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# Risk assessment of water inrush in karst tunnels based on a modified grey evaluation model: Sample as Shangjiawan Tunnel

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**Abstract.** A modified grey clustering method is presented to systematically evaluate the risk of water inrush in karst tunnels. Based on the center triangle whitenization weight function and upper and lower limit measure whitenization weight function, the modified grey evaluation model doesn't have the crossing properties of grey cluster and meets the standard well. By adsorbing and integrating the previous research results, seven influence factors are selected as evaluation indexes. A couple of evaluation indexes are modified and quantitatively graded according to four risk grades through expert evaluation method. The weights of evaluation indexes are rationally distributed by the comprehensive assignment method. It is integrated by the subjective factors and the objective factors. Subjective weight is given based on analytical hierarchy process, and objective weight obtained from simple dependent function. The modified grey evaluation model is validated by Jigongling Tunnel. Finally, the water inrush risk of Shangjiawan Tunnel is evaluated by using the established model, and the evaluation result obtained from the proposed method is agrees well with practical situation. This risk assessment methodology provides a powerful tool with which planners and engineers can systematically assess the risk of water inrush in karst tunnels.

**Keywords:** karst tunnel; water inrush; risk assessment; triangular whitenization weight function; grey clustering method

## 1. Introduction

High-risk and deep buried tunnels often go with the important infrastructure projects. The water inrush is one of the major geological hazards in tunnel construction. It brings about difficulty and safety risk to construction. There were 97 safety accidents occurred in the construction of tunnel in traffic and hydropower field of China from 2001 to 2010. Geological hazards caused by water inrush accounted for 77.3%, that caused nearly one thousand people killed, a large amount of mechanical equipment was scrapped, and part of the tunnels were forced to stop or relocate. The geological hazard of water inrush also caused serious environmental damage and huge economic

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losses.

The risk assessment of water inrush is an effective way to ensure the safety of tunnel construction. It also can be used to reduce and control the major disaster. In practice, water inrush in karst tunnels is a very complex dynamic catastrophe phenomenon of groundwater. The water inrush can be influenced by many factors such as the internal influencing factors and the external influence factors. The formation lithology and unfavorable geology are included in the internal influencing factors. Meanwhile, the hydrogeology, topography and geomorphology, and excavation disturbance are included in the external influencing factors (Zhao *et al*. 2013, Li *et al*. 2013a, 2011a, Li and Li 2014). In practice, a number of methods have been used for risk assessment analysis (Beard 2010). The following are ways of doing so.

(1) Expert system method (Han *et al*. 2004). Expert system method is commonly used to evaluate the risk of water inrush, however, it is complicated to implement. (2) Fuzzy comprehensive evaluation (Kuang *et al*. 2010). Fuzzy comprehensive evaluation overemphasize the role of extremum, however, it is easy to lose effective information. (3) Analytical hierarchy process (Xu *et al*. 2011b). Analytical hierarchy process is a qualitative analysis method with simulating the intelligence activities of human brain. With the increasing of evaluation indexes, the weights of indexes become difficult to determine, and the eigenvectors are more complex. (4) Attribute synthetic assessment (Zhou *et al*. 2013). The solving procedure of attribute measure function in attribute synthetic assessment method is too complicated, and it is easy to cause deflections. (5) Geographic information system (GIS) technology (Li and Li 2014). MAPGIS geographic information system software is selected as the basic platform of GIS, and VC++ is selected as the development tool. It is tested in water inrush cases in the auxiliary tunnel in Jinping II Hydropower Station. (6) Bayesian Networks (Sousa and Einstein 2012, Schubert *et al*. 2012). The Bayesian Networks have ability to combine domain knowledge with data, encode dependencies among variables, and their ability to learn causal relationships.

In the 1980s, the grey system theory was proposed by Deng (1982). Then Liu *et al*. (2010) has achieved some progress from a lot of study. The theory can extract positive, worthwhile information from the minimal amounts of information based on the uncertain systems. The theory indicated that the running orderliness can be predicted correctly and controlled properly, and it has been widely used to analyze the slope stability (Ding 2011), flood disaster risk assessment (Li *et al*. 2013b), qualified identification (Liu *et al*. 2014) and rock explosion prediction (Xie and Pan 2007). At the same time, the study of risk assessment of water inrush in karst tunnels based on the grey system theory has been reported rarely.

The water inrush hazard in karst tunnel is the result of synthetic effect of multi-factors. Some factors play an important role in the preparation process of water inrush. And some factors are not very clear. Therefore, the complex relationship between water inrush and its influence factors is considered as a grey system, and it can be analyzed by using the grey clustering method.

The influence factors of water inrush are used to establish the modified grey evaluation model for water inrush prediction with the optimization of triangular whitenization weight function. Compared with the traditional grey evaluation model the modified grey evaluation model doesn’t have the crossing properties of grey cluster. And it well satisfies the standard. The modified grey evaluation model is adopted to predict the risk and classification of water inrush. The prediction results of the modified grey evaluation model are close to the practical records. It proved that the model is effective and available. The proposed method provides a practical way to accurately predict the risk and classification of water inrush in karst tunnel.

## 2. Basic principle of grey evaluation model

Based on triangular winterization weight function, the grey clustering method is one of the foundation and core components of grey system theory. According to the winterization weight function values of the assessment indexes, the evaluated object can be summarized based on the prior gray classes (Liu *et al.* 2010). The class of the evaluated object can be identified accordingly.

In this paper, the evaluated object is $n$, evaluation index is $m$ and grey class is $s$. $x_{ij}$ is the actual value of index $j$ of object $i$ ($i$ = 1, 2, …, $n$; $j$ = 1, 2, …, $m$). $f_j^k(\cdot)$ is the triangular whitenization weight function of grey class $k$ of index $j$ ($k$ = 1, 2, …, $s$). $\omega_j$ is the weight of the index $j$ in comprehensive clustering. According to the object $n$ and the index $m$, the sample matrix of actual value can be obtained as

$$L = (x_{ij})_{nm} \tag{1}$$

The related object of $i$ can be evaluated and classified according to the actual value of $x_{ij}$ and the triangular whitenization weight function. The main process can be summarized as

(1) The value range of index $j$ ($[a_1, a_{s+1}]$) is divided into $s$ grey classes ($[a_1, a_2]$, $\cdots$, $[a_{k+1}, a_k]$, …, $[a_{s-1}, a_s]$, $[a_s, a_{s+1}]$). The $a_k$ can be derived from the engineering practice or the qualitative research achievement.

(2) Calculating the weights of evaluation indexes $\omega_j$ and establishing the triangular whitenization weight function $f_j^k(\cdot)$.

(3) The comprehensive clustering coefficients of grey class $k$ of the evaluated object $i$ can be calculated by the following form

$$\sigma_i^k = \sum_{j=1}^{k} f_j^k(x_{ij})\omega_j \tag{2}$$

(4) The grey class $k^*$ of a certain object can be calculated as

$$\max_{1\le k\le s}\{\sigma_i^k\} = \sigma_i^{k^*} \tag{3}$$

## 3. Risk assessment of water inrush based on grey evaluation model

The risk assessment model of water inrush is an important component for the comprehensive evaluation system of water inrush hazard in karst tunnel. Based on the previous research results, seven major influencing factors such as the formation lithology ($I_1$), the unfavorable geological conditions ($I_2$), the groundwater level ($I_3$), the landform and physiognomy ($I_4$), the attitude of rock formation ($I_5$), the contact zones of dissolvable and insoluble rock ($I_6$), and the layer and interlayer fissures ($I_7$) are used in the present paper to establish the evaluation system of water inrush in karst tunnels (Beard 2010, Xu *et al.* 2011a, b, Zhou *et al.* 2013, Mao *et al.* 2010, Li *et al.* 2011b, 2013b). The hazard grade of water inrush is divided into 4 levels, such as high risk ($C_1$), moderate risk ($C_2$), low risk ($C_3$) and no risk ($C_4$). These indexes covered the internal and external factors of water inrush, and the evaluation index system is systematic and integrated.

Table 1 Grading standards of evaluation indexes of water inrush

| Evaluation indexes | Risk grade of water inrush | | | |
|---|---|---|---|---|
| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
| Formation lithology ($I_1$) | > 85 | 70~85 | 60~70 | < 60 |
| Unfavorable geological conditions ($I_2$) | > 85 | 70~85 | 60~70 | < 60 |
| Groundwater level (/m) ($I_3$) | > 60 | 30~60 | 10~30 | < 10 |
| Landform and physiognomy (Proportion of Negative landform area) (/%) ($I_4$) | > 60 | 40~60 | 20~40 | < 20 |
| Attitude of rock formation (Modified strata inclination) (/°) ($I_5$) | 25~45 | 10~25 | 0~10 | 0~10 |
| Contact zones of dissolvable and insoluble rock ($I_6$) | > 85 | 70~85 | 60~70 | < 60 |
| Layer and interlayer fissures ($I_7$) | > 85 | 70~85 | 60~70 | < 60 |

The evaluation indexes and grading standards for risk assessment of water inrush in karst tunnels are referenced (Li *et al*. 2015a, b, Zhou *et al*. 2013, Li *et al*. 2013a, b), as shown in Table 1. In practical engineering, the evaluation index values are determined by using expert scoring method. They include the formation lithology, the unfavorable geological conditions, the contact zones of dissolvable and insoluble rock, and the layer and interlayer fissures.

(1) The grading standards of formation lithology, unfavorable geological conditions, contact zones of dissolvable and insoluble rock, layer and interlayer fissures are divided by expert scoring method.
(2) The grading method of attitude of rock formation quoted Zhou *et al*. (2013). The parameters ($\varphi'$, $\varphi_1'$ and $\varphi_2'$) are the modified strata inclination, and the methods of solution are presented in the research results of Li *et al*. (2011a) and Xu *et al*. (2011a).
(3) Groundwater level is the height difference between real groundwater levels and tunnel invert.
(4) According to section 2, the evaluation indexes of water inrush $m$ are 7, and the grey classes $s$ are 4. Meanwhile, the water inrush risk grade will be determined by the grey class $k$, such as no risk ($k$ = 1), low risk ($k$ = 2), moderate risk ($k$ = 3) and high risk ($k$ = 4).

### *3.1 Risk assessment of water inrush based on traditional grey evaluation model*

In recent years, the grey evaluation method has been established by the triangular whitenization weight function. It is commonly used in practice to assess the engineering safety. For example, Cai *et al*. (2004) applied the evaluation method to the construction technology in subway construction; Duan *et al*. (2006) established a risk assessment model for information system based on grey theory and analytic hierarchy process (AHP); Xie and Pan (2007) applied the grey whitenization weight function cluster theory to forecast the rockburst disaster.

The traditional grey evaluation model has been established by the end-point triangular whitenization weight function. The crossing properties of grey cluster of end-point triangular whitenization weight function as shown in Fig. 1.

In Fig. 1, $a_1$ ~ $a_5$ are the threshold values of related evaluation indicators. The $a_0$ is the continuous extension of evaluation indicators on the left side and $a_6$ is the continuous extension of evaluation indicators on the right side. The parameter $\lambda_k$ ($k$ = 1, 2, 3, 4) is the midpoint between $a_k$

and $a_{k+1}$. It can be calculated by Eq. (4).

$$\lambda_k = \left(a_k + a_{k+1}\right)/2 \tag{4}$$

The traditional risk assessment of water inrush by grey evaluation model which has the following problems as shown in Fig. 1.

(1) If the number of grey classes over 3. And the end-point triangular whitenization weight function has been used in the grey evaluation model. There will be the crossover phenomenon exists.
The cross region of grey class 1 and grey class 2 is $U_1$, the cross region of grey class 2 and grey class 3 is $U_2$. The value range of grey class 1 is $[a_0, a_3]$, the value range of grey class 2 is $[a_1, a_4]$, and the value range of grey class 3 is $[a_2, a_5]$ (as shown in Fig. 1).
The $U_1$ and $U_2$ can be expressed in the following forms

$$U_1 = [a_0,\ a_3] \ \cap \ [a_1,\ a_4] = [a_1,\ a_3] \tag{5}$$

$$U_2 = [a_1,\ a_4] \ \cap \ [a_2,\ a_5] = [a_2,\ a_4] \tag{6}$$

The cross region of grey class 1, grey class 2, and grey class 3 is $U_0$. It can be expressed in the following form

$$U_0 = U_1 \ \cap \ U_2 = [a_2,\ a_3] \neq \varnothing \tag{7}$$

(2) The traditional risk assessment of water inrush derived from grey evaluation model does not satisfy the standard.
When $x \in [\lambda_{k-1}, \lambda_k]$

$$\sum_{k=1}^{4} f_j^k(x) \neq 1 \tag{8}$$

If we assume that the grey class $s$ is 2, $x \in [\lambda_{k-1}, \lambda_k]$, then

$$\begin{aligned} f_j^{k-1}(x) + f_j^k(x) &= \frac{a_{k+1} - x}{a_{k+1} - \lambda_{k-1}} + \frac{x - a_{k-1}}{\lambda_k - a_{k-1}} \\ &> \frac{a_{k+1} - x}{\lambda_k - \lambda_{k-1}} + \frac{x - a_{k-1}}{\lambda_k - a_{k-1}} \\ &> \frac{\lambda_k - x}{\lambda_k - \lambda_{k-1}} + \frac{x - \lambda_{k-1}}{\lambda_k - \lambda_{k-1}} \\ &= 1 \end{aligned} \tag{9}$$

It is concluded that the traditional grey evaluation model has the crossing properties of grey cluster and does not well satisfy the standard.

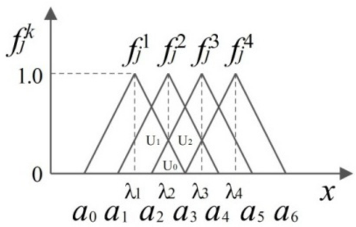



Fig. 1 Whitenization weight function of traditional grey evaluation model

### *3.2 Risk assessment of water inrush based on modified grey evaluation model*

This paper presents a modified grey evaluation model which based on the center triangle whitenization weight function and upper and lower limit measure whitenization weight function. Assuming that $\lambda_1$, $\lambda_2$, …, $\lambda_k$, …, $\lambda_s$ are center points of related grey classes. Extending the range of $f_j^1(x)$ to $\lambda_0$ by the lower limit measure whitenization weight function. Extending the range of $f_j^4(x)$ to $\lambda_5$ by the upper limit measure whitenization weight function. A mixed center triangle whitenization weight function is established as shown in Fig. 2. The extension value of number field of evaluation indices $\lambda_0 = 0$, $\lambda_5 = 100$ or 70 are based on the grading standards of evaluation indices of water inrush and practice engineering situation. The whitenization weight functions are shown in Fig. 3.

The grey class $k$ of the index value $x$ can be calculated by Eqs. (10)-(13). They are expressed in the following form

$$f_j^1 = \begin{cases} 0 & x \notin [\lambda_0, \lambda_2] \\ 1 & x \in [\lambda_0, \lambda_1] \\ \dfrac{\lambda_2 - x}{\lambda_2 - \lambda_1} & x \in [\lambda_1, \lambda_2] \end{cases} \tag{10}$$

$$f_j^2 = \begin{cases} 0 & x \notin [\lambda_1, \lambda_3] \\ \dfrac{x - \lambda_1}{\lambda_2 - \lambda_1} & x \in [\lambda_1, \lambda_2] \\ \dfrac{\lambda_3 - x}{\lambda_3 - \lambda_2} & x \in [\lambda_2, \lambda_3] \end{cases} \tag{11}$$

$$f_j^3 = \begin{cases} 0 & x \notin [\lambda_2, \lambda_4] \\ \dfrac{x - \lambda_2}{\lambda_3 - \lambda_2} & x \in [\lambda_2, \lambda_3] \\ \dfrac{\lambda_4 - x}{\lambda_4 - \lambda_3} & x \in [\lambda_3, \lambda_4] \end{cases} \tag{12}$$

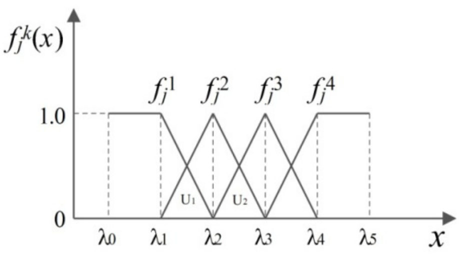
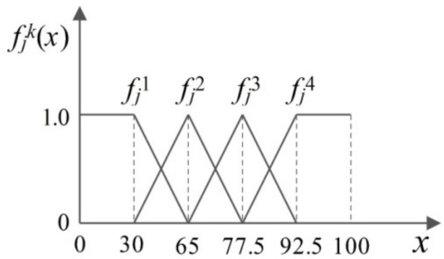
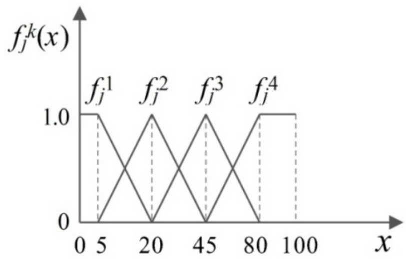


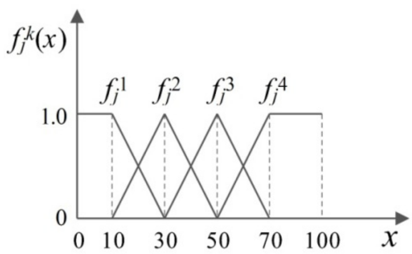
Fig. 2 Whitenization weight function of modified grey evaluation model



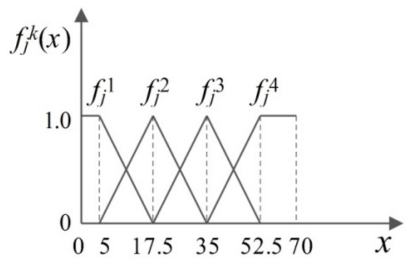
(a) Whitenization weight function of $I_1$, $I_2$, $I_6$ and $I_7$



(b) Whitenization weight function of $I_3$



(c) Whitenization weight function of $I_4$



(d) Whitenization weight function of $I_5$

Fig. 3 Whitenization weight function of each evaluation index

$$f_j^4 = \begin{cases} 0 & x \notin [\lambda_3, \lambda_5] \\ \dfrac{x-\lambda_3}{\lambda_4-\lambda_3} & x \in [\lambda_3, \lambda_4] \\ 1 & x \in [\lambda_4, \lambda_5] \end{cases} \tag{13}$$

Compared to the traditional grey assessment model, the risk assessment of water inrush derived from the modified grey evaluation model which has some innovations.

(1) The crossing properties of grey clustering disappeared.

The cross region of grey class 1, grey class 2 and grey class 3 is $U_0$, and $U_0 = \varnothing$.

(2) The modified grey evaluation model satisfies the standard.

The center triangle whitenization weight function and upper and lower limit measure whitenization weight function are adopted in the modified grey evaluation model. The modified grey evaluation model is satisfies the standard very well, as shown in Figs. 4 and 5.

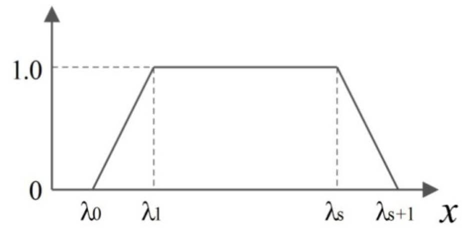



Fig. 4 Clustering coefficient and function of pure center triangular whitenization weight function

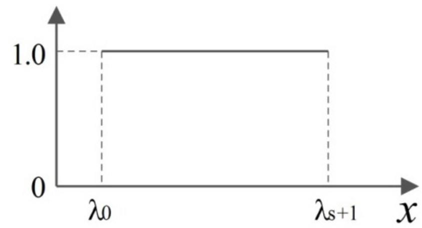



Fig. 5 Clustering coefficient and function of mixed center triangle whitenization weight function

### *3.3 Weights analysis of evaluation indexes of grey evaluation model*

In the process of water inrush risk evaluation, the extent of evaluation indexes impact of water inrush can be measured by weights. Considering the actual situation and the uncertain historical data, the weight of every index is determined by using the comprehensive weight method which integrates the subjective factors and the objective factors. It is expressed in the following from

$$\left.\begin{aligned} \omega_j &= \omega_{j1}\psi_1 + \omega_{j2}\psi_2 \\ \psi_1 + \psi_2 &= 1 \end{aligned}\right\} \tag{14}$$

where $\omega_j$ is the comprehensive weight of evaluation index; $\omega_{j1}$ is the subjective weight vector; $\omega_{j2}$ is the objective weight vector. And $\psi_1$ and $\psi_2$ are distribution of the weights between subjective weight and objective weight. The subjective weight is estimated from the analytical hierarchy process (AHP) (Tongji University 2003, Saaty 1990), and the objective weight is derived from the simple dependent function (Xue *et al*. 2010).

(1) Subjective weights
The subjective weights of evaluation indexes are determined by using AHP method (Li *et al*. 2013a, b). The judgment matrix of evaluation model is established by the 1~9 scale of AHP (Saaty 1990). $\omega_{j2}$ = {$\omega$ ($I_1$), $\omega$ ($I_2$), $\omega$ ($I_3$), $\omega$ ($I_4$), $\omega$ ($I_5$), $\omega$ ($I_6$), $\omega$ ($I_7$)} = {0.178, 0.350, 0.178, 0.098, 0.058, 0.098, 0.038}

(2) Objective weights
There are some subjective factors in calculating the weight vector. The weight of grey evaluation is usually given in the light of experts' experience, which is inevitably subjective. Through the determination of factor weight by using the simple dependent function (Huang and

She 2006). The calculation of the dependent degree, objective and reasonable results are obtained.

$$r_{ji}(V_i,V_{ji})=\begin{cases}\dfrac{2\left(V_i-a_{ji}\right)}{b_{ji}-a_{ji}},V_i\geq\dfrac{a_{ji}+b_{ji}}{2}\\ \dfrac{2\left(b_{ji}-V_i\right)}{b_{ji}-a_{ji}},V_i\leq\dfrac{a_{ji}+b_{ji}}{2}\end{cases} \tag{15}$$

Where $V_i \in V_{pi}$ ($i$ = 1, 2, …, $n$). The $V_{p_i}$ is joint domain and $p$ is the classification of appraisal object (Huang and She 2006).

The $r_{ji\max}$ ($V_i$, $V_{ji}$) is expressed in the following form

$$r_{ji\max}\left(V_i,V_{ji}\right)=\max_{j=1}^{m}\left\{r_{ji}\left(V_i,V_{ji}\right)\right\} \tag{16}$$

① The bigger the value of the index $i$ the higher the category. The index weights of the object is much greater. The $r_i$ is expressed in the following form

$$r_i=\begin{cases}j_{\max}\times\left\{1+r_{ji\max}\left(V_i,V_{ji}\right)\right\} & r_{ji\max}\left(V_i,V_{ji}\right)\geq-0.5\\ j_{\max}\times0.5 & r_{ji\max}\left(V_i,V_{ji}\right)<-0.5\end{cases} \tag{17}$$

② The bigger the value of the index $i$ the higher the category. The index weights of the object are much smaller. The $r_i$ is expressed in the following form

$$r_i=\begin{cases}(m-j_{\max}+1)\times\left\{1+r_{ji\max}\left(V_i,V_{ji}\right)\right\} & r_{ji\max}\left(V_i,V_{ji}\right)\geq-0.5\\ (m-j_{\max}+1)\times0.5 & r_{ji\max}\left(V_i,V_{ji}\right)<-0.5\end{cases} \tag{18}$$

The weight of evaluation index is $\omega_{j2}$. It is expressed in the following form

$$\omega_{j2}=r_i/\sum_{i=1}^{n}r_i \tag{19}$$

## 4. Risk assessment process of water inrush in karst tunnel

It starts with various factors that influence water inrush in karst tunnels, building modified grey assessment model according to their contents, characteristics and species. The assessment model is modified by the mixed center triangle whitenization weight function. The mixed center triangle whitenization weight function is established by the center triangle whitenization weight function and the upper and lower limit measure whitenization weight function. The risk evaluation process for water inrush is shown in Fig. 6.

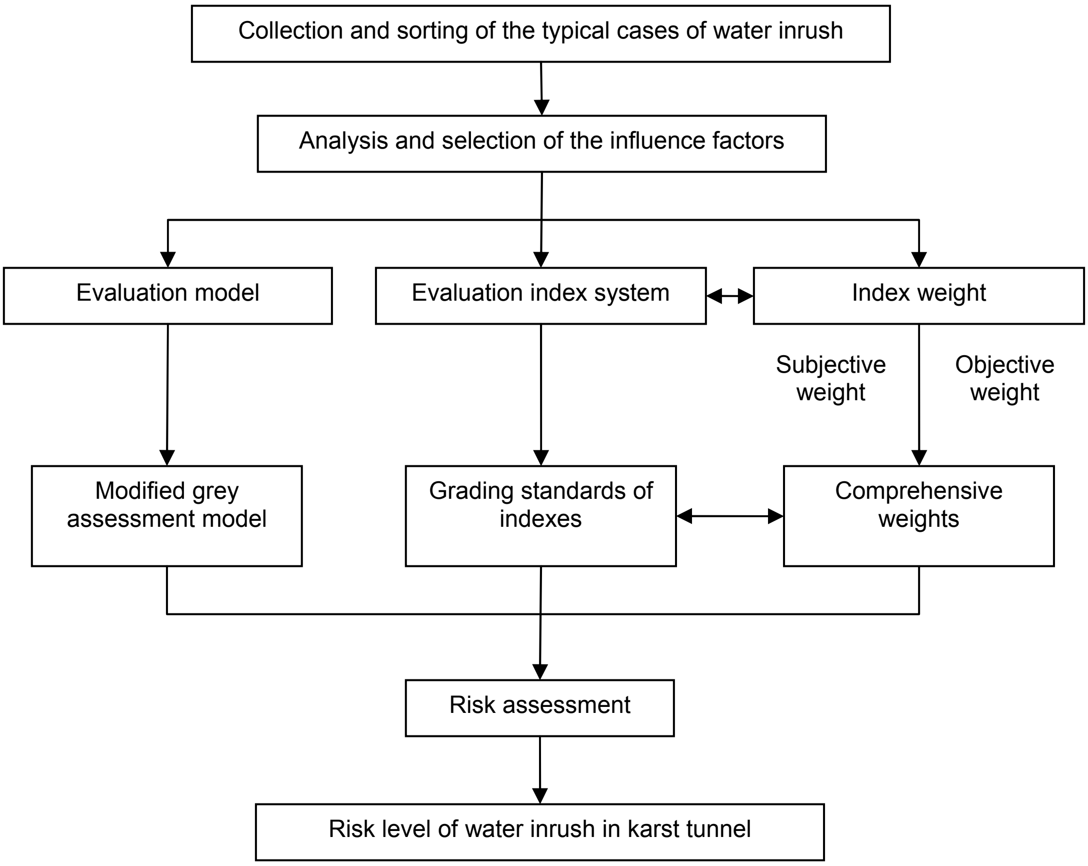



Fig. 6 Flow chart of grey system for karst water inrush

Table 2 Valuation indexes of Jigongling Tunnel

| Evaluation indexes | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|
| Jigongling Tunnel | 65 | 80 | > 60 | 40 | 13° | 70 | 75 |

## 5. Model validation

In order to verify the modified grey evaluation model, a practical engineering is assessed. Jigongling Tunnel is located in karst mountain area of Hubei Province. It is 4.5 km long, with a maximum overburden thickness of 388 m. The risk of water inrush from K19+509 to K19+539 is evaluated. The values of indexes are from the literature (Li *et al*. 2013a). They are shown in Table 2.

According to the Eqs. (10)-(13) and Fig. 3, the whitenization weight functions of evaluation indexes are calculated.

(1) Whitenization weight functions of $I_1$, $I_2$, $I_6$ and $I_7$

$$f_j^1 = \begin{cases} 0 & x \notin [0,65] \\ 1 & x \in [0,30] \\ \dfrac{65-x}{65-30} & x \in [30,65] \end{cases} \tag{20}$$

$$f_j^2 = \begin{cases} 0 & x \notin [30,77.5] \\ \dfrac{x-30}{65-30} & x \in [30,65] \\ \dfrac{77.5-x}{77.5-65} & x \in [65,77.5] \end{cases} \tag{21}$$

$$f_j^3 = \begin{cases} 0 & x \notin [65,92.5] \\ \dfrac{x-65}{77.5-65} & x \in [65,77.5] \\ \dfrac{92.5-x}{92.5-77.5} & x \in [77.5,92.5] \end{cases} \tag{22}$$

$$f_j^4 = \begin{cases} 0 & x \notin [77.5,100] \\ \dfrac{x-77.5}{92.5-77.5} & x \in [77.5,92.5] \\ 1 & x \in [92.5,100] \end{cases} \tag{23}$$

(2) Whitenization weight functions of $I_3$

$$f_j^1 = \begin{cases} 0 & x \notin [0,20] \\ 1 & x \in [0,5] \\ \dfrac{20-x}{20-5} & x \in [5,20] \end{cases} \tag{24}$$

$$f_j^2 = \begin{cases} 0 & x \notin [5,45] \\ \dfrac{x-5}{20-5} & x \in [5,20] \\ \dfrac{45-x}{45-20} & x \in [20,45] \end{cases} \tag{25}$$

$$f_j^3 = \begin{cases} 0 & x \notin [20,80] \\ \dfrac{x-20}{45-20} & x \in [20,45] \\ \dfrac{80-x}{80-45} & x \in [45,80] \end{cases} \tag{26}$$

$$f_j^4 = \begin{cases} 0 & x \notin [45,100] \\ \dfrac{x-45}{80-45} & x \in [45,80] \\ 1 & x \in [80,100] \end{cases} \tag{27}$$

(3) Whitenization weight functions of $I_4$

$$f_j^1 = \begin{cases} 0 & x \notin [0,30] \\ 1 & x \in [0,10] \\ \dfrac{30-x}{30-10} & x \in [10,30] \end{cases} \tag{28}$$

$$f_j^2 = \begin{cases} 0 & x \notin [10,50] \\ \dfrac{x-10}{30-10} & x \in [10,30] \\ \dfrac{50-x}{50-30} & x \in [30,50] \end{cases} \tag{29}$$

$$f_j^3 = \begin{cases} 0 & x \notin [30,70] \\ \dfrac{x-30}{50-30} & x \in [30,50] \\ \dfrac{70-x}{70-50} & x \in [50,70] \end{cases} \tag{30}$$

$$f_j^4 = \begin{cases} 0 & x \notin [50,100] \\ \dfrac{x-50}{70-50} & x \in [50,70] \\ 1 & x \in [70,100] \end{cases} \tag{31}$$

(4) Whitenization weight functions of $I_5$

$$f_j^1 = \begin{cases} 0 & x \notin [0,17.5] \\ 1 & x \in [0\ ,\ 5] \\ \dfrac{17.5-x}{17.5-5} & x \in [5,17.5] \end{cases} \tag{32}$$

$$f_j^2 = \begin{cases} 0 & x \notin [5,35] \\ \dfrac{x-5}{17.5-5} & x \in [5,17.5] \\ \dfrac{35-x}{35-17.5} & x \in [17.5,35] \end{cases} \tag{33}$$

$$f_j^3 = \begin{cases} 0 & x \notin [17.5,52.5] \\ \dfrac{x-17.5}{35-17.5} & x \in [17.5,35] \\ \dfrac{52.5-x}{52.5-35} & x \in [35,52.5] \end{cases} \tag{34}$$

$$f_j^4 = \begin{cases} 0 & x \notin [35,70] \\ \dfrac{x-35}{52.5-35} & x \in [35,52.5] \\ 1 & x \in [52.5,70] \end{cases} \tag{35}$$

According to the Eqs. (15)-(19), the objective weights of evaluation indexes are calculated by using the simple dependent function method, as shown in Table 3.

Table 3 Objective weights of evaluation indexes

| Evaluation indexes | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|
| Objective weights | 0.142 | 0.177 | 0.142 | 0.106 | 0.150 | 0.106 | 0.177 |

Table 4 Comprehensive weights of evaluation indexes

| Evaluation indexes | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|
| Comprehensive weights | 0.160 | 0.264 | 0.160 | 0.102 | 0.104 | 0.102 | 0.108 |

Table 5 Cluster coefficients and corresponding grey cluster of all indicators Jigongling Tunnel

| Grey cluster | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x$ |
|---|---|---|---|---|---|---|---|---|
| No risk ($C_4$) | 0 | 0 | 0 | 0 | 0.36 | 0 | 0 | 0.037 |
| Low risk ($C_3$) | 1 | 0 | 0 | 0.50 | 0.64 | 0.60 | 0.20 | 0.360 |
| Moderate risk ($C_2$) | 0 | 0.833 | 0.571 | 0.50 | 0 | 0.40 | 0.80 | **0.490** |
| High risk ($C_1$) | 0 | 0.167 | 0.429 | 0 | 0 | 0 | 0 | 0.113 |

According to the Eqs. (14)-(19), the weights of evaluation indexes can be calculated by using the comprehensive weight method. It is shown in Table 4. The proportion of the weights between subjective weight and objective weight is the same ($\psi_1 = 0.5$ and $\psi_2 = 0.5$).

The risk of water inrush from K19+509 to K19+539 in Jigongling Tunnel is evaluated by the grey evaluation method described in Section 2. The cluster coefficients and corresponding grey cluster of all indicators are shown in Table 5.

In conclusion, the risk of water inrush from K19+509 to K19+539 in Jigongling Tunnel is in moderate risk which agrees well with practical situation of K19+509–K19+539 (Li *et al*. 2013a). Validity of the modified grey evaluation model is verified.

## 6. Engineering applications

### 6.1 Engineering situation

Shangjiawan Tunnel is located in karst mountain area of Hubei Province in China. It is 3.8 km long. The surface and underground karst considerably developed in the areas of karst depression, which have closely hydraulic affiliation since mutually linkage into a karst spatial system. Shangjiawan Tunnel is the typical karst developing tunnel. The water inrush of karst tunnel influenced the stability of adjacent rock and the safety of tunnel. The hydrological geological map of Shangjiawan Tunnel as shown in Fig. 7.

The outcrop of bedrock in tunnel site is sedimentary rock. Sedimentary rock of quaternary system is mainly at the foot of valley slopes, and along the river's valley. The faulted structure of Tongcheng River is extremely well-developed with a crisscross of faults in tunnel site, and the average width of fault zone is 1.5 km. A lot of northeast faults develop in the main body of tunnel site, in particular the fault zone of Zengjiagou and Shangjiawan. The engineering geological section of Shangjiawan Tunnel is presented in Fig. 8.

### 6.2 Risk assessment of water inrush

To ensure the safety of tunnel construction and reduce the risk of water inrush, we use the grey cluster assessment theory as described earlier in this article to evaluate the water inrush risks. Grey cluster assessment of the grey system theory is used to predict the water inrush of karst tunnel so as to have effective prevention and control on this kind of accident.

Based on the geological and hydrogeological conditions, risk of water inrush from K64+855 to K64+980 is evaluated in design stage.

(1) Formation lithology ($I_1$)

The formation lithology in the tunnel site mainly includes argillaceous siltstone, conglomerate,

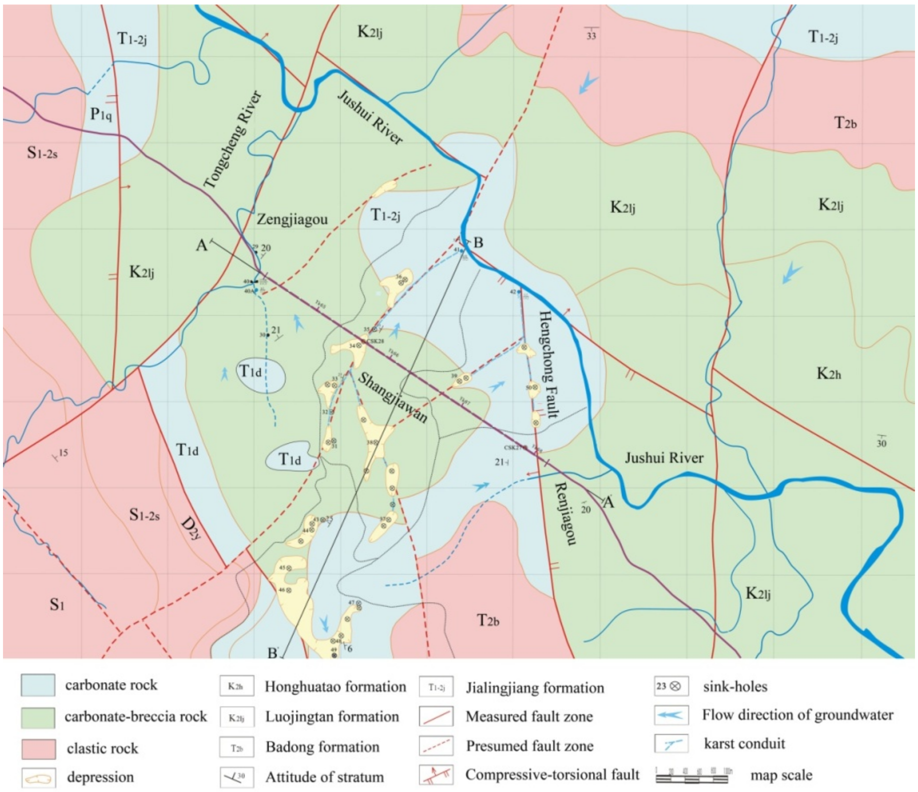



Fig. 7 Hydrological geological map of Shangjiawan Tunnel

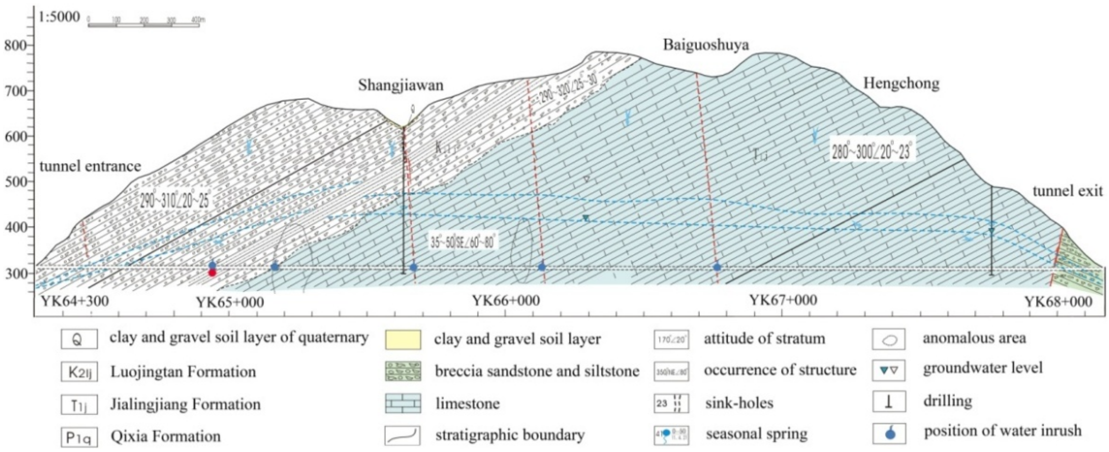



Fig. 8 Engineering geological section of Shangjiawan Tunnel

(muddy) siltstone, dolomitic limestone and thin limestone (see Figs. 7 and 8). The formation lithology of rock strata at K64+920 is conglomerate, mudstone and argillaceous siltstone. The rock dissolubility belonged to middle level.

(2) Unfavorable geological conditions ($I_2$)

The geological situation of K64+855-K64+980 in front of the tunnel face is forecasted by comprehensive geological forecast. The forecast result indicates that the surrounding rock of K64+910-K64+916 has bad geological phenomena such as high elastic modulus ratio, joint fissures and large faults etc. There is abundant karst fissure water in the conglomerate and limestone of Cretaceous and Triassic. The geological condition at K64+916-K64+926 is very complex, rock weathering is severe and the stability of adjoining rock of tunnel is bad. There is high risk of water inrush at K64+910-K64+926.

(3) Groundwater level ($I_3$)

There is a river in the regional scale of Shangjiawan Tunnel crossing. The water table is above the tunnel face level at a minimum of 100 m. Groundwater recharge mainly comes from infiltration of rain and seepage of surface water. Water inrush disaster easy to touch off during tunnel construction, which affects the construction or operation safety greatly.

(4) Landform and physiognomy ($I_4$)

The typical landforms produced by the plate tectonic processes and the subsequent erosion. Average height of landforms are 333 m to 802 m above sea level. The route of tunnel will pass through a total of several mountains, and the landform undulates terribly. The slopes of terrain in this area are all gentle, and the high land of the karst is located in the seepage flow zone and subsurface flow zone. The negative landform area belongs to medium-sized, and the range of negative landform area values are between 45% and 50% depend on the research results of Li *et al*. (2011a).

(5) Attitude of rock formation ($I_5$)

The Cretaceous in surrounding outcrop areas of tunnel site is in direct unconformity contact with the Triassic. The variation of attitude of rock formation is little along the tunnel, which is 290°~320°∠22°~30°. The attitude of rock formation in the evaluated region is 300°∠30°. The value of $\varphi'$ is adopted as 30° which was achieved by the developing conditions of fractures in rock mass.

(6) Contact zones of dissolvable and insoluble rock ($I_6$)

The flow conduit is easy to be formed in dissolvable rock which caused by the underground water erosion and the resistance by insoluble rock. Therefore, the location of contact zones of dissolvable and insoluble rock will affect the safety of tunneling, the location of contact zones of dissolvable rock (mudstone with siltstone and pelitic siltstone) and insoluble rock (dolomitic limestone) was K64+920, and the development of karst was strong.

(7) Layer and interlayer fissures ($I_7$).

The major ingredient of Cretaceous conglomerate is limestone, and some conglomerate with calcareous cementation. The different mechanisms of surface water and hydro-static pressure accelerate the development of compound karstification compared with the karstification developed in single soluble rocks. The condition of surrounding rock in the process of tunnel construction is shown in Fig. 9. There were some mud-water mixtures flow out from the advancing hole, It is shown in Fig. 9(a). It can be concluded that the cave and fissure were well developed and strong permeability. Karst is developed intensely in this area.

(a) Mud-water mixtures flow out from the advancing hole

(b) Karst fissure water

Fig. 9 Condition of surrounding rock in the process of tunnel construction

Table 6 Valuation indexes of Shangjiawan Tunnel

| Evaluation indexes | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|
| Shangjiawan Tunnel | 80 | 90 | 75 | 45 | 30° | 90 | 85 |
| Objective weights | 0.104 | 0.139 | 0.179 | 0.112 | 0.149 | 0.138 | 0.179 |
| Comprehensive weights | 0.141 | 0.245 | 0.179 | 0.105 | 0.104 | 0.118 | 0.108 |

Table 7 Cluster coefficients and corresponding grey cluster of all indicators Shangjiawan Tunnel

| Grey cluster | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x$ |
|---|---|---|---|---|---|---|---|---|
| No risk ($C_4$) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Low risk ($C_3$) | 0 | 0 | 0 | 0.250 | 0.286 | 0 | 0 | 0.056 |
| Moderate risk ($C_2$) | 0.833 | 0.167 | 0.143 | 0.750 | 0.714 | 0.167 | 0.500 | 0.411 |
| High risk ($C_1$) | 0.167 | 0.833 | 0.857 | 0 | 0 | 0.833 | 0.500 | **0.533** |

Based on the geological and hydrogeological conditions, and with the expert scoring method obtains a comprehensive evaluated score to make the values of indexes. The values of risk assessment indexes are shown in Table 6.

The weight of every evaluation index is derived from the comprehensive weight method (see Eqs. (14)-(19)). It is shown in Table 6. The triangular whitenization weight function $f_i^k(\cdot)$ is derived from Fig. 3 and Eqs. (10)-(13). The grey cluster of Shangjiawan Tunnel at the section located between 64.855 km and 64.980 km (K64+855-K64+980) of the tunnel is estimated from the weights of evaluation indexes and the triangular whitenization weight function by using the integrated cluster coefficients. The grey cluster of risk of water inrush is determined by Eq. (3). The Shangjiawan Tunnel was in high risk. It is shown in Table 7.

### *6.3 Practical situation of K64+920-K64+940*

Before the excavation of K64+920-K64+940 the water seepage appeared at K64+915. During the drilling of boreholes at K64+917, water inrush with a certain pressure is gushed forth from the

(a) Water inrush caused water wave

(b) Average water level is 30~60 centimeters

(c) The scene of water inrush disaster

(d) Water poured out of the tunnel's entrance

Fig. 10 Water inrush at K64+920

boreholes. With the excavation of tunnel working face, the total water inrush rate increased significantly at K64+920. A total water inrush rate of 7,700 $m^3$/min from the karst cave. The scope of water level fluctuation is 30 cm ~ 60 cm (see Figs. 10(a) and (b)). The average water is 50 centimeters deep. The realistic water inrush case is shown in Figs. 10(c) and 10(d).

Therefore, the evaluation result obtained from the modified method is consistent with the excavation inspection.

## 7. Discussions

The relatively compact framework for risk assessment of water inrush in karst tunnel is not perfect, and the comprehensive water inrush evaluation indicator system is not completed yet. It still lacks detailed evaluation indexes and some other standard conveniences. The method reported in this paper provides a useful strategy and robust tool in the risk assessment of water inrush. This paper does valuable explore on creating index systems, confirming the weights of indexes and establishing the comprehensive evaluating model for water inrush in karst tunnel. It still has much space for further study, such as quantify the evaluating indicators and establish the comprehensive evaluating model. There are many un-certainties and complex problems in the fields of geology and engineering for risk assessment of water inrush under complex geological conditions. The selection and quantification of evaluation indexes are too complicated. However, in this research, we'll still have to explore more to improve the norm, method, interval, quantification and efficiency of risk evaluation in karst tunnel based on the engineering practice and expert experience.

## 8. Conclusions

This paper establishes a modified grey evaluation model based on the center triangle whitenization weight function and upper and lower limit measure whitenization weight function. The modified grey evaluation model doesn't have the crossing properties of grey cluster and meets the standard well.

- Based on grey clustering theory, a modified grey evaluation model is presented to systematically evaluate the risk of water inrush in karst tunnels. The methodology consists of mixed center triangle whitenization weight function. The proposed method provides a scientific and reliable means for the risk assessment of water inrush in karst tunnels.
- The weights of evaluation indices are rationally distributed by using comprehensive assignment method which integrate the subjective factors and the objective factors. Subjective weight is given based on analytical hierarchy process, and objective weight obtained from simple dependent function.
- Evaluation of engineering practice are carried through with Shangjiawan Tunnel at K64+855-K64+980 as a case study, and the evaluation results obtained from the proposed method are generally in good agreement with the excavation inspection.

Each method has its limitations, and the proposed method is no exception. The evaluation indices should be quantitatively graded, and the values of some indexes are derived from expert evaluation method with a certain subjective.

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