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Spatial variability analysis of soil strength to slope stability assessment

Mara Lombardi [1a], Monica Cardarilli [*2], and Giuseppe Raspa [2b]

[1] *Department of Chemical Materials Environment Engineering, Sapienza University of Rome, Safety Engineering, via Eudossiana 18 - Rome, Italy*
[2] *Department of Chemical Materials Environment Engineering, Sapienza University of Rome, via Eudossiana 18 - Rome, Italy*

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Abstract. Uncertainty is a fact belonging to engineering practice. An important uncertainty that sets geotechnical engineering is the variability associated with the properties of soils or, more precisely, the characterization of soil profiles. The reason is due largely to the complex and varied natural processes associated with the formation of soil. Spatial variability analysis for the study of the stability of natural slopes, complementing conventional analyses, is able to incorporate these uncertainties. In this paper the characterization is performed in back-analysis for a case of landslide occurred to verify afterwards the presence of the conditions of shear strength at failure. This approach may support designers to make more accurate estimates regarding slope failure responding, more consciously, to the legislation dispositions about slope stability evaluation and future design. By applying different kriging techniques used for spatial analysis it has been possible to perform a 3D-slope reconstruction. The predictive analysis and the areal mapping of the soil mechanical characteristics would support the definition of priority interventions in the zones characterized by more critical values as well as slope potential instability. This tool of analysis aims to support decision-making by directing project planning through the efficient allocation of available resources.

Keywords: soil uncertainty; spatial variability; shear strength; kriging; slope stability

1. Introduction

When referring to a natural soil, it is associated with the word *uncertainty*. Proper tools for estimating the consequences of natural hazardous events on human lives and built environment and thus to propose optimal and sustainable allocation of public resources for economy are required (Lagaros 2014). Since last few decades, therefore, a growing propensity of considering uncertainty in optimization process has been undertaken (Bhattacharjya *et al*. 2015).

The focus on technology has made it possible to increase the efficiency of the technical execution of the surveys and reduce the time of data acquisition. It also has improved and integrated the best experience with the empirical transformations of different variables characterizing the soil.

*Corresponding author, Ph.D. Student, E-mail: monica.cardarilli@uniroma1.it
[a] Ph.D., E-mail: mara.lombardi@uniroma1.it
[b] Professor, E-mail: giuseppe.raspa@uniroma1.it

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Hacking (1975) identified two classes of uncertainty associated with the soil properties: the first *epistemic*, which reflects lack of information, reducible perfecting survey instruments and knowledge; the second *aleatory*, not reducible, related to natural inherent randomness and, as such, which represents a function of the spatial variability of the property (Lacasse and Nadim 1996).

Excellent authors (Vanmarcke 1977a, Harr 1987, Rethati 1988, Tang 1993, Thorne and Quine 1993, Christian *et al*. 1994, Jaska *et al*. 1997, Phoon and Kulhawy 1999b, Uzielli 2008, Bond and Harris 2008, Wu 2009, Griffiths *et al*. 2009, among others) focusing on need to develop new methods about the spatial variability treatment of the acquired data which aim of optimizing usage and provide a soil characterization as complete as simple to implement. Both types of uncertainty contribute to defining the soil stability concept.

Instability may result due to rainfall, increase in groundwater table and change in stress conditions. In this way, the evaluation of slope stability conditions becomes a primary concern to save human lives, reduce property damages and provide continuous services.

The engineering solutions to slope failure problems should be able to identify the existing safety conditions and suggest for technically viable solutions which taking into account all the uncertainties.

This paper presents a procedure using the probabilistic approach of geostatistics to characterize the aleatory soil variability, in particular about shear strength parameter, to perform a spatial prediction representative of the sampled data, for the evaluation of the slope stability conditions.

2. Probabilistic analysis and scale of inherent soil variability

The evaluation of the role of inherent soil variability necessarily requires the implementation of probability concepts and methods. Numerous authors have been undertaken in the last time to perform a probabilistic slope stability analysis that deals with the uncertainties of soil properties in a systematic manner (Alonso 1976, Vanmarcke 1977b, Li and Lumb 1987, Mostyn and Li 1993, Christian *et al*. 1994, El-Ramly *et al*. 2002, Baecher and Christian 2003, Griffiths and Fenton 2004).

Before introducing the geostatistical analysis, the concept of regionalised random variables and random functions must be introduced (Guarascio *et al*. 2009).

In the *Theory of Regionalised Random Variable* (Matheron 1963), the true measurement *z* of

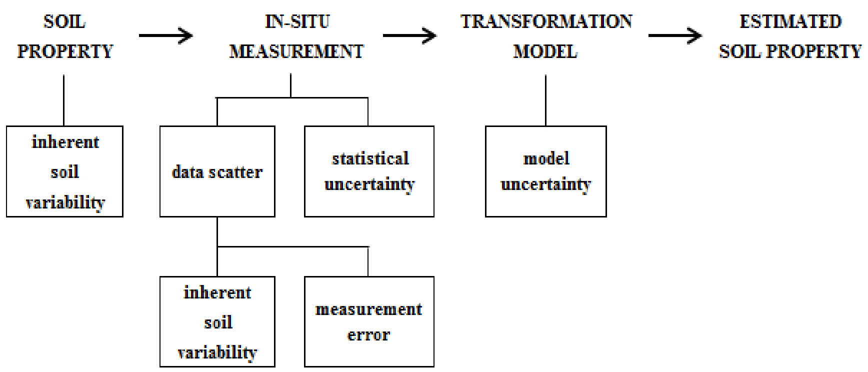


Fig. 1 Uncertainty in soil property estimates (after Kulhawy 1992)

some attribute is assumed to be the value of an Aleatory Variable (AV) Z. An AV is a variable that may assume multiple values and whose values are randomly generated according to some probabilistic mechanism. The RV is usually location-dependent, hence the notation $Z(x)$, with x being the location coordinates vector, and hence the term regionalised. The AV $Z(x)$ is also information-dependent, in the sense that its probability distribution changes as more data about the un-sampled value $z(x)$ become available. The family of AV is called a Aleatory Function (AF) defined over a given domain S, the set of values

$$\left\{ Z(x), \quad x \in S \right\} \tag{1}$$

The set of functions represents the spatial law of the AF Z.

In general, to simplify the analysis, analytical and transformation models are used to interpret results of site investigation using simplified assumptions and approximations. But, in reality due to the complexity of soil formation and depositional processes, the complete spatial law of soil behaviour is not possible to estimate especially by a limited number of points (measured shear strength parameter). Therefore the necessary to formulate the *weak stationarity hypothesis*, where the first two moments entirely characterise data distribution to make the study exemplified.

As frequent as typical of natural phenomena, soil properties do not have mean and/or variance value (so also variogram function) constant in the field, namely they may be function of the location in space (not translation invariant). The *non-stationary* methods are introduced to treat a spatial parameter that may not modelled by AF stationary. The most common situation, of non-stationarity is when variable values fluctuate around one value that, within individual *neighbourhood of estimate*, may not be considered constant.

For many years, the heterogeneity of soil has been recognized as the result of factors and causes operating and interacting at various spatial and temporal scales (Burrough 1993). It demands for more accurate knowledge on spatial distribution of soils the evaluation of the spatial dependence and scale (Godwin and Miller 2003). This is because the variation at some scales may be much greater or smaller than others.

Soils clearly differ on regional scale (Brejda *et al*. 2000, Guimaraes Couto *et al*. 1997), and a great variability may be expected as due to a wide variation of soil characteristics. The characterization of the spatial variability of soil attributes is essential to achieve a better understanding of complex relations between soil properties and other factors (Goovaerts 1998), and to determine appropriate management practices for soil use (Bouma *et al*. 1999).

Depending on the scale of the study, several direct and/or indirect field investigation methods are thus applied in order to define the main characteristics of the variability of the soil.

Some may delineate large scale features such as permeable channels, whereas others may detect finer scale transitions. It may be clearly seen that there are different scales of variability, ranging from the micro level at the grain size scale to the geological scale of several tens and hundreds of meters.

3. Dichotomous approach: Drift and residues

If the phenomenon manifests with evidence, at the correct scale of observation, a gradual increase or decrease, this implies the need to identify an AF model with variable mean (non-translation invariant). Empirically, this trend might be described by the tendency of mean at the scale in which must observe the phenomenon for geostatistical study, because it is evident, certain

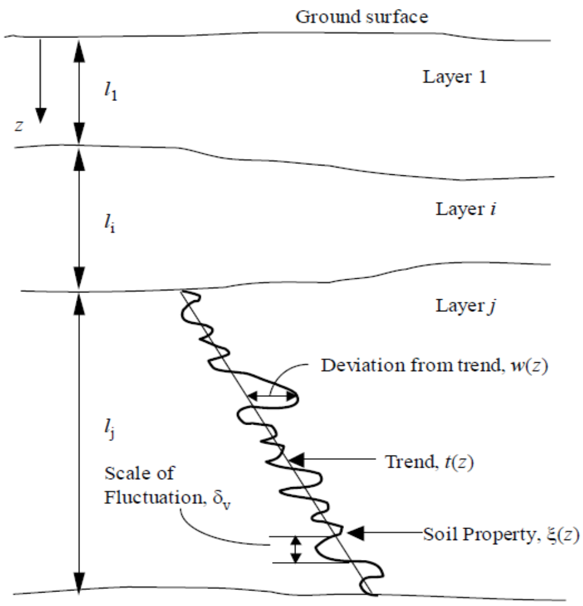


Fig. 2 Inherent soil variability (after Phoon and Kulhawy 1999a)

frequencies appear or do not appear depending on the scale of observation (non-stationary at the large-scale and weak stationary at the small one).

This influence by the scale of observation of the phenomenon, appears to introduce a strong element of subjectivity. In truth, this is related to the natural soil characteristics, so the result is just the expression of the regional component with a physical meaning.

In this study has been used the *dichotomous approach* inspired by the observation of natural processes and by the scale of soil variability.

The data have been decomposed to extrapolate the component responsible for the non-stationary condition: the spatial trend or *drift*. In a model with drift, the hypothesis consists of the AF Z (indicated as *target variable*) may be considered, in each point x, as the sum of two components

$$Z(x) = m(x) + Y(x) \qquad \text{with} \qquad E[Z(x)] = m(x) \qquad \text{and} \qquad E[Y(x)] = 0 \tag{2}$$

The first term, the trend, is considered deterministic component of variability on a regional scale, term to indicate a systematic spatial variation due to lithology and sedimentary processes, while the *residues* as stochastic local fluctuations at the small-scale with zero mean which satisfy the condition of weak stationarity.

The model with drift derives from the probabilistic treatment of everything is not *intrinsic characteristic* of the phenomenon. Nevertheless, even if the model is inspired by the physical property of the RV, it does not have a precise physical meaning since the drift is not directly observable or deducible theoretically. Therefore, the dichotomous model remains a theoretical construct, functional only to operation estimated, in limited situations of non-stationarity those in which the variable of the study presents small fluctuations around a geometric shape knowable and moldable with a regular and continuous function.

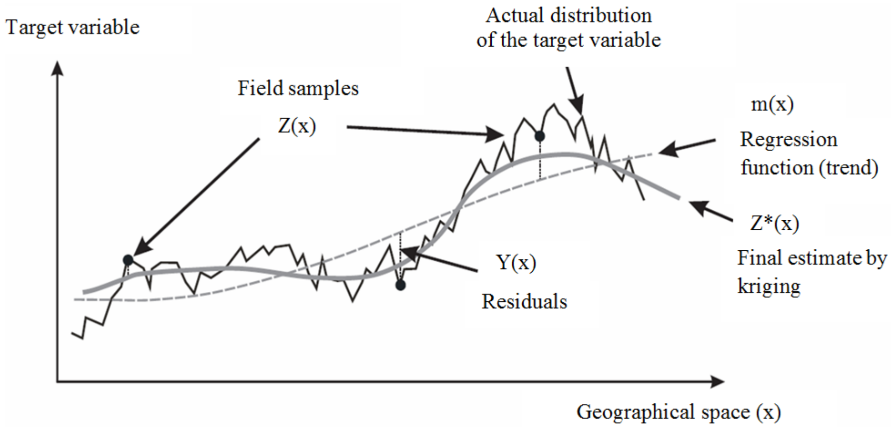


Fig. 3 Schematic representation and mathematical modeling of spatial variation with main components of target variable (Hengl *et al*. 2007)

3.1 Trend removal

The drift may be removed by well-defined mathematical functions to sampled points along the preferential direction (the study has considered the vertical one); the easiest way to do this is by regression analysis (least squares method) and subtracting it from the original data generating the residues variables through the difference.

Typically the deterministic component of the Z is modelled with a polynomial function where the degree represents the drift shape

$$m(x) = \sum_{k=0}^{n} a_k f^k(x) \tag{3}$$

where a_k are the coefficients of the polynomial and $f^k(x)$ the monomials of degree k with x the location coordinates vector.

For the purposes, the scale at which watching is crucial (what appears as trend to a scale may appear residue at a wider scale). Its choice has the meaning and implication which must be taken into account, in particular: the trend represents the regular component of RV and this must appear, at least, at the scale of the neighbourhood of estimate where its size is usually conditioned by the density of sampling; the drift model influences the spatial variability of residues so it is necessary that there is consistency between the two entities; then the dichotomous model must be valid at the scale at which it will be used.

3.2 Residual variogram model

The residues remaining after detrending, commonly manifests *spatial correlation* showing the presence of reciprocal statistical dependence. It displays a degree of association among themselves as function of their distance in space (*autocorrelation*). The residues Y have, for definition, a zero constant mean

$$E[Y(x)] = E[Y(x+h)] = 0 \tag{4}$$

and the respectively existing variogram satisfying the weak stationarity hypothesis

$$Var[Y(x+h)-Y(x)]=2\gamma(h) \tag{5}$$

So described, the aleatory component (variability about the trend) may be modelled with a stationary AF with a variogram function translation-invariant.

In particular the mean and variance do not change with the location and the correlation between the residual increments at two different points is a function only of their separation distance, rather than their absolute positions.

The variance of the residues reflects uncertainty about the difference between the interpolated trend and the actual value of soil properties at unobserved locations; by changing the drift model both the residual variance and their autocorrelation function are changed.

The AF description of residual spatial variability of sampled points needs to fit a theoretical mathematical function, with specific properties, which describes the empirical variogram as well as possible: a permissible model that captures the major spatial features of the attribute under study, defined at the origin and over the entire range, which enables to quantify the spatial autocorrelation.

4. Kriging: Spatial prediction method

The main application of this study is the estimation and mapping of soil mechanical properties in un-sampled area for evaluation of slope stability conditions.

The Kriging method, as the *best linear unbiased prediction* (Viscarra Rossel *et al.* 2010), represents a generalized linear (least-squares) regression algorithms used for determining the optimal weighting of the measurements points to obtain, for the scope, a reasonable spatial prediction representative at all un-sampled locations in the region of interest.

The general formula, applied to original dataset, consists of weighted sum of the data

$$Z^*(x_0)=\sum_{i=1}^{n}\lambda_i Z(x_i) \tag{6}$$

where:

$Z(x_i)$ = measured value at i^{th} location;
λ_i = unknown weight for measured value at i^{th} location;
x_0 = prediction location;
$Z\times(x_0)$ = estimated value at x_0 location;
n = number of measured values.

The Kriging technique due its goodness prediction to the following properties, it is: (i) *exact*, estimated value coincides with the real value; (ii) *local*, predicts a value using the known data falling in its neighbourhood of estimate; (iii) *linear*, estimated value using a linear combination (weighted average) of the variable values known in the surroundings; (iv) *stochastic*, considers the statistical properties of the data and the spatial autocorrelation.

In Kriging the weights are based not only on the Euclidean distance between the measured points and the prediction location, but also on the overall spatial arrangement among the measured points. These optimal weights depend, in fact, on spatial arrangement and autocorrelation quantified.

Furthermore, the Kriging may estimate the error associated with each prediction and its *correctness*, meaning that in a sampled point the estimate value is equal to the observed one, is such as so the mean estimation error is null; in this way Kriging provides also the minimum estimation variance of the error, for which it is defined an *accurate* method.

It appears evident that Kriging variance as measure of precision, relies on the correctness of the theoretical variogram model assumed.

As with any method, if the assumptions do not hold or in case of no spatial dependence, Kriging interpolation might be not representative of the data points.

4.1 Kriging of the Drift and Simple Kriging (*Universal Kriging*)

The regionalized variable theory assumes that the spatial variation of any variable Z may be expressed, as in this study, as the sum of two dichotomous components.

Universal Kriging (UK) estimates both global trend and Kriging weights in one step incorporating them into the system of simultaneous equations. It may produce good local evaluations in the presence of non-constant drift, especially in situations where estimate is extrapolated rather than interpolated from the local sample values. UK fits the drift as a function of the site coordinates preserving the universality conditions (unbiasedness) and optimal sample pattern minimizing the error variance estimation.

The spatial reconstruction resulting from the UK, which includes both two components, is representative of the original variable Z.

For the purpose of the study, it has been considered useful and appropriate to provide a spatial prediction of individual components making a separation of the UK method: the use of *Simple Kriging* (SK) to the spatial residuals estimation (because of zero-mean) and subtracting it from the original data obtaining the spatial drift prediction (*Kriging of the drift*) through the difference.

Therefore, this provides a split representation of contributions so each component characterizes the inherent soil variability: both for predicted values and estimation errors variance.

Afterwards, the assessments may highlight aspects and characteristics that with Z-predicted mapping might not be possible to see so appreciate.

5. Case study

The study area is located in Celano, a municipality in the Province of L’Aquila (Italy). Celano rises along Apennines chain and is characterized by peaks also over 1000 meters of altitude.

Particularly, the investigated site is located on the north-west of the center of Celano and constituted by a slope portion of about 60 meters of length and 50 meters of width bounded by ways and terraced buildings. The angle of natural slope ranges between 20 and 30 degrees.

The various lithotechnic characteristics of this region have determined different geomorphological features. These conditions, often in conjunction with other factors, have generated the genesis of movements due to deformation of the slopes or wide landslides.

The large valley bordered by the mountains contrast with the typical landscape of the chain, where geological and climatic factors have been developed the deposition and modeling of continental sedimentary sequences.

The deposits, outcropping in the area, display complex stratigraphic correlations among local Plio-Pleistocene lacustrine and alluvial deposits and Holocene alluvial debris. Altogether such deposits are in contact with the Meso-Cenozoic limestone and Neogene sandstone relieves that also represent the bedrock units.

In particular the site is founded on recent (Pleistocene-Holocene) alluvial fan deposits bereft of groundwater.

5.1 Sampled data processing

Note the nature of the soils, 10 tests have been performed in-situ by the use of Dynamic Probing Medium (DPM). These tests have been used to determine the strength of soils investigated with measurement every 10 cm of depth.

In a preliminary observation of surveys, along depth, one test has been excluded from the following analysis for a discrepancy of its value (highly homogeneous) presumably due to its execution near the highway downstream slope so affected by the effects of an anthropic soil mixture or, possibly, by the presence of loose-fill materials. Therefore, to not alter or affect the results of the following analysis, the study do not include the test characterized by this trend.

The remaining tests have been interpreted with empirical and indirect relations, in particular: (i) Liao and Whitman (1986) for the independence from overburden lithostatic stresses; (ii) Terzaghi and Peck (1948), Skempton (1986) to assess the relative density index of soil; (iii) Schmertmann (1978) to estimate the angle of friction at peak for sand and gravel.

The chosen methods to estimate the shear strength of soils has taken account of the information derived from geological characterization: in particular, the granular nature of the soils involved

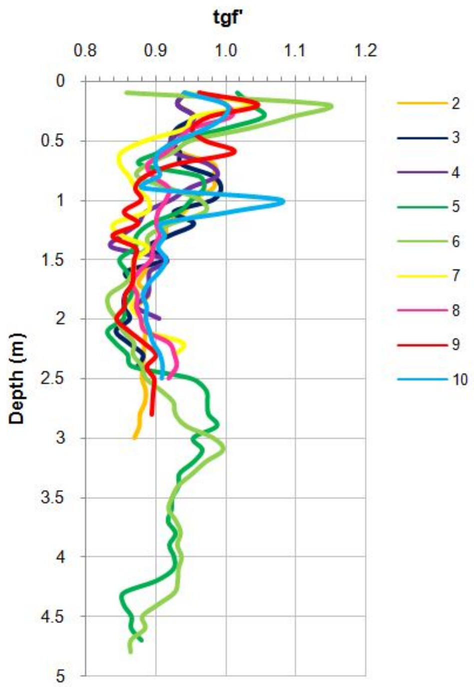


Fig. 4 Vertical profiles of peak friction angle (inherent variability) obtained by tests performed in situ

and the absence of groundwater, moreover the type of in-situ soil testing and the extrapolated data which identified the bedrock presence as abrupt change along the stretches of the latest depth.

Whereas the nature, state and condition of soil, the friction angle at peak has been considered the representative parameter to entirely define the shear strength at the effective stress of the slope. In particular, to conform to Italian regulations (NTC08) prescription, the angle has been converted to its tangent value.

The existence of many experimental relations also involves a clear sign of the uncertainties and approximations inherent in the empirical estimation procedures.

Note this one and the heterogeneity of the soil, the values obtained in the same layer may also be significantly different from themselves, with also incorrect and unreliable values.

The study has not performed an assessment of the epistemic uncertainty: not conditioning the obtained data so their representativeness of investigated points.

Vertical profiles have been represented below for the tests to allow a better stratigraphic interpretation revealing mechanical behavior of the soil, its thicknesses and characteristics.

5.2 Characterization of spatial variability

The application problem, when trying to model experimental variograms on data, consists in the different sampling scales for the vertical and horizontal directions due to the sedimentary nature of a flat soil deposit. In particular, as show below, the locations separated by vertical distance are in order of few meters (small-scale) while the distance in a horizontal direction is representative of the large-scale at more than 10 meters (270 punctual sampled values have been used for carrying out the spatial analyses).

The variogram cloud, plotted in the Fig. 5 below, clearly shows the pairs of values as a function of the sample distance and the different variability scale of shear strength related to horizontal and vertical directions necessary to determine the empirical variograms.

When more different scales of the same variable are not recognized (generally the smallest), experimental variogram does not show a clear structure near the origin of axes: this may be unjustly associated with a nugget effect as measurement error (no spatial relationship) for small extents. It is therefore essential to make a experimental variogram changing the distance (width),

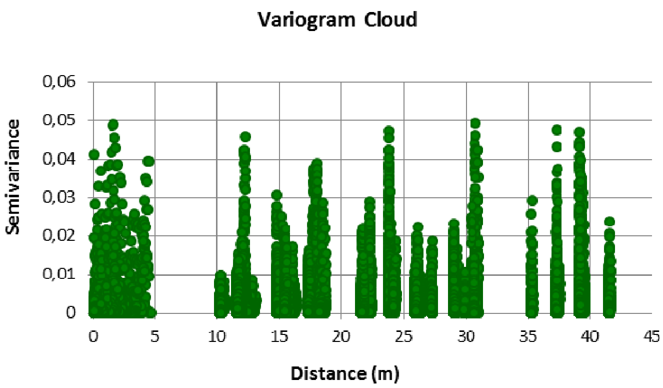


Fig. 5 Variogram Cloud: Scales of variability relate to sampled points. The concentration of pairs of values for distances below 5 m reveals a scale of variability along the vertical direction. The remaining pairs indicate for distances over 10 m a widespread variability along the horizontal plane

its tolerance and the number of lags over which the variogram will be calculated by allowing to appreciate adequately the spatial variability. Hicks and Samy (2002) has been observed that the scale of fluctuation in the horizontal direction is much larger (less variability), due to natural processes, than in the vertical direction.

To underline the presence of spatial correlation between sampled values, they have been analyzed separately in the two scales of variability corresponding to the horizontal and the vertical ones: for each one the slope has been separated into multiple layers parallel respectively to the direction analyzed.

The analysis on horizontal plane shows the presence of directional zonal anisotropy where variability increases along the longitudinal section of the slope and reduces transversely reaching stable sill; while the vertical highlights, from the first distances, a spatial trend that involves moving average of the data (parabolic) along the depth after an initial and temporary achievement of a constant sill so, the non-stationarity of the *Z*-variable along the same direction (over 2 meters the data values return to be more correlated). Since the depth constitutes a single direction and not a plane, the variogram at the small-scale is unique. The directional anisotropy condition there is not applying (isotropy).

The presence of a systematic variability is evident even by the performance of the variable for the different tests along the depth-direction.

The smoothing factor (R^2) depends on the number and spatial distribution of sample points. There is no universally accepted method for selecting an R^2 value (Aguilar *et al*. 2005). In this application, by comparing verticals value of observed shear strength parameter, the trend has been fitted by a cubic polynomial where the smoothing factor approximates about 40 percent of the variability due essentially to stratigraphic processes. The logic suggests using a polynomial of degree higher as a representation of the trend but because the interest for us is to characterize the aleatory variability (residue), therefore too accurate description of the trend would encompass inevitably also part of the residual variability.

The residual experimental variogram along the depth, obtained from trend removal of original data, shows as the spatial correlation progressively reaches a sill remaining constant. It validates

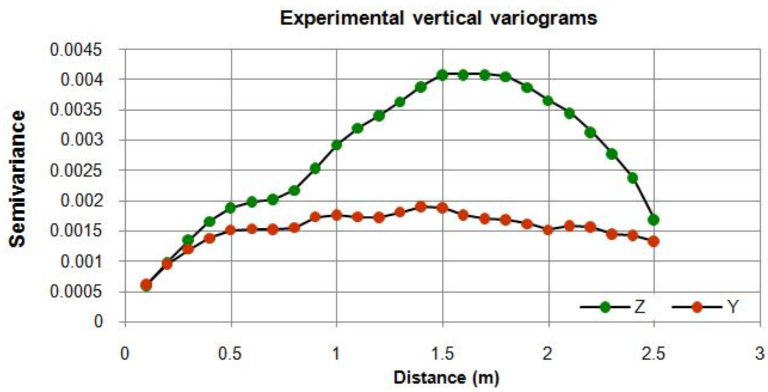


Fig. 6 Vertical variograms (before and after detrending). The green curve identifies the variability of spatial distances between the known points on the small scale. The parabolic trend is due to the presence of a trend in the vertical direction. The red curve represents, for the same pairs, the residual component obtained by removing the trend (detrending values) which gives a constant sill to the experimental variogram

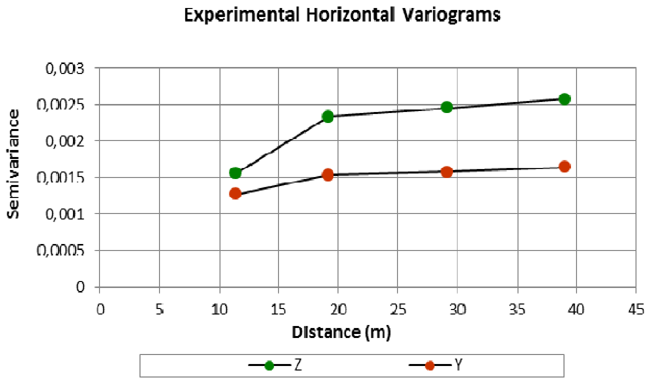


Fig. 7 Horizontal variograms (before and after detrending). The green curve represents the semivariance of the distances on the large scale. The detrended punctual values are identified by the red curve. Also in this case, the residual component gives to the experimental variogram a constant sill value

the hypothesis that *Y* is weakly stationary. The horizontal scale retains a slight zonal anisotropy such that the omnidirectional variogram may be considered representative of the variability characterization. The different degree of variability, almost entirely absent from the residual values, then indicates how the variability of the horizontal shear strength is closely related to the sedimentary phenomenon particularly to the main directions of deposition.

As in the original data, directional geometric anisotropy remains between the two scales of fluctuation. This means that fitted variogram model in the horizontal and vertical directions reach the same degree of variability (sill) but at different distance of separation (range).

5.3 Anisotropic variability modelling

The theoretical variogram model is used for interpolation purposes (Cressie 1990, Davis 2002). In this case a three-dimensional variogram model has been built to reflect the aleatory residual and their spatial correlation in each investigated direction to the variability modelling.

The applying model which best fits the two variability scales (horizontal and vertical) of residuals data is the Exponential variogram function (with zero nugget) whose optimal and representative parameters are at following reported.

The spatial variogram model represents, as well as the two-dimensional correlation properties, also the ratio of the geometric anisotropy of ranges expressing the different variability at the two scales.

5.4 Kriging spatial mapping: Prediction and uncertainty

The geostatistical literature uses many different terms for what are essentially the same or at least very similar techniques. Stochastic (kriging) predictions provide explicit functions to represent the relationships between the inputs and outputs of a linear or nonlinear system, which is a desirable advantage for response estimation and parameter identification in design and model updating problem.

Several forms of kriging do exist, the most common three are; simple kriging, where the mean value is known prior to calculations and is constant; ordinary kriging, where the mean is assumed

Table 1 Exponential variogram properties

	Vertical	Horizontal
Nugget	0	
Partial sill	0.0016	
Practical range (m)	0.27	7.49

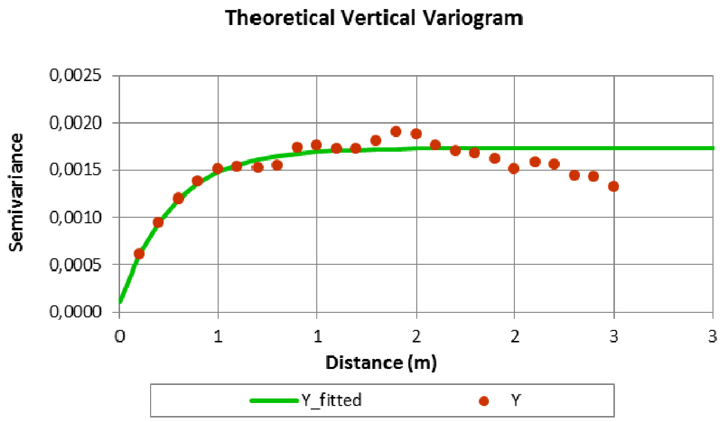


Fig. 8 Vertical fitted model. The exponential theoretical model (green curve) fits the variability at small-scale of residual values (in red)

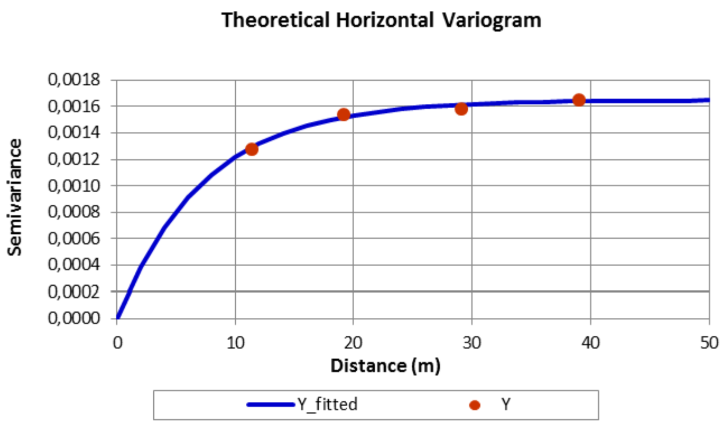


Fig. 9 Horizontal fitted model. The exponential theoretical model (blue curve) fits the variability at large-scale related to detrended values (in red)

constant, but is unknown; and universal kriging, where the mean is unknown and not constant.

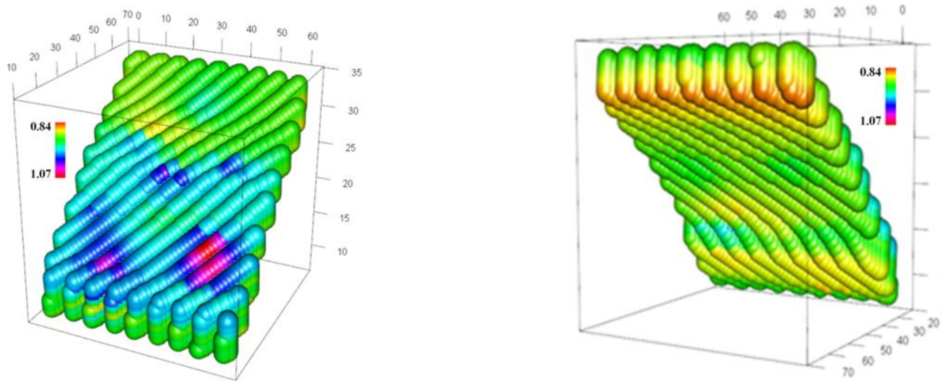
Universal kriging (UK) model was introduced by Matheron (1969) and that is by many statisticians considered to be the best linear unbiased prediction model of spatial data (Gotway and Stroup 1997, Stein 1999, Christensen 2001).

The results of a Kriging normally consist, for a given configuration of estimation, in unknown weight for the measured value and in the local precision of estimates. They depend essentially by the number of measuring points, their geometrical position, mutual respect and the entity to estimate, and not at last the variogram fitting function.

Originally, UK was intended as a generalized case of kriging where the trend, as deterministic part, is modelled as a function of coordinates, within the kriging system. Thus, many authors (Deutsch and Journel 1998, Wackernagel 2013, Papritz and Stein 1999) reserve the term Universal Kriging for this case. Once the deterministic component of spatial variation has been modelled, the stochastic residues can be interpolated with kriging and then added to the estimated trend as suggested by Ahmed and de Marsily (1987). If the residuals show no spatial autocorrelation (pure nugget effect), we proceed with OLS estimation of the regression coefficients. Otherwise, if the residuals show spatial auto-correlation, we can run regression-kriging.

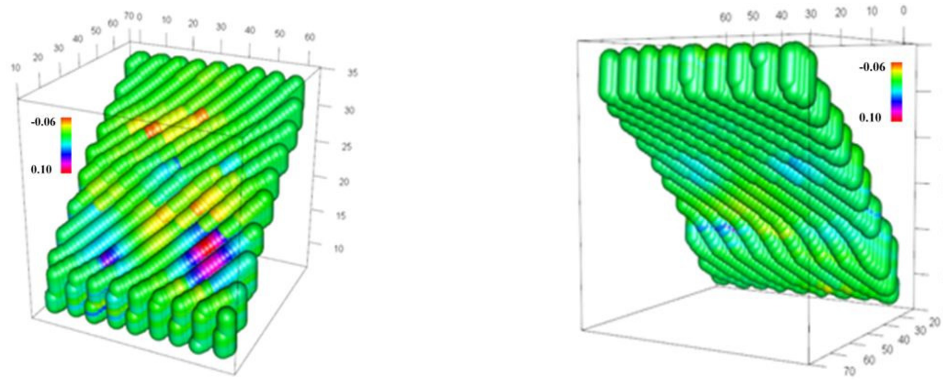
By implementing the 3D model of exponential variogram obtained from the variability characterization of residual values, to the initial variable prediction indicative of the sampled data (UK), the following estimation map has resulted.

The variability characterized by predictive map relative to original data detects particular high values downstream localized. Upstream, instead, the interpolator has provided a prediction



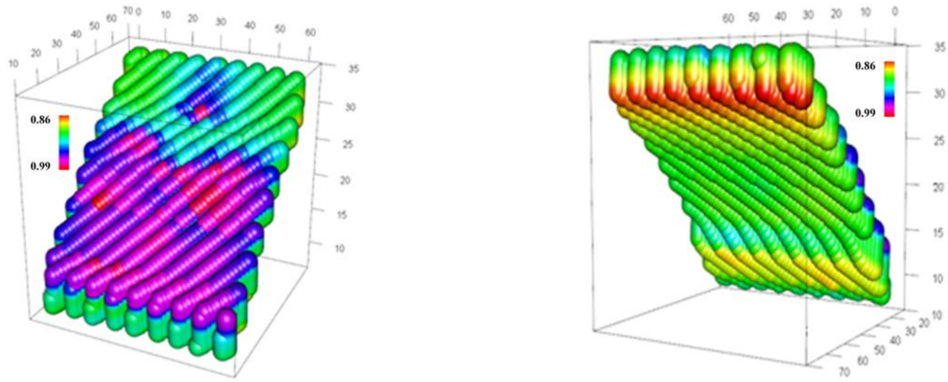
(a) Upper slope profile (b) Lower slope profile

Fig. 10 Variability map of UK predictions obtained by the characterization of residual values. The estimate of the target variable values ranging from minimum to maximum resistance



(a) Upper slope profile (b) Lower slope profile

Fig. 11 Aleatory variability map (SK predictions) obtained by the characterization of residual component. Stochastic predictions of values oscillate around value close to zero



(a) Upper slope profile (b) Lower slope profile

Fig. 12 Deterministic variability map (predicted drift) obtained by the modeling along the vertical trend. The predicted values vary in a smaller range than the UK estimates

Table 2 Ranges of predicted variables

	Target (Z^*)	Residual (Y^*)	Drift (m^*)
Min	0.84	-0.06	0.86
Max	1.07	0.10	0.99

Table 3 Range of predicted target values variance

	Variance
Min	0.00034
Max	0.00259

characterized by lower values reaching the minimum strength in the slope area in contact with bedrock. The spatial estimation maps of the two dichotomous components of original data variability are severally illustrated below, where the map reflecting spatial drift has been derived from the difference between UK and SK prediction values.

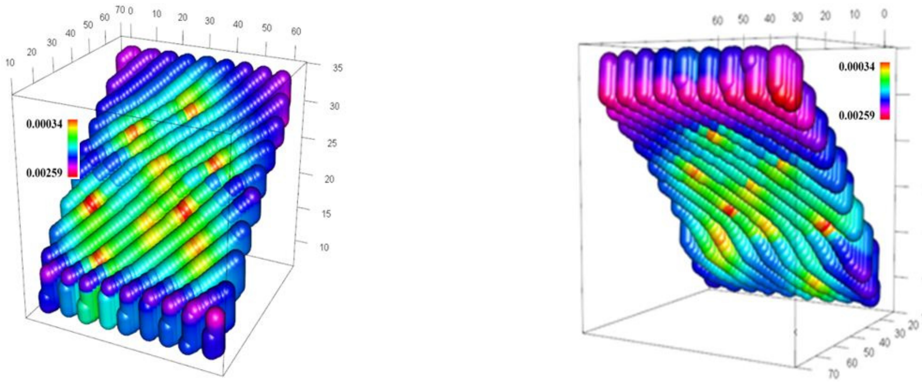
The predictions of aleatory component defined with residues as local fluctuations of original data, shows a localization of minimum values fairly extended along superficial layer upstream and downstream, and a slight majority of higher values at even greater depths in the central area.

The structural variability, mathematically reconstructed, assumes a dominant role for the spatial reconstruction of the original variable; it highlights extreme maximum values focused mainly downstream slope while the minimum ones are located on bedrock layer with less prominent.

The geostatistical approach is perhaps most useful in that it provides not only an estimate of the unknown value, as discussed above, but also an estimate of the uncertainty refer to a specific spatial location (Goovaerts 1997) of the predicted shear strength value: the *Kriging variance*.

The three-dimensional spatial maps below illustrate the spatial distribution of errors associated with the interpolation method.

The map illustrates how the increase of the variance values of the predicted locations at the edge of the interesting area, reflects the increasing of distance from measured points. So the



(a) Upper slope profile (b) Lower slope profile

Fig. 13 Variability map of UK variances. The predictions of variance reflect the new position of estimated locations whose values are very close to the null value

estimation variance allows evaluating the ability of the Kriging method to estimate uncertainty accurately with respect to the true data. Thus the UK approach provides a very accurate ranking of the spatial distribution of the estimation uncertainty.

5.5 Error analysis: *Cross-validation method*

The several measure used to compare the relative performance of the interpolation method with respect to the true data estimation is the Cross-Validation (CV). It is a useful method allows to establish how well the theoretical model predicts values at unknown locations as measure of the discrepancy. It removes each data location and predicts the associated value using the data at the rest of the locations. Thus, repeating this for all measured points, CV compares the predicted value to the observed value and obtains useful information about the quality of Kriging predictions so validates the goodness of fitted variogram model, parameters and neighbourhood.

Associated with any estimate, derived from a finite number of observations, is the *Error of Estimate*. As Aleatory Variable, its amplitude is used to assess the goodness of the same estimate defining the error as the difference between the estimate and its true (but unknown and aleatory) value. The estimated error is characterized by a law of probability density that should ideally represented a very narrow, symmetric and with zero-centered shape (i.e., the mean of the errors equals to zero), that reflect the lack of a significant and systematic error in the estimations. The histogram, referring to original data, is below plotted.

There is no apparent bias, no significant negative average error so it may not represent a systematic overestimation (Wackernagel 2013).

The combination of data by a weighted linear sum tends away from low and high estimates; there are more near the mean. The characteristic to note is the *smoothing effect*; Kriging surface

Table 4 Estimation error parameters

Estimation Error	
Mean	-0.00046
Variance	0.000746

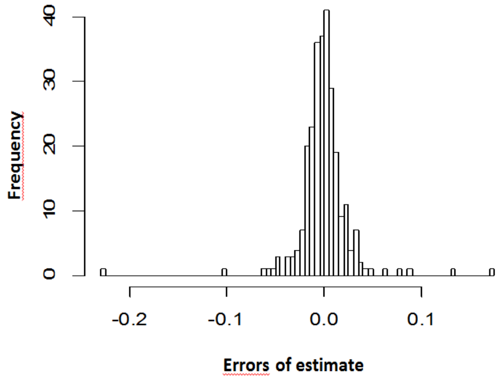


Fig. 14 Estimation error histogram

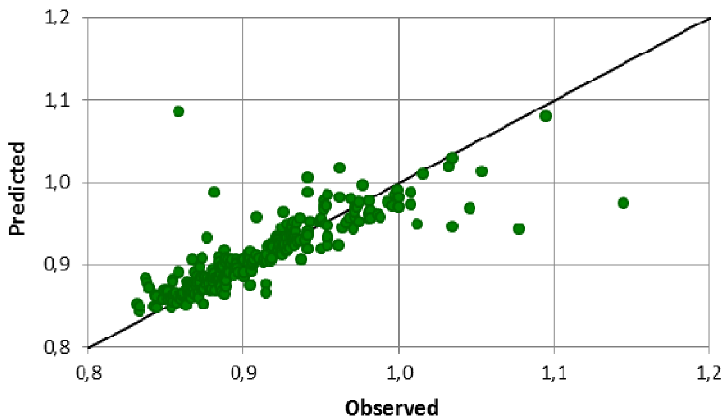


Fig. 15 Scatterplot between predicted and observed original values

Table 5 Range of predicted and observed values

	Observed (*Z*)	Predicted (*Z**)
Min	0.83	0.84
Max	1.15	1.09

will basically be as smooth as possible given the constraints of the data; thus, the estimated maps represent smoother outputs than real variable (Pyrcz and Deutsch 2014).

In the following Fig. 15 and Table 5 there are showed a comparison between original data input and the relative estimated output values.

In particular, the results of the CV show that the smoothness of Kriging has involved an appreciable underestimation of higher peaks and a slight overestimation of low soil strength values. The average of the squared standardized errors, as the ratio of squared estimation error and the predicted value variance, has given, once again, a measurement of the adequacy of the model and its predictions obtaining a slightly overestimated value of 15%. It allows comparing the magnitudes of both the predicted and the actual error.

Table 6 Performance criteria between predicted and observed values

RMSE	0.027
MAE	0.015

Comparing the estimation error variance and the variance of the predicted values (its average value) obtained from CV sampled locations, the different dispersion value indicates a overestimation of the error variance of about 25%.

Root Mean Square Error (RMSE) is a quadratic scoring rule which measures the average magnitude of the error. The difference between forecast and corresponding observed values are each squared and then the square root of the average is taken over the sample. RMSE gives the standard deviation of the model prediction error. Since the errors are squared before they are averaged, the RMSE gives a relatively high weight to large errors. This means the RMSE is most useful when large errors are particularly undesirable.

Mean Absolute Error (MAE) measures the average magnitude of the errors in a set of forecasts, without considering their direction. The MAE is the average over the verification sample of the absolute values of the differences between forecast and the corresponding observation. The MAE is a linear score which means that all the individual differences are weighted equally in the average.

The MAE and the RMSE may be used together to diagnose the variation in the errors in a set of forecasts. Both the MAE and RMSE may range from zero to infinity: lower values indicates better model performance thus for this case study the comparative error assessment gives acceptable results.

5.6 Analysis of the results

Spatial distribution of shear strength is quite heterogeneous and asymmetric, as previously illustrated. The task of the estimated variability from experimental data is thus very chellenging indeed and it allows appreciating each contribution to the overall spatial variability for indicative values of critical soil strength conditions both local and global. However the estimated data provided an ideal situation to examine the true prediction errors associated with spatial interpolation and thus these true errors were used to evaluate interpolation model performance.

The results indicate that for relatively uniform, dense sampling locations, methods appear to be optimal. We hypothesize that it is a consequence of the relatively large number of observations, which lessens the influence of extreme values on model calibration and spatial interpolation.

The spatial prediction carried out based on the data available has enabled interpretation of the variability of the shear strength of the ground to be used in the stability assessment.

It is related to: (i) increase of the available information on the measured values; (ii) local and punctual assessment; (iii) spatial aleatory uncertainty of the parameters and extrapolation of the real behaviour of the soil (systematic tendency, fluctuations, anisotropy) all with regards to disposition, density and reliability of surveys measurement.

The analysis of the uncertainty may have possible effects on the same stability analysis for the individuation of the failure conditions, based on accurate, correct and local estimated un-sampled values; selection of slope sections with more criticality for the estimation of the stability conditions (choice more cautious and conscious) related to spatial distribution and correlation model (estimations sensitivity) previously defined.

The results have showed the presence of strength values particularly critical and locally circumscribed representing the likely predisposing factor the instability condition of the slope that has really occurred implying the soil failure with landslide downstream.

Relevant outcomes have showed that the proposed technique is endowed with good correlation and accuracy, even when measurement and estimation errors are present. The Kriging mappings highlight that methods are applicable with relatively limited data. In addition, the prediction of uncertainty may be described at various levels and scales of soil variability.

6. Conclusions

The Italian legislation does not provide a clear guidance in the selection of characteristic geotechnical parameters to be used in slope stability analysis and a support in the choice of representative shear strength parameter at failure. The reason is due largely to complex and varied natural processes associated with soil formation. Spatial variability analysis for the study of the stability of natural slopes may incorporate uncertainties complementing conventional analyses.

In this study, the carried out analyzes and the used methodologies want to verify the presence of the conditions of shear strength at failure using back-analysis.

This approach would be an incentive for designers to improve soil characterization by considering the inherent variability a resource which needs to be exploited for a more accurate as well as conscious assessment of shear strength, consequently of potential slope stability conditions so future intervention design.

Regarding further developments, the methods applied and implemented in this paper may have useful future applications for: (i) planning functional investigations by increasing number, density and spatial distribution regularity; (ii) data availability and correctness of measurements with representative estimates of spatial variability and model of more accurate and correct application; (iii) punctual prevision of least strength values for instance on potential cinematic failure surface definition; (iv) map of the respective probabilities of exceedance (i.e., fractiles map); (v) punctual forecast of the Factor of Safety minimum values and (vi) map of the respective probabilities of failure.

References

Aguilar, F.J., Agüera, F., Aguilar, M.A. and Carvajal, F. (2005), "Effects of terrain morphology, sampling density and interpolation methods on grid DEM accuracy", *Photogramm. Eng. Remote Sensing*, **71**(7), 805-816.

Ahmed, S. and de Marsily, G. (1987), "Comparison of geostatistical methods for estimating transmissivity using data on transmissivity and specific capacity", *Water Resour. Res.*, **23**(9), 1717-1737.

Alonso, E.E. (1976), "Risk analysis of slopes and its application to slopes in Canadian sensitive clays", *Geotechnique*, **26**(3), 453-472.

Baecher, G.B. and Christian, J.T. (2003), *Reliability and Statistics in Geotechnical Engineering*, John Wiley & Sons, Chichester, UK.

Bhattacharjya, S., Chakraborti, S. and Das, S. (2015), "Robust optimization of reinforced concrete folded plate and shell roof structure incorporating parameter uncertainty", *Struct. Eng. Mech.*, *Int. J.*, **56**(5), 707-726.

Bond, A. and Harris, A. (2008), *Decoding Eurocode 7,* Taylor and Francis, London, UK, 616 p.

Bouma, J., Stoorvogel, J., Van Alphen, B.J. and Booltink, H.W.G. (1999), "Pedology, precision agriculture, and the changing paradigm of agricultural research", *Soil Sci. Soc. Am. J.*, **63**(6), 1763-1768.

Brejda, J., Moorman, J., Smith, T.B., Karlen, J.L., Allan, D.L. and Dao, T.H. (2000), "Distribution and

variability of surface soil properties at a regional scale”, *Soil Sci. Soc. Am. J.*, **64**(3), 974-982.
Burrough, P.A. (1993), “Soil variability: A late 20th century view”, *Soils Fertiliz.*, **56**(5), 529-562.
Christian, J.T. and Ladd, C.C. and Baecher, G.B. (1994), “Reliability applied to slope stability analysis”, *J. Geotech. Eng. Div.,* ASCE, **120**(12), 2180-2207.
Cressie, N. (1990), “The origins of Kriging”, *Math. Geol.*, **22**(3), 239-252.
Christensen, R. (2001), “Best Linear Unbiased Prediction of Spatial Data: Kriging”, In: *Linear Models for Multivariate*, *Time Series, and Spatial Data*, (Ed. R. Cristensen), Springer.
Davis, J.C. (2002), *Statistics and Data Aanalysis in Ggeology*, John Wiley & Sons, New York, NY, USA.
Deutsch, C. and Journel, A. (1998), *GSLIB: Geostatistical Software and User’s Guide*, (2nd Ed.), Oxford University Press, New York, NY, USA.
Do, N.A., Oreste, P., Dias, D., Antonello, C., Djeran-Maigre, I. and Livio, L. (2014), “Stress and strain state in the segmental linings during mechanized tunneling”, *Geomech. Eng.*, *Int. J.*, **7**(1), 75-85.
El-Ramly, H., Morgenstern, N.R. and Cruden, D.M. (2002), “Probabilistic slope stability analysis for practice”, *Can. Geotech. J.*, **39**(3), 665-683.
Goovaerts, P. (1997), *Geostatistics for Natural Resource Evaluation*, Oxford University Press, New York, NY, USA.
Griffiths, D.V. and Fenton, G.A. (2004), “Probabilistic slope stability analysis by finite elements”, *J. Geotech. Eng. Div.*, *ASCE*, **130**(5), 507-518.
Griffiths, D.V., Huang, J. and Fenton, G.A. (2009), “Influence of spatial variability on slope reliability using 2-D random fields”, *ASCE*, *J. Geotech. Geoenviron. Eng.,* **135**(10), 1367-1378.
Godwin, R.J. and Miller, P.C.H. (2003), “A review of the technologies for mapping withi-field variability”, *Biosyst. Eng*., **84**(4), 393-407.
Goovaerts, P. (1998), “Geostatistical tools for characterizing the spatial variability of microbiological and physico-chemical soil properties”, *Biol. Fertil. Soils*, **27**(4), 315-334.
Gotway, C.A. and Stroup, W.W. (1997), “A generalized linear model approach to spatial data analysis and prediction”, *J. Agric. Biol. Environ. Statist.,* **2**, 157-178.
Guarascio, M., Lombardi, M., Rossi, G. and Sciarra, G. (2009), “Geostatistics/reliability based risk analysis of the Vajont landslide”, *Safety and Security Engineering III*, WIT Transactions on the Built Environment, **108**, 607-615.
Guimaraes Couto, E., Stein, A. and Klamt, E. (1997), “Large area spatial variability of soil chemical properties in central Brazil”, *Agric. Ecosyst. Environ*., **66**(2), 139-152.
Hacking, I. (1975), *The Emergency of Probability*: *A Philosophical Study of Early Ideas about Probability*, *Induction and Statistical Inference*, Cambridge University Press, Cambridge, UK.
Harr, M.E. (1987), *Reliability-Based Design in Civil Engineering,* McGraw Hill.
Hengl, T., Heuvelink, G.B.M. and Rossiter, D.G. (2007), “About regression-kriging: From equations to case studies”, *Comput*. *Geosci*., **33**(10), 1301-1315.
Hicks, M.A. (2013), “An explanation of characteristic values of soil properties in Eurocode 7”, In: *Modern Geotechnical Design Codes of Practice*, (Eds. P. Arnold, G.A. Fenton, M.A. Hicks, T. Schweckendiek and B. Simpson), *IOS Press*, 36-45.
Hicks, M.A. and Samy, K. (2002), “Influence of anisotropic spatial variability on slope reliability”, *Proceedings of the 8th International Symposium on Numerical Models in Geomechanics* (Eds. G.N. Pande and S. Piettruszczak), Rome, Italy, April, pp. 535539.
Jaksa, M.B., Brooker, P.I. and Kaggwa, W.S. (1997), “Inaccuracies associated with estimation of random measurement errors”, *J. Geotech. Geoenv. Eng*., *ASCE*, **123**(5), 393-401.
Kalenchuk, K.S., Hutchinson, D.J. and Diederichs, M.S. (2009), “Application of spatial prediction techniques to defining three-dimensional landslide shear surface geometry”, *Landslides*, **6**(4), 321-333.
Kulhawy, F. H. (1992), “On the evaluation of soil properties”, *ASCE Geotech. Spec. Publ*., **31**, 95-115.
Lacasse, S. and Nadim, F. (1996), “Uncertainties in characterizing soil properties”, In: *Uncertainty in the Geologic Environment: From Theory to Practice*, (Eds. C.D. Shackleford, P.P. Nelson, M.J.S. Roth), ASCE Geotechnical Special Publication, **58**, 49-75.
Lagaros, N.D. (2014), “Risk assessment of steel and steel-concrete composite 3D buildings considering

sources of uncertainty", *Earthq. Struct., Int. J.*, **6**(1), 19-43.
Li, K.S. and Lumb, P. (1987), "Probabilistic design of slopes", *Can. Geotech. J.*, **24**(4), 520-535.
Liao, S.S.C. and Whitman, R.V. (1986), "Overburden correction factors for SPT in sand", *J. Geotech. Eng., ASCE*, **112**(3), 373-377.
Matheron, G. (1963), "Principles of geostatistics", *Econom. Geol.*, **58**(8), 1246-1266.
Matheron, G. (1969), *Le Krigeage Universel,* Cahiers de Centre de Morphologie Mathematique, **1**, Fontainbleau, France.
Mostyn, G.R. and Li, K.S. (1993), "Probabilistic slope stability-state of play", *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, (Eds. Li and Lo), Canberra, Australia, February, pp. 89-110.
Pachepsky, Y., Radcliffe, D.E. and Selim, H.M. (2003), *Scaling Methods in Soil Physics*, CRC Press.
Papritz, A. and Stein, A. (1999), "Spatial Prediction by Linear Kriging", In: *Spatial Statistics for Remote Sensing*, (Eds. A. Stein, F. van der Meer, and B. Gorte), Kluwer Academic Publishers, Dodrecht, Netherlands, pp. 83-113.
Phoon, K.K. and Kulhawy, F.H. (1999a), "Characterization of geotechnical variability", *Can. Geotech. J.,* **36**(4), 612-624.
Phoon, K.K. and Kulhawy, F.H. (1999b), "Evaluation of geotechnical property variability", *Can. Geotech. J.,* **36**(4), 625-639.
Pyrcz, M.J. and Deutsch, C.V. (2014), *Geostatistical Reservoir Modeling*, Oxford University Press.
Rethati, L. (1988), "Probabilistic Solutions in Geotechnics"*,* In: *Developments in Geotechnical Engineering,* Elsevier, Amsterdam, Netherlands, 46 p.
Rodriguez, R.F., Nicieza, C.G., Gayarre, F.L. and Lopez, F.L.R. (2015), "Application of hydraulic cylinder testing to determine the geotechnical properties of earth-filled dams", *Geomech. Eng., Int. J.*, **9**(4), 483-498.
Rota, M. (2007), "Estimating uncertainty in 3D models using geostatistics", Ph.D. Thesis; European School for Advanced Studies in Reduction of Seismic Risk (Rose School), University of Pavia, Italy.
Sarma, D.D. (2009), *Geostatistics with Applications in Earth Sciences*, (Second edition), Springer Science & Business Media, pp. 78-80.
Schmertmann, J.H. (1978), "Guidelines for Cone Penetration Test, Performance and Design", Report FHWA-TS-78-209; U.S. Department of Transportation, Washington, D.C., USA.
Schneider, H.R. and Schneider, M.A. (2013), "Dealing with uncertainties in EC7 with emphasis on determination of characteristic soil properties", In: *Modern Geotechnical Design Codes of Practice*, (Eds. P. Arnold, G.A. Fenton, M.A. Hicks, T. Schweckendiek and B. Simpson), *IOS Press*, pp. 87-101.
Skempton, A.W. (1986), "Standard penetration test procedures and the effects in sands of overburden pressure, relative density, particle size, ageing and overconsolidation", *Geotechnique*, **36**(3), 425-447.
Stein, M.L. (1999), *Interpolation of Spatial Data: Some Theory for Kriging,* Series in Statistics, Springer, New York, NY, USA.
Stuedlein, A.W. and Holtz, R.D. (2013), "Bearing capacity of spread footings on aggregate pier reinforced clay", *J. Geotech. Geoenviron. Eng. ASCE*, **139**(1), 49-58.
Sung, E.C. (2007), "Effects of spatial variability of soil properties on slope stability", *Eng. Geol.*, **92**(3), 97-109.
Tang, W.H. (1993), *Recent Developments in Geotechnical Reliability,* Balkema, ISBN 9054103035.
Terzaghi, K. and Peck, R.B. (1948), *Soil Mechanics in Engineering Practice,* New York, John Wiley and Sons.
Thorne, C.P. and Quine, M.P. (1993), "How Reliable are Reliability Estimates and Why Soils Engineers Rarely Use Them", In: *Probabilistic Methods in Geotechnical Engineering*, (Eds. Li and Lo), Balkema, Rotterdam, Netherlands. ISBN: 9054103035, 325-332
Tutmez, B. and Dag, A. (2013), "Mapping water chemical variables with spatially correlated errors", *Environ. Ecol. Statist.*, **20**(1), 19-35.
Tutmez, B. and Yuceer, M. (2013), "Regression Kriging analysis for longitudinal dispersion coefficient", *Water Resour. Manage.*, **27**(9), 3307-3318.

Tutmez, B., Cengiz, A.K. and Sarici, D.E. (2013), "Use of uncertain numbers for appraising tensile strength of concrete", *Struct. Eng. Mech., Int. J.*, **46**(4), 447-458.
Uzielli, M. (2008), "Statistical analysis of geotechnical data", In: *Geotechnical and Geophysical Site Characterization,* (Eds. Huang and Mayne), Taylor and Francis Group.
Vanmarcke, E.H. (1977a), "Probabilistic modeling of soil profiles", *J. Geotech. Eng. Div., ASCE*, **103**(11), 1227-1246.
Vanmarcke, E.H. (1977b), "Reliability of earth slopes", *J. Geotech. Eng. Div., ASCE*, **103**(11), 1247-1265.
Viscarra Rossel, R., McBratney, A. and Minasny, B. (2010), *Proximal Soil Sensing*, Springer Science & Business Media.
Wackemagel, H. (2013), *Multivariate Geostatistics*: *An Introduction with Applications*, Springer Science & Business Media.
Wu, T.H. (2009), "Reliability of Geotechnical Predictions", In: *Geotechnical Risk and Safety*, (Eds. Honjo *et al.*), Taylor and Francis Group.
Yemefack, M., Rossiter, D.G. and Njomgang, R. (2005), "Multi-scale characterization of soil variability within an agricultural landscape mosaic system in southern Cameroon", *Geoderma*, **125**(1), 117-143.
Zhu, Q. and Lin, H.S. (2010), "Comparing ordinary kriging and regression kriging for soil properties in contrasting landscapes", *Pedosphere*, **20**(5), 594-606.

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