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Dynamic compaction of cold die Aluminum powders

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Abstract. In this paper, process of dynamic powder compaction is investigated experimentally using impact of drop hammer and die tube. A series of test is performed using aluminum powder with different grain size. The energy of compaction of powder is determined by measuring height of hammer and the results presented in term of compact density and rupture stress. This paper also presents a mathematical modeling using experimental data and neural network. The purpose of this modeling is to display how the variations of the significant parameters changes with the compact density and rupture stress. The closed-form obtained model shows very good agreement with experimental results and it provides a way of studying and understanding the mechanics of dynamic powder compaction process. In the considered energy level (from 733 to 3580 J), the relative density is varied from 63.89% to 87.41%, 63.93% to 91.52%, 64.15% to 95.11% for powder A, B and C respectively. Also, the maximum rupture stress are obtained for different types of powder and the results shown that the rupture stress increases with increasing energy level and grain size.

Keywords: aluminum powder; compaction; drop hammer machine; impact loading; powder metallurgy

1. Introduction

The powder compaction subjected to both static and dynamic loading has been comprehensively investigated by manufactures (Majzoobi *et al*. 2015a, b, Sukegawa *et al*. 2000, Vivek *et al*. 2014, Vogler *et al*. 2007). In these studies, the effects of grain size, loading conditions, temperature and pressure history on mechanical properties of metal powders were studied. Although, less experimental work has been performed to realize the low velocity compaction of aluminum powder. So, a good comprehension of the effects of loading conditions on the compaction process is missing. In addition to manufacturing matters, the mechanical behavior of aluminum powder subjected to impact loading is important to realize the geological materials behavior for planetary science applications, penetration and seismic coupling (Housen and Holsapple 2003).

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Low velocity compaction is a new manufacturing technique with low cost and more safety (Majzoobi *et al*. 2015a, b). This method includes shock waves propagation through particles of material to persuade quick densification. In manufacturing process, the shock wave is produced by giving the hammer or projectile a specified impact velocity by drop hammer system and hydraulic press subjected to dynamic and static loading respectively (Babaei *et al*. 2015a, c). The primary impact velocity along with the hammer mass results in an energy of compaction that adjusts the dynamic production process. In recent years, high and low velocity impact has been studied both experimentally and numerically, however less works have been perform for low velocity compaction (Khan *et al*. 2013, Yan *et al*. 2013, Yin *et al*. 2013).

Modelling of processes and system identification using input-output data have always attracted many research efforts (Babaei *et al*. 2015b, d). In fact, system identification techniques are applied in many fields in order to model and predict the behaviours of unknown and/or very complex systems based on given input-output data (Natke 2014). Theoretically, in order to model a system, it is required to understand the explicit mathematical input-output relationship precisely. Such explicit mathematical modelling is, however, very difficult and is not readily tractable in poorly understood systems. Alternatively, soft-computing methods (Jamali *et al*. 2014, Porkhial *et al*. 2015), which concern computation in imprecise environment, have gained significant attention. The main components of soft computing, namely, fuzzy-logic, neural network, and genetic algorithm have shown great ability in solving complex non-linear system identification and control problems. Several research efforts have been expended to use evolutionary methods as effective tools for system identification (Iba *et al*. 1993, Kristinsson and Dumont 1992, Porter and Zadeh 1995). Among these methodologies, the Group Method of Data Handling (GMDH) algorithm is self-organizing approach by which gradually more complicated models are generated based on the evaluation of their performances on a set of multi-input-single-output data pairs. The GMDH was firstly developed by (Ivakhnenko 1971) as a multivariate analysis method for complex systems modelling and identification. In this way, GMDH was used to circumvent the difficulty of having a priori knowledge of a mathematical model of the process being considered. The main idea of GMDH is to build an analytical function in a feed forward network based on a quadratic node transfer function (Farlow 1984) whose coefficients are obtained using a regression technique. GMDH, which is an inductive learning method, is similar to neural approach but is bounded in nature. In neural networks, the output of each unit passes through a threshold logic unit which can be linear or nonlinear transfer function such as sigmoid functions. Each unit depends on the state of many other units which creates different level of interactions in such unbounded network structure. The error of output is back-propagated in order to re-tune the connection weights adaptively. However, in inductive GMDH algorithms, a bounded network structure with all combinations of input pairs is trained by scanning the measure of threshold objective function through the optimal connection weights for all inputs-output data pairs. In recent years, the use of such self-organizing network leads to successful application of the GMDH type algorithm in a broad range area in engineering, science, and economics (Darvizeh *et al*. 2003, Madandoust *et al*. 2010, Nariman-Zadeh *et al*. 2003, Nariman-Zadeh and Jamali 2007).

The main purpose of this paper is to characterize the relative density and rupture stress of the aluminium compacts and to realize how they correspond to powder morphology. The experimental procedure and set-up will be explain first, next, the experimental results will be described in detail. In third section, the results are shown that GMDH-type neural network can effectively model and predict the relative density and rupture stress of compacts, each as a function of important input parameters in aluminum powder compaction process. Moreover, it is shown that such GMDH-type

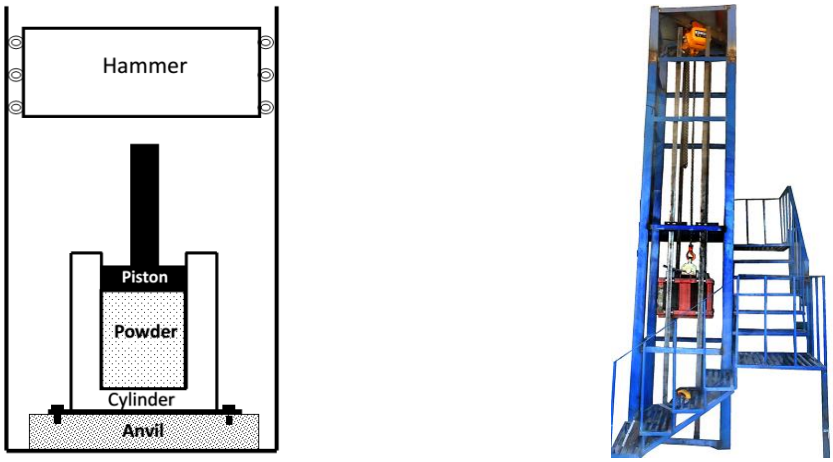

Fig. 1 Schematic and photograph of experimental set-up

neural network modelling leads to a better and simpler polynomial representation of the compaction process when a set of dimensionless parameters is used as input variables rather than the physical input parameters. Consequently, GMDH-type neural networks for modelling the data obtained from low velocity compaction process can be effectively constructed and designed.

2. Experimental study

The drop hammer testing system is used to study the mechanical behavior of aluminum powder compaction subjected to impact loading at low strain rate. Fig. 1 demonstrates photographs of the drop hammer rig.

The device is 4 m long and is guided by four steel tracks with winch system, an electrical magnetic release and a hammer. Also, this device has a box to raise energy level by appending more masses. After dropping hammer from maximum height, the device can attain drop velocity of $8ms^{-1}$. The different components of the system and its performance has been studied extensively in previous researches (Babaei *et al*. 2015a, c).

The compaction mechanism involves the transmission of a pressure pulse from impact loads causing densification of the powder mass. When the hammer was dropped, the hammer potential energy is converted into the piston kinetic energy quickly. By impacting the piston on the surface of powder, the piston kinetic energy is transferred to pressure energy in the powder and the piston is decelerated. The pressure pulse will move to the inside surface of the cylinder wall where most of the energy of the pulse will be reflected back into the cylinder wall as a tensile pulse. The reflected tensile pulse will continue to be reflected from the cylinder surfaces, alternately in compression and in tension, until it is attenuated to zero. Thus, the shock pulse from the impact load will not be transmitted to the powder mass but will, instead, cause the rapid acceleration of the cylinder wall towards its axis. Those powder particles in contact with the cylinder wall will be accelerated inwards, causing impacts with adjacent particles. This will give rise to inter particle shearing, resulting in particles being broken up and oxide layers being ruptured. The cleaned particle surfaces will, thus, be capable of welding together, resulting in a dense, coherent mass of compact. Compaction will be completed when the velocity of the powder is reduced to zero. The

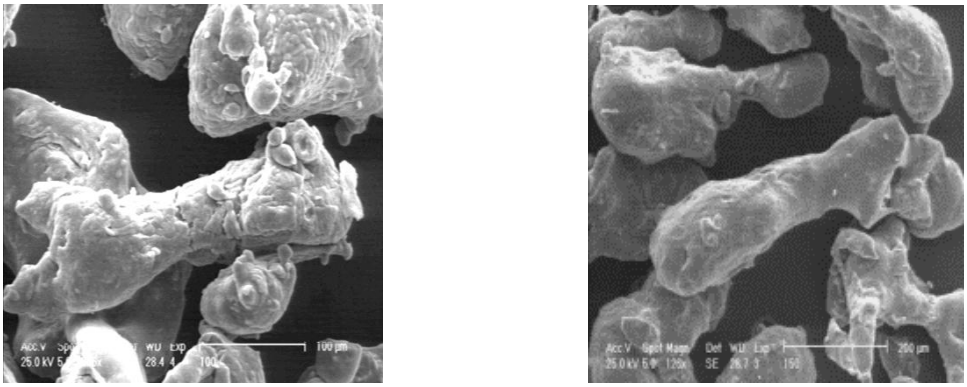

Fig. 2 SEM micrograph of aluminum powder before compaction process

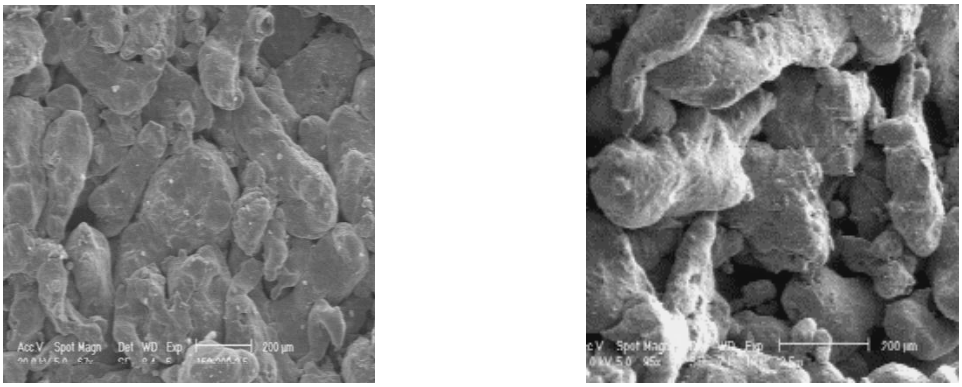

Fig. 3 SEM micrograph of aluminum powder after compaction process

density achieved during this sequence of the events will depend on the initially potential energy, a fully dense compact resulting from the correct amount of energy. Too low a compaction energy results in compacts exhibiting central porosity, while an excess of energy gives rise to melting of the compact center.

To perform the test, the aluminum powder with a grain size less than 250μm that has a purity of 99%, is used. Figs. 2 and 3 show the aluminum powder before and after compaction process which is provided by Scanning Electron Microscopy (SEM). In these figures, the decrease in the empty space among particles which is along with density increase, is totally clear.

To investigate the effect of grain size on the mechanical properties of compacts, particles divided into three types with grain size of 100 μm to 150 μm (Powder A), 150 μm to 200 μm (Powder B) and 200 μm to 250 μm (Powder C). In this procedure, 125 gr of chosen powder is put into the cylinder to be under impact loads. Different energy levels have been created for tested samples by raising the height of the hammer and increasing its mass. The hydraulic jack has been used in order to remove the compacts after compaction.

The obtained experimental results describe amount of compact density and rupture stress subjected to low velocity impact loading. The effects of variation of grain size and loading conditions are considered in these investigations.

Tensile testing is so hard to carry out directly on brittle materials. In these cases, the material

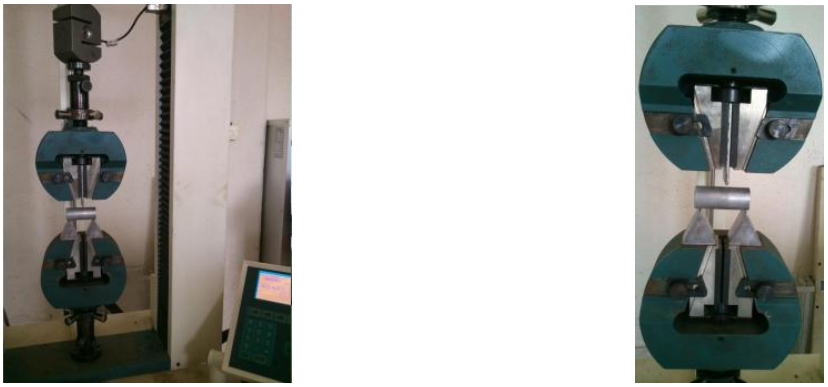

Fig. 4 Photographs of the three point bending tests

Table 1 Summary of experimental results for aluminum powder compacts subjected to impact loading. (ρ_{Al} = 2700 kg/m^3 and σ_{Al} = 120 MPa for all cases)

Compact label	Energy (J)	Compact height (mm)	Compact density (kg/m^3)	Relative density (%)	Rupture force P (N)	Rupture stress σ_{rup} (Mpa)
A-1	733	102.5	1725	63.89	473	1.91
A-2	1100	95.3	1855	68/70	862	3.48
A-3	1467	89.0	1987	73.59	1640	6.63
A-4	1834	86.3	2050	75.95	1820	7.35
A-5	2200	81.9	2160	80	2230	9.01
A-6	2568	80.9	2207	81.74	2540	10.26
A-7	2920	78.4	2257	83.59	2960	11.96
A-8	3580	74.9	2360	87.41	3470	14.02
B-1	733	102.5	1726	63.93	505	2.04
B-2	1100	94.5	1871	69.30	955	3.86
B-3	1467	88.2	2006	74.30	1680	6.79
B-4	1834	84.1	2102	77.85	1880	7.60
B-5	2200	81.0	2184	80.89	2375	9.60
B-6	2568	78.5	2252	83.41	2620	10.59
B-7	2920	75.8	2333	86.41	3010	12.16
B-8	3580	71.6	2471	91.52	3540	14.30

tensile strength can be calculated by carrying out a bending test in which compressive stress develops on one side and corresponding tensile stress develops on the opposite side. If the material is considerably stronger in tension than compression, fracture begins on the compressive side of the part and, therefore, provides the essential data to obtain the tensile strength of material.

In the three point bending test (TPB), the upper bar produces compression stress concentration at the point of contact. As shown in Fig. 4, the specimen (D = 30.5 mm) is located above two simply support beams. The load is imported on the upper surface of specimen through the above bar during the bending test. It is necessary to note that the distance between two beams is L = 45 mm. The results of the experiments on aluminum powders are shown in Table 1.

Table 1 Summary of experimental results for aluminum powder compacts subjected to impact loading. (ρ_{Al} = 2700 kg/m^3 and σ_{Al} = 120 MPa for all cases)

Compact label	Energy (J)	Compact height (mm)	Compact density (kg/m^3)	Relative density (%)	Rupture force P (N)	Rupture stress σ_{rup} (Mpa)
C-1	733	102.1	1732	64.15	520	2.10
C-2	1100	94.4	1873	69.37	732	2.96
C-3	1467	84.7	2089	77.37	1670	6.75
C-4	1834	80.8	2189	81.07	1940	7.84
C-5	2200	78.8	2243	83.07	2490	10.06
C-6	2568	76.7	2307	85.44	2700	10.91
C-7	2920	72.7	2431	90.04	3210	12.97
C-8	3580	68.9	2568	95.11	3750	15.15

* $\sigma_{rup} = \dfrac{8PL}{\pi D^3}$

Fig. 5 Aluminum powder compacts

Figs. 6 and 7 show the compact density and rupture stress versus applied energy respectively. The decrease in porosity space which occurs in the first phase of compaction, causes a decrease in the volume occupied by powder and leads to an increase in density. Also, the results show that the increase in the grain size causes an increase in the rupture stress.

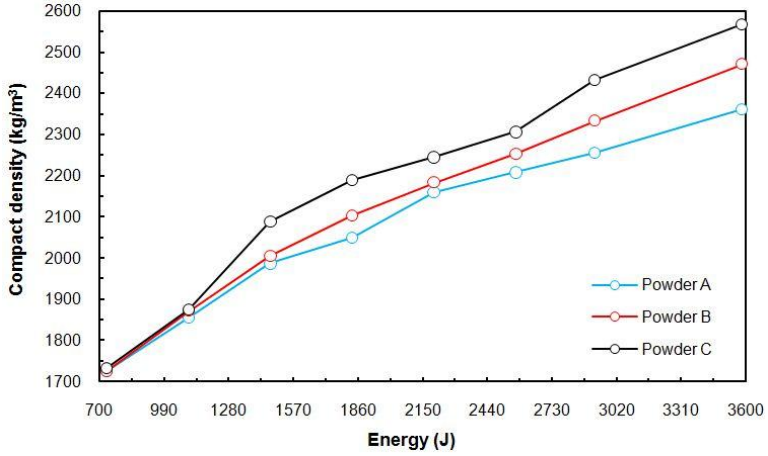

Fig. 6 Compact density versus compaction energy

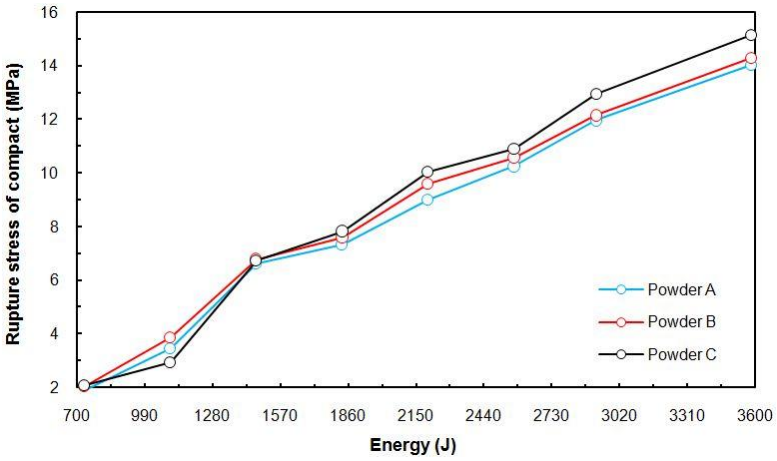

Fig. 7 Rupture stress of compact versus compaction energy

3. Modelling using GMDH-type neural networks

The classical GMDH algorithm can be represented as a set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. Such representation can be used in modelling to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ which can be approximately used instead of actual one, *f*, in order to predict output $\hat{y}$ for a given input vector $X = (x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output *y*. Therefore, given *M* observation of multi-input-single-output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \qquad (i = 1,2,3, \dots, M) \tag{1}$$

It is now possible to train a GMDH-type neural network to predict the output values $\hat{y}_i$ for any given input vector $X_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$, that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \qquad (i = 1,2,3, \dots, M) \tag{2}$$

The problem is now to determine a GMDH-type neural network so that the square of difference between the actual output and the predicted one is minimized, that is

$$\sum_{i=1}^{M} \left[\hat{f}\left(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}\right) - y_i \right]^2 \rightarrow \min \tag{3}$$

General connection between inputs and output variables can be expressed by a complicated polynomial of the form

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} x_i x_j x_k \tag{4}$$

General connection between inputs and output variables can be expressed by a complicated polynomial of the form

$$\hat{y} = G\left(x_i, x_j\right) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^{\,2} + a_5 x_j^{\,2} \tag{5}$$

To predict the output y. The coefficient a_i in Eq. (5) are calculated using regression techniques, so that the difference between actual output, y and the calculated one, $\hat{y}$, for each pair of x_i, x_j as input variables is minimized. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in Eq. (5) whose coefficients are obtained in a least-squares sense. In this way, the coefficients of each quadratic function G_i are obtained to optimally fit the output in the whole set of input-output data pair, that is

$$r^2 = \frac{\sum_{i=1}^{M}\left(y_i - G_i(\ )\right)^2}{\sum_{i=1}^{M} y_i^{\,2}} \tag{6}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of total n input variables are taken in order to construct the regression polynomial in the form of Eq. (5) that best fits the dependent observations ($y_{i,}$ i = 1, 2, ..., M) in a least-squares sense. Consequently, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will be built up in the second layer of the feed forward network from the observations $\{(y_{i,}\, x_{ip},\, x_{iq});\ (i = 1, 2, ..., M)\}$ for different p, $q \in \{i = 1, 2, ..., M\}$. In other words, it is now possible to construct M data triples $\{(y_{i,}\, x_{ip},\, x_{iq});\ (i = 1, 2, ..., M)\}$ from observation using such p, $q \in \{i = 1, 2, ..., M\}$ in the form $\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ \vdots & \vdots & \vdots \\ x_{Mp} & x_{Mq} & y_M \end{bmatrix}$.

Using the quadratic sub-expression in the form of Eq. (5) for each row of M data triples, the following matrix equation can be readily obtained as

$$A\mathbf{a} = \boldsymbol{Y} \tag{7}$$

Where **a** is the vector of unknown coefficients of the quadratic polynomial in Eq. (5).

$$\mathbf{a} = \{a_0, a_1, a_2, a_3, a_4, a_5\} \tag{8}$$

and

$$\boldsymbol{Y} = \{y_1, y_2, y_3, \ldots, y_M\}^T \tag{9}$$

Where $\boldsymbol{Y}$ is the vector of output's value from observation. It can be readily seen that

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^{\,2} & x_{1q}^{\,2} \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^{\,2} & x_{2q}^{\,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^{\,2} & x_{Mq}^{\,2} \end{bmatrix} \tag{10}$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equations in the form of

$$\mathbf{a} = \left(A^T A\right)^{-1} A^T \mathbf{Y} \tag{11}$$

Which determines the vector of the best coefficients of the quadratic Eq. (5) for the whole set of M data triples. However, such solution directly from normal equations is rather susceptible to round off error and, more importantly, to the singularity of these equations.

SVD is the method for solving most linear least squares problems where some singularities may exist in the normal equations. The SVD of a matrix, $A \in \Re^{M\times 6}$, is a factorization of the matrix into the product of three matrices, column-orthogonal matrix $U \in \Re^{M\times 6}$, diagonal matrix $W \in \Re^{M\times 6}$ with non-negative elements (singular values), and orthogonal matrix $V \in \Re^{6\times 6}$ such that

$$A = UWV^T \tag{12}$$

The most popular technique for computing the SVD was originally proposed in (Babaei and Darvizeh 2011, Darvizeh *et al*. 2003, Gharababaei *et al*. 2010, Nariman-Zadeh *et al*. 2002b). The problem of optimal selection of vector of the coefficients in Eqs. (8) and (11) is firstly reduced to finding the modified inversion of diagonal matrix W, in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal **a** are calculated using the following relation.

$$\mathbf{a} = V\left[diag\left(\frac{1}{w_j}\right)\right]U^T \mathbf{Y} \tag{13}$$

Such procedure of SVD approach of finding the optimal coefficients of quadratic polynomials, **a**, improves the performance of self-organizing GMDH type algorithms that is employed to build networks based on input-output observation data triples.

4. Structure identification of GMDH-Type networks

For simultaneous determination of structure and parametric identification of GMDH-type neural networks, the numbers of layers as well as the number of neurons in each layer are determined according to a threshold for error Eq. (6). In addition, unlike two previous approaches, some of input variables or generated neurons in different layers can be included in subsequent layers. The main steps of this approach are described as follows:

Step 1: Set $K = 1$; Set Threshold.

Step 2: Construct $N'_k = \frac{N_k(N_k - 1)}{2}$ neurons according to all possibilities of connection by each pair of neurons in the layer. This can be achieved by forming the quadratic expression $G(x_i, x_j)$ which approximates the output y in Eq. (5) with least-squares errors of Eq. (6) either by solving the normal Eq. (11) or by SVD approach Eq. (13).

Step 3: Select the single best neuron out of these N'_k neurons, x', according to its value of r^2. If (Error < Threshold) Then END; Otherwise Set Vec_of_Var = $\{x_1, x_2, x_3, ..., x_n, x'\}$.

Step 4: Set $N_k = N_k + 1$; goto 2.

5. GMDH-Type neural network modelling and prediction of behavior of Aluminum powder compaction subjected to impact loading

The method described in previous sections is now used to design GMDH-type network systems for a set of dimensionless parameters constructed upon experimental input-output data in a series of compaction tests given in Table 1. Accordingly, the set of output-inputs variables used to train the GMDH-type neural network is a dimensionless set, $\Pi = \{\pi_1, \pi_2, \pi_3, ..., \pi_k\}$, rather than the set of real physical variables $\{y, X\} = \{y, x_1, x_2, x_3, ..., x_n\}$. Hence, given M observation of multi-input-single-output data pairs which have been converted to the equivalent dimensionless parameters (Babaei and Darvizeh 2011, Darvizeh *et al*. 2003, Gharababaei *et al*. 2010).

So that

$$\pi_{1i} = f\left(\pi_{2i}, \pi_{3i}, \pi_{4i}, ..., \pi_{ki}\right)\left(i = 1, 2, 3, ..., M\right) \tag{14}$$

It is now possible to train a GMDH-type neural network to predict the output values $\hat{\pi}_{1i}$ for any given input vector (π_{2i}, π_{3i}, π_{4i}, ..., π_{ki}), that is

$$\hat{\pi}_{1i} = \hat{f}\left(\pi_{2i}, \pi_{3i}, \pi_{4i}, ..., \pi_{ki}\right)\left(i = 1, 2, 3, ..., k\right) \tag{15}$$

The problem is now to determine a GMDH-type neural network so that the square of difference between the actual dimensionless output and the predicted one is minimized, that is

$$\sum_{i=1}^{M}\left[\hat{f}\left(\pi_{2i}, \pi_{3i}, \pi_{4i}, \ldots, \pi_{ki}\right) - \hat{\pi}_{1i}\right]^2 \rightarrow \min \tag{16}$$

Again, the quadratic form of only two variables is used in the form of Eq. (17) to predict the output π_1.

$$\hat{\pi}_1 = G\left(\pi_i, \pi_j\right) = a_0 + a_1\pi_i + a_2\pi_j + a_3\pi_i\pi_j + a_4{\pi_i}^2 + a_5{\pi_j}^2 \tag{17}$$

In order to construct such independent dimensionless parameters in the case of modelling of compact density $\rho_{compact}$ (kg/m^3) and rupture stress of compact σ_{rup} (Pa), height of hammer H (m), compact height h (m), grain size A (m), cylinder diameter D (m), mass of hammer M (kg), mass of powder m (kg), static yield stress of pure aluminum σ_{Al} (Pa) and density of that ρ_{Al} (kg/m^3) have been considered as input parameters in neural network, that is

$$\rho_{compact} = \sigma_{rup} = f\left(H, h, A, D, M, m, \sigma_{Al}, \rho_{Al}\right) \tag{18}$$

From this set of inputs-output parameters, 4 independent dimensionless parameters have been constructed according to 3 main dimensions (M, L, T), as follows

$$\pi_1 = \frac{\rho_{compact}}{\rho_{Al}} \quad \text{(for modelling of relative density)} \tag{19a}$$

$$\pi_1 = \frac{\sigma_{rup}}{\sigma_{Al}} \quad \text{(for modelling of relative rupture stress)} \tag{19b}$$

$$\pi_2 = \frac{h}{A} \tag{20}$$

$$\pi_3 = \frac{H}{D} \tag{21}$$

$$\pi_4 = \frac{M}{m} \tag{22}$$

So that

$$\pi_1 = f\left(\pi_2, \pi_3, \pi_4\right) \tag{23}$$

It should be noted that the simplest possible dimensionless parameters have been considered according to the involved physical parameters.

In order to model, based on experimental data presented in Table 1, the multi-input-single-output set of constructed dimensionless data according to Eqs. (19a)-(22), the method previously mentioned was used separately in conjunction with SVD approach for the coefficient of the quadratic polynomials.

In order to demonstrate the prediction ability of such GMDH-type neural networks in the case of dimensionless modelling, the data have been randomly divided into two different sets, namely, training and testing sets. The training set, which consists of 18 out of 24 inputs-output data pairs, is used for training the GMDH-type neural network models using SVD approach for the coefficients of the quadratic polynomials. The predicting set, which consists of 6 unforeseen inputs-output data samples during the training process, is merely used for predicting to show the prediction ability of such GMDH-type neural networks models during the training process.

Accordingly, Figs. 8 and 9 show the relative density of aluminum powder compacts, using GMDH-type network model constructed with singular value decomposition approach for the coefficients of the quadratic polynomials.

Also, the coefficient of determination (R^2) for training and prediction data are reported in these figures, which show that GMDH-type network model can predict well. The structures of GMDH-type network are depicted in Fig. 10.

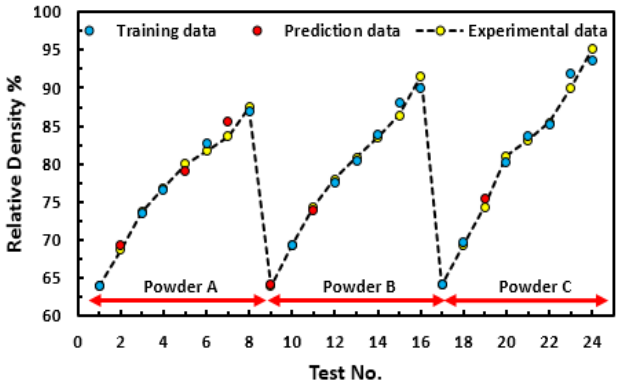

Fig. 8 Variation of compact relative density with input data samples: comparison of experimental values with computed/predicted values

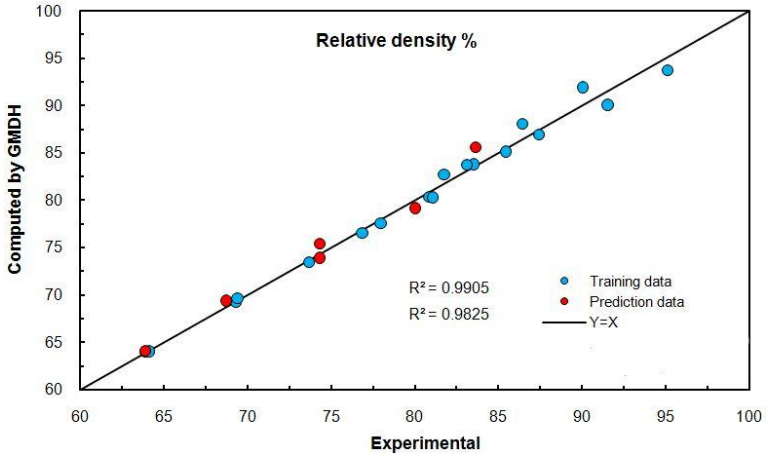

Fig. 9 Comparison of experimental values with computed/predicted values by GMDH-type network

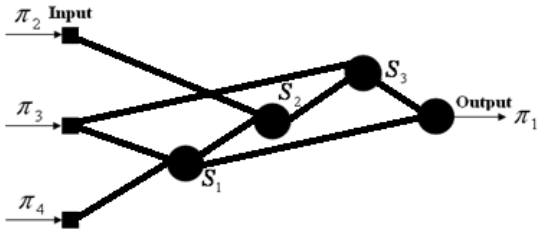

Fig. 10 GMDH-type network for relative density

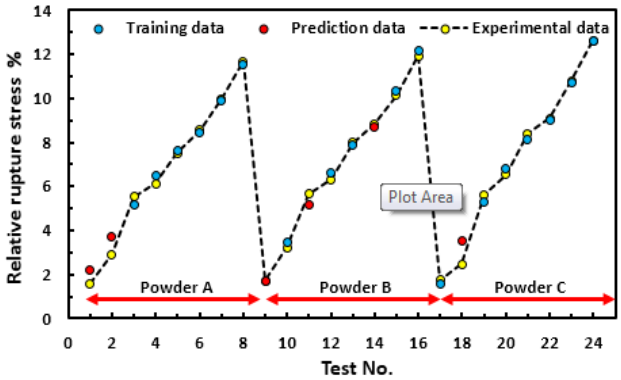

Fig. 11 Variation of compact relative rupture stress with input data samples: comparison of experimental values with computed/predicted values

Consequently, it is now possible to present the obtained polynomial equations for relative density based on the structure of the GMDH-type neural network depicted in Fig. 10 using SVD approach for the coefficient of the quadratic polynomials in the form of

$$S_1 = 0.002286 + 0.3748\pi_3 + 0.1275\pi_4 - 0.0002131\pi_3^2 - 0.00006134\pi_4^2 - 0.0001114\pi_3\pi_4 \qquad (24a)$$

$$S_2 = 0.02573\text{-}0.03385\pi_2 + 1.361S_1 + 0.00003924\pi_2^{\,2} - 0.002104S_1^{\,2} - 0.0002315\pi_2 S_1 \quad (24b)$$

$$S_3 = 0.1030\text{-}1.577\pi_3 + 2.677S_2 - 0.01061\pi_3^{\,2} - 0.03209S_2^{\,2} - 0.04149\pi_3 S_2 \quad (24c)$$

$$\pi_1 = 0.06864\text{+}1.848S_1 - 0.8265S_3 + 0.06219S_1^{\,2} + 0.08517S_3^{\,2} - 0.1477S_1 S_3 \quad (24d)$$

Similarly, Figs. 11 and 12 show the relative rupture stress of aluminum powder compacts subjected to impact loading. Also, the structures of GMDH-type network are depicted in Fig. 13.

The obtained polynomial equations for relative rupture stress based on the structure of the GMDH-type neural network depicted in Fig. 13 are presented in the form of

$$P = 0.000145\text{+}0.0207\pi_3 - 0.001529\pi_4 - 0.00035\pi_3^{\,2} + 0.000004378\pi_4^{\,2} - 0.0002\pi_3\pi_4 \quad (25a)$$

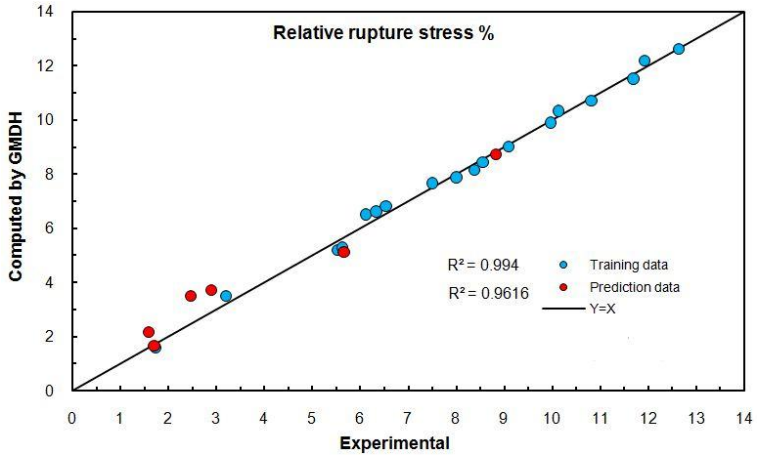

Fig. 12 Comparison of experimental values with computed/predicted values by GMDH-type network

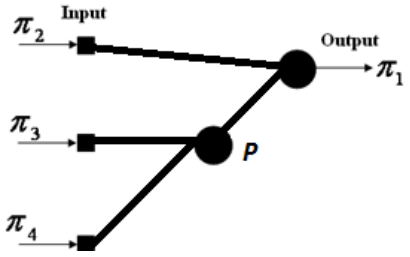

Fig. 13 GMDH-type network for relative rupture stress

Table 2 Values of modelling and prediction error

Data	Training data		Prediction data	
Model	Relative density	Relative rupture stress	Relative density	Relative rupture stress
Error	0.010710072	0.108391391	0.013769102	0.137088282

$$\pi_1 = 0.07141\text{-}0.002512\pi_2 - 1.251P + 0.000003745\pi_2^{\,2} - 0.006039P^2 - 0.0003586\pi_2 P \quad (25b)$$

Also, the comparison results of modelling and prediction error which have been calculated by Eq. (6) are demonstrated in Table 2.

6. Conclusions

The aim of this study was to investigate the behavior of aluminum powder compaction subjected to impact loading. Three various types of powders have been tested by drop hammer system. Also, the three point bending test has been done for each compacts until fracture to calculate the rupture stress. The method for designing GMDH-type networks have been proposed and successfully used for the modelling and prediction of the process parameters of the very complex process of compaction of aluminum powder under low velocity impact loads.

- In the considered energy level (from 733 to 3580 J), the relative density is varied from 63.89% to 87.41%, 63.93% to 91.52%, 64.15% to 95.11% for powder A, B and C respectively.
- There is a linear relation between compaction energy and relative density.
- The maximum rupture stress are obtained for different types of powder and the results shown that the rupture stress increases with increasing energy level and grain size.
- It has been shown that GMDH-type networks provide effective means to model and predict both relative density and rupture stress of compact according to different inputs.
- It has been shown that (SVD) can effectively improve the performance of such GMDH-type networks over the traditional use of normal equations, especially in cases when all inputs-output data pairs have been used for the modelling.

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