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Comparison of measurement uncertainty calculation methods on example of indirect tensile strength measurement

Bulent Tutmez*

Department of Mining Engineering, Inonu University, Malatya, Turkey

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Abstract. Indirect measure of the tensile strength of laboratory samples is an important topic in rock engineering. One of the most important tests, the Brazilian strength test is performed to obtain the tensile strength of rock, concrete and other quasi brittle materials. Because the measurements are provided indirectly and the inspected rock materials may have heterogeneous properties, uncertainty quantification is required for a reliable test evaluation. In addition to the conventional measurement evaluation uncertainty methods recommended by the Guide to the Expression of Uncertainty in Measurement (GUM), such as Taylor's and Monte Carlo Methods, a fuzzy set-based approach is also proposed and resulting uncertainties are discussed. The results showed that when a tensile strength measurement is measured by a laboratory test, its uncertainty can also be expressed by one of the methods presented.

Keywords: measurement uncertainty; Brazilian test; GUM; fuzzy set; Monte Carlo method

1. Introduction

Rocks are heterogeneous materials and they contain numerous microcracks. As novel approaches, holographic interferometry and digital image processing techniques can be used to determine the fracture process zone of rock and concrete (Castro-Montero *et al*. 1995, Yue *et al*. 2013). Since rocks are much weaker in tension than in compression or shear, tensile failure also serve a function in rock engineering in the field of drilling, blasting and cutting of rocks, exploitation of rock slopes (Rojek *et al*. 2011, Wan *et al*. 2016). Laboratory methods to measure the tensile strength of rocks contain the direct uniaxial tensile test and indirect tensile tests (Hudson and Harrison 1997). Although the direct uniaxial method seems to be a suitable method, it is difficult to perform it in practice for rock materials.

The most commonly used test is the diametrical compression of thin discs, referred to as the Brazilian test (Rocco *et al*. 1999). To provide the indirect tensile strength from the Brazilian method, one must know the principal tensile stress, in particular at the rock disc center, where a crack initiates (Wosatko *et al*. 2011). From a metrological perspective, an indirect indication is a measurement in which the value of the unknown quantity sought is provided using measurements of other quantities related to the measurand as stated by Rabinovich (2013).

Because tensile strength is closely related to the stress threshold for fracture initiation in compression, recently alternative tensile testing methods such as sleeve fracturing test, the beam

*Corresponding author, Professor, E-mail: bulent.tutmez@inonu.edu.tr

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bending test, and the modified tension test have been also suggested (Perras and Diederichs 2014). Measuring the tensile strength of rock is conducted by the International Society for Rock Mechanics standard Bieniawski and Hawkes (2007) which summarizes both direct and indirect Brazilian test methods (Erarslan *et al*. 2012, Bieniawski and Hawkes 2007).

As in various engineering sciences, in rock mechanics laboratory, there are two main sources of uncertainties can be distinguished: due to natural variability and due to laboratory measure (device, method etc.) (Adl-Zarrabi *et al*. 2009). In addition to these factors, indirect characteristics of the Brazilian test is that the stress state at the centre of the testing disc is not a purely tensile mode (Chen and Hsu 2001), can be added. Thus, a big amount of uncertainties can be mentioned. Therefore, evaluation of uncertainties in test measurements and making reliable uncertainty quantification is primarily required. Measurement uncertainties have to decide whether new theories or applications should be accepted or discarded. Up to present, to implement the measurement uncertainty analyses in rock engineering laboratory, some limited availability of studies has been presented. In these studies, it is aimed to evaluate uncertainties by conventional methods (Kuhinek *et al*. 2011, Chen 2012).

In the GUM (Guide to the Expression of Uncertainty in Measurement) framework, two propagation methods for measurement uncertainties are expressed: Taylor's and Monte Carlo (MC) methods (BIPM 2008a, b). In the first approach, to appraise the random uncertainty, a combined uncertainty model which is based on the relative contribution of the errors and combined variations are used. In the second approach, the MC method integrates and generates distributions rather than propagating uncertainties (Pavese 2009). Both the methods calculate the potential uncertainties on the ground o probabilistic point of view. On the other hand, fuzzy set theory may cover a wide spectrum to focus on the problem. Following the fundamental sources on fuzzy sets, uncertainty and information (Klir 2005), fuzzy set-based measurement uncertainty evaluation has been discussed in different works (Mauris *et al*. 2001, Salicone 2007, Müller 2009).

The true value of a physical quantity (tensile strength) in question is assessed by a suitably constructed estimator. In this paper, in addition to the conventional measurement uncertainty quantification tools such as Taylor's and MC methods, a fuzzy set based measurement uncertainty evaluation is also practiced. By using the fuzzy membership functions, fuzzy arithmetic and alpha-cut approaches, lack of knowledge and extended uncertainties are expressed on the ground of possibility-probability transformation.

In the next section, the problem and solution methodologies are presented. Section 3 addresses the measurement uncertainty measures. Applications and a brief discussion are also given in the section. Section 4 concludes the paper.

2. Methodology

2.1 A reminder on Brazilian indirect test method

Tensile strength is among the most important parameters influencing deformability results. In general, rocks are heterogeneous materials and they include numerous microcracks. Therefore, rocks show different behaviour under tensile and compressive conditions (Gui *et al*. 2015). The modulus E_c and the compressive strength σ_c are easy to measure in the laboratory by uniaxial compression tests. However, the tensile strength σ_t is difficult to obtain by direct tension tests (Claesson and Bohloli 2002). The Bieniawski and Hawkes (2007) suggested the Brazilian test for determining the tensile strength of rock materials. In this indirect testing method, a disc is

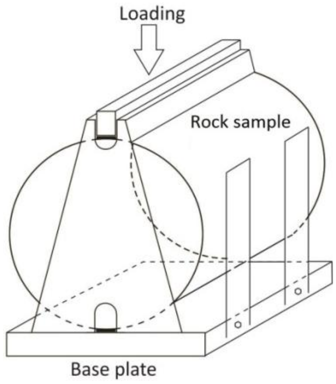


Fig. 1 Brazilian test method

compressed by forces applied at opposite ends of a diameter to failure. Because many rock types are anisotropic, it is necessary to find an uncertainty evaluation method for appraising the tensile strength of the rocks from the Brazilian test.

Fig. 1 illustrates the general mechanism of the test. Even though the Brazilian test is the famous conventional indirect method used in rock and concrete sciences, the main shortcoming of this test is that the stress state at the centre of the testing disc is not a purely tensile mode (Chen and Hsu 2001). The specific forms of measurement equations can be considered as mathematical models of specific indirect measurements. The tensile strength of rocks σ_t has been obtained by the following expression (Li and Wong 2013)

$$\sigma_t = \frac{0.636xF}{Dxt} \; (MPa) \tag{1}$$

where, F denotes the failure load (N), D (mm) and t (mm) represent diameter and thickness, respectively. Eq. (1) is based on the theory of elasticity for isotropic media. The expression presents the tensile stress perpendicular to the loaded diameter at the center of the disc at the time of failure when the applied force is F (Claesson and Bohloli 2002).

From a general evaluation perspective, to calculate the uncertainty of tensile strength measurement, firstly the sources of uncertainty in measurements are identified. Then the amount of the uncertainty from each source is estimated. As a consequence, to give an overall figure the individual uncertainties are combined.

2.2 GUM-Taylor's method

The guide to the expression of uncertainty in measurement (GUM) is concerned with the expression of uncertainty in the measurement of a well-defined physical quantity, the measurand. The guide mainly suggests two conventional methods such as Taylor's and Monte Carlo methods for evaluating and expressing uncertainty in experimental measurements.

The conventional method, ISO-GUM (Taylor's) model obtains a unique relationship in most cases between the measurand (analytical result) Y and N input quantities X_1, X_2,..., X_N, given as

$$Y = f(X_1, X_2, \ldots, X_N). \tag{2}$$

The Taylor's method considers the standard uncertainties of input quantities and the relative contribution of the errors. The estimated variation connected with the measurand *y*, is described as the combined standard uncertainty and represented by $u_c(y)$, and it is calculated from the estimated standard deviation connected with each input estimate $u(x_i)$. Propagation of uncertainty is carried out on the ground of a first order Taylor series approximation as follows

$$Y = f(X_1, X_2, \dots, X_N) = y(x_1, x_2, \dots x_N) + \sum_{i=1}^{N} c_i X_i \tag{3}$$

where c_i denote the sensitivity coefficients (partial derivatives), which present how the estimate *y* varies with changes in the values of the factors (inputs) x_i. In particular, the change in *y* produced by the standard uncertainty of the estimate x_i, is presented by $(\Delta y)_i = (\partial f / \partial x_i)(\Delta x_i)$. This change is generated by the standard uncertainty of the estimate x_i, and the corresponding variation in *y* is $(\partial f / \partial x_i) u(x_i)$. When the input quantities are correlated, the appropriate expression should be structured by the combined variance $u_c^2(y)$ and the estimated correlation coefficient *r*. (BIPM 2008a).

The combined uncertainty $u_c(y)$ is linear sum of terms representing the variation of *y*. It can be obtained from the positive square root of the combined variance $u_c^2(y)$

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) \tag{4}$$

The combined uncertainty $u_c(y)$ can be used to define the uncertainty of a measurement result. It is essential to describe a measure of uncertainty that expresses an interval concerning the measurand that may be anticipated to cover a large fraction of the distribution of values. For this purpose, GUM Uncertainty Framework suggests an interval-based uncertainty that is an expanded uncertainty, *U*. The expanded uncertainty is provided by multiplying $u_c(y)$ by a coverage factor *k*

$$U = k u_c(y), \quad y - U \le Y \le y + U. \tag{5}$$

If there is no enough degrees of freedom to set $k = 2.0$ (95% confidence level), it is necessary to estimate the degrees of freedom for the total uncertainty. The GUM proposes the Welch-Satterthwaite formula and *t*-distribution to estimate the effective degrees of freedom (BIPM 2008a).

2.3 Monte Carlo Method

From an analytical approach, when a probability density function (PDF) for each input variable is known, a PDF for the measured value *Y* can be determined. This process can be performed using a reliable simulation approach in general cases. The MCM can be used to determine a PDF for the target variable based on PDFs of input variables (Desenfant *et al*. 2009). In this approach, for each error source, an appropriate PDF function is chosen. The method does not require the computation of derivatives. In addition, it does not rely on the capacity of a linear approximation to the measurement function *f* in a neighbourhood of the measurand whose size is comparable with the measurement uncertainties of the input quantities (Müller *et al*. 2009).

BIPM (2008b) is concerned with the propagation of probability distributions using a Monte Carlo method. M vectors x_r, r = 1, 2,....,M, are drawn from the PDFs $g_{X_i}(\varepsilon_i)$ for the N input quantities X_i. The expected (model) values and their averages can be calculated as follows

$$y_r = f(x_r), r = 1,2, \dots, M. \tag{6}$$

$$\tilde{y} = \frac{1}{M}\sum_{r=1}^{M} y_r. \tag{7}$$

The overall standard uncertainty $u(y)$ is estimated by the sample standard deviation of the draws, that is

$$u^2(\tilde{y}) = \frac{1}{M-1}\sum_{r=1}^{M}(y_r - \tilde{y})^2. \tag{8}$$

In the MC method, the process is recurrent until a converged value is obtained for the standard deviation (Steele and Douglas 2009). This deviation is then the estimate of the combined standard uncertainty u_c. Based on the simulation results, confidence intervals for y can be provided and other statistical information can be drawn from the sample $y^{(1)}, y^{(2)}, \dots, y^{(M)}$.

2.4 Fuzzy set-based method

Fuzzy set m provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables (Zimmermann 2010). As discussed in Tutmez (2009), the fuzzy methodologies can have a large impact in analyzing of mechanical properties of construction and building materials.

A fuzzy variable X is described by its membership function $\mu_X(x)$ which satisfies the condition $0 \le \mu_X(x) \le 1$. A membership function can also be expressed with regard to α-cuts at different vertical levels α. An alpha-cut X_α is described as

$$X_\alpha = \{x | \mu_X(x) \ge \alpha\}. \tag{9}$$

Fig. 2 illustrates a graphical representation for triangular membership function which is also a subset. In the graph, F_0 denotes the support and F_1 called the kernel. F_α addresses an interval at a presumption level.

Although various types encountered in fuzzy set literature, the membership functions comprising of straight segments are often used in practice for their simplicity. In the present study, because we have limited number of measurements, the trapezoidal membership functions were preferred. They can be described by a minimal amount of information. In addition, data relating to the corner points of a function are hereby sufficient.

Suppose, a quantity X is described as the sum of two independent quantities X_1 and X_2. The distribution for X is a symmetric trapezoidal distribution Trap (a, b, β) with lower bound a and upper bound b, and a parameter β equal to the ratio of the semi-width of the top of the trapezoid to that of the base (BIPM 2008b). Fig. 3 illustrates the trapezoidal probability density function constructed by rectangular independent quantities. The parameters of the trapezoidal distribution

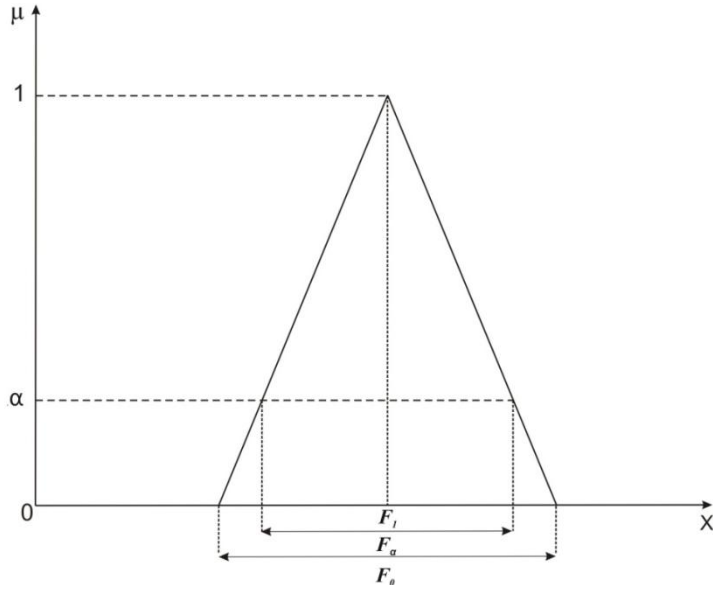


Fig. 2 A representation for triangular function and alpha-cut

can be calculated as

$$\beta = \frac{\lambda_1}{\lambda_2}, \qquad \lambda_1 = \frac{|(b_1 - a_1)(b_2 - a_2)|}{2}, \qquad \lambda_2 = \frac{b - a}{2} \tag{10}$$

In measurement uncertainty analysis, to specify a region of confidence and to structure the data for a trapezoidal membership variable, variance of the quantity X is be utilized. The variance derived from distribution can be formulated as follows

$$V(X) = \frac{(b - a)^2}{24}(1 + \beta^2). \tag{11}$$

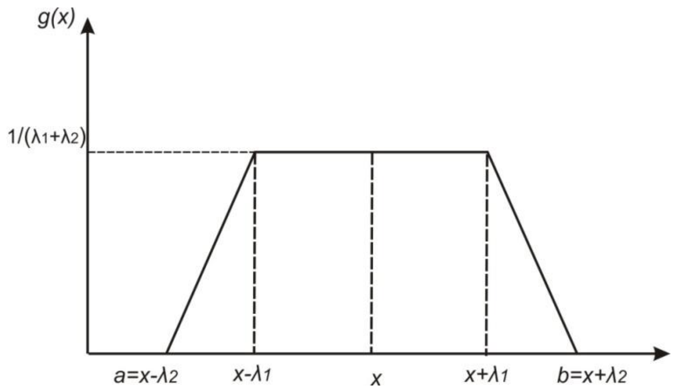


Fig. 3 Trapezoidal density function and critical parameters (BIPM 2008b)

In Eq. (11), $0 \leq \beta \leq 1$ and as $\beta \to 1$, this trapezoidal distribution approaches the rectangular distribution, while for $\beta = 0$, it is a triangular distribution (BIPM 2008b). The other parameters *a* and *b* represent upper and lower limits, respectively. Following the construction of membership functions, integration is required. The arithmetic operations utilized in this study such as sum, difference and division have been discussed in Hanss (2005). In a similar manner, the level of confidence is defined on the ground of fuzzy *α-cut* level. The method perceives, for example, each confidence interval level of *1-α*, with each *α-cut* of a fuzzy subset (Müller 2009).

3. Results and discussion

To apply the conventional and fuzzy set-based uncertainty evaluation methods for rock test measurement, the sample data set given in Ulusay *et al*. (2011) was considered. The data set consists of 10 laboratory measurements applied to sandstones sampled from different depths. In addition to geometrical parameters (diameter and thickness), the data also include the failure loads observed from the loading test unit.

The tests had been carried out in accordance with the Bieniawski and Hawkes (2007) standard test method stated in Bieniawski and Hawkes (2007). To determine the tensile strength of rock specimens, first 54 mm diameter NX-size cylindrical core samples, having a thickness less than 27 mm had been prepared. The surfaces of the specimens had been made free from any irregularities across the thickness using polishing machine. The samples were loaded into the Brazilian test apparatus across its diameter. The load at the speed of 200 *N*/*s* was applied continuously at a constant such that failure materializes within 15-30 seconds.

First all covered quantities *F*, *D*, *t* require to be described by their first two moments, standard deviation and mean value, for the GUM uncertainty framework. As explained in Section 2, all the parameters of the sampled rocks are considered as trapezoidal distributions.

Next, providing the sensitivity coefficients of the measurement function some partial derivations have been performed as suggested in the standard GUM uncertainty framework. For example, the sensitivity coefficient of *F* has been provided as follows

$$c_F = \frac{\partial}{\partial F}\frac{F}{Dt} = \frac{159}{250 * D * t} = 0.00043 \text{ (MPa)}. \tag{12}$$

The expected values of the distributions, the standard uncertainties, and the uncertainty budgets (sensitivity coefficients) obtained from the measurements are outlined in Table 1. As seen in Fig. 4, some weak correlations were recorded. The amount of Pearson's *r* values, which measures the strength and direction of the linear relationships between the variables, were recorded as smaller than 0.4. Therefore, the correlations have not been considered to provide the combined uncertainties.

Table 1 Input quantities

Input quantity	Expected value	Uncertainty	Budget (*c*)
F	15.790	0.417	0.00043
D	54.030	0.387	-0.00013
t	27.087	0.671	-0.00026

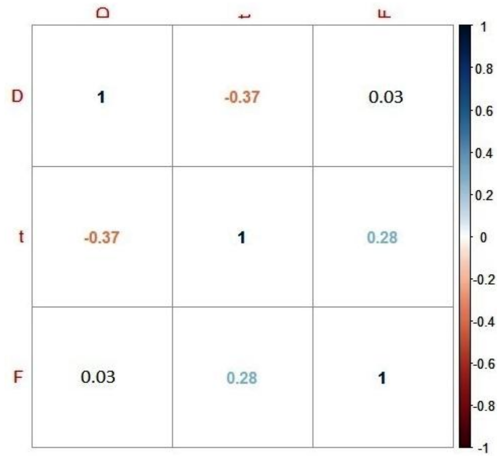


Fig. 4 Correlations between input variables

Table 2 Estimations and measurement uncertainties

ISO Method	Strength (MPa)	Combined uncertainty
GUM	6.979	0.256
MC	6.984	0.257

Table 3 Approximation of MC method

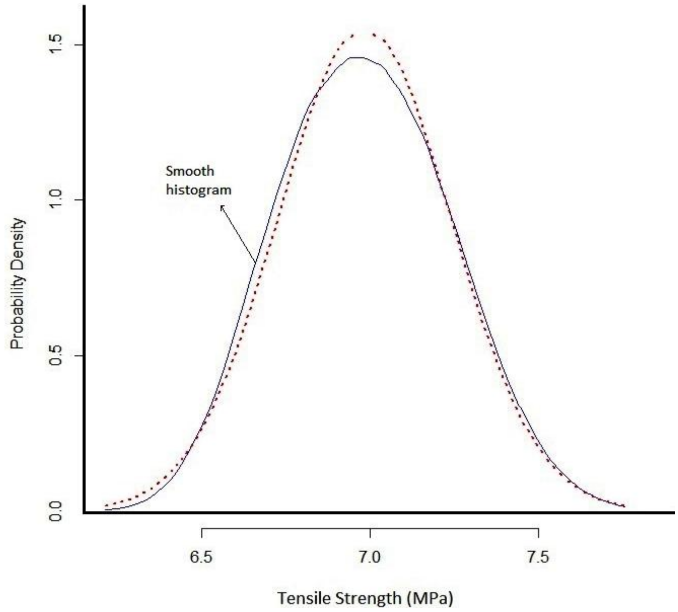
Confidence level	Coverage factor-*k*	Coverage interval
99%	2.58	[6.321-7.647]
95%	1.96	[6.481-7.488]
90%	1.64	[6.563-7.406]
68%	1.0	[6.727-7.241]

The major parts of the computer implementations were conducted using NIST Uncertainty Machine (Lafarge and Possolo 2015) and R packages such as 'metRology' (Ellison 2015). Table 2 indicates the results obtained for both the GUM and the MC methods. The MC simulation was performed using 10^6 replicates. Table 3 summarizes the coverage intervals produced by MC method at different levels with the coverage factors. For example, at 99% confidence level upper limit has been determined as follows

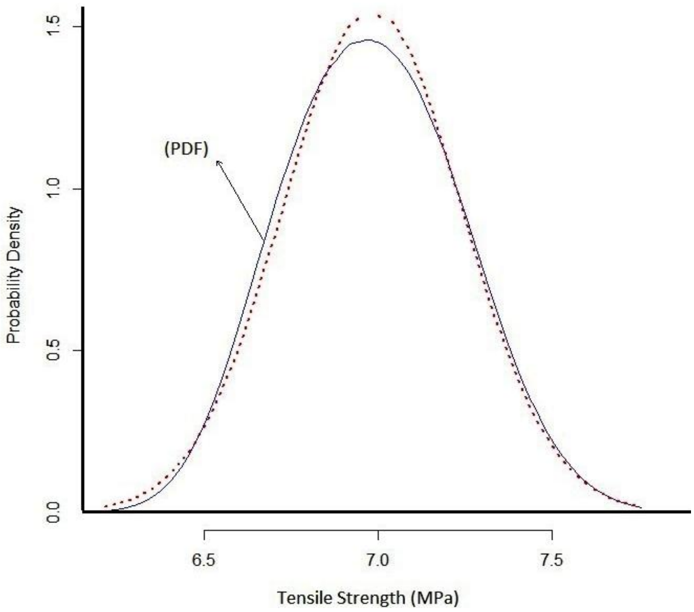
$$7.647 = 6.984 + 2.58 * 0.257 \tag{13}$$

Both the computer implementations produced Gaussian type output function in accordance with the GUM theoretical framework. The density functions addressed similar smooth histograms as presented in Figs. 5(a)-(b).

Fuzzy set-based applications were made by fuzzy trapezoidal membership functions. Fuzzy arithmetic operations and alpha-cut identification was also performed by a code written in MATLAB Environment (MATLAB-R2009b 2009). The estimate of the resulting function, the



(a)



(b)

Fig. 5 Density functions for tensile strength: (a) GUM-Taylor method; (b) MC method

tensile strength of rock, can be considered as the maximum (peak) of the membership function. The midpoint of the resulting interval corresponds to the estimate. Fig. 6 illustrates the final membership function. The alpha-cut at a level of 0.33 identifies a confidence interval with a level of confidence of 1 – 0.33 = 0.67. This α-cut has been considered to calculate a confidence interval

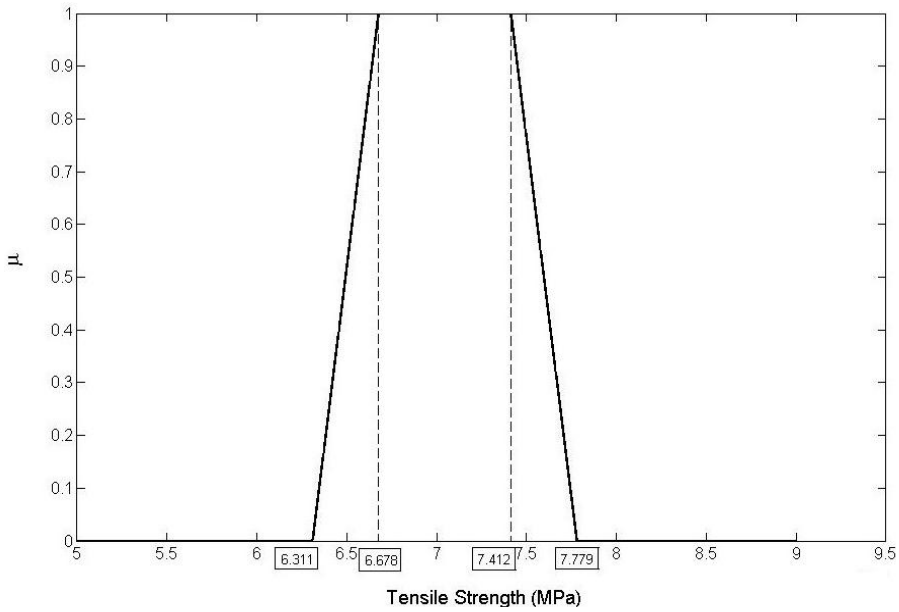


Fig. 6 Final membership function for tensile strength

which can be compared to the standard uncertainty. An interval containing approximately 67% of all possible values of [6.678, 7.412] is given in Fig. 6. The half-interval width is determined as (7.412 – 6.678)/2 = 0.367.

Even though the MC approach has produced [6.72-7.25] coverage interval for 67% confidence level, a larger interval [6.678-7.412] has been recorded for the fuzzy approach. The main reason for the larger confidence interval by fuzzy set approach can be explained by the transformation between probability and possibility. Although a possibility measure exhibits an inexact but consistent knowledge, a probability measure addresses precise but varied knowledge. For the evaluation of uncertainties by an alternative way, the use of probability density function-based fuzzy approach provided some satisfactory results. The method has transparency and flexibility. It can also be applied by a limited number of data.

In other respects, the correlations have not considered in the application due to amount of the correlation coefficients. The correlations in a different data set among the input variables F, D and t may lead to the measurement uncertainty in a direct way. This difficulty, to represent correlations in fuzzy method should be stressed as a disadvantage of the fuzzy method.

4. Conclusions

Rocks fail due to compression, shear, tensile and/or a combination of these stresses. In particular, the tensile strength is a key measure that influences rock crushing, deformability and blasting results. In this paper, the Brazilian test measures on rock samples taken from sandstones have been appraised by different uncertainty evaluation methods.

In addition to conventional Taylor’s and MC methods, a fuzzy set-based measurement uncertainty evaluation is practiced. The results showed that the conventional Taylor’s and MC approaches produce relatively narrow coverage intervals comparing to the fuzzy-set based method. The explanation of the results can be found in the information theory. Against to precise and varied knowledge, inexact and consistent knowledge produced by this study can also be considered by the users.

In an engineering application such as geotechnical work, measurement uncertainties can obtain an almost unlimited diversity of engaged dependencies which have critical importance for functioning. In addition to an observation recorded in a rock mechanics test, its uncertainty should also be expressed by one/all of the methods discussed in this study.

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