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Theoretical solutions for displacement and stress of a circular opening reinforced by grouted rock bolt

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Abstract. This paper presented solutions of displacement and stress for a circular opening which is reinforced with grouted rock bolt. It satisfies the Mohr-Coulomb (M-C) or generalized Hoek-Brown (H-B) failure criterion, and exhibits elastic-brittle-plastic or strain-softening behavior. The numerical stepwise produce for strain-softening rock mass reinforced with grouted rock bolt was developed with non-associative flow rules and two segments piecewise linear functions related to a principle strain-dependent plastic parameter, to model the transition from peak to residual strength. Three models of the interaction mechanism between grouted rock bolt and surrounding rock proposed by Fahimifar and Soroush (2005) were adopted. Based on the axial symmetrical plane strain assumption, the theoretical solution of the displacement and stress were proposed for a circular tunnel excavated in elastic-brittle-plastic and strain-softening rock mass compatible with M-C or generalized H-B failure criterion, which is reinforced with grouted rock bolt. It showed that Fahimifar and Soroush's (2005) solution is a special case of the proposed solution for n = 0.5. Further, the proposed method is validated through example comparison calculated by MATLAB programming. Meanwhile, some particular examples for M-C or generalized H-B failure criterion have been conducted, and parametric studies were carried out to highlight the influence of different parameters (e.g., the very good, average and very poor rock mass). The results showed that, stress field in plastic region of surrounding rock with considering the supporting effectiveness of the grouted rock bolt is more than that without considering the effectiveness of the grouted rock bolt, and the convergence and plastic radius are reduced.

Keywords: strain-softening; support effectiveness; numerical stepwise produce; circular opening; grouted rock bolt

1. Introduction

Scientific and reasonable design of support system in tunnel is more important for the construction and operation and management of the tunnel. From 1873, grouted rock bolt is widely used in many fields of civil engineering in practice because of the advantages of simple structure, convenient construction, low cost and engineering adaptability etc. Grouted rock bolt can effectively improve the strength and stabilization of surrounding rock by convergence control, and

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the reinforcement effect is more obvious, especially in crack and weaken rock mass. Lots of scholars have studied and solved lots of technical problem in practice (Sharan 2003, 2005, 2008, Park 2009, Wang 2010, 2012, Pan and Dias 2015, 2016, Showkati *et al*. 2015). However, the theoretical study on the mechanism and design method does not meet the demands of engineering practice, especially for the strain-softening rock mass. For examples, Serranoa and Olallab (1999) used the Euler's variational method and assumption of Hoek and Brown failure criterion and obtained the tensile resistance of rock anchors. Cai *et al*. (2004a) developed an analytical model for rock bolts based on an improved Shear-Lag Model. Cai (2004) proposed an analytical solution for rock bolting design of soft rock. Three analytical models for grouted rock bolt are presented by Li (1999) based on the mechanical coupling model at the interface between the bolt and the grout medium. Osgoui and Oreste (2010) presented an elastic-plastic analytical solution for a circular tunnel reinforced by grouted bolts based on the non-linear Hoek-Brown yield criterion. Bobet and Einstein (2011) presented a closed-form solution for a tunnel supported with DMFC and with CMC or Continuously Frictionally Coupled (CFC) rock bolt based on several assumptions. An analytical model is proposed by Cai *et al*. (2004b) to predict the axial force of grouted rock bolt in the tunneling design. Ma *et al*. (2013) presented an analytical model for fully encapsulated rock bolts subjected to tensile loading in pull-out tests. Shin *et al*. (2011) carried out theoretical analyses on the ground reaction curve reinforced with grouted bolts considering seepage forces. The closed-form solution is obtained by Carranza-Torres (2009) for the rock bolt design which regularly spaced around the tunnel. Ahmad and Hamed (2005) developed the rock-support interaction concepts, derived the ground response curve of tunnel reinforced with grouted rock bolt based on the H-B failure criterion.

Zou (Zou and Li 2015, Zou and Yu 2015, Zou and He 2016, Zou *et al*. 2016) focused on the stress and displacement of a circular opening that is excavated in a strain-softening or elastic-brittle-plastic rock mass by considering out-of-plane stress. However, the influence of the grouted rock bolt has not been investigated in those papers. The main objective of this paper is to develop the solution and the numerical stepwise procedure which considering the reinforcement of grouted rock bolt for a circular opening excavated in elastic-brittle-plastic and strain-softening rock mass compatible with M-C or generalized H-B failure criterion, respectively. The paper consists of fourth parts. The first part deals with the definition of the problem, failure criterion and grouted rock bolt behavior. In the second part**,** the system of first-order ordinary differential equations is obtained from the stress equilibrium, constitutive, and consistency equations. The similarity solution considering the reinforcement of grouted rock bolt is developed for circular opening excavated in elastic-brittle-plastic rock mass using generalized H-B or M-C failure criterion, respectively. In the third part, the solution with reinforcement of grouted rock bolt for the circular opening excavated in strain-softening rock mass are proposed by incorporating with M-C and generalized H-B failure criterion. In the final part**,** the numerical implementation of the similarity solution for elastic-brittle-plastic rock mass is discussed. The numerical stepwise procedures using generalized H-B or M-C failure criterion, which is modified from Brown *et al*. (1983) methods, for a circular opening excavated in strain-softening rock mass are described, respectively. The accuracy and practical application of the solution and numerical procedure are illustrated by some examples considering the reinforcement of grouted rock bolt with two categories rock mass (e.g., very good and very poor rock mass). The influence of the reinforcement of grouted rock bolt on ground reaction curves for circular opening is investigated.

2. Definition of the problem

2.1 Assumption

Fig. 1 shows a circular opening excavated in a continuous, homogenous, isotropic, initially elastic rock mass subjected to a hydrostatic stress σ_0. The opening surface is subjected to an internal pressure p_0. Neglecting gravity field, stress and displacement are only the functions of radius under the polar coordinates. Grouted rock bolts are considered to be installed systematically over the periphery of the opening. The number of grouted rock bolt is assumed to be distributed uniformly on the tunnel surface both in the circumferential and longitudinal directions to keep the model symmetry. The horizontal distance between two adjacent anchors is S_c. The longitudinal spacing along the axial of the tunnel is S_l. Length of grouted rock bolt is L_b and is more than the plastic radius without the reinforcement of grouted rock bolt. Under assumption of rigid connection among the rock bolt, grouted and surrounding rock, there is no relative shear displacement among them. In an axi-symmetrical problem, the tangential bolt spacing around the opening is defined by the product of the tunnel radius and the angle between two adjacent bolts.

In the paper, M-C and generalized H-B failure criterion are adopted to deduce the stress, displacement, and plastic radius with considering the reinforcement of grouted rock bolt for elastic-brittle-plastic and strain-softening surrounding rock, respectively.

2.2 Failure criterion

M-C failure criterion is given by

$$\sigma_1 = \sigma_3 N + Y \tag{1}$$

where σ_1 is the major principle stress, σ_3 is the minor principle stress, Y and N are the rock mass strength parameters, $N = (1 + \sin\varphi)/(1 - \sin\varphi)$, $Y = 2c\cos\varphi/(1 - \sin\varphi)$. c and φ are the cohesion and internal friction angle.

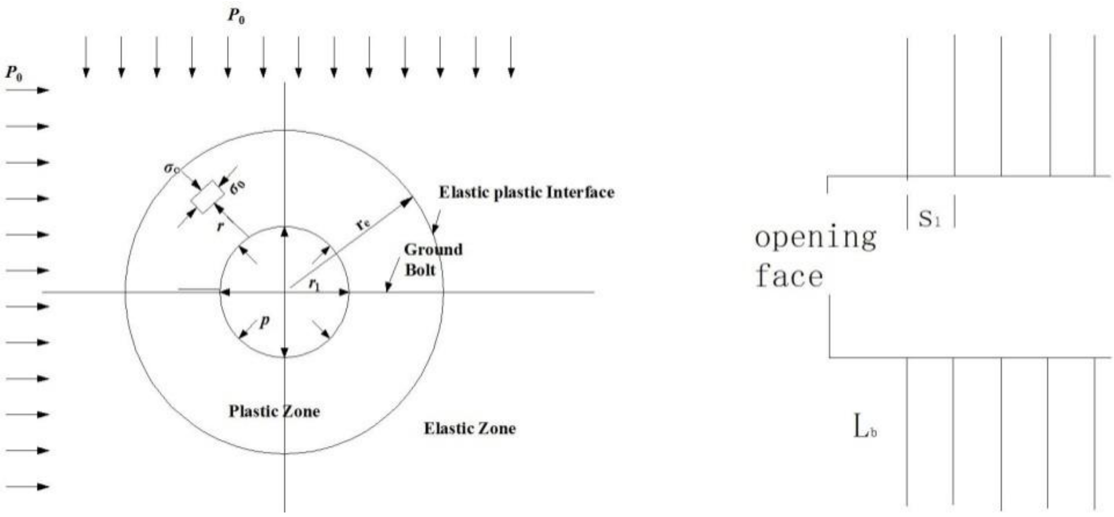


Fig. 1 Definition of the model used in this study

Generalized H-B failure criterion is expressed by

$$\sigma_1 = \sigma_3 + \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s \right)^n \quad (2)$$

where σ_c is the unconfined compressive strength of the rock; m, s, and n in Eq. (2) are the generalized H-B constants for the rock mass before yielding, which are given by $m = m_i \exp[(GSI - 100)/(28 - 14D)]$, $s = \exp[(GSI - 100)/(9 - 3D)]$, $n = 1/2 + [\exp(-GSI/15) - \exp(-20/3)]/6$. D is a factor that depends on the degree of disturbance to which the rock has been subjected in terms of blast damage and stress relaxation, and it varies between 0 and 1; GSI is the geological strength index of the rock mass, and it varies between 10 and 100.

2.3 Elastic-brittle-plastic and strain-softening material behavior model

2.3.1 Elastic-brittle-plastic material behavior model

The relationship of stress and strain for elastic-brittle-plastic rock mass is shown in Fig. 2. The rock mass will be dropt from peak to residual strength after yielding. Under the assumption of the plane strain condition, $\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_r$. σ_1 and σ_3 are the major and minor principle stress. $\varepsilon_1 = \varepsilon_\theta$ and $\varepsilon_3 = \varepsilon_r$. ε_1 and ε_3 are the major and minor principle strain. ε_{1e} is the elastic strain of the interface between plastic and elastic region. ε_1^p and ε_3^p are the major and minor plastic principle strain.

Based on the elastic-brittle-plastic model, M-C and generalized H-B failure criteria, the stress

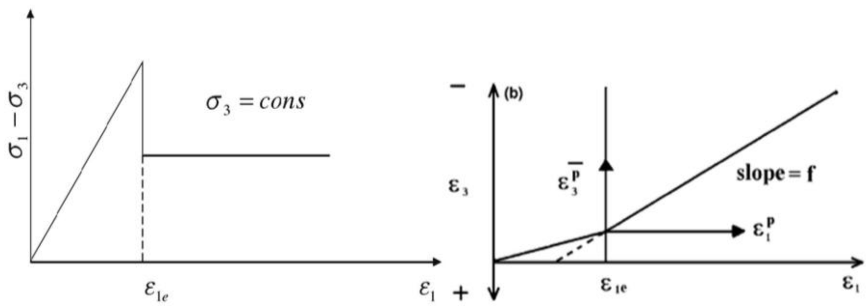


Fig. 2 Elastic-brittle-plastic material behavior model

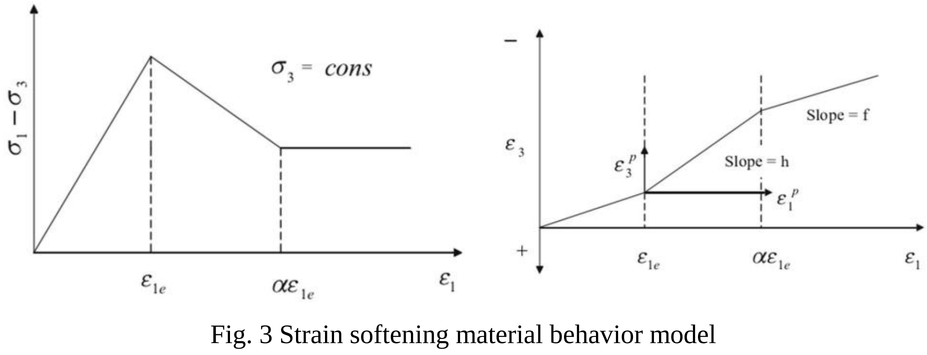


Fig. 3 Strain softening material behavior model

and displacement of tunnel considering the rock bolt effectiveness for two interaction cases are deduced, respectively.

2.3.2 Strain-softening material behavior model

Strain-softening model is shown in Fig. 3. The peak strength of strain-softening rock mass will be transformed to the residual strength after yielding. ε_{1e} is elastic circumferential strain, α is the width of the softening region and equals 1 for the brittle material. f and h are the material parameters for softening and plastic region.

In the paper, based on the strain-softening material behavior model, a numerical stepwise produce considering rock bolt effectiveness is proposed.

2.4 Grouted rock bolt behavior

Grouted rock bolt behavior model is shown in Fig. 4. According to Fahimifar and Soroush's theory (2005): behaviors of grouted rock bolt will change from elastic to plastic states with the deformation of surrounding rock developing, and the grouted rock bolt behaviors are characterized by three cases:

- *Case a*: Rock bolt and rock mass both behave elastically. As surrounding rock is better, strength is bigger. Because the elastic deformation is small, the reinforcement of grouted rock bolt can be neglected when surrounding rock is in the elastic state.
- *Case b*: Rock bolt behaves elastically, but rock mass behaves plastically. At this time, the plastic region of surrounding rock appears. The tensile loading of rock bolt does not exceed its ultimate tensile strength. Therefore, deformation of rock bolt in plastic region of surrounding rock is still in the stage of elastic state.
- *Case c*: Rock bolt and rock mass both behave plastically. As surrounding rock is very poor, the plastic region of surrounding rock expands gradually. Tensile loading of grouted rock bolt in the plastic region exceeds the ultimate tensile strength in the part of the inner wall near the surrounding rock and plastic deformation occurs. However, deformation of grouted rock bolt in the elastic region is still in the elastic stage.

For these categories, the solutions of stress and displacement for elastic-brittle-plastic and strain-softening rock mass are presented in the following parts, respectively.

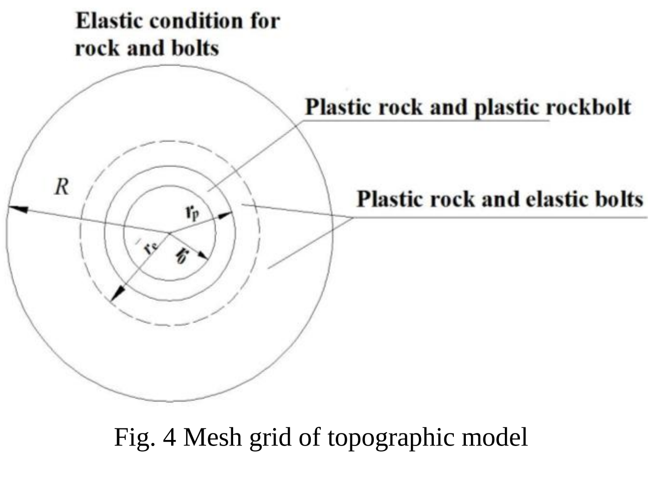


Fig. 4 Mesh grid of topographic model

3. Solutions for elastic-brittle-plastic rock mass

3.1 Rock bolt and rock mass both behave elastically

As rock bolt and rock mass both behave elastically, the rock bolt can be neglected because of the small elastic deformation.

According to Lame's solution, stress and displacement in elastic region are determined by

$$\sigma_r = \sigma_0 + \left(p_0 - \sigma_0\right)\left(\frac{r_0}{r}\right)^2 \tag{3}$$

$$\sigma_\theta = \sigma_0 - \left(p_0 - \sigma_0\right)\left(\frac{r_0}{r}\right)^2 \tag{4}$$

$$u = -\frac{r_0^{\,2}}{r}\left(\frac{1+v}{E}\right)\left(\sigma_0 - p_0\right) \tag{5}$$

where, p_0 is the support pressure. σ_0 is the hydrostatic stress. r_0 is the radius of circular opening. r is the radius at an arbitrary point. v and E are the Poisson's ratio and elasticity modulus.

3.2 Rock bolt behave elastically, but rock mass behaves plastically

As rock bolt behaves elastically and rock mass behaves plastically, there is no relativity shear displacement between rock bolt and surrounding rock mass. Radial stress at the interface between elastic and plastic region can be obtained by Eqs. (6) and (7) for M-C and generalized H-B failure criterion, respectively.

$$\sigma_R = \left(2\sigma_0 - Y\right)/\left(1+N\right) \tag{6}$$

$$\sigma_R = \sigma_0 - \frac{1}{2}\sigma_c\left(m\frac{\sigma_R}{\sigma_c} + s\right)^n \tag{7}$$

where σ_0 is the hydrostatic stress and R is the plastic radius.

Under the small strain condition, the relationship of strain and displacement are described by

$$\begin{cases} \varepsilon_r = -du/dr \\ \varepsilon_\theta = -u/r \end{cases} \tag{8}$$

The total strain can be decomposed elastic and plastic strain and expressed by

$$\begin{cases} \varepsilon_r = \varepsilon_r^e + \varepsilon_r^p \\ \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p \end{cases} \tag{9}$$

where, subscript e and p is the elastic and plastic part.

The non-associate flow rules is adopted in the paper, and

$$\varepsilon_r^p + h\varepsilon_\theta^p = 0 \tag{10}$$

where h is the coefficient for dilatation angle, φ , $h = (1 + \sin\varphi)/(1 - \sin\varphi)$.

Combining Eqs. (8), (9) and (10), we have

$$\frac{du}{dr} + h\frac{u}{r} = f(r) \tag{11}$$

As the reinforcement of grouted rock bolt is not taken into consideration, the displacement in plastic region can be obtained by Sharan's theory (2005)

$$u = r^{-h}\int_R^r r^h f(r)dr + u_R\left(\frac{R}{r}\right)^h \tag{12}$$

and elastic strain can be related to the stress based on Hook's law.

Combining Eq. (12) and Hooke's law (Dan *et al*. 2015), displacement and radial strain in plastic region are obtained.

In the plane strain problem, stress equilibrium differential equation is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{13}$$

Considering the reinforcement of grouted rock bolt, radial stress is

$$\sigma_r' = \sigma_r - T / C \tag{14}$$

$$T = A_b E_s \varepsilon_r \tag{15}$$

where, T is tensile loading of grouted rock bolt. C is the effective cross sectional area, $C = S_l$. S_c. S_l is the axial spacing between the mounting bolt. S_c is the circumferential spacing between the mounting rock bolts. A_b is cross section area of grouted rock bolt. E_s is the elastic modulus of grouted rock bolt. ε_r is the strain of grouted rock bolt.

Considering the reinforcement of grouted rock bolt, circumferential stresses based on the M-C and generalized H-B failure criterion is obtained by Eqs. 16(a)-(b).

$$\sigma_\theta = \sigma_r' N_r + Y_r \tag{16a}$$

$$\sigma_\theta = \sigma_r' + \sigma_c\left(m\frac{\sigma_r'}{\sigma_c} + s\right)^n \tag{16b}$$

With the assumption that there is no relativity shear displacement between bolt and surrounding rock mass, elastic strain of grouted rock bolt equals to the strain of surrounding rock. Combining Eqs. (14)-(15), we have

$$\sigma_r' = \sigma_r - \left(A_b E_s \varepsilon_r\right) / C \tag{17}$$

Then, radial stresses based on the M-C and generalized H-B failure criteria are obtained by

$$\sigma_\theta - \sigma_r = \sigma_r\left(N_r - 1\right) + Y_r - TN_r / C \tag{18a}$$

$$\sigma_\theta - \sigma_r = \sigma_c\left(\frac{m\left(\sigma_r - T/C\right)}{\sigma_c} + s\right)^n - T/C \tag{18b}$$

Substituting Eqs. 18(a)-(b) into Eq. (13), Eqs. 19(a)-(b) are obtained for M-C and generalized H-B failure criterion failure criterion, respectively.

$$\frac{d\sigma_r}{dr} = \frac{\sigma_r\left(N_r - 1\right) + Y_r - TN_r / C}{r} \tag{19a}$$

$$\frac{d\sigma_r}{dr} = \frac{\sigma_c\left(\frac{m\left(\sigma_r - T/C\right)}{\sigma_c} + s\right)^n - T/C}{r} \tag{19b}$$

Combining Eqs. (6), (7), (12), (14), (15), (16), (19) and Hook's law, σ_r, σ_r', σ_θ, T, ε_r and R are given by MATLAB programing with the boundary condition: $\sigma_r|_{r=r0} = p_0$.

As $n = 0.5$, the proposed solutions is yielded to the solution in Fahimifar and Soroush (2005).

3.3 Rock mass and grouted rock bolt both behaves plastically

3.3.1 Rock mass behaves plastically, but rock bolt part behaves plastically

As rock bolt behaves elastically, tensile loading of rock bolt can be calculated with the method in Section 3.2. As rock bolt behaves plastically, the tensile loading of rock bolt is the yield loading, T_{max}, and the corresponding strain is the yield strain, ε_{max}.

$$\varepsilon_r = \varepsilon_{max} \tag{20}$$

$$\sigma_r' = \sigma_r - T_{max} / C \tag{21}$$

$$T = A_b E_s \varepsilon_{max} = T_{max} \tag{23}$$

where, T_{max} is the maximum tensile loading. As the tensile loading is up to the maximum load, T_{max} is a constant value.

3.3.2 Rock mass and grouted rock bolt both behaves plastically

As rock bolt behaves plastically, tensile strain is up to the maximum and $T = T_{max}$. Combining Eqs. (6), (7), (12), (14), (15), (16), (17), (18), (21), (22) and Hook's law, σ_r, σ_r', σ_θ, T, ε_r and R can be obtained by MATLAB programing.

Accordingly, the solutions of stress and displacement for the two interaction cases in elastic-brittle-plastic surrounding rock tunnel are given.

4. Solutions for strain-softening rock mass

4.1 Solution procedures

There is no analytical solution of stress and displacement for the strain-softening rock mass obeyed generalized H-B rock mass. Numerical stepwise produce considering the rock bolt effectiveness is presented. The numerical stepwise produce is shown in the following.

(1) It is assumed that the total plastic region is divided into n concentric annuli as shown in Fig. 5, which is bounded by two circles of normalized radii $\lambda_{(j\text{-}1)} = r_{(j\text{-}1)}/R$, R and $\lambda_{(j)} = r_{(j)}/R$.

(2) It should be noted that the thickness of each annulus is not equal in general because it is determined automatically during the numerical process to satisfy the equilibrium condition as explained later in this section. On the outer boundary of plastic zone, $\rho_{(0)}$ equals to 1.

(3) The jth annulus, adjacent to the elastic region, with inner radius $r_{(j\text{-}1)}$ and outer radius r_j is illustrated in Fig. 5. The small circumferential strain increment, $\Delta\varepsilon_{\theta(j)} = 0.01\varepsilon_{\theta(j\text{-}1)}$, is regarded as the known increment. The stress, the outer radius and displacement in the jth annulus can be determined using analytical solution for the elastic-brittle-plastic medium.

(4) If the pressure decreased further, strain-softening deformation will continue in the other j-1 plastic annuli until residual strength is reached during the circumferential strain increment increase.

(5) n-1 times of brittle-plastic analysis will be carried out in the second plastic annuli. The nth plastic annulus with thickness experiences strain-softening in the small circumferential strain increment $\Delta\varepsilon_{\theta(n)}$ and only one brittle-plastic analysis is performed.

(6) Taking the above process, stress, strain, displacement and plastic radius in softening region are obtained.

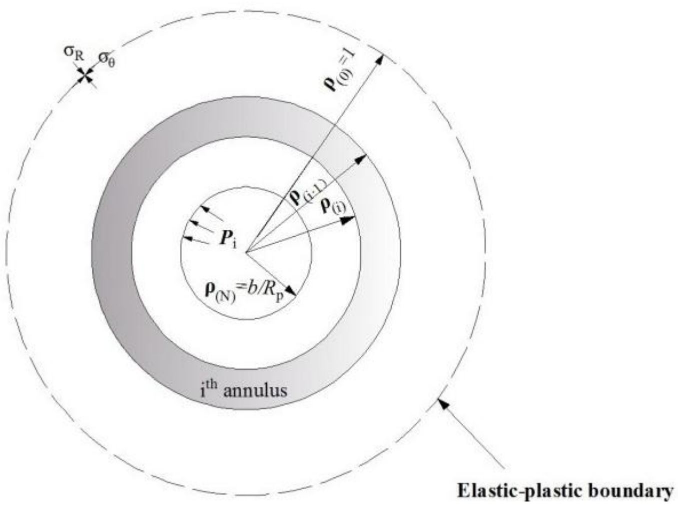


Fig. 5 Normalized plastic region with finite number of annuli

4.2 Rock bolt and rock mass both behave elastically

Stress, strain and displacement for surrounding rock are obtained with the method in Section 3.1 as the rock bolt and rock mass both behave elastically.

4.3 Rock bolt behave elastically, but rock mass behaves plastically

As rock bolt behaves elastically and rock mass behaves plastically, the plastic region occurs in surrounding rock with the plastic radius R. The total plastic region is divided into n annuli for the internal radius with $\lambda_{(j-1)} = r_{(j-1)}/R$ and outer radius with $\lambda_{(j)} = r_{(j)}/R$ for ith annulus. As surrounding rock is critical state, $\lambda_{(1)} = 1$ for the outer radius of outer annulus. Stress and displacement in the interface are given by

$$\begin{cases} \sigma_{r(1)} = \sigma_R \\ \sigma_{\theta(1)} = 2\sigma_0 - \sigma_R \end{cases} \tag{23}$$

$$\begin{cases} \varepsilon_{r(1)} = -\left(\sigma_R - \sigma_0\right)\dfrac{1}{2G} \\ \varepsilon_{\theta(1)} = -\left(\sigma_0 - \sigma_R\right)\dfrac{1}{2G} \end{cases} \tag{24}$$

Stress and strain for the jth annulus are

$$\Delta\varepsilon_{\theta(j)} = 0.01\varepsilon_{\theta(j-1)} \tag{25}$$

$$\varepsilon_{\theta(j)} = \Delta\varepsilon_{\theta(j)} + \varepsilon_{\theta(j-1)} \tag{26}$$

$$\varepsilon_{r(j)} = \varepsilon_{r(j-1)} - f\Delta\varepsilon_{\theta(j)} \tag{27}$$

$$\varepsilon_{r(j)} = \varepsilon_{r(j-1)} + \Delta\varepsilon_{r(j)} \tag{28}$$

where, f is the ratio value of circumferential and radial strain (Brown et al. 1983), and f = 2 in elastic state and f =1 in plastic region.

The radial stresses for jth annulus based on the M-C and generalized H-B failure criteria are obtained by Eqs. 19(a)-(b).

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \left\{\frac{1}{2}\left(\sigma_{r(j)} + \sigma_{r(j-1)}\right)\left(N_{(j)} - 1\right) + Y_{(j)} - N_{(j)}\left(\frac{T_j + T_{j-1}}{2C}\right)\right\} \times \frac{2}{r_j + r_{j-1}} \tag{29a}$$

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \left\{\sigma_c\left[\frac{m_j}{2}\left[\left(\sigma_{r(j)} + \sigma_{r(j-1)}\right) - \frac{T_j + T_{j-1}}{C}\right] + s_j\right]^n - \left(\frac{T_j + T_{j-1}}{2C}\right)\right\} \times \frac{2}{r_j + r_{j-1}} \tag{29b}$$

$$T_j = A_b E_s \varepsilon_{r(j)} \tag{30}$$

Substituting Eq. (30) into Eq. (29), radial stresses considering rock bolt effectiveness for M-C and generalized H-B failure criterion are expressed by

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \left\{ \frac{1}{2}\left(\sigma_{r(j)} + \sigma_{r(j-1)}\right)\left(N_r - 1\right) + Y_r - N_r \left(\frac{A_b E_s \varepsilon_{r(j)} + A_b E_s \varepsilon_{r(j-1)}}{2C} \right) \right\} \times \frac{2}{r_j + r_{j-1}} \tag{31a}$$

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \left\{ \begin{array}{l} \left[\dfrac{m_j \sigma_c}{2} \left[\left(\sigma_{r(j)} + \sigma_{r(j-1)}\right) - \dfrac{A_b E_s \left(\varepsilon_{r(j)} + \varepsilon_{r(j-1)}\right)}{C} \right] + s_j \sigma_c^2 \right]^n \\ - \left(\dfrac{A_b E_s \left(\varepsilon_{r(j)} + \varepsilon_{r(j-1)}\right)}{2C} \right) \end{array} \right\} \times \frac{2}{r_j + r_{j-1}} \tag{31b}$$

$$\sigma'_{r(j)} = \sigma_{r(j)} - \left(A_b E_s \varepsilon_{r(j)}\right) / C \tag{32}$$

Circumferential stresses are given by using M-C and generalized H-B failure criteria.

$$\sigma_{\theta(j)} = \sigma'_{r(j)} N_{(j)} + Y_{(j)} \tag{33a}$$

$$\sigma_{\theta(j)} = \sigma'_{r(j)} + \sigma_{c(j-1)} \left(\frac{m_{(j-1)} \sigma'_{r(j)}}{\sigma_{c(j-1)}} + s_{(j-1)} \right)^n \tag{33b}$$

For the every annulus, the radial and circumferential stresses are obtained by programming with MATLAB.

For the small annulus, the relationship between displacement and strain is

$$\frac{r_j}{r_{j-1}} = \frac{2\varepsilon_{\theta(j-1)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}{2\varepsilon_{\theta(j)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}} \tag{34}$$

Define

$$\lambda_{(j)} = \frac{r_{(j)}}{R} \tag{35}$$

We have

$$\frac{\lambda_{(j)}}{\lambda_{(j-1)}} = \frac{2\varepsilon_{\theta(j-1)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}{2\varepsilon_{\theta(j)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}} \tag{36}$$

$$u_{(j)} = -\varepsilon_{\theta(j)}\rho_{(j)} \tag{37}$$

The displacement and plastic radius are obtained with numerical stepwise produce by MATLAB programming.

For the deterioration process of material properties, Brown *et al*. (1983) assumed that the material properties decrease linearly with strain from their peak values, m_p, s_p and n_p, at $\varepsilon_{r(j)} = \varepsilon_{r(0)}$ to the residual ones, m_r, s_r and n_r, such as

$$\omega_{(j)} = \omega + (\omega_r - \omega)\frac{\left(\varepsilon_{r(j)} - \varepsilon_{r(0)}\right)}{(\alpha - 1)\varepsilon_{r(0)}} \tag{38}$$

where, ω is the constant related to m and n. α is the constant defining strain at which residual strength is reached and 1 for the brittle material.

Taking the above numerical stepwise produce, the stress, stain, displacement and plastic radius in softening are obtained.

As n = 0.5, the Fahimifar and Soroush's (2005) solution is a special case of the proposed solutions for strain-softening surrounding rock.

4.4 Rock bolt and rock mass both behaves plastically

4.4.1 Rock mass behaves plastically, but rock bolt part behaves plastically

In plastic region of surrounding rock, the tensile loading is the maximum uplift force as the rock bolt strain exceeds its yielding strain, ε_{max}. At this time, tensile loading is constant as the strain increasing. Then, rock bolt in the plastic region of surrounding rock can be divided into elastic and plastic parts. As rock bolt is in elastic states, the stress and strain are obtained with the method in Section 4.3. As rock bolt is in plastic states, the stress and strain are obtained with the condition: $T = T_{max}$.

4.4.2 Rock mass and grouted rock bolt both behave plastically

As rock bolt behaves plastically, $T = T_{max}$. Stress, strain and plastic radius are obtained with the method in Section 3.3.

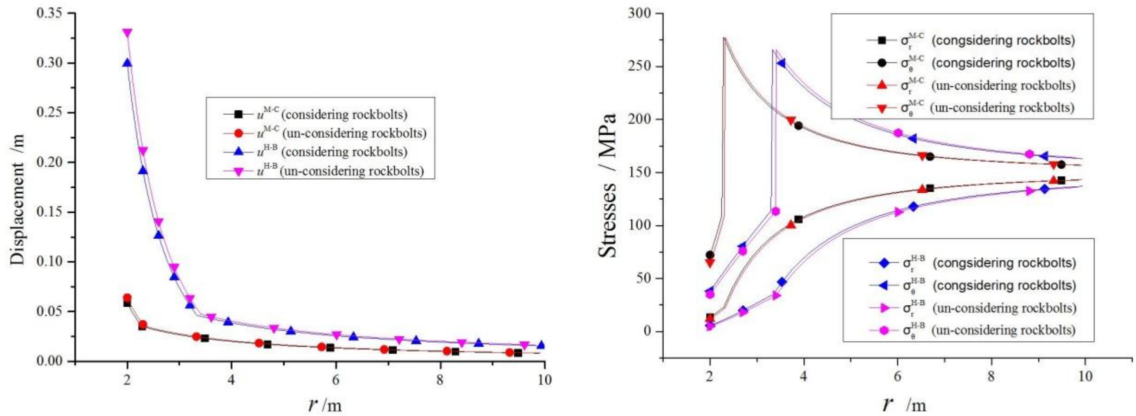
5. Numerical calculations and discussions

In order to analysis the regulation of stress and strain with the rock bolt effectiveness for elastic-brittle-plastic and strain-softening rock mass based on the M-C and generalized H-B failure criterion, calculation parameters for very good and very poor rock mass from Sharan (2005) are adopted. A technique of equivalent M-C and generalized H-B strength parameters is adopted for the examples to validate the presented approach. In the following Figures, $\sigma_r^{M\text{-}C}$, $\sigma_\theta^{M\text{-}C}$ and $u^{M\text{-}C}$ are the radial and circumferential stress and displacement based on the M-C failure criterion. $\sigma_r^{H\text{-}B}$, $\sigma_\theta^{H\text{-}B}$ and $u^{H\text{-}B}$ are the radial and circumferential stress and displacement based on the H-B failure criterion.

5.1 Very good rock mass

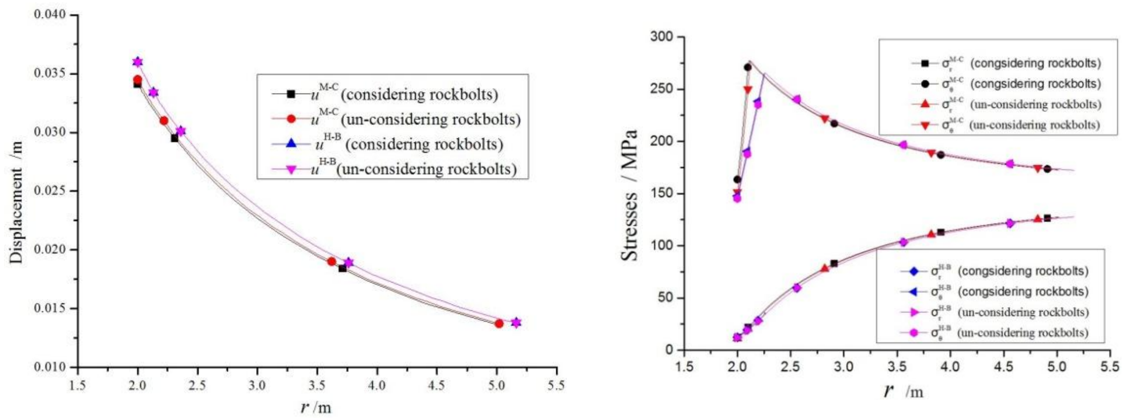
In order to validate the proposed method for elastic-brittle-plastic rock mass in the paper, the parameters of very good rock mass for generalized H-B failure criterion from Sharan (2005) are adopted as following: r_0 = 2.0 m, $\sigma_c = \sigma_0$ = 150 MPa, E = 10 GPa, ν = 0.2, m_p = 10.2, s_p = 0.062, n = 0.5, n_r = 0.51, m_{br} = 2.27, s_r = 0.0002, h = 3.0 m, p_0 = 12 MPa, r_w = 0.01, ξ = 1, E_b = 200 GPa, $\varepsilon_{\max}$ = 0.005, C = 0.0628 m^2. The evolution of strength parameters for M-C failure criterion are as following: E = 10 GPa, r_0 = 2.0 m, ν = 0.2, σ_0 = 150 MPa, c_p = 6.70 MPa, φ_p = 55.57°, c_r = 2.53 MPa, φ_r = 39.67, p_0 = 12 MPa, h = 3.0 m, A_b = 0.00049 m^2, C = 0.628 m^2, E_b = 200 GPa, $\varepsilon_{\max}$ = 0.005. The results of stress and displacement whether considering the rock bolt effectiveness or not are shown in Figs. 6 and 7 for elastic-brittle-plastic and strain-softening rock mass, respectively.

It can be seen from Figs. 6 and 7 that the results of stress is more than that without reinforcement of grouted rock bolt, and the results of displacement and plastic radius are on the contrary. The results of displacement based on the generalized H-B failure criterion are more



(a) Displacements (b) Stresses

Fig. 6 Results of stress and displacement for M-C and generalized H-B failure criterion in very good rock mass for elastic-brittle-plastic model



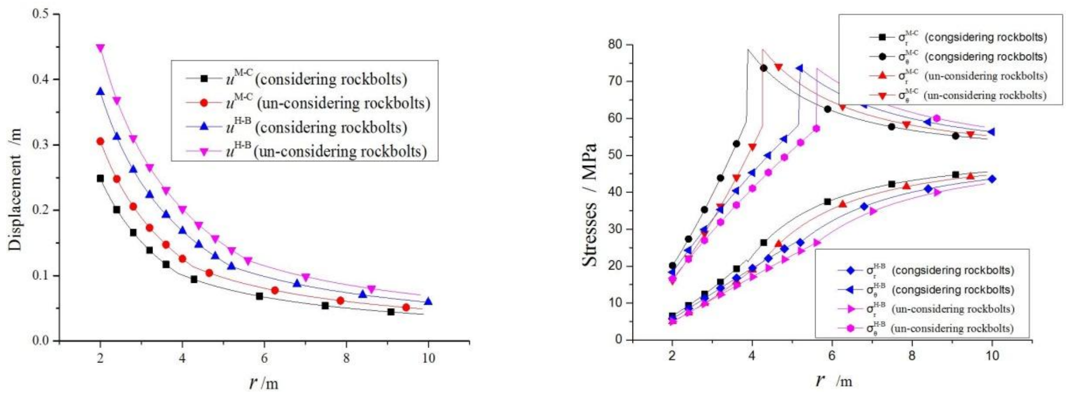
(a) Displacements (b) Stresses

Fig. 7 Results of stress and displacement for M-C and generalized H-B failure criterion in very good rock mass for strain-softening model than that based on the M-C failure criterion, and the results of stress are on the contrary. For example, without the reinforcement of grouted rock bolt for the elastic-brittle-plastic rock mass, the plastic radius is 4.323 m. The plastic radius is 3.754 m with considering the reinforcement of grouted rock bolt.

On the other hand, the reinforcement of grouted rock bolt for stress, displacement and plastic radius is not obvious.

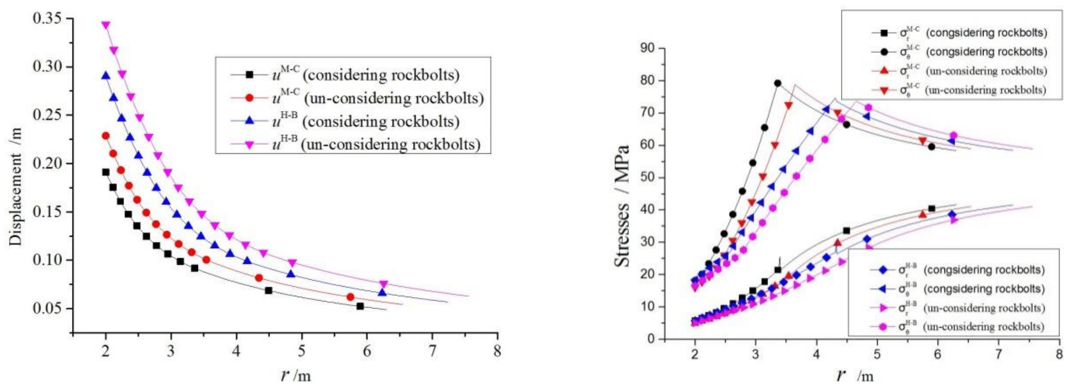
5.2 Very poor rock mass

The parameters of very poor rock mass for generalized H-B failure criterion from Sharan (2005) are adopted as following: r_0 = 2 m, $\sigma_c = \sigma_0$ = 50 MPa, E = 1.4 GPa, ν = 0.3, m_b = 1.7, s_p = 0.0039, n = 0.52, n_r = 0.52, m_{br} = 1.14, s_r = 0.0019, h = 1.0 m, p_0 = 5 MPa, r_w = 0.01, ξ = 1, E_b = 200 GPa, $\varepsilon_{\max}$ = 0.005, C = 0.628 m^2. The evolutions of strength parameters for M-C failure criterion are as



(a) Displacements (b) Stresses

Fig. 8 Results of stress and displacement for M-C and generalized H-B failure criterion in very poor rock mass for elastic-brittle-plastic model



(a) Displacements (b) Stresses

Fig. 9 Results of stress and displacement for M-C and generalized H-B failure criterion in very poor rock mass for strain-softening model following: E = 1.4 GPa, r_0 = 2.0 m, v = 0.3, σ_0 = 50 MPa, c_p = 1.68 MPa, φ_p = 33.21°, c_r = 1.00 MPa, φ_r = 26.23°, p_0 = 5 MPa, h = 1.0 m, A_b = 0.00049 m^2, C = 0.628 m^2, E_b = 200 GPa, $\varepsilon_{\max}$ = 0.005.

The results of stress and displacement whether considering the support effectiveness of rock bolt or not are shown in Figs. 8 and 9 for elastic-brittle-plastic and strain-softening rock mass, respectively.

It can be found from Figs. 8 and 9 that the grouted rock bolt on the tunnel stabilization is one of the best methods for the convergence control, especially in plastic region. Radial and circumferential stress increased and displacement and convergence reduced as a result of grouted rock bolt effectiveness. For displacement and plastic radius, the results based on the generalized H-B failure criterion are more than that based on M-C failure criterion and the regulations of stress are on the contrary. The properties of rock mass are poorer and the above regulations are more obvious. The reason is that the extent of the plastic region is directly related to the properties of this rock mass and any improvement in the rock strength will reduce the extent of the overstressed rock region. A reduction in the apparent plastic region, in turn, curtails tunnel surface displacements. The extent of the plastic region is influenced by the strength parameters of the yielded rock mass. For examples, without the reinforcement of grouted rock bolt, the plastic radius is 8.013 m, on the contrary, the plastic radius is 5.688 m.

On the other hand, the reinforcement of grouted rock bolt for very poor rock mass is the better one among the two types rock mass.

The rules of stress, displacement and plastic radius for the strain-softening rock mass are more similarity to those for the elastic-brittle-plastic rock mass with considering the grouted rock bolt effects.

6. Conclusions

In this paper, a simple numerical approach, which simplifies the strain-softening process into a series of brittle-plastic ones, is presented to obtain the ground reaction curves in elastic-brittle-plastic and strain-softening rock mass reinforced by grouted rock bolt. On the basis of the elastic-brittle-plastic model, numerical procedures for strain-softening solution of the problem are proposed, and the numerical method gives the ground reaction curves, the radius for the elastic, softening and residual regions. The strain-softening solution is degenerated to the elastic-brittle-plastic solution as the critical shear plastic strain approaches zero and the elastic-plastic solution as the critical shear plastic strain becomes large enough. The proposed method is validated by Fahimifar and Soroush's theory (2005) and Fahimifar and Soroush's solution is the special case of the presented method as n = 0.5. The comparison results show that stress will increase and the convergence will reduce considering the grouted rock bolt, and the results of converge based on the generalized H-B failure criterion is more than those based on the M-C failure criterion and the stress is on the contrary.

Although the proposed method looks similar to that from Fahimifar and Soroush's theory (2005), the procedure is completely different from them. Compared with Fahimifar and Soroush's (2005) only considering the H-B failure criterion, the proposed solutions are based on the generalized H-B and M-C failure criterion. Significantly, the proposed method has been validated by Fahimifar and Soroush's theory (2005) which is the special case of the presented method as n = 0.5.

According to practical tunnel engineering, the proposed method can also be used to judge whether the reinforcement is sufficient or not. Future work includes comparing numerical results with in situ monitoring data.

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