_1'(x), x] = \sigma_t - \left(\frac{1-m}{m}\right)\left(\eta_1 c_0\right)^{\frac{m}{m-1}}\left(m\sigma_t\right)^{\frac{1}{1-m}} f_1'(x)^{\frac{m}{m-1}} + \rho\left(r_u - 1\right)f_1\left(x\right) \tag{29}$$

$$\psi_2[f_2(x), f_2'(x), x] = \sigma_t - \left(\frac{1-m}{m}\right)\left(\eta_2 c_0\right)^{\frac{m}{m-1}}\left(m\sigma_t\right)^{\frac{1}{1-m}} f_2'(x)^{\frac{m}{m-1}} - \rho f_2\left(x\right) \tag{30}$$

In order to get the extremum of the objective function Λ, it is necessary to search for the extremum values of objective functions ψ_1 and ψ_2. In other words, the analytical solutions of collapse dimension could be obtained by seeking for the minimum values of objective functions

$\psi_1$and ψ_2 with the method of variational principle in the realm of plasticity theory. As a consequence the expressions of ψ_1 and ψ_2 should be turned into two Euler's equations through the variational method. The expressions of variational equations of ψ_1 and ψ_2 on stationary conditions can be written as

$$\frac{\partial \psi_1}{\partial f(x)} - \frac{\partial}{\partial x}\left[\frac{\partial \psi_1}{\partial f'(x)}\right] = 0 \tag{31}$$

$$\frac{\partial \psi_2}{\partial g(x)} - \frac{\partial}{\partial x}\left[\frac{\partial \psi_2}{\partial g'(x)}\right] = 0 \tag{32}$$

By the variational calculation the explicit forms of the two Euler's equations for the Eqs. (29) and (30) can be obtained as

$$\left(r_u - 1\right)\rho - \left(\eta_1 c_0\right)^{\frac{m}{m-1}} \left(m\sigma_t\right)^{\frac{1}{1-m}} \left(m-1\right)^{-1} f_1'(x)^{\frac{2-m}{m-1}} f_1''(x) = 0 \tag{33}$$

$$-\rho - \left(\eta_2 c_0\right)^{\frac{m}{m-1}} \left(m\sigma_t\right)^{\frac{1}{1-m}} \left(m-1\right)^{-1} f_2'(x)^{\frac{2-m}{m-1}} f_2''(x) = 0 \tag{34}$$

Obviously, Eqs. (33) and (34) are two nonlinear second-order homogeneous differential equations. By the integral calculation process the expressions of velocity discontinuity surface of lower and upper soil layers are

$$f_1(x) = -k_1\left[\frac{n_0}{\left(1-r_u\right)\rho} - x\right]^m - n_1 \tag{35}$$

$$f_2(x) = -k_2\left[\frac{n_2}{\rho} - x\right]^m - n_3 \tag{36}$$

in which

$$k_1 = \frac{\sigma_t}{\eta_1 c_0}\left[\frac{\left(1-r_u\right)\rho}{\eta_1 c_0}\right]^{m-1} \tag{37}$$

$$k_2 = \frac{\sigma_t}{\eta_2 c_0}\left[\frac{\rho}{\eta_2 c_0}\right]^{m-1} \tag{38}$$

where n_0, n_1, n_2 and n_3 stand for the integration constants coefficients determined by mechanical and geometric boundary conditions, respectively.

From what mentioned above, given the detaching curves are supposed symmetric with respect to the y-axis. It can be seen from the Fig. 2 the shear stress on the ground surface equals to zero at the point which its x-height is 2.5i.

$$\tau_{xy}(x = 2.5i,\, y = 0) = 0 \tag{39}$$

Furthermore, the explicit expressions of the function of velocity discontinuity surface should fulfill other boundary conditions. Such as

$$f_2(x = 2.5i) = 0 \tag{40}$$

$$f_1(x = L_2) = f_2(x = L_2) = -nH \tag{41}$$

$$c(x = L_1) = f_1(x = L_1) \tag{42}$$

where n indicates the ratio of the upper height of roof collapse to the depth from the center of tunnel to ground surface H. Substituting Eqs. (35) and (36) into above expressions, they turn into

$$f_2(x) = -k_2(2.5i - x)^m \tag{43}$$

$$n_1 = -k_1\left(\frac{n_0}{(1-r_u)\rho} - L_2\right)^m + nH \tag{44}$$

For the purpose of keeping the whole curve look smooth, another boundary condition should be satisfied

$$f_1'(x = L_2) = f_2'(x = L_2) \tag{45}$$

Building on this result, the expressions of the function of velocity discontinuity surface can be obtained

$$f_1(x) = -k_1(Z - x)^m + k_1(Z - L_2)^m - nH \tag{46}$$

in which

$$Z = \left(\frac{\eta_1}{\eta_2}\right)^{\frac{m}{m-1}}\left(\frac{2.5i - L_2}{1 - r_u}\right) + L_2 \tag{47}$$

On the basis of the expression of the profile of the circular tunnel, the piece of external work can be calculated by integrating $c(x)$ over the interval $[0, L_1]$

$$\int_0^{L_1} c(x)dx = -HL_1 - \frac{R^2}{2}\left[\arccos\frac{L_1}{R} - \frac{\pi}{2} - \frac{L_1}{R^2}\sqrt{R^2 - L_1^{\,2}}\right] \tag{48}$$

Therefore, the objective function Λ can be calculated, which is

$$\begin{aligned}\Lambda = v\Big\{&\left[k_1(r_u - 1)\rho(Z - L_2)^m - (r_u - 1)\rho Hn\right](L_2 - L_1) + \sigma_t(2.5i - L_1) \\ &+ \frac{1}{m+1}\left[\left(\frac{1-m}{m}\right)(\eta_1 c_0)^{\frac{m}{m-1}}(m\sigma_t)^{\frac{1}{1-m}}(mk_1)^{\frac{m}{m-1}} + k_1(r_u - 1)\rho\right]\cdot\left[(Z - L_2)^{m+1} - (Z - L_1)^{m+1}\right] \\ &- \frac{1}{m+1}\left[\left(\frac{1-m}{m}\right)(\eta_2 c_0)^{\frac{m}{m-1}}(m\sigma_t)^{\frac{1}{1-m}}(mk_2)^{\frac{m}{m-1}} - k_2\rho\right]\cdot(2.5i - L_2)^{m+1} - r_u\rho L_2 Hn\end{aligned} \tag{49}$$

$$\begin{aligned}&+(r_u-1)\rho\left[(H+R)L_1-\frac{L_1}{2}\sqrt{R^2-L_1^2}-\frac{R^2}{2}\arcsin\frac{L_1}{R}\right]-Rq\arcsin\frac{L_1}{R}\\&+2.5i\sigma_S+\sqrt{2\pi}iS_m\rho/2\Big\}\end{aligned} \tag{49}$$

For the purpose of getting the explicit forms of detaching curve profile consisting of $f_1(x)$ and $f_2(x)$, the values of L_1 and L_2 must be obtained by combining and solving Eqs. (41) and (42). Then it is not difficulty to draw the shape of failure surface by Eqs. (43) and (46).

4.4 Effects of different dilation coefficients on shape of roof collapse

In the process of investigating the influence of dilation coefficient η on collapse mechanism in soil layer with considering water level, the cases that only soil under water level follows non-associated flow rule and both layers follow non-associated flow rule are discussed. According to the failure mechanism, the strength parameters are considered to be the same with ignoring the pore water effect made by the underground water.

(1) The influence of dilation coefficient η on collapse mechanism in saturated soils

According to the Fig. 3, the numerical results with dilation coefficient η varying from 0.6 to 0.8 are obtained corresponding to m = 1.5, σ_t = 60 kPa, C_0 = 0.1 Mpa, ρ = 18 kN/m^3, σ_s = 80 kPa, R = 5 m, H = 10 m, s_m = 0.2 m, r_u = 0.1. With the decrease of dilation coefficient the size of the potential failure blocks increases. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in saturated soil need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

(2) Only soil under water level follows non-associated flow rule

According to the Fig. 4, the numerical results with dilation coefficient η_1 varying from 0.7 to 0.9 are obtained corresponding to m = 1.6, σ_t = 50 kPa, C_0 = 0.1 Mpa, σ_s = 80 kPa, R = 5 m, H = 10 m, s_m = 0.2 m, r_u = 0.2, ρ = 18 kN/m^3, and n = 0.35. With the decrease of dilation coefficient the size of the potential failure blocks increases. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in soil under water level need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

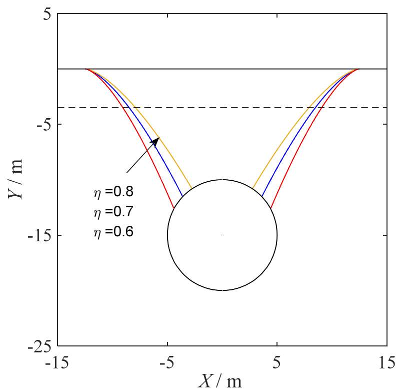


Fig. 3 Effects of dilation coefficient η on failure mechanisms of shallow tunnels

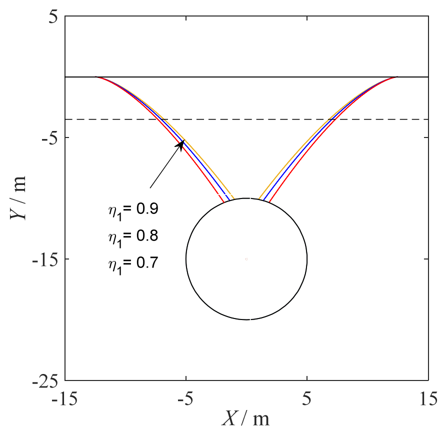
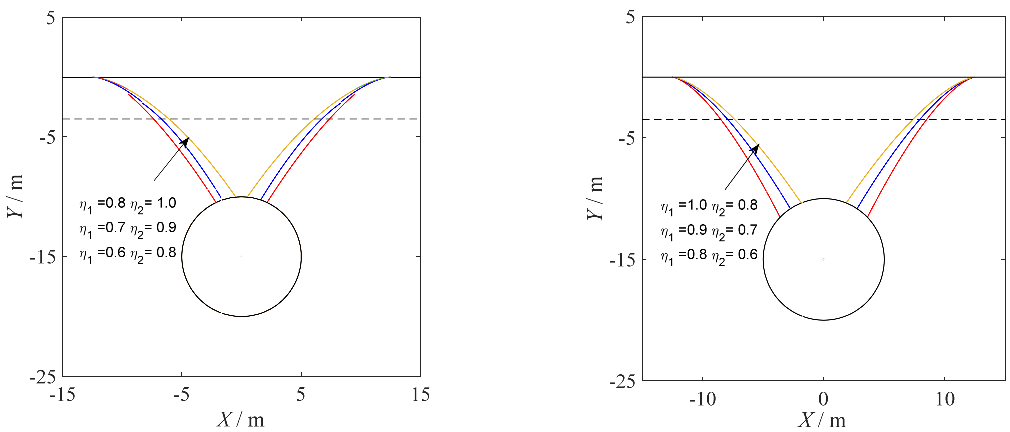


Fig. 4 Effects of dilation coefficient η_1 on failure mechanisms of shallow tunnels

(3) Both layer soils follow non-associated flow rule

For investigating the influences of dilation coefficients η_3 and η_4 on the range of collapse block, the numerical results are obtained and shown in Fig. 5 when $\eta_1 < \eta_2$ and $\eta_1 > \eta_2$. When $m = 1.6$, $\sigma_t = 50$ kPa, $C_0 = 0.1$ Mpa, $r_u = 0.2$, $\sigma_s = 80$ kPa, $R = 5$ m, $H = 10$ m, $s_m = 0.2$ m, $\rho = 18$ kN/m^3, $n = 0.35$ with both dilation coefficients decrease, the total scope of collapse block reduce whenever $\eta_1 < \eta_2$ or $\eta_1 > \eta_2$. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in both layer soils need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

From the perspective of engineering, the position of the water table will contribute to controlling the size of collapse block. The shallower the underground water lever is buried the scope of the failure block will be larger during the process of excavation. In consequence, supporting system for shallow ground water table should be intensified to avoid collapse and more measurements should be taken to protect the soil saturated in the underground water.



(a) $\eta_1 < \eta_2$ (b) $\eta_1 > \eta_2$

Fig. 5 Effects of dilation coefficients η_1 and η_2 on failure mechanisms of shallow tunnels

5. Collapse mechanism for shallow tunnels in layered soils

According to the introduction of the previous works which are on the basis of the condition that the tunnel is buried in single soil layer with different water levels, some kinds of tunnels excavated in layered soils determined by different material parameters should not be ignored in engineering. Furthermore the seepage forces and surface settlement should be regarded as external loading in the limit analysis. So the new failure mechanism consisting of two different functions which describe two soil layers is suggested in this work. As shown in Fig. 6, the failure mechanism is made up of two curves, $y = g_1(x)$ and $y = g_2(x)$ in the symmetrical coordinate system. The 1 and 2 in the subscript of parameters m, C_0, σ_t and ρ are geotechnical parameters. L_3 is the width of collapse block around the circumference of tunnel lining, $L_3 + L_4$ equals the width of the collapse block of lower soil layer. h_1 is the height of the collapse block in the upper rock, R is the radius of the circular tunnel. Based on the study by Yang *et al*. (2016), this work is to explore the influences of the different dilation coefficients on shallow tunnel in the layered soils.

With considering different soil layers, during the process of the impending collapse in layered soils, the dissipation densities of the internal forces on the detaching surface, $\dot{D}_{i3}$ and $\dot{D}_{i4}$, are

$$\dot{D}_{i3} = \sigma_{n3}\dot{\varepsilon}_{n3} + \tau_{n3}\dot{\gamma}_{n3} = \frac{\dot{u}}{w'}[1+g_1{}'(x)^2]^{-\frac{1}{2}}\left[\sigma_{t1} - \left(\frac{1-m_1}{m_1}\right)(\eta_3 c_{01})^{\frac{m_1}{m_1-1}}(m_1\sigma_{t1})^{\frac{1}{1-m_1}}g_1{}'(x)^{\frac{m_1}{m_1-1}}\right] \tag{50}$$

$$\dot{D}_{i4} = \sigma_{n4}\dot{\varepsilon}_{n4} + \tau_{n4}\dot{\gamma}_{n4} = \frac{\dot{u}}{w'}[1+g_2{}'(x)^2]^{-\frac{1}{2}}\left[\sigma_{t2} - \left(\frac{1-m_2}{m_2}\right)(\eta_4 c_{02})^{\frac{m_2}{m_2-1}}(m_2\sigma_{t2})^{\frac{1}{1-m_2}}g_2{}'(x)^{\frac{m_2}{m_2-1}}\right] \tag{51}$$

where $\dot{\varepsilon}_{n3}$ and $\dot{\varepsilon}_{n4}$ are normal plastic strain rates in upper and lower soils, respectively, $\dot{\gamma}_{n3}$ and $\dot{\gamma}_{n4}$ are shear plastic strain rates in upper and lower soils, respectively, w' is the thickness of the plastic detaching zone, and $\dot{u}$ is the velocity of the collapse block. So the Eq. (18) takes the form

$$\begin{aligned} D &= \int_{L_3}^{L_3+L_4}\dot{D}_{i3}w'[1+g_1{}'(x)^2]^{\frac{1}{2}}dx + \int_{L_3+L_4}^{2.5i}\dot{D}_{i4}w'[1+g_2{}'(x)^2]^{\frac{1}{2}}dx \\ &= \dot{u}\int_{L_3+L_4}^{2.5i}\left[\sigma_{t2} - \left(\frac{1-m_2}{m_2}\right)(\eta_4 c_{02})^{\frac{m_2}{m_2-1}}(m_2\sigma_{t2})^{\frac{1}{1-m_2}}g_2{}'(x)^{\frac{m_2}{m_2-1}}\right]dx \\ &+ \dot{u}\int_{L_3}^{L_3+L_4}\left[\sigma_{t1} - \left(\frac{1-m_1}{m_1}\right)(\eta_3 c_{01})^{\frac{m_1}{m_1-1}}(m_1\sigma_{t1})^{\frac{1}{1-m_1}}g_1{}'(x)^{\frac{m_1}{m_1-1}}\right]dx \end{aligned} \tag{52}$$

With reference to the work rate of failure block produced by weight, Eq. (19) takes the form

$$\begin{aligned} W_e' &= \dot{u}\left\{\rho_2'\int_{L_3+L_4}^{2.5i}g_2(x)dx + \rho_2'\int_0^{L_3+L_4}g_1(L_3+L_4)dx + \rho_1'\int_{L_3}^{L_3+L_4}[g_1(x) - g_1(L_3+L_4)]dx \right. \\ &\left. + \rho_1'\int_0^{L_3}[c(x) - g_1(L_3+L_4)]dx - \rho_2'\int_0^{2.5i}s(x)dx\right\} \\ &= \dot{u}\left\{\rho_2'\int_{L_3+L_4}^{2.5i}g_2(x)dx + \rho_1'\int_{L_3}^{L_3+L_4}g_1(x)dx + \rho_1'\int_0^{L_3}c(x)dx - \rho_2'\int_0^{2.5i}s(x)dx \right. \\ &\left. + (\rho_2' - \rho_1')(L_3+L_4)g_1(L_3+L_4)\right\} \end{aligned} \tag{53}$$

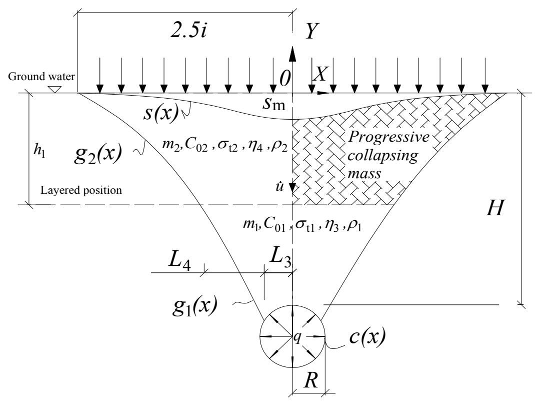


Fig. 6 Curved failure mechanism of shallow tunnels buried in the layered soils

where ρ_1' and ρ_2' are the buoyant weight per unit volume of the upper and lower soils, respectively. Therefore the work rate produced by seepage forces along the velocity discontinuity surface takes the form

$$\begin{aligned} W_u &= \dot{u}\left\{\int_{L_3+L_4}^{2.5i}(\rho_w - r_u'\rho_2)g_2(x)dx + \int_0^{L_3+L_4}(\rho_w - r_u'\rho_2)g_1(L_3+L_4)dx - \int_0^{2.5i}(\rho_w - r_u'\rho_2)s(x)dx\right. \\ &\left.+\int_{L_3}^{L_3+L_4}(\rho_w - r_u'\rho_1)[g_1(x)-g_1(L_3+L_4)]dx + \int_0^{L_3}(\rho_w - r_u'\rho_1)[c(x)-g_1(L_3+L_4)]dx\right\} \\ &= \dot{u}\left\{\int_{L_3+L_4}^{2.5i}(\rho_w - r_u'\rho_2)g_2(x)dx + \int_{L_3}^{L_3+L_4}(\rho_w - r_u'\rho_1)g_1(x)dx + \int_0^{L_3}(\rho_w - r_u'\rho_1)c(x)dx\right. \\ &\left.-\int_0^{2.5i}(\rho_w - r_u'\rho_2)s(x)dx + r_u'(\rho_1-\rho_2)(L_3+L_4)g_1(L_3+L_4)\right\} \end{aligned} \tag{54}$$

5.1 Upper bound solutions of the size of collapse block

By the variational calculation the explicit forms of the two Euler's equations for the Eqs. (33) and (34) take the form

$$\left(\eta_3 c_{01}\right)^{\frac{m_1}{m_1-1}}\left(m_1\sigma_{t1}\right)^{\frac{1}{1-m_1}}\left(1-m_1\right)^{-1} g_1'(x)^{\frac{2-m_1}{m_1-1}}\ g_1''(x) + (1-r_u')\rho_1 = 0 \tag{55}$$

$$\left(\eta_4 c_{02}\right)^{\frac{m_2}{m_2-1}}\left(m_2\sigma_{t2}\right)^{\frac{1}{1-m_2}}\left(1-m_2\right)^{-1} g_2'(x)^{\frac{2-m_2}{m_2-1}}\ g_2''(x) + (1-r_u')\rho_2 = 0 \tag{56}$$

where r_u' stands for pore pressure coefficient in layered soils.

The explicit failure surfaces of circular tunnel profile in layered soils can be drawn according to the analytical solutions of velocity discontinuity surfaces and which are shown as Eqs. (57) and (58).

$$g_1(x) = -k_3(Z'+L_3+L_4-x)^{m_1} + k_3\left(Z'\right)^{m_1} - k_4(2.5i-L_3-L_4)^{m_2} \tag{57}$$

in which

$$g_2(x) = -k_4(2.5i - x)^{m_2} \tag{58}$$

$$k_3 = \frac{\sigma_{t1}}{\eta_3 c_{01}}\left[\frac{\left(1-r_u'\right)\rho_1}{\eta_3 c_{01}}\right]^{m_1-1} \tag{59}$$

$$k_4 = \frac{\sigma_{t2}}{\eta_4 c_{02}}\left[\frac{\left(1-r_u'\right)\rho_2}{\eta_4 c_{02}}\right]^{m_2-1} \tag{60}$$

$$Z' = \left[\frac{k_4 m_2}{k_3 m_1}\right]^{\frac{1}{m_1-1}}(2.5i - L_3 - L_4)^{\frac{m_2-1}{m_1-1}} \tag{61}$$

5.2 Effects of different dilation coefficients on shape of roof collapse in layered soils

In general, the deeper the depth of the buried circular tunnel is, the better the nature of the soil is. So the relationships of the parameters between two soil layers are $m_1 < m_2$, $c_{01} \geq c_{02}$ and $\rho_1 > \rho_2$. In the process of investigating the influence of dilation coefficient η on collapse mechanism in layered soil, the cases that only upper soil follows non-associated flow rule, only lower soil follows and both layers follow non-associated flow rule are discussed.

(1) Only lower soil follows non-associated flow rule

According to the Fig. 7, the numerical results with dilation coefficient η_3 varying from 0.6 to 0.8 are obtained corresponding to σ_s = 80 kPa, R = 5 m, H = 10 m, s_m = 0.2 m, h_1 = 3.5 m, m_1 = 1.4, m_2 = 1.6, σ_{t1} = 60 kPa, σ_{t2} = 50 kPa, C_{01} = 100 kPa, C_{02} = 85 kPa, r_u' = 0.2, ρ_1 = 17 kN/m^3, ρ_2 = 15 kN/m^3. With the decrease of dilation coefficient the size of the potential failure blocks increases. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in lower soil need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

(2) Only upper soil follows non-associated flow rule

According to the Fig. 8, the numerical results with dilation coefficient η_4 varying from 0.6 to 0.8 are obtained corresponding to σ_s = 80 kPa, R = 5 m, H = 10 m, s_m = 0.2 m, h_1 = 3.5 m, m_1 = 1.4, m_2 = 1.6, σ_{t1} = 60 kPa, σ_{t2} = 50 kPa, C_{01} = 100 kPa, C_{02} = 85 kPa, r_u' = 0.2, ρ_1 = 17 kN/m^3 and ρ_2 = 15 kN/m^3. With the decrease of dilation coefficient the size of the potential failure blocks increases. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in upper soil need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

(3) Both layer soils follow non-associated flow rule

For investigating the influences of dilation coefficients η_3 and η_4 on the range of collapse block, the numerical results are obtained and shown in Fig. 9. When $\eta_3 < \eta_4$ and $\eta_3 > \eta_4$. When σ_s = 80 kPa, R = 5 m, H = 10 m, s_m = 0.2 m, h_1 = 3.5 m, m_1 = 1.4, m_2 = 1.6, σ_{t1} = 60 kPa, σ_{t2} = 50 kPa, C_{01}

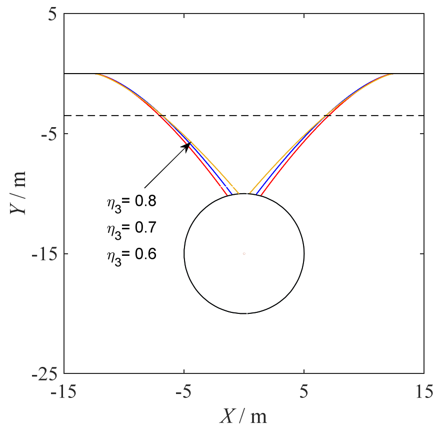


Fig. 7 Effects of dilation coefficient η_3 on failure mechanism of shallow tunnel

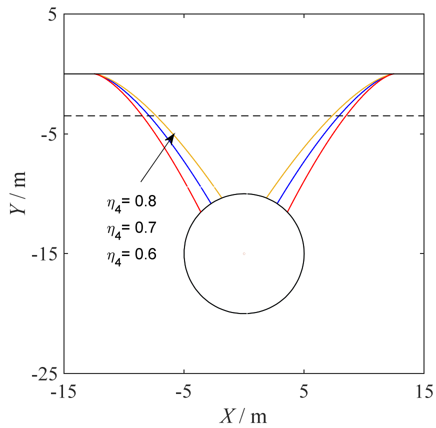
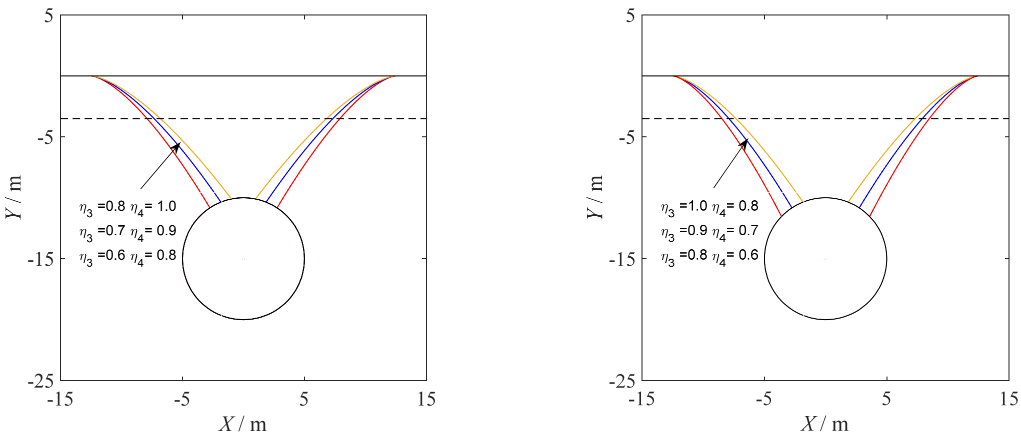


Fig. 8 Effects of dilation coefficient η_4 on failure mechanism of shallow tunnel



(a) $\eta_3 < \eta_4$ (b) $\eta_3 > \eta_4$

Fig. 9 Effects of dilation coefficients η_3 and η_4 on failure mechanisms of shallow tunnels

= 100 kPa, C_{02} = 85 kPa, r'_u = 0.2, ρ_1 = 17 kN/m^3 and ρ_2 = 15 kN/m^3, with both dilation coefficients decrease, the total scope of collapse block reduce whenever $\eta_3 < \eta_4$ or $\eta_3 > \eta_4$. From the perspective of engineering, the shallow-buried circular tunnels with smaller dilation coefficient in both layer soils need larger supporting forces and supporting system of shallower tunnels should be intensified to keep the stability.

6. Conclusions

On the basis of previous work which has focused the efforts on the collapse mechanism in deep tunnels, a new curved failure mechanism of circular shallow tunnel subjected to seepage considering the joined effects of surface settlement and dilation is proposed to estimate the stability of tunnel roof under a limit state. Considering the different water levels, this work investigated the effects of different dilation coefficients on the scope of the failure blocks in predicting the stability of the tunnel roof. With nonlinear failure criterion and non-associated flow rule, the numerical solution for the shape of collapse mechanism is obtained by setting an objective function consisted of energy dissipation rate and the external work rate. Some conclusions are drawn as follows:

(1) It is found that dilation coefficients have significant influence on the possible collapse scope of shallow tunnel subjected to seepage with varying water table. The range of falling blocks tends to decrease with increase of dilation coefficient. When only the layer under the water level follows non-associated flow rule, the scope of collapse block increases with the decrease of dilation coefficient η_1. When both layers under and above the water level follow non-associated flow rule and both dilation coefficients decrease, the potential range of collapse block decreases in soils.

(2) It is found that dilation coefficients have significant influence on the possible collapse scope of shallow tunnel in layered soils. When only the upper soil follows non-associated flow rule, the range of collapse block increases with the decrease of dilation coefficient η_4. When only the lower layer follows non-associated flow rule, the scope of collapse block increases with the decrease of dilation coefficient η_3. When both layers follow non-associated flow rule and both dilation coefficients decrease, the potential range of collapse block decreases in layered soils.

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