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A predicting model for thermal conductivity of high permeability-high strength concrete materials

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Abstract. The high permeability-high strength concrete belongs to the typical of porous materials. It is mainly used in underground engineering for cold area, it can act the role of heat preservation, also to be the bailing and buffer layer. In order to establish a suitable model to predict the thermal conductivity and directly applied for engineering, according to the structure characteristics, the thermal conductivity predicting model was built by resistance network model of parallel three-phase medium. For the selected geometric and physical cell model, the thermal conductivity forecast model can be set up with aggregate particle size and mixture ratio directly. Comparing with the experimental data and classic model, the prediction model could reflect the mixture ratio intuitively. When the experimental and calculating data are contrasted, the value of experiment is slightly higher than predicting, and the average relative error is about 6.6%. If the material can be used in underground engineering instead by the commonly insulation material, it can achieve the basic requirements to be the heat insulation material as well.

Keywords: high permeability; high strength concrete; porous materials; thermal resistance; heat conductivity; cold high altitudes

1. Introduction

There are widely distributed of the permafrost and seasonal frozen soil are widely distributed at high altitude. A study has discussed that the Qinghai Tibet Plateau in China has a most widely and thickness permafrost distributing, also the temperature is the lowest all of China. The annual average temperature in these areas were –2°C to –6°C (Luo *et al.* 2012). In all kinds of geotechnical engineering, either in constructing or operating period, the temperature must be strictly controlled, especially for the residency underground engineering. Unless to do so, the internal temperature can meet to the requirements for inhabiting, also the frozen soils (rock) can be ensured in a station of stabilization when the temperature was changed. Kwon *et al.* (2009), Wei *et al.* (2011) and Chen *et al.* (2014) considered that if the high permeability-high strength concrete material was chosen to be the filling material which was played a role of bailing and absorption of vibrating in underground space, the key parameter to measure the heat-insulating property was the effective coefficient of heat conductivity. So it is important to choose a suitable predicting model which can meet its own character to calculate the thermal conductivity. So far as to get the direct relationship

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type whichc can reflect the mixture and the effective coefficient of heat conductivity directly, it umst be a significance trying in theorizing and practicing.

At present, colleagues have developed the thermal conductivity of porous materials deeply, and they have also got so much excellent results. For instance, Bozomolov and Chudnovsky (1941) built a theory equation to describe the thermal conductivity for porous materials. This model was built from the bulk factor for solid phase and fluid phase. Kunni and Smith (1960) had gotten a mathematical model to predict the thermal conductivity of porous rocks through the total parameters theory. But the model had lost the sight of the impact with porosity, so it was limited to be using in practical application.

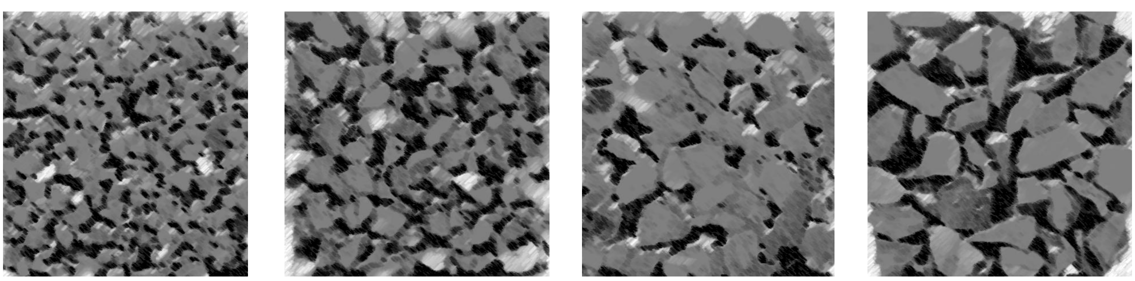
After that, through the solid-gas grain feature of porous materials Liu *et al*. (2010) has done some experiments. After the porosity was introduced, the Kunni and Smith model was amendment, and the model can be used well like some classical formulas. Park (Park and James 1992, Park and Hartley1999) had assumed that the solid particles can be simplified into a cylinder with spherical cavity, so the thermal conductivity of different particles was obtained. Florez *et al*. (2013) had simplified the calculation method also, while the particles diametric should be in 20~50 μm. So the sintered porous materials' thermal conductivity calculation model was built, where the model could contain the influence factors for sintering shell. Other counterparts (Kan *et al*. 2012, Go *et al*. 2014) also used the fractal theory and artificial neural network theory to calculate thermal conductivity where the method can describe the micro scale spatial structure of porous media.

2. The structure composition with high permeability-high strength concrete

The structure of high permeability-high strength concrete material is cemented the uniform size aggregates by gelatinization materials, which are composed of cement, polyacrylate and water. The porous material is a structure of frame-ventage work, which is named skeleton pore structure in engineering.

When the material is cut by the cutting machine, the specimen slice maps have been obtained, then the real slices of high permeability-high strength concrete are shown in Fig. 1.

The effective thermal conductivity of porous materials is contacted with porosity closely (Pabst and Gregorová 2014, Li *et al*. 2001). According to the prior experiment, a phenomenon was found that the porosity of the high permeability-high strength concrete was determined by the size of aggregate and thickness of polymer slurry which were packed around the uniform size aggregates. When the material was produced by the way of non-vibration or plane-vibration, the strength was



(a) Particle size = 10 mm (b) Particle size = 15 mm (c) Particle size = 20 mm (d) Particle size = 25 mm

Fig. 1 The real slices forhigh permeability-high strength concrete of different particle sizes

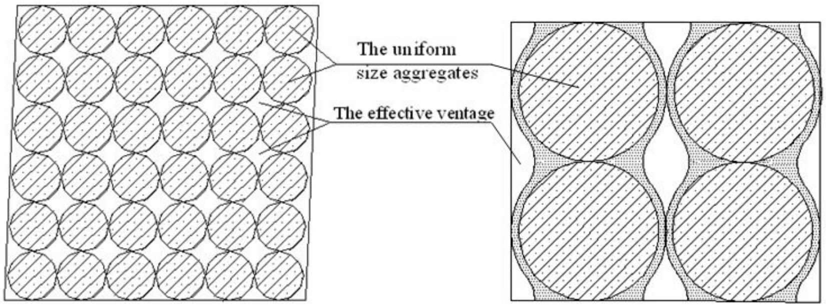


Fig. 2 The simplified skeleton model ofhigh permeability-high strength

mainly supplied by the force of friction and bond strength. Then from the character of structure and stress, the perfect frame must be simplified in Fig. 2 below.

On the match designing stage, the complete porosity n_0 of high permeability-high strength concrete can be obtained by the following formula

$$n_0 = 1 - \frac{\rho_s}{\rho_t} \tag{1}$$

where n_0 is the complete porosity (%), ρ_s is the bulk density (g/cm^3), ρ_t is the theoretical density (g/cm^3).

From the documentations (Zheng *et al*. 2007), the cement can react with the water, where about 25% of the mass will be took into total mass for cement materials. Then the chemistry bound moisture would shrink after reacting. After the volume shrinkage, the volume accounted for only 3/4 of the original free water. Except for bound moisture, the other free water only pays the role of helping the hydrating of cement and evaporating into aerosphere. So the theoretical density of the high permeability-high strength concrete can be calculated by the formula below.

$$\rho_t = \frac{100 + P_c + P_c \times (1/4)}{(100/\rho_c) + (P_c/\rho_g) + P_c \times (1/4) \times (3/4)} \times \rho_w \tag{2}$$

Where, P_c is the mass ratio between cement and aggregate, ρ_c is the relative density of cement (g/cm^3), ρ_g is the apparent density (g/cm^3), ρ_w is the density of water (g/cm^3).

When the researching of lightweight aggregate porous and vegetation concrete by Guan (2014), the space occupied of volume for solid material was amended. This method could be considered by the space randomness and confectioning way. Then the theoretical and geometrical porosity can be expressed when the compensation factor was lead into the simplified skeleton model of high permeability-high strength in Fig. 2.

$$n_0 = 1 - \frac{\pi}{6\alpha}\left(1 + \frac{\delta}{r}\right) \tag{3}$$

Where α is the coefficient of correction which is a parameter related to the space randomness and confectioning way, it has an experimental span between 0.85 and 0.95. The parameter δ is expressed of the pulp thickness.

Combining with equations with Eqs. (1), (2) and (3), the paste coated thickness of the aggregate δ can be expressed like this

$$\delta=(\chi-1)r \tag{4}$$

3. Mathematical model

When the unit body of the high permeability-high strength concrete is desiccation, the material is composed by spheric aggregate, polymer slurry and air. Due to the calculating model is a symmetry structure, it can be sampled for 1/4 to be analyzed. By the way, before calculating it should be assumed below: (1) The heat transfer model can be simplified for one dimension stationary state. (2) There is no air convection among the skeletons. (3) Ignored the effect of radialization. (4) Heat transfer is only occurred along the direction of $-y$ axis.

So the unit calculating model should be established in Fig. 3, where R_s, R_c and R_a are the thermo-resistance of aggregate, polymer slurry and air. According to the knowledge of heat transfer theory, the total thermo-resistance can be calculated by the parallel network for each unit thermo-resistance (Gong 1999) like Fig. 4. Then the total thermo-resistance can be expressed by

$$\frac{1}{R}=\frac{1}{R_s}+\frac{1}{R_c}+\frac{1}{R_a} \tag{5}$$

Assuming λ is the thermal conductivity of one calculating element, the total thermo- resistance can also be expressed by

$$R=\frac{r+\delta}{A\lambda}=\frac{1}{r\lambda} \tag{6}$$

Then take the infinitesimal part dy to analysis, the differential format of the total thermo-resistance can be expressed by

$$dR=\frac{dy}{\lambda_s A_s(y)+\lambda_c A_c(y)+\lambda_a A_a(y)} \tag{7}$$

From the unit calculating model, the configuration equation of the 1/4 spheric aggregate can be expressed by

$$x^2+y^2=r^2,\quad 0\le y\le r,\quad 0\le x\le r \tag{8}$$

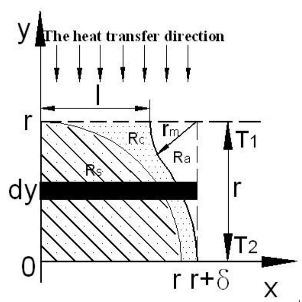


Fig. 3 The unit calculating model

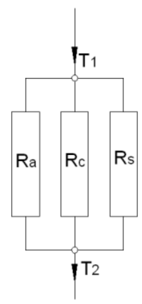


Fig. 4 The network of schematic diagram

Yovanovich *et al*. (1978) and Florez *et al*. (2013) have thought that the surface energy can be formatted among spheric solid particles. When they have the same radius, the equation could be obtained by

$$r_m = \frac{1}{2}\frac{l^2}{r-l} \tag{9}$$

From the known geometrical relationship $l + r_m = r + \delta$, combined with Eqs. (4) and (9), then the equation can be changed for

$$l = r\left(\chi + 1 - \sqrt{\chi^2 + 1}\right) \tag{10}$$

Taking $t = r - r_m \sin\theta$, the 1/4 spheric aggregate outer pulp geometric equation can be expressed by

$$\begin{cases} x^2 + y^2 = \left(r+\delta\right)^2, & 0 \le y \le t \\ \left(x-r-\delta\right)^2 + \left(y-r\right)^2 = r_m^2, & t \le y \le r \end{cases} \tag{11}$$

So, the equation for the area of cross-section about the aggregate, polymer pulp and air in pore space A_s, A_c and A_a can be expressed by

$$A_s\left(y\right) = \pi\left(r^2 - y^2\right) \tag{12}$$

$$\begin{cases} A_{c1}\left(y\right) = \pi\left(r+\delta\right)^2 - \pi r^2, & 0 \le y \le t \\ A_{c2}\left(y\right) = \pi\left[\sqrt{r_m^2 - \left(y-r\right)^2} + l + r_m\right]^2 - \pi r^2, & t \le y \le r \end{cases} \tag{13}$$

$$\begin{cases} A_{a1}\left(y\right) = \left(4-\pi\right)\left(r+\delta\right)^2 + \pi y^2, & 0 \le y \le t \\ A_{a2}\left(y\right) = 4\left(r+\delta\right)^2 + \pi y^2 - \pi\left[\sqrt{r_m^2 - \left(y-r\right)^2} + l + r_m\right]^2, & t \le y \le r \end{cases} \tag{14}$$

Taking the Eqs. (4), (12), (13) and (14) into Eq. (7), and integrate the equation, the simplified expression of total thermo-resistance for unit body can be written by

$$\begin{aligned} R &= \int_0^t \frac{1}{\lambda_s A_s\left(y\right) + \lambda_c A_{c1}\left(y\right) + \lambda_a A_{a1}\left(y\right)} dy + \int_t^r \frac{1}{\lambda_s A_s\left(y\right) + \lambda_c A_{c2}\left(y\right) + \lambda_a A_{a2}\left(y\right)} dy \\ &= -\frac{1}{2\pi\sqrt{AB}} \ln\left|\frac{\sqrt{A}t - \sqrt{B}}{\sqrt{A}t + \sqrt{B}}\right| + \frac{1}{\pi F} \ln\left|\frac{\left(2Cr + D - F\right)\left(2Ct + D + F\right)}{\left(2Cr + D + F\right)\left(2Ct + D - F\right)}\right| \end{aligned} \tag{15}$$

The expressions and codes in upper Eq. (15) can be seen in Table 1.

Take the Eq. (15) into Eq. (6), the effective coefficient of heat conductivity λ for the high permeability-high strength concrete could be expressed by

$$\frac{1}{\lambda} = -\frac{r}{2\pi\sqrt{AB}} \ln\left|\frac{\sqrt{A}t - \sqrt{B}}{\sqrt{A}t + \sqrt{B}}\right| + \frac{1}{\pi F} \ln\left|\frac{\left(2Cr + D - F\right)\left(2Ct + D + F\right)}{\left(2Cr + D + F\right)\left(2Ct + D - F\right)}\right| \tag{16}$$

Table 1 The table of expressions and codes

Codes	Expressions	Codes	Expressions
A	$\lambda_s - \lambda_a$	D	$2r(\lambda_c - \lambda_a)$
B	$r^2 \{\lambda_s - \lambda_c + \chi^2 [(-1+4/\pi)\lambda_a - \lambda_c]\}$	E	$(\lambda_c - \lambda_a)[r^2 + (l + 2r_m)^2] + r^2 [\lambda_s - \lambda_c + 4\chi^2\lambda_a/\pi]$
C	$2\lambda_a - \lambda_s - \lambda_c$	F	$\sqrt{D^2 + 4CE}$

4. The verification and discussion of thermal conductivity for high permeability- high strength concrete materials

In order to analyze that formula with the thermal conductivity, the grush aggregate should be chosen for non-contamination, hardness and roughness to meet the requirements of the mixture ratio design for high permeability-high strength concrete materials high permeability-high strength concrete materials. So the uniform size aggregate was prepared on four styles of 10 mm, 15 mm, 20 mm and 25 mm. The cement was chosen in Portland cement of grade 42.5, the dust-flue ratio should be allocated of 1/8, 1/7, 1/6 and 1/5. Then the table of mixture ratios can be listed in Table 3 below.

From the engineering and thermal physics, the thermal conductivity has close connection with temperature, and different temperature has different thermal conductivity when the materials in the environment of high temperature. But in this article, the high permeability-high strength concrete materials was used in room temperature, the influence of temperature could be ignored. So some calculated parameters were chosen in the status of 20°C. Due to applying in engineering of directly and compare to other heat insulation materials, the calculating parameters were collected by Zhu (1999) who studied in the temperature stress for mass concrete and temperature controlling. The materials thermal conductivity selections were listed in Table 2.

When the parameters in Tables 2 and 3 were taken into Equation (16), the thermal conductivity of high permeability-high strength concrete materials in different grain sizes and fust-flue ratios could be obtained. After calculating, in order to prove the results, the specimen in a size of 250 mm × 250 mm × 25 mm was poured in different mixture ratios, and then took the specimen on the plate thermal conductivity apparatus of SK-DR300A+ type to get the true value of the thermal conductivity for all specimens. Before the determination, the temperature of the cold plate is set on 15°C, the temperature of the hot plate is set on 35°C. When the temperature of the specimen is stable enough for a long time, the instrument automatically could stop running and output the testing results. Also, the parameters were taken into the Kunni and Smith model (Kunii and Smith 1960) to calculate the effective thermal conductivity equation, and it could be expressed by

$$k_e = k_f \cdot \left(\frac{2}{1 - k_f / k_s} \right) \cdot \left[\frac{\ln\left(k_s / k_f\right)}{1 - k_f / k_s} - 1 \right] \tag{17}$$

Table 2 The materials selection

Materials name	Thermal conductivity [W/(m.K)]
Granite aggregate	2.92
Polymer pulp	1.23
Air (room temperature)	0.0258

Where the thermal conductivity of fluid is k_f, k_s is the thermal conductivity of solid particle.

That's all, the calculated results of the Kunni and Smith model and predicting model can be seen in Table 3 below.

From the Table 3, the coefficient of thermal conductivity value with predicting and experimenting were equivalence, that is displayed the high permeability-high strength concrete materials have the same capacity to other commonly insulating materials which are using in engineering widely. The thermal conductivity is only 1/10~1/3 of ordinary concrete material, that is to say, the high permeability-high strength concrete materials can achieve the basic requirements as normal temperature insulation materials. Then the curves that could compare with the thermal conductivity by experimental and theory calculating on different particle sizes and cement aggregate ratios could be drawn in Fig. 5.

From Fig. 5, it is found that the thermal conductivity of high permeability-high strength concrete materials can be influenced obviously by the grain size of aggregate and fust-flue ratios. All the way, in a certain range, the greater the grain size of aggregate is, the lower of the thermal conductivity, the smaller of fust-flue ratios, the acker of porosity and the thicker of the pulp are, the higher of the thermal conductivity is. But comparing with predicting value and experiment value, the value of experiment is slightly higher than predicting, and the average relative error is about 6.6%.The thermal conductivity increased may be resulted by the pulp filled of partialaperture gap when the specimen was confectioning.

Otherwise, it is found that the predicting curve and experiment value have the same change trend when comparing with the two curve-banks, and also the results could meet each other. So it

Table 3 The mixture ratio and thermal conductivity calculating results

Aggregate (kg)	Fust-flue ratios	Actual density (kg/m^3)	Thermal conductivity [W/(m.K)]			Relative error (%)
			Kunni and Smith model	Experiment value	Predicting model	
1523 (10 mm)	1:8	1712.6	0.1963	0.1536	0.1176	23.4
	1:7	1745.2	0.1963	0.1728	0.1426	17.5
	1:6	1798.5	0.1963	0.1899	0.1892	0.37
	1:5	1813.3	0.1963	0.2573	0.2609	1.38
1517 (15 mm)	1:8	1683.0	0.1963	0.1790	0.1720	4.07
	1:7	1683.0	0.1963	0.2378	0.2080	14.3
	1:6	1783.7	0.1963	0.2777	0.2751	0.94
	1:5	1875.6	0.1963	0.3523	0.3775	6.68
1473 (20 mm)	1:8	1647.4	0.1963	0.2423	0.2235	8.41
	1:7	1783.7	0.1963	0.2754	0.2698	2.08
	1:6	1822.2	0.1963	0.3412	0.3556	4.05
	1:5	1869.6	0.1963	0.4336	0.4555	4.81
1458 (25 mm)	1:8	1641.5	0.1963	0.2822	0.2723	3.64
	1:7	1765.9	0.1963	0.3319	0.3281	1.16
	1:6	1780.7	0.1963	0.4115	0.4309	4.50
	1:5	1914.1	0.1963	0.5096	0.5556	8.28

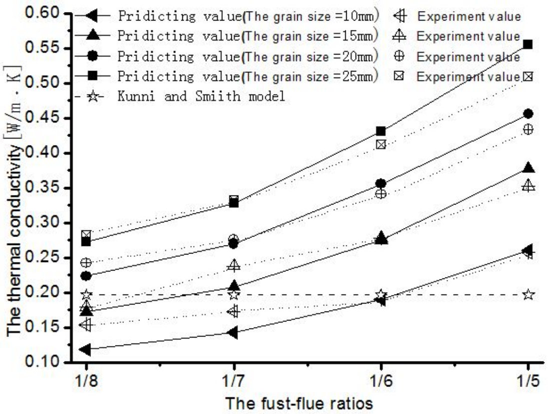


Fig. 5 Comparing with the thermal conductivity by experimental and theory calculating on different particle sizes and cement aggregate ratios

proves that the predicting model in this article has a great degree of belief. While the Kunni and Smith model neglected the influenced of porosity changing, the calculating value can't expound the effect of thermal conductivity by change of grain size of aggregate and fust-flue ratios. So the Kunni and Smith model is inapplicable in using for high permeability-high strength concrete materials, it is inaccuracy in using.

5. Conclusions

In order to establish a suitable model to forecast the thermal conductivity and directly applied for engineering, the thermal conductivity predicting model was built by resistance network model of parallel three-phase medium by the structure characteristics of this material. Through the experiment, it is proved that the thermal conductivity predicting model could be expressed by grain size of aggregate and fust-flue ratios directly. Comparing with the experimental data and Kunni and Smith model, it can be shown that the prediction model can reflect the mixture ratio intuitively. Then the experimental and calculating data are near equivalent, the value of experiment is slightly higher than predicting, and the average relative error is about 6.6%. If the material can be used in underground engineering instead by the commonly insulation material, it can achieve the basic requirements to be the heat insulation material as well.

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