Geomechanics and Engineering, *Vol. 11, No. 2 (2016) 289-307*
DOI: http://dx.doi.org/10.12989/gae.2016.11.2.289

A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations

Hayat Saidi [1], Abdelouahed Tounsi [*1,2,3] and Abdelmoumen Anis Bousahla [2,4]

[1] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[2] *Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria*
[3] *Algerian National Thematic Agency of Research in Science and Technology (ATRST), Algeria*
[4] *Département de Génie Civil, Centre Universitaire de Relizane, Algeria*

(*Received June 28, 2015, Revised April 03, 2016, Accepted April 29, 2016*)

Abstract. A simple hyperbolic shear deformation theory taking into account transverse shear deformation effects is proposed for the free flexural vibration analysis of thick functionally graded plates resting on elastic foundations. By considering further supposition, the present formulation introduces only four unknowns and its governing equations are therefore reduced. Hamilton's principle is employed to obtain equations of motion and Navier-type analytical solutions for simply-supported plates are compared with the available solutions in literature to check the accuracy of the proposed theory. Numerical results are computed to examine the effects of the power-law index and side-to-thickness ratio on the natural frequencies.

Keywords: shear deformation theory; vibration; functionally graded plate

1. Introduction

Functionally graded materials (FGMs) are a type of advanced composite materials that contain higher mechanical response than homogeneous material made of identical constituents. Such kind material are obtained by a continuously graded distribution of the volume fractions of the constituents (Koizumi 1997), the FGM is thus suitable for diverse applications, such as thermal coatings of barrier for ceramic engines, gas turbines, nuclear fusions, spacecraft heat shields, heat exchanger tubes, optical thin layers, biomaterial electronics, etc (Akbaş 2015, Bennai *et al.* 2015, Arefi 2015, Ait Atmane *et al.* 2015a, Belkorissat *et al.* 2015, Ebrahimi and Dashti 2015, Darılmaz *et al.* 2015, Bouguenina *et al.* 2015, Boukhari *et al.* 2016, Ebrahimi and Habibi 2016, Hadji *et al.* 2016, Kar *et al.* 2016, Moradi-Dastjerdi 2016, Trinh *et al.* 2016).

Currently, many functionally graded (FG) plate structures which have been employed for engineering fields led to the development of various plate models to accurately examine the static, buckling and vibration responses of FG structures (Benachour *et al.* 2011, Jha *et al.* 2013a, Hadji

*Corresponding author, Professor, E-mail: tou_abdel@yahoo.com

http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

et al. 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Klouche Djedid *et al*. 2014, Nguyen *et al*. 2015, Tagrara *et al*. 2015, Pradhan and Chakraverty 2015, Hadji and Adda Bedia 2015a, b, Kar and Panda 2015, Hamidi *et al*. 2015, Bennoun *et al*. 2016, Eltaher *et al*. 2016). The classical plate theory (CPT) is based on the supposition that straight lines which are normal to the neutral surface before deformation remain straight and normal to the neutral surface after deformation. Since the transverse shear deformation is neglected (Feldman and Aboudi 1997, Mahdavian 2009, Chen *et al*. 2006, Baferani *et al*. 2011a), it cannot be suitable for the investigating of moderately thick or thick plates in which transverse shear deformation effects are more important. For FG thick and moderately thick plates; the first-order shear deformation theory (FSDT) has been employed (Praveen and Reddy 1998, Croce and Venini 2004, Efraim and Eisenberger 2007, Zhao *et al*. 2009). In such formulation, in-plane displacements are linearly varied within the thickness and need a shear correction coefficient to correct the unrealistic distribution of the transverse shear stresses and shear strains across the thickness. To avoid the employment of the shear correction coefficient and predict a better distribution of the transverse shear deformation in FG plates, higher-order shear deformation plate theories (HSDTs) have been developed (Reddy 2000, 2011, Pradyumna and Bandyopadhyay 2008, Jha *et al*. 2013b, Neves *et al*. 2013, Talha and Singh 2010, Chen *et al*. 2009, Mantari and Soares 2012, 2013, Houari *et al*. 2013, Matsunaga 2008, Tounsi *et al*. 2013, Mahi *et al*. 2015, Bellifa *et al*. 2016, Ait Yahia *et al*. 2015, Al-Basyouni *et al*. 2015, Bourada *et al*. 2015, Attia *et al*. 2015, Merazi *et al*. 2015, Abdelhak *et al*. 2015). However, some of these HSDTs are computational costs due to of number of additional variables incorporated to the theory (Pradyumna and Bandyopadhyay 2008, Jha *et al*. 2013a, b, Neves *et al*. 2013, Reddy 2011, Talha and Singh 2010). Thus, a simple higher-order shear deformation theory presented in this article is necessary.

This article aims to propose a simple higher-order shear deformation theory for free vibration response of FG plates resting on elastic foundation. By considering a further assumption to the existing higher-order shear deformation model, the proposed theory involves only four unknowns and their governing equations are consequently reduced. Pasternak model is employed to simulate the interactions between the plate and elastic foundation. Equations of motion are obtained via Hamilton's principle. Analytical solutions of simply supported plates are proposed. The computed results are compared with the existing solutions to check the accuracy of present formulation in predicting the vibration behavior of FG plates resting on elastic foundation.

2. Theoretical formulations

A FG rectangular plate with length, width and uniform thickness equal to a, b and h respectively is shown in Fig. 1. The FG plate is composed by a mixture of ceramic and metal components whose material characteristics change across the plate thickness with a power law distribution of the volume fractions of the constituents of the two materials as (Akavci 2015, Ahouel *et al*. 2016)

$$E(z) = E_m + \left(E_c - E_m\right)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1a}$$

$$\rho(z) = \rho_m + \left(\rho_c - \rho_m\right)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1b}$$

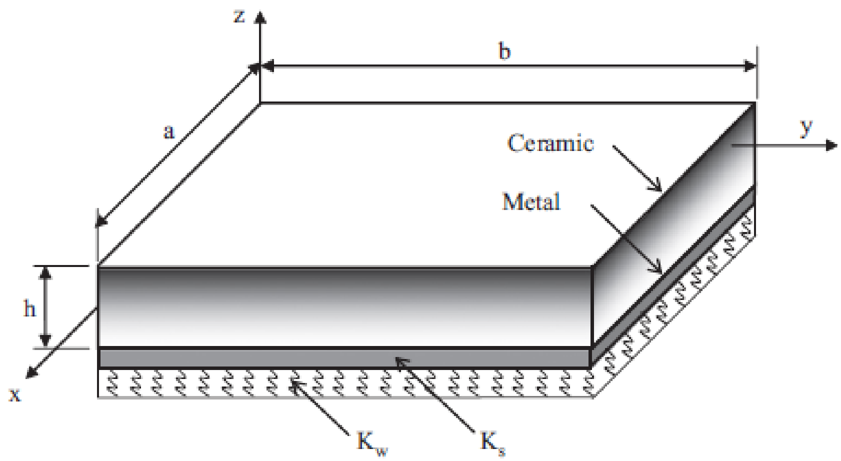

Fig. 1 Schematic representation of a rectangular FG plate resting on elastic foundation

where the subscripts *m* and *c* denote the metallic and ceramic components, respectively; and *p* is the power law exponent. The value of *p* equal to zero indicates a fully ceramic plate, whereas infinite *p* represents a fully metallic plate. Since the influences of the variation of Poisson's ratio *v* on the behavior of FG plates are very small (Yang *et al*. 2005, Kitipornchai *et al*. 2006, Bourada *et al*. 2012, Ould Larbi *et al*. 2013, Saidi *et al*. 2013, Bousahla *et al*. 2014, Fekrar *et al*. 2014, Bouchafa *et al*. 2015, Larbi Chaht *et al*. 2015, Sallai *et al*. 2015, Zemri *et al*. 2015, Meradjah *et al*. 2015, Bouderba *et al*. 2016, Laoufi *et al*. 2016), it is supposed to be constant for convenience.

2.1 Kinematics and strains

In this work, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y,t) \tag{2a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y,t) \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

where u_0; v_0; w_0, θ_x, θ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\theta_x = -\partial\varphi(x, y) / \partial x$ and $\theta_y = -\partial\varphi(x, y) / \partial y$, the displacement field of the present model can be expressed in a simpler form as (Draiche *et al*. 2014)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \tag{3a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} - f(z)\frac{\partial \varphi}{\partial y} \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{3c}$$

where the shape function $f(z)$ is chosen according to Mahi *et al*. (2015) as

$$f(z) = \frac{h}{2}\tanh\left(2\frac{z}{h}\right) - \frac{4}{3\cosh^2(1)}\left(\frac{z^3}{h^2}\right) \tag{4}$$

Clearly, the displacement field in Eq. (3) considers only four unknowns (u_0, v_0, w_0 and φ). The nonzero strains associated with the displacement field in Eq. (3) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 \varphi}{\partial x^2} \\ -\dfrac{\partial^2 \varphi}{\partial y^2} \\ -2\dfrac{\partial^2 \varphi}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \tag{6a}$$

and

$$g(z) = -\frac{df(z)}{dz} \tag{6b}$$

For elastic and isotropic FGMs, the constitutive relations can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (1), stiffness coefficients, C_{ij}, can be written as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \quad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{8}$$

2.2 Equations of motion

Hamilton's principle is herein employed to determine the equations of motion

$$0 = \int_0^t (\delta U + \delta V - \delta K)\, dt \tag{9}$$

where δU is the variation of strain energy; δV is the variation of work done; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is computed by

$$\begin{aligned}
\delta U &= \int_V \left[\sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \tau_{xy} \delta\, \gamma_{xy} + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz}\right] dV \\
&= \int_A \left[N_x \delta\, \varepsilon_x^0 + N_y \delta\, \varepsilon_y^0 + N_{xy} \delta\, \gamma_{xy}^0 + M_x^b \delta\, k_x^b + M_y^b \delta\, k_y^b + M_{xy}^b \delta\, k_{xy}^b \right. \\
&\quad \left. + M_x^s \delta\, k_x^s + M_y^s \delta\, k_y^s + M_{xy}^s \delta\, k_{xy}^s + S_{yz}^s \delta\, \gamma_{yz}^s + S_{xz}^0 \delta\, \gamma_{xz}^0 \right] dA = 0
\end{aligned} \tag{10}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz\,, \quad (i = x, y, xy) \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{11}$$

The variation of the potential energy of elastic foundation can be calculated by

$$\delta V = \int_A f_e \delta\, w_0 dA \tag{12}$$

where f_e is the density of reaction force of foundation. For the Pasternak foundation model (Bounouara *et al*. 2016, Abdelbari *et al*. 2016, Ait Atmane *et al*. 2016, Chikh *et al*. 2016, Bakora and Tounsi 2015, Tebboune *et al*. 2015, Meksi *et al*. 2015, Ait Amar Meziane *et al*. 2014, Zidi *et al*. 2014, Khalfi *et al*. 2014, Bouderba *et al*. 2013)

$$f_e = K_W w - K_{S1} \frac{\partial^2 w}{\partial x^2} - K_{S2} \frac{\partial^2 w}{\partial y^2} \tag{13}$$

where K_W is the modulus of subgrade reaction (elastic coefficient of the foundation) and K_{S1} and K_{S2} are the shear moduli of the subgrade (shear layer foundation stiffness). If foundation is homogeneous and isotropic, we will get $K_{S1} = K_{S2} = K_S$. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation.

The variation of kinetic energy of the plate can be expressed as

$$\begin{aligned}
\delta K &= \int_V \left[\dot{u}\delta\,\dot{u} + \dot{v}\delta\,\dot{v} + \dot{w}\delta\,\dot{w}\right]\rho(z)\,dV \\
&= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&\quad - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\,\dot{v}_0\right) \\
&\quad - J_1\left(\dot{u}_0\frac{\partial\delta\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{\varphi}}{\partial y} + \frac{\partial\dot{\varphi}}{\partial y}\delta\,\dot{v}_0\right) \\
&\quad + I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right) + K_2\left(\frac{\partial\dot{\varphi}}{\partial x}\frac{\partial\delta\,\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial y}\frac{\partial\delta\,\dot{\varphi}}{\partial y}\right) \\
&\quad + J_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{\varphi}}{\partial y} + \frac{\partial\dot{\varphi}}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right)\Big\} dA
\end{aligned} \tag{14}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1b); and (I_i, J_i, K_i) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right)\rho(z)dz \tag{15a}$$

$$\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(f, z\,f, f^2\right)\rho(z)dz \tag{15b}$$

Substituting Eqs. (10), (12), and (14) into Eq. (9), integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_0, and $\delta\varphi$; the following equations of motion are obtained

$$\begin{aligned}
\delta\,u_0 &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} - J_1\frac{\partial\ddot{\varphi}}{\partial x} \\
\delta\,v_0 &: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} - J_1\frac{\partial\ddot{\varphi}}{\partial y} \\
\delta\,w_0 &: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - f_e = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 - J_2\nabla^2\ddot{\varphi} \\
\delta\,\varphi &: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = J_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - J_2\nabla^2\ddot{w}_0 - K_2\nabla^2\ddot{\varphi}
\end{aligned} \tag{16}$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial x^2$ is the Laplacian operator in two-dimensional Cartesian coordinate system.

Substituting Eq. (5) into Eq. (7) and the subsequent results into Eqs. (11), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s \gamma, \tag{17}$$

in which

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t, \tag{18a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}^t, \tag{18b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{18c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{18d}$$

$$S = \{S_{xz}^s, S_{yz}^s\}^t, \quad \gamma = \{\gamma_{xz}^0, \gamma_{yz}^0\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{18e}$$

and stiffness components are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz, \tag{19a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{19b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{19c}$$

Introducing Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (δu_0, δv_0, δw_0, $\delta \varphi$) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + (A_{12} + A_{66})d_{12}v_0 - B_{11}d_{111}w_0 - (B_{12} + 2B_{66})d_{122}w_0 \\ &\quad - (B_{12}^s + 2B_{66}^s)d_{122}\varphi - B_{11}^s d_{111}\varphi = I_0\ddot{u}_0 - I_1 d_1\ddot{w}_0 - J_1 d_1\ddot{\varphi},\end{aligned} \tag{20a}$$

$$\begin{aligned}&A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + (A_{12} + A_{66})d_{12}u_0 - B_{22}d_{222}w_0 - (B_{12} + 2B_{66})d_{112}w_0 \\ &\quad - (B_{12}^s + 2B_{66}^s)d_{112}\varphi - B_{22}^s d_{222}\varphi = I_0\ddot{v}_0 - I_1 d_2\ddot{w}_0 - J_1 d_2\ddot{\varphi},\end{aligned} \tag{20b}$$

$$\begin{aligned}&B_{11}d_{111}u_0 + (B_{12} + 2B_{66})d_{122}u_0 + (B_{12} + 2B_{66})d_{112}v_0 + B_{22}d_{222}v_0 \\ &- D_{11}d_{1111}w_0 - 2(D_{12} + 2D_{66})d_{1122}w_0 - D_{22}d_{2222}w_0 - D_{11}^s d_{1111}\varphi \\ &- 2(D_{12}^s + 2D_{66}^s)d_{1122}\varphi - D_{22}^s d_{2222}\varphi - f_e = I_0\ddot{w}_0 \\ &+ I_1(d_1\ddot{u}_0 + d_2\ddot{v}_0) - I_2(d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0) - J_2(d_{11}\ddot{\varphi} + d_{22}\ddot{\varphi})\end{aligned} \tag{20c}$$

$$\begin{aligned}&B_{11}^s d_{111}u_0 + (B_{12}^s + 2B_{66}^s)d_{122}u_0 + (B_{12}^s + 2B_{66}^s)d_{112}v_0 + B_{22}^s d_{222}v_0 - D_{11}^s d_{1111}w_0 \\ &- 2(D_{12}^s + 2D_{66}^s)d_{1122}w_0 - D_{22}^s d_{2222}w_0 - H_{11}^s d_{1111}\varphi \\ &- 2(H_{12}^s + 2H_{66}^s)d_{1122}\varphi - H_{22}^s d_{2222}\varphi + A_{44}^s d_{11}\varphi + A_{55}^s d_{22}\varphi = J_1(d_1\ddot{u}_0 + d_2\ddot{v}_0) \\ &- J_2(d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0) - K_2(d_{11}\ddot{\varphi} + d_{22}\ddot{\varphi})\end{aligned} \tag{20d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \ (i, j, l, m = 1, 2). \tag{21}$$

2.3 Closed-form solution for simply-supported FG plates

Based on Navier method, the following expansions of generalized displacements are taken to automatically respect the simply supported boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}e^{i\omega t}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}e^{i\omega t}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{22}$$

where $\alpha = m\pi / a$ and $\beta = n\pi / b$, ω is the frequency of free vibration of the plate, $\sqrt{i} = -1$ the imaginary unit.

Substituting Eqs. (22) into Eq. (20) and collecting the displacements and acceleration for any values of m and n, the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

where

$$S_{11} = A_{11}\alpha^2 + A_{66}\beta^2, \quad S_{12} = \alpha\beta\left(A_{12} + A_{66}\right), \quad S_{13} = -\alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right),$$
$$S_{14} = -\alpha\left(B_{11}^s\alpha^2 + B_{12}^s\beta^2 + 2B_{66}^s\beta^2\right), \quad S_{22} = A_{66}\alpha^2 + A_{22}\beta^2,$$
$$S_{23} = -\beta\left(B_{11}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right), \quad S_{24} = -\beta\left(B_{11}^s\beta^2 + B_{12}^s\alpha^2 + 2B_{66}^s\alpha^2\right),$$
$$S_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 + K_w + K_s(\alpha^2 + \beta^2),$$
$$S_{34} = D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4, \tag{24}$$
$$S_{44} = H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2$$
$$m_{11} = m_{22} = I_0, \quad m_{13} = -\alpha\, I_1, \quad m_{14} = -\alpha\, J_1, \quad m_{23} = -\beta\, I_1, \quad m_{24} = -\beta\, J_1,$$
$$m_{33} = I_0 + I_2(\alpha^2 + \beta^2), \quad m_{34} = J_2(\alpha^2 + \beta^2), \quad m_{44} = K_2(\alpha^2 + \beta^2)$$

Eq. (24) is a general form for buckling and free vibration analysis of FG plates resting on elastic foundations under in-plane loads. The stability problem can be carried out by neglecting the mass matrix while the free vibration problem is achieved by omitting the in-plane loads.

3. Numerical examples and discussions

In this section the accuracy of the present theory for vibrational analysis of simply supported FG plates is verified. FG plates made of a material combination of metal and ceramic (Al/Al_2O_3) are considered. The material properties of FG plates are given in Table 1. For convenience, the following non-dimensional parameters are employed

$$\hat{\omega} = \omega\, h\sqrt{\rho_m / E_m}, \quad \overline{\omega} = \omega\frac{a^2}{h}\sqrt{\rho_c / E_c}, \quad k_w = \frac{K_w a^4}{D_m}, \quad k_s = \frac{K_s a^2}{D_m}, \quad D_m = \frac{E_m h^3}{12(1-\nu^2)} \tag{25}$$

Table 2 aims to prove the accuracy of the present formulation in predicting the free vibration response of Al/Al_2O_3 plate resting on elastic foundations. By considering, different values of thickness ratio h / a, power law exponent p and foundation parameters k_w; k_s, the non-dimensional $\hat{\omega}$ of square plates are listed in Table 2 and compared with those given by Baferani *et al*. (2011b) using a third-order shear deformation theory (TSDT) and Thai and Choi (2014) using a zeroth-order shear deformation theory (ZSDT). It can be observed that the computed results are in very

Table 1 Material properties employed in the FG plates

Propriétés	Aluminium (Al)	Alumina (Al_2O_3)
Young's modulus (GPa)	70	380
Poisson's ratio	0.3	0.3
Mass density kg/m^3	2702	3800

Table 2 Dimensionless fundamental frequency $\hat{\omega}$ of square plates

k_w	k_s	h / a	Theory	Power law index (p)				
				0	0.5	1	2	5
0	0	0.05	Ref [(a)]	0.0291	0.0249	0.0227	0.0209	0.0197
			Ref [(b)]	0.0291	0.0246	0.0222	0.0202	0.0191
			Present	0.0291	0.0247	0.0222	0.0202	0.0191
		0.10	Ref [(a)]	0.1134	0.0975	0.0891	0.0819	0.0767
			Ref [(b)]	0.1134	0.0963	0.0868	0.0788	0.0740
			Present	0.1134	0.0964	0.0870	0.0790	0.0740
		0.15	Ref [(a)]	0.2454	0.2121	0.1939	0.1778	0.1648
			Ref [(b)]	0.2452	0.2090	0.1885	0.1706	0.1589
			Present	0.2450	0.2090	0.1880	0.1710	0.1600
		0.20	Ref [(a)]	0.4154	0.3606	0.3299	0.3016	0.2765
			Ref [(b)]	0.4150	0.3551	0.3205	0.2892	0.2667
			Present	0.4152	0.3551	0.3205	0.2892	0.2665
0	100	0.05	Ref [(a)]	0.0406	0.0389	0.0382	0.0380	0.0381
			Ref [(b)]	0.0406	0.0386	0.0378	0.0374	0.0377
			Present	0.0406	0.0386	0.0378	0.0374	0.0376
		0.10	Ref [(a)]	0.1599	0.1540	0.1517	0.1508	0.1515
			Ref [(b)]	0.1597	0.1526	0.1494	0.1478	0.1487
			Present	0.1597	0.1526	0.1494	0.1478	0.1487
		0.15	Ref [(a)]	0.3515	0.3407	0.3365	0.3351	0.3362
			Ref [(b)]	0.3512	0.3369	0.3304	0.3269	0.3286
			Present	0.3513	0.3369	0.3303	0.3270	0.3285
		0.20	Ref [(a)]	0.6080	0.5932	0.5876	0.5861	0.5879
			Ref [(b)]	0.6075	0.5857	0.5753	0.5694	0.5722
			Present	0.6076	0.5856	0.5752	0.5692	0.5720
100	0	0.05	Ref [(a)]	0.0298	0.0258	0.0238	0.0221	0.0210
			Ref [(b)]	0.0298	0.0255	0.0233	0.0214	0.0204
			Present	0.0298	0.0255	0.0232	0.0214	0.0205
		0.10	Ref [(a)]	0.1162	0.1012	0.0933	0.0867	0.0821
			Ref [(b)]	0.1161	0.0999	0.0910	0.0836	0.0795
			Present	0.1162	0.0999	0.0910	0.0837	0.0796
		0.15	Ref [(a)]	0.2519	0.2204	0.2036	0.1889	0.1775
			Ref [(b)]	0.2516	0.2173	0.1982	0.1818	0.1716
			Present	0.2517	0.2173	0.1982	0.1818	0.1716
		0.20	Ref [(a)]	0.4273	0.3758	0.3476	0.3219	0.2999
			Ref [(b)]	0.4269	0.3702	0.3381	0.3097	0.2901
			ZSDT	0.4272	0.3702	0.3380	0.3096	0.2898

Table 2 Continued

k_w	k_s	h / a	Theory	Power law index (p)				
				0	0.5	1	2	5
100	100	0.05	Ref [(a)]	0.0411	0.0395	0.0388	0.0386	0.0388
			Ref [(b)]	0.0411	0.0392	0.0392	0.0381	0.0384
			Present	0.0411	0.0392	0.0384	0.0381	0.0384
		0.10	Ref [(a)]	0.1619	0.1563	0.1542	0.1535	0.1543
			Ref [(b)]	0.1617	0.1549	0.1519	0.1505	0.1515
			Present	0.1617	0.1549	0.1519	0.1505	0.1515
		0.15	Ref [(a)]	0.3560	0.3460	0.3422	0.3412	0.3427
			Ref [(b)]	0.3557	0.3421	0.3359	0.3329	0.3349
			Present	0.3558	0.3420	0.3360	0.3327	0.3348
		0.20	Ref [(a)]	0.6162	0.6026	0.5978	0.5970	0.5993
			Ref [(b)]	0.6156	0.5950	0.5852	0.5800	0.5834
			Present	0.6156	0.5948	0.5852	0.5800	0.5832

[(a)] Baferani *et al*. (2011b)
[(b)] Thai and Choi (2014)

Table 3 Dimensionless fundamental frequency $\overline{\varpi}$ of rectangular plates ($k_w = k_s = 100$)

a / b	a / h	Theory	Power law index (p)					
			0	0.5	1	2	5	10
0.5	5	Ref [(a)]	11.3952	11.2331	11.1780	11.2018	11.3593	11.4558
		Present	11.3959	11.2335	11.1783	11.2019	11.3587	11.4557
	10	Ref [(a)]	11.7257	11.4992	11.4270	11.4530	11.6243	11.7093
		Present	11.7259	11.4993	11.4271	11.4529	11.6239	11.7092
	20	Ref [(a)]	11.8246	11.5780	11.5005	11.5273	11.7054	11.7886
		Present	11.8246	11.5781	11.5005	11.5272	11.7053	11.7885
1	5	Ref [(a)]	15.3904	14.8757	14.6305	14.5004	14.5843	14.6636
		Present	15.3923	14.8768	14.6313	14.5006	14.5830	14.6635
	10	Ref [(a)]	16.1728	15.4895	15.1887	15.0455	15.1497	15.2045
		Present	16.1735	15.4898	15.1890	15.0455	15.1488	15.2043
	20	Ref [(a)]	16.4249	15.6851	15.3663	15.2209	15.3414	15.3929
		Present	16.4251	15.6852	15.3663	15.2209	15.3411	15.3928
2	5	Ref [(a)]	28.6467	26.8009	25.7640	24.9077	24.5036	24.4352
		Present	28.6591	26.8086	25.7703	24.9109	24.4983	24.4367
	10	Ref [(a)]	32.3893	29.7133	28.3322	27.2931	26.8741	26.6994
		Present	32.3937	29.7163	28.3346	27.2932	26.8675	26.6951
	20	Ref [(a)]	33.8869	30.8606	29.3467	28.2628	27.9294	27.7426
		Present	33.8882	30.8614	29.3474	28.2627	27.9267	27.7419

[(a)] Thai and Choi (2014)

good agreement with those calculated by TSDT (Baferani *et al*. 2011b) and by ZSDT (Thai and Choi 2014). It is also concluded from Table 2 that the increase of the foundation parameters k_w; k_s, leads to an increase of non-dimensional fundamental frequency. Compared to the Winkler parameter k_w, the Pasternak foundation parameter k_s has dominant influence on increasing the non-dimensional frequency.

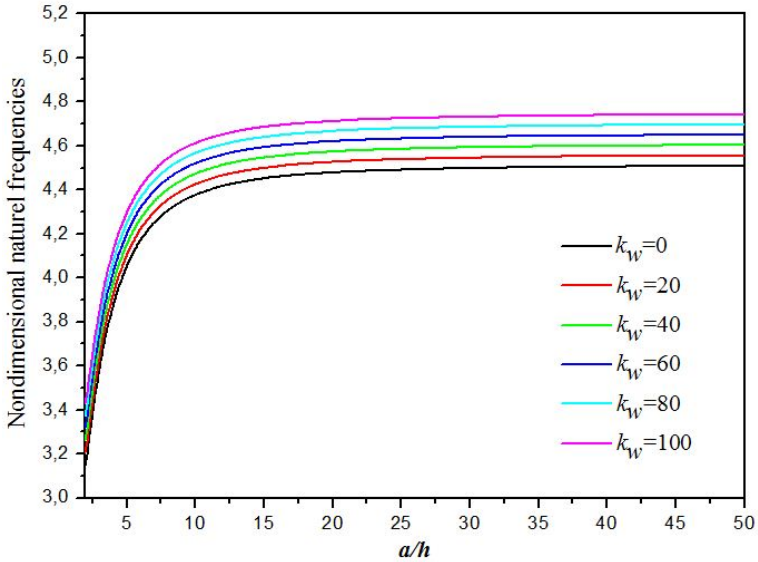

Fig. 2 Effect of winkler parameter (k_w) and the length-to-thickness ratio a/h on the natural frequency $\overline{\omega}$ of Al/Al$_2$O$_3$ square plates (p = 3, k_s = 10)

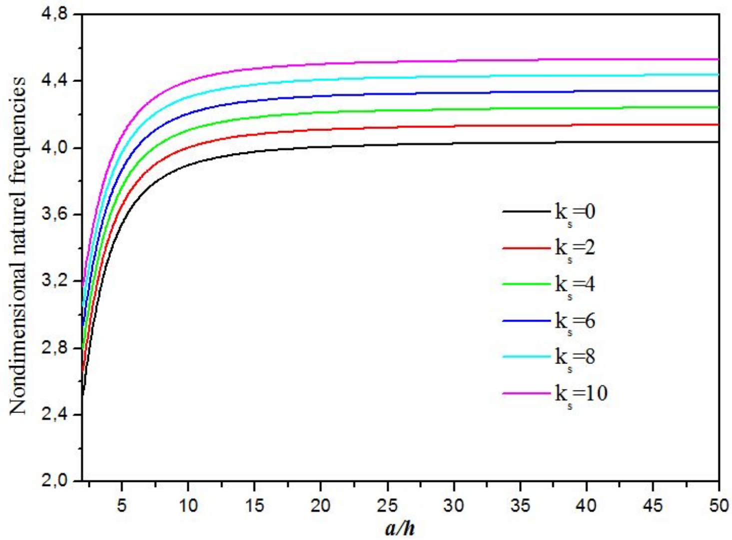

Fig. 3 Effect of pasternak parameter (k_s) and the length-to-thickness ratio a/h on the natural frequency $\overline{\omega}$ of Al/Al$_2$O$_3$ square plates, (p = 3, k_w = 10)

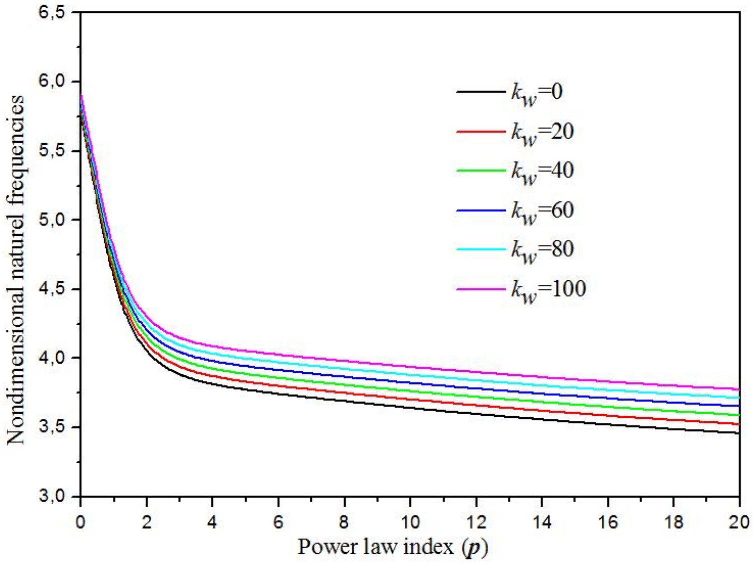

Fig. 4 Effect of winkler parameter (k_w) and the power law index p on the natural frequency $\overline{\omega}$ of Al/Al_2O_3 square plates, (a / h = 10)

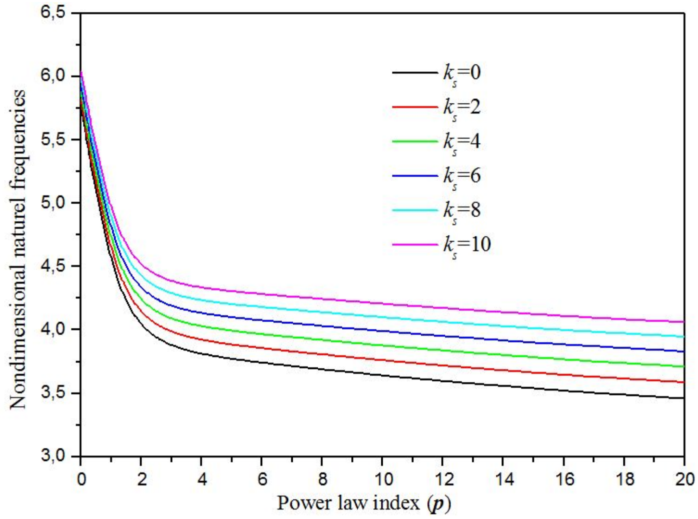

Fig. 5 Effect of pasternak parameter (k_s) and the power law index p on the natural frequency $\overline{\omega}$ of Al/Al_2O_3 square plates, (a / h = 10 and k_w = 10)

Table 3 presents non-dimensional fundamental frequencies $\overline{\omega}$ of FG rectangular plates resting on elastic foundation. In this example, the non-dimensional fundamental frequencies computed by present method are compared with those predicted by Thai and Choi (2014) based on ZSDT. The non-dimensional foundation parameters (k_w, k_s) are considered to be 100. The reliability of the presented formulation for FG plates can be concluded from Table 3; where the results are in an excellent agreement as values of non-dimensional fundamental frequency are consistent with those

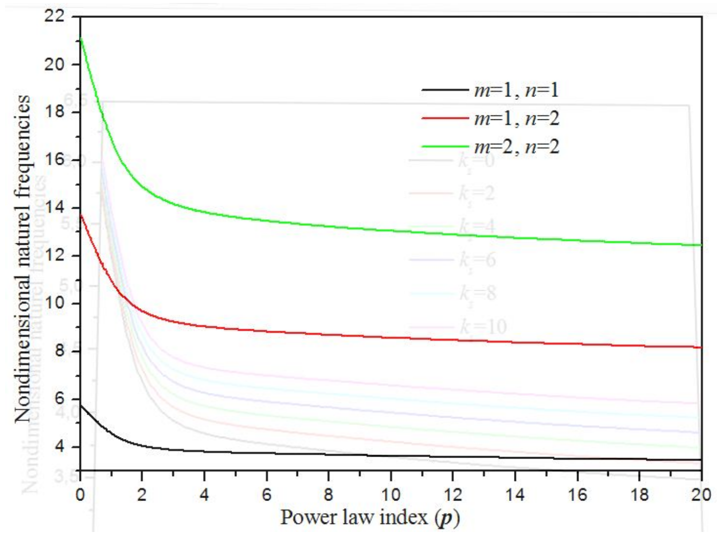

Fig. 6 Effect of the power- law index p on the natural frequency $\bar{\omega}$ of Al/Al_2O_3 square plates, ($a / h = 10$ and $k_w = k_s = 0$)

predicted by Thai and Choi (2014).

The variation of non-dimensional frequencies $\bar{\omega}$ with various values of length-to-thickness ratios are plotted in Figs. 2 and 3 for different values of Winkler parameter k_w and Pasternak foundation parameter k_s, respectively. It can be seen from this figure, that for thin plates where a / h takes high values, the frequencies become almost constants. It can be also observed that, increasing value of Winkler and Pasternak parameters cause the increase in the natural frequency. The figures demonstrate also, that Pasternak parameter of foundation has more significant influence than Winkler parameter on the fundamental frequency of plate.

The variations of non-dimensional fundamental frequencies of square FG plates with respect to power law index p and for different values of Winkler and Pasternak parameters are plotted in Figs. 4 and 5, respectively. It is observed from the figures that, increasing value of power law index causes a reduction of the fundamental frequency. It is due to the fact that a higher value of p corresponds to lower value of volume fraction of the ceramic phase, and thus makes the plates become the softer ones.

The variation of non-dimensional frequencies in terms of the power-law index is presented in Fig. 6 for different mode number. It can be observed from this figure that the frequencies diminish with the decrease of the mode number.

4. Conclusions

In the current investigation, analytical formulation for free vibration response of FG plates resting on elastic foundation is developed on the supposition that transverse shear displacements vary as a hyperbolic function within the thickness of plate. The proposed model contains only four unknowns and equations of motion are obtained from Hamilton's principle. Navier-type solutions are determined for simply-supported boundary conditions and compared with the existing solutions to check the validity of the proposed theory. The material properties are estimated by power-law form. It has been demonstrated that the present analytical formulation can accurately predict natural frequencies of FG plates resting on elastic foundation.

Acknowledgments

This research was supported by the Algerian National Thematic Agency of Research in Science and Technology (ATRST) and university of Sidi Bel Abbes (UDL SBA) in Algeria.

References

Abdelbari, S., Fekrar, A., Heireche, H., Saidi, H., Tounsi, A. and Adda Bedia, E.A. (2016), "An efficient and simple shear deformation theory for free vibration of functionally graded rectangular plates on Winkler–Pasternak elastic foundations", *Wind Struct*., *Int*. *J*., **22**(3), 329-348.

Abdelhak, Z., Hadji, L., Daouadji, T.H. and Bedia, E.A. (2015), "Thermal buckling of functionally graded plates using a n-order four variable refined theory", *Adv*. *Mater*. *Res*., *Int*. *J*., **4**(1), 31-44.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos*. *Struct*., *Int*. *J*., **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J*. *Sandw*. *Struct*. *Mater*., **16**(3), 293-318.

Ait Atmane, H., Tounsi, A. and Bernard, F. (2015a), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int*. *J*. *Mech*. *Mater*. *Des*., 1-14.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015b), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos*. *Struct*., *Int*. *J*., **19**(2), 369-384.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct*. *Eng*. *Mech*., *Int*. *J*., **53**(6), 1143-1165.

Akavci, S.S. (2015), "An efficient shear deformation theory for free vibration of functionally graded thick rectangular plates on elastic foundation", *Compos*. *Struct*., **108**, 667-676.

Akbaş, Ş.D. (2015), "Wave propagation of a functionally graded beam in thermal environments", *Steel Compos*. *Struct*., *Int*. *J*., **19**(6), 1421-1447.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos*. *Struct*., **125**, 621-630.

Arefi, M. (2015), "Elastic solution of a curved beam made of functionally graded materials with different cross sections", *Steel Compos*. *Struct*., *Int*. *J*., **18(3)**, 659-672.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos*. *Struct*., *Int*. *J*., **18**(1), 187-212.

Baferani, A.H., Saidi, A.R. and Jomehzadeh, E. (2011a), "An exact solution for free vibration of thin functionally graded rectangular plates", *Proc*. *Inst*. *Mech*. *Eng*. *Part C*, **225**, 526-536.

Baferani, A.H., Saidi, A.R. and Ehteshami, H. (2011b), "Accurate solution for free vibration analysis of functionally graded thick rectangular plates resting on elastic foundation", *Compos*. *Struct*., **93**(7), 1842-1853.

Bakora, A. and Tounsi, A. (2015)," Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations", *Struct. Eng. Mech., Int. J.*, **56**(1), 85-106.
Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.: Part B*, **60**, 274-283.
Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4). [In press]
Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.
Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.
Benachour, A., Daouadji, H.T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B*, **42**(6), 1386-1394.
Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams" *Steel Compos. Struct., Int. J.*, **19**(3), 521-546.
Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.
Bouchafa, A., Bachir Bouiadjra, M., Houari, M.S.A. and Tounsi, A. (2015), "Thermal stresses and deflections of functionally graded sandwich plates using a new refined hyperbolic shear deformation theory", *Steel Compos. Struct., Int. J.*, **18**(6), 1493-1515.
Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.
Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech., Int. J.*, **58**(3), 397-422.
Bouguenina, O., Belakhdar, K., Tounsi, A. and Adda Bedia, E.A. (2015), "Numerical analysis of FGM plates with variable thickness subjected to thermal buckling", *Steel Compos. Struct., Int. J.*, **19**(3), 679-695.
Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.
Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.
Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.
Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.
Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Computat. Method.*, **11**(6), 1350082.
Chen, C.S., Chen, T.J. and Chien, R.D. (2006), "Nonlinear vibration of initially stressed functionally graded plates", *Thin-Wall. Struct.*, **44**(8), 844-851.
Chen, C., Hsu, C. and Tzou, G. (2009), "Vibration and stability of functionally graded plates based on a higher-order deformation theory", *J. Reinf. Plast. Compos.*, **28**(10), 1215-1234.
Chikh, A., Bakora, A., Heireche, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2016), "Thermo-

mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(4), 617-639.
Croce, L.D. and Venini, P. (2004), "Finite elements for functionally graded Reissner–Mindlin plates", *Comput. Methods Appl. Mech. Eng.*, **193**(9-11), 705-725.
Darılmaz, K. (2015), "Vibration analysis of functionally graded material (FGM) grid systems", *Steel Compos. Struct., Int. J.*, **18**(2), 395-408.
Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos. Struct., Int. J.*, **17**(1), 69-81.
Ebrahimi, F. and Dashti, S. (2015), "Free vibration analysis of a rotating non-uniform functionally graded beam", *Steel Compos. Struct., Int. J.*, **19**(5), 1279-1298.
Ebrahimi, F. and Habibi, S. (2016), "Deflection and vibration analysis of higher-order shear deformable compositionally graded porous plate", *Steel Compos. Struct., Int. J.*, **20**(1), 205-225.
Efraim, E. and Eisenberger, M. (2007), "Exact vibration analysis of variable thickness thick annular isotropic and FGM plates", *J. Sound Vib.*, **299**(4-5), 720-738.
Eltaher, M.A., Khater, M.E., Park, S., Abdel-Rahman, E. and Yavuz, M. (2016), "On the static stability of nonlocal nanobeams using higher-order beam theories", *Adv. Nano Res., Int. J.*, **4**(1), 51-64.
Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**(4), 795-810.
Feldman, E. and Aboudi, J. (1997), "Buckling analysis of functionally graded plates subjected to uniaxial loading", *Compos. Struct.*, **38**(1-4), 29-36.
Hadji, L., Daouadji, T.H., Tounsi, A. and Bedia, E.A. (2014), "A higher order shear deformation theory for static and free vibration of FGM beam", *Steel Compos. Struct., Int. J.*, **16**(5), 507-519.
Hadji, L. and Adda Bedia, E.A. (2015a), "Influence of the porosities on the free vibration of FGM beams", *Wind Struct., Int. J.*, **21**(3), 273-287.
Hadji, L. and Adda Bedia, E.A. (2015b), "Analyse of the behavior of Functionally graded beams based on neutral surface position", *Struct. Eng. Mech., Int. J.*, **55**(4), 703-717.
Hadji, L., Hassaine Daouadji, T., Ait Amar Meziane, M., Tlidji, Y. and Adda Bedia, E.A. (2016), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(2), 315-325.
Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.
Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**(2), 374-383.
Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int. J. Mech. Sci.*, **76**, 102-111.
Jha, D.K., Kant, T. and Singh, R.K. (2013a), "A critical review of recent research on functionally graded plates", *Compos. Struct.*, **96**, 833-849.
Jha, D.K., Kant, T. and Singh, R.K. (2013b), "Free vibration response of functionally graded thick plates with shear and normal deformations effects", *Compos. Struct.*, **96**, 799-823.
Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct., Int. J.*, **18**(3), 693-709.
Kar, V.R. and Panda, S.K. (2016), "Nonlinear thermomechanical deformation behaviour of P-FGM shallow spherical shell panel", *Chinese J. Aeronaut.*, **29**(1), 173-183.
Khalfi, Y., Houari, M.S.A. and Tounsi, A. (2014), "A refined and simple shear deformation theory for thermal buckling of solar functionally graded plates on elastic foundation", *Int. J. Comput. Meth.*, **11**(5), 135007.
Kitipornchai, S., Yang, J. and Liew, K.M. (2006), "Random vibration of the functionally graded laminates in

thermal environments", *Comput. Method. Appl. Mech. Eng.*, **195**(9-12), 1075-1095.

Klouche Djedid, I., Benachour, A., Houari, M.S.A., Tounsi, A. and Ameur, M. (2014), "A *n*-order four variable refined theory for bending and free vibration of functionally graded plates", *Steel Compos. Struct., Int. J.*, **17**(1), 21-46.

Koizumi, M. (1997), "FGM activities in Japan", *Compos. Part B*, **28**(1-2), 1-4.

Laoufi, I., Ameur, A., Zidi, M., Adda Bedia, E.A. and Bousahla, A.A. (2016), "Mechanical and hygrothermal behaviour of functionally graded plates using a hyperbolic shear deformation theory", *Steel Compos. Struct., Int. J.*, **20**(4), 889-912.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct., Int. J.*, **18**(2), 425-442.

Mahdavian, M. (2009), "Buckling analysis of simply-supported functionally graded rectangular plates under non-uniform inplane compressive loading", *J. Solid Mech.*, **1**(3), 213-225.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Modelling*, **39**(9), 2489-2508.

Mantari, J. and Soares, C.G. (2012), "Generalized hybrid quasi-3D shear deformation theory for the static analysis of advanced composite plates", *Compos. Struct.*, **94**(8), 2561-2575.

Mantari, J. and Soares, C.G. (2013), "A novel higher-order shear deformation theory with stretching effect for functionally graded plates", *Compos. Part B*, **45**(1), 268-281.

Matsunaga, H. (2008), "Free vibration and stability of functionally graded plates according to a 2D higher-order deformation theory", *Compos. Struct.*, **82**(4), 499-512.

Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), "A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations", *Struct. Eng. Mech., Int. J.*, **53**(6), 1215-1240.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(3), 793-809.

Merazi, M., Hadji, L., Daouadji, T.H., Tounsi, A. and Adda Bedia, E.A. (2015), "A new hyperbolic shear deformation plate theory for static analysis of FGM plate based on neutral surface position", *Geomech. Eng., Int. J.*, **8**(3), 305-321.

Moradi-Dastjerdi, R. (2016), "Wave propagation in functionally graded composite cylinders reinforced by aggregated carbon nanotube", *Struct. Eng. Mech., Int. J.*, **57**(3), 441-456.

Mouaici, F., Benyoucef, S., Ait Atmane, H. and Tounsi, A. (2016), "Effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory", *Wind Struct., Int. J.*, **22**(4), 429-454.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Cinefra, M., Roque, C.M.C., Jorge, R.M.N. and Soares, C.M.M. (2013), "Static, free vibration and buckling analysis of isotropic and sandwich functionally graded plates using a quasi-3D higher-order shear deformation theory and a meshless technique", *Compos. Part B*, **44**(1), 657-674.

Nguyen, K.T., Thai, T.H. and Vo, T.P. (2015), "A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 91-120.

Ould Larbi, L, Kaci, A., Houari, M.S.A. and Tounsi, A. (2013), "An efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams", *Mech. Based Des. Struct. Mach.*, **41**(4), 421-433.

Pradhan, K.K. and Chakraverty, S. (2015), "Free vibration of functionally graded thin elliptic plates with various edge supports", *Struct. Eng. Mech., Int. J.*, **53**(2), 337-354.

Praveen, G.N. and Reddy, J.N. (1998), "Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates", *Int. J. Solids Struct.*, **35**(33), 4457-4476.

Pradyumna, S. and Bandyopadhyay, J.N. (2008), "Free vibration analysis of functionally graded curved

panels using a higher-order finite element formulation", *J. Sound Vib.*, **318**(1-2), 176-192.
Reddy, J.N. (2000), "Analysis of functionally graded plates", *Int. J. Numer. Methods Eng.*, **47**(1-3), 663-684.
Reddy, J.N. (2011), "A general nonlinear third-order theory of functionally graded plates", *Int. J. Aerosp. Lightweight Struct.*, **1**(1), 1-21.
Saidi, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2013), "Thermo-mechanical bending response with stretching effect of functionally graded sandwich plates using a novel shear deformation theory", *Steel Compos. Struct., Int. J.*, **15**(2), 221-245.
Sallai, B., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2015), "Analytical solution for bending analysis of functionally graded beam", *Steel Compos. Struct., Int. J.*, **19**(4), 829-841.
Tagrara, S.H., Benachour, A., Bachir Bouiadjra, M. and Tounsi, A. (2015), "On bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams", *Steel Compos. Struct., Int. J.*, **19**(5), 1259-1277.
Talha, M. and Singh, B.N. (2010), "Static response and free vibration analysis of FGM plates using higher order shear deformation theory", *Appl. Math. Model.*, **34**(12), 3991-4011.
Tebboune, W., Benrahou, K.H., Houari, M.S.A. and Tounsi, A. (2015), "Thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory", *Steel Compos. Struct., Int. J.*, **18**(2), 443-465.
Thai, H.T. and Choi, D.H. (2014), "Zeroth-order shear deformation theory for functionally graded plates resting on elastic foundation", *Int. J. Mech. Sci.*, **78**, 35-43.
Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.
Trinh, T.H., Nguyen, D.K., Gan, B.S. and Alexandrov, S. (2016), "Post-buckling responses of elastoplastic FGM beams on nonlinear elastic foundation", *Struct. Eng. Mech., Int. J.*, **58**(3), 515-532.
Yang, J., Liew, K.M. and Kitipornchai, S. (2005), "Stochastic analysis of compositionally graded plates with system randomness under static loading", *Int. J. Mech. Sci.*, **47**(10), 1519-1541.
Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech., Int. J.*, **54**(4), 693-710.
Zhao, X., Lee, Y.Y. and Liew, K.M. (2009), "Free vibration analysis of functionally graded plates using the element-free kp-Ritz method", *J. Sound Vib.*, **319**(3-5), 918-939.
Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

CC