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Estimating model parameters of rockfill materials based on genetic algorithm and strain measurements

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Abstract. The hyperbolic stress–strain model has been shown to be valid for modeling nonlinear stress–strain behavior for rockfill materials. The Duncan-Chang nonlinear constitutive model was adopted to characterize the behavior of the modeled rockfill materials in this study. Accurately estimating the model parameters of rockfill materials is a key problem for simulating dam deformations during both the dam construction period and the dam operation period. In order to estimate model parameters, triaxial compression experiments of rockfill materials were performed. Based on a genetic algorithm, the constitutive model parameters of the rockfill material were determined from the triaxial compression experimental data. The investigation results show that the predicted strains provide satisfactory precision when compared with the observed strains and the strains forecasted by a gradient-based optimization algorithm. The effectiveness of the proposed inversion procedure of model parameters was verified by experimental investigation in a laboratory.

Keywords: parameter estimation; rockfill materials; genetic algorithm; strain measurement; triaxial compression experiment; Duncan-Chang model

1. Introduction

Rockfill dams are constructed using mostly quarried rockfill materials obtained by blasting rock and have been increasingly used for purposes such as irrigation, power generation, and flood control. Nonlinear constitutive models are often adopted to characterize the deformation behavior of the rockfill materials. Accurately estimating the model parameters of rockfill materials is a significant problem of interest for scientific investigators and engineers. The inverse problem of parameter estimation is often ill-posed. The ill-posed quality of the inverse problem can be characterized by the nonuniqueness and instability of the identified parameters. Avril *et al*. (2008a) classified constitutive parameter identification procedures as the finite element model updating method, the constitutive equation gap method, the virtual fields method, the equilibrium gap method, and the reciprocity gap method. In the most intuitive approach, it consists of performing iteratively finite element simulations of the test to find constitutive parameters that achieve the

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best match between the computed and actual measurements. Avril and Pierron (2007) reviewed several approaches available in the literature for identifying the constitutive parameters of linear elastic materials from full-field measurements. Avril *et al*. (2008b) dealt with the identification of elastoplastic constitutive parameters from deformation fields measured over the surface of thin flat specimens with the grid method. The approach for recovering the constitutive parameters was the virtual fields method. Varadarajan *et al*. (2003) performed test investigations and proposed an estimation procedure for the model parameters of rockfill materials. The constitutive relation of rockfill materials is crucial for the formulation of the finite element method mode. For most rockfill materials, the strain–stress relation has characteristics of hyperbolic form. The commonly used constitutive models in rockfill materials include the Duncan–Chang model and the elastoplastic model. Because of its simplicity and easy implementation in practice, the Duncan–Chang model has been widely used in FEM applications (Duncan and Chang 1970). Fahey and Carter (1993) performed a finite element study of the pressuremeter test in sand using a nonlinear elastic plastic model. Varadarajan *et al*. (2006) developed the relationships between the material constants of the modeled rockfill materials and the characteristics of the particles; these were used to predict the material constants for the prototype-size rockfill materials. A series of isotropically consolidated–drained and consolidated–undrained triaxial tests were performed on freshly deposited silt and clayey silts to provide guidance for selecting the hyperbolic parameters for these materials (Timothy *et al*. 1994). Because observation errors exist in measuring data, the local optimization problem is induced in solving the inverse problem of model parameter identification. Traditional gradient-based optimization algorithms cannot solve this problem. A genetic algorithm (GA) is an optimization procedure inspired by genetics and evolution. These algorithms may incorporate stochastic mutation, exchange of information between members of a population of solutions, and selection of possible solutions in subsequent generations based on relative fitness. Although GA optimizations cannot be guaranteed to find the global optimum, they are effective techniques for finding good solutions to high-dimensional problems with difficult search spaces consisting of many local optima. GAs have been widely used in inverse analysis (Cropper *et al*. 2012), parameter estimation (Harrouni *et al*. 1996), the estimation of thermophysical properties (Adili *et al*. 2010), nonlinear least-squares estimates (Mitra and Mitra 2012), and modeling for aluminum stress–strain prediction (Yang *et al*. 1996). Suched *et al*. (2013) estimated geotechnical parameters from pressuremeter tests for the MRT Blue Line extension in Bangkok. Bagheripour *et al*. (2011) developed the Hoek–Brown failure criterion for strength prediction in anisotropic rock, and Silvestri and Abou-Samra (2009) presented an analytical solution procedure of the stress–strain relationship of modified Cam clay in undrained shear. The aim of this paper is to propose an inverse procedure for estimating the model parameters of rockfill materials based on a GA and strain measurements in triaxial compression experiments of rockfill materials; to deal with the local minima problem in the inverse problem of parameter identification; and to increase the prediction accuracy of computing models in numerical simulation.

2. Constitutive model and triaxial compression test of rockfill materials

The constitutive model developed by Desai (2001) based on the disturbed state concept has been adopted to characterize the behavior of rockfill materials in the present study. This model allows for a number of factors such as irreversible plastic strains, plastic hardening, and nonassociative aspects. The disturbance referred to in this model includes microcracking, decay, and degradation observed in many natural systems. The degradation of rockfill materials due to the breakage of particles has been used to formulate the disturbance function by Varadarajan *et al*. (2006) in depicting the behavior of two rockfill materials using the model. The hyperbolic relation between stress and strain developed by Duncan and Chang (1970) is defined in terms of an initial modulus, the material shear strength, and a reduction factor R_f. The reduction factor defines the deviatoric stress on the hyperbolic curve. The initial modulus is defined by the following equation (Duncan and Chang 1970)

$$E_i = kP_a(\frac{\sigma_3}{P_a})^n \tag{1}$$

where k is the modulus number; n is the modulus exponent; σ_3 is effective confining pressure; and p_a is atmospheric pressure. The tangent modulus E_t for a given iteration is accordingly defined as

$$E_t = kP_a(\frac{\sigma_3}{P_a})^n[1-\frac{R_f\sigma_d(1-\sin\varphi)}{2c\cos\varphi+2\sigma_3\sin\varphi}]^2 \tag{2}$$

where σ_d is the principal stress difference between axial pressure and confining pressure; $\sigma_d = \sigma_1 - \sigma_3$; and σ_1 is axial pressure. If the Mohr–Coulomb failure envelope is nonlinear, the following expression (Duncan and Chang 1970) can be used to model the variation of φ with σ_3

$$\varphi = \varphi_0 - \Delta\varphi \lg(\frac{\sigma_3}{p_a}) \tag{3}$$

where φ_0 and $\Delta\varphi$ are constants. When the E-B model is applied, the tangent bulk modulus is calculated by

$$B_t = k_b P_a(\frac{\sigma_3}{P_a})^m \tag{4}$$

where B_t is the tangent bulk modulus and k_b and m are the coefficient and exponent of the tangent bulk modulus, respectively. According to the relationship between the Poisson's ratio and

Fig. 1 Rockfill material

voluminal modulus and the relationship between the Poisson's ratio and elastic modulus, the tangent Poisson's ratio can be calculated after the voluminal modulus and elastic modulus are known.

$$\mu_t = \frac{1}{6}\frac{3B_t - E_t}{B_t} \tag{5}$$

where μ_t is the tangent Poisson's ratio. The rockfill material, as shown in Fig. 1, was used as the cushion zone of the Pushihe concrete-face rockfill dam. The largest size of the rockfill particles was 100 mm, as shown in Fig. 2; the smallest size was 0.1 mm. Triaxial compression tests of the

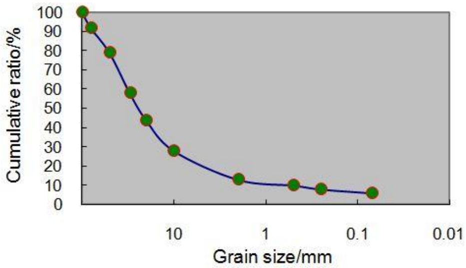

Fig. 2 Particle size distribution for rockfill material

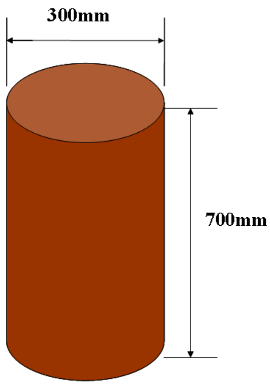

Fig. 3 Triaxial compression tests of rockfill material

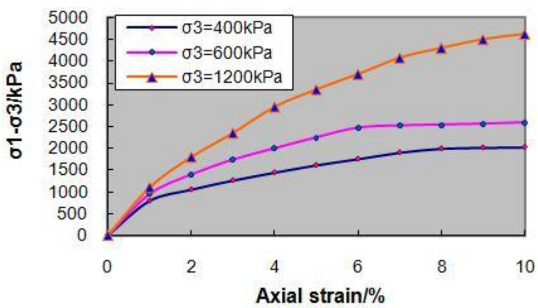

Fig. 4 Variation of principal stress difference versus axial strain in triaxial compression tests of rockfill material

rockfill material were performed in a laboratory. The diameter of the test model was 300 mm, and the height was 700 mm, as shown in Fig. 3. The typical variation curves of the principal stress difference versus axial strain in the triaxial compression tests of the rockfill material are shown in Fig. 4.

3. Identification procedures of model parameters of rockfill material using GA

Parameter identification estimates model parameters by calculating the difference between the observed strain and computed strain minimization in triaxial compression tests of rockfill materials. The output residual method has hence been developed as an alternative approach. It consists of minimizing the objective function of the form (Adili *et al*. 2010)

$$\min J(\mathrm{p}) = \left\| \varepsilon_m - \varepsilon_c(p) \right\|_2^2 \tag{6}$$

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$$p = [k, n, R_f, c, \varphi, \Delta\varphi, k_b, m]^T \tag{7}$$

There are two kinds of solving procedures for estimating model parameters. One kind of approach is a gradient-based optimal search algorithm, such as the BFGS optimization method, the Gauss–Newton (G–N) algorithm (Yeh 1981), and the Powell optimization method, which have fast computational efficiency and do not ensure global optimization. Other kinds of approaches, such as GA, the ant colony algorithm (Abbaspour 2001), the simulated annealing algorithm (Li and Liu 2008), and the Tabu search algorithm, are able to find global optimization. Because there are observation errors during experiments, the objective function of parameter inversion is non-convex, as shown in Fig. 5.

The G–N algorithm has proven to be an effective algorithm to perform minimization in gradient-based optimal search algorithms. The popularity of this algorithm stems from the fact that it does not require calculation of the Hessian matrix, as is required by the Newton method, and the rate of convergence is superior when compared to traditional gradient searching procedures. The

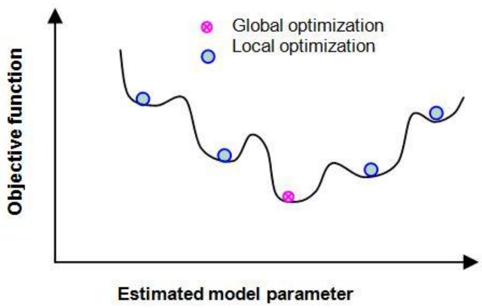

Fig. 5 Non-convex characteristics of objective function of parameter inversion

G–N algorithm generates the following parameter sequence for an unconstrained minimization problem (Yeh 1986).

$$p^{i+1} = p^i - \rho^i d^i \tag{8}$$

where

$$A^i d^i = g^i \tag{9}$$

$$A^i = [J_J(p^i)]^T [J_J(p^i)] \tag{10}$$

$$g^i = [J(p^i)]^T e^i \tag{11}$$

$$e^i = [\varepsilon_c(p) - \varepsilon_m] \tag{12}$$

where e is the error vector; J_J is the Jacobian matrix of strain with respect to model parameter p; ρ^i is step size; d^i is the G–N direction vector; and i denotes the number of iterations. The estimated model parameters based on the G–N optimization algorithm are listed in Table 3.

Because some local optimizations exist for the objective function, some gradient-based optimal search algorithms cannot deal with the global optimization problem, as shown in Fig. 5. Therefore, a GA is used to solve the inverse problem of parameter identification. A simple GA works on the chromosomes of the population and not on the decision variables. Individuals in the population are coded as strings (known as chromosomes) so that genotypes (chromosome values) are uniquely mapped to the decision variables (known as phenotypes). A chromosome needs to be decoded into its phenotypic values, and the objective function (fitness) of the chromosome can then be evaluated.

The mapping from a binary string to a real number in domain [c_j, d_j] for variable p_j is completed as follows

$$p_j = a_j + p_j^d \times \frac{d_j - c_j}{2^{m_j} - 1} \tag{13}$$

where m_j denotes the required bits for variable p_j, and c_j and d_j are the lower and upper bounds of variable p_j, respectively.

$$p_j^d = \sum_{i=0}^{m_j - 1} (b_i \times 2^i) \tag{14}$$

where p_j^d is the decimal number of variable p_j and [b_{mj-1}, , , b_2, b_1, b_0] is the binary substring of variable p_j.

The search process in GAs operates on the encoding of the decision variables rather than on the decision variables themselves, except in cases where the genotypes are identical to the phenotypes, as in the case of a real-value coding. It starts by generating chromosomes representing the initial population with a specific number of individuals. The fitness values of the individuals are evaluated on the basis of the objective function of the optimization problem. Hereafter, genetic evolution proceeds from generation to generation. In producing a new generation, the parents needed for breeding a new child are first selected according to their fitness values and put into the mating pool. The fitness of the individual in the population is related to its value of the objective function

$$f_j = \frac{1}{J_j} \tag{15}$$

where f_j is the fitness of some jth individual.

The extraction probability of each individual is proportional to the ratio of its fitness to the average fitness of all the individuals. In the selection process, the reproduction probabilities of individuals are given by their relative fitness (Li and Liu 2006)

$$pro_j = \frac{f_j}{\sum_{j=1}^{j=M} f_j} \tag{16}$$

where pro_j is the reproduction probability of a jth individual and M is population size.

The selection will ensure that high-fitted individuals have a high probability of being selected. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, every chromosome is evaluated by measuring its fitness function in the population and assigning it a score. Based on their fitness, multiple individuals are stochastically selected from the current population to form a new population. To create the next generation, new individuals—called *offspring*—are formed by either merging two chromosomes from the current generation using a crossover operator or modifying a chromosome using a mutation operator. The crossover operator takes two selected individuals and combines them about a crossover point, thereby creating two new individuals. The mutation operator randomly modifies the genes of a chromosome, introducing further randomness into the population. The cycle restarts by the formulation of a new generation by selection according to the fitness values. Some of the best parents and offspring are kept—the others are rejected to keep the population size constant.

A one-point crossover operator randomly selects one crossover point and then copies everything before this point from the first parent and then copies everything after the crossover point from the second parent. For example, two binary strings before single-point crossover operation are expressed as [1100101] and [0011011]. The crossover site is randomly selected on the fifth gene. After single-point crossover operation, two new binary strings are changed to [1100011] and [0011101].

Recombination takes place to produce children from the selected parents, often via some kind of crossover operation. The generated children can go through mutations, with some bits in the chromosome being mutated. The purpose of mutation, which is generally a background operator with a small probability, is to prevent premature convergence of the population. The GA sequence ends when either the maximum generations have been produced or other predefined termination conditions are satisfied. The average performance of individuals in the population is expected to increase during the evolution of the population as good individuals are preserved and bred with one another and less-fit individuals die out (Mitra and Mitra. 2012).

Mutation, with a preassigned low mutation probability, is further applied to the new chromosomes produced by the crossover process. Mutation is a genetic operator that ensures that the probability of searching any given string belonging to the parameter space will never be zero and thus has the effect of avoiding the possibility of convergence of the procedure to a local optimum. For the binary chromosomes, mutation causes a single bit to change its state from 0 to 1 and vice versa (Mitra and Mitra 2012). For example, a binary string before mutation is expressed as [1100011]. The mutation site is randomly selected on the fourth gene. After mutation operation,

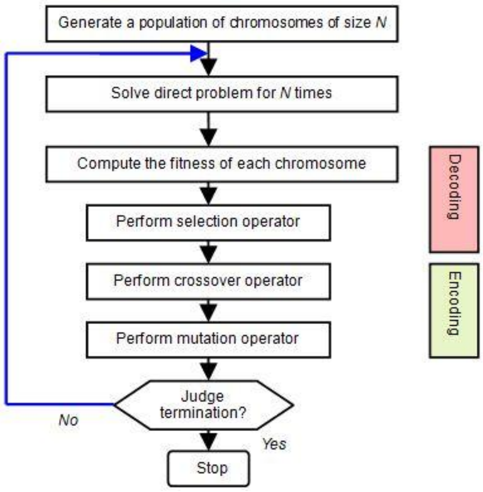

Fig. 6 Block diagram of a simple GA for solving the inverse problem

Table 1 General parameters of a GA for solving the inverse problem

GA parameters	Value
Population size	80
Bits of precision	64
Cross probability	0.86
Mutation probability	0.06
Maximum number of generations	100

Table 2 Solution domains of identified model parameters of rockfill materials

Parameters	C/kPa	φ_0/°	$\Delta\varphi$/°	k	n	R_f	K_b	m
Minimum value	0.00	45.0	3.0	100	0.4	0.5	100	0.15
Maximum value	10.0	60.0	10.0	800	0.95	0.95	500	0.45

a new binary string is changed to [11001011].

The GA terminates when either a maximum number of generations has been produced or a satisfactory fitness level has been reached (Adili *et al*. 2010). A block diagram of a simple GA is depicted in Fig. 6. Table 1 lists the general parameters of a GA for solving the inverse problem.

After investigating the model parameters of the rockfill materials of other dams (Zheng *et al*. 2013, Zhou *et al*. 2010, 2011, Xu *et al*. 2012), the solution domains of identified model parameters of rockfill materials were determined and are listed in Table 2.

4. Experimental verification of identification procedure of model parameters

In order to verify the effectiveness of the proposed parameter identification of rockfill material, three-dimensional compression tests were performed in different confining pressures. The material

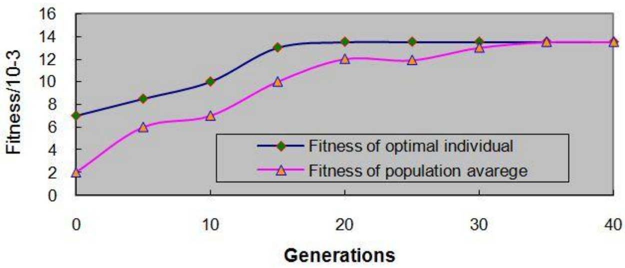

Fig. 7 Variation of fitting function versus evolution generation

Table 3 Identified model parameters of rockfill materials

Parameters	C/kPa	φ_0/°	$\Delta\varphi$/°	k	n	R_f	K_b	m
Estimated by G–N algorithm	0.00	50.3	7.7	347	0.64	0.82	195	0.23
Estimated by GA	0.02	54.4	4.8	253	0.65	0.72	163	0.26

parameters of the Duncan–Chang constitutive model were identified based on a GA and measured strain data. Table 3 presents the hyperbolic stress–strain parameters obtained from the triaxial compression tests by using the GA and G–N optimization algorithm. Fig. 7 shows the variation of the fitting function for the average population value and the optimal individual value versus evolution generation. From this figure, we can observe that the average population value of the fitting function approaches the optimal individual value after 35 evolution generations.

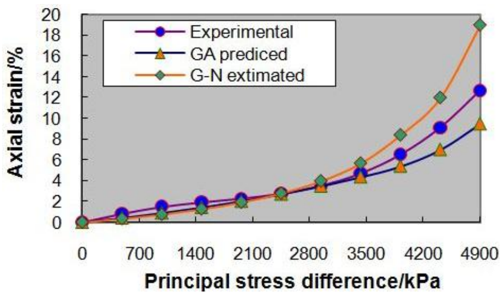

Fig. 8 Comparison of predicted strains with observed ones (confining pressure of 1200 kPa)

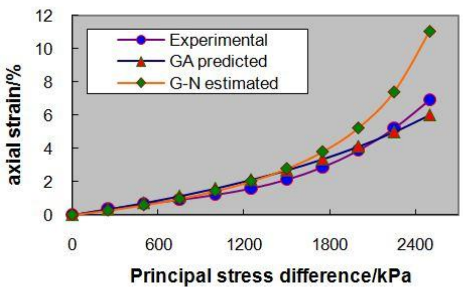

Fig. 9 Comparison of predicted strains with observed ones (confining pressure of 600 kPa)

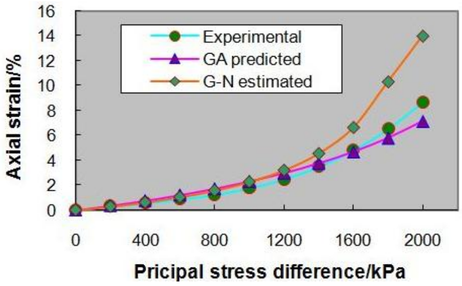

Fig. 10 Comparison of predicted strains with observed ones (confining pressure of 400 kPa)

Figs. 8-10 show the comparison of predicted axial strains with observed ones in different confining pressures. From Fig. 8, in the case of confining pressure of 1200 kPa, we can observe that the maximum relative error of the axial strain forecasted by the GA falls to 25% from 50% forecasted by initial estimation using the G–N optimization algorithm.

From Fig. 9, in the case of confining pressure of 600 kPa, we can observe that the maximum relative error of the axial strain forecasted by the GA falls to 15% from 59% forecasted by the G–N optimization algorithm.

From Fig. 10, in the case of confining pressure of 400 kPa, we can observe that the maximum relative error of the axial strain forecasted by the GA falls to 17% from 63% forecasted by the G–N optimization algorithm.

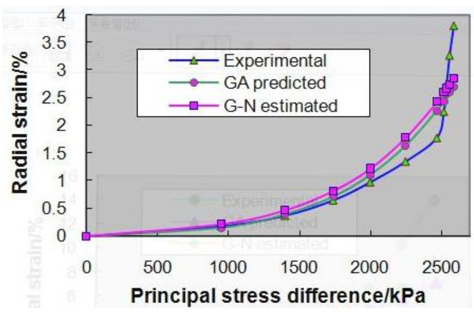

Fig. 11 Comparison of forecast radial strains with observed ones (confining pressure σ_3 = 600 kPa)

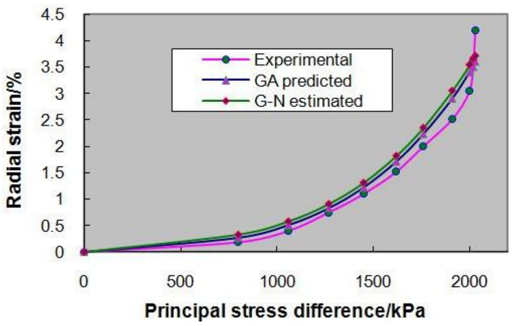

Fig. 12 Comparison of forecast radial strains with observed ones (confining pressure σ_3 = 400 kPa)

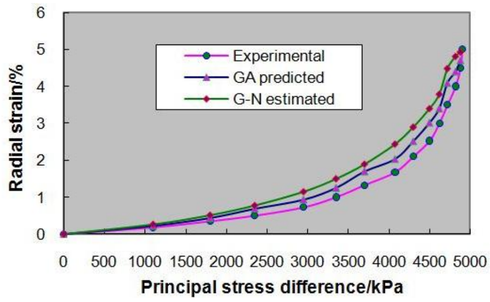

Fig. 13 Comparison of forecast radial strains with observed ones (confining pressure σ_3 = 1200 kPa)

From Figs. 11-13, we can observe that the maximum relative errors of the radial strains forecasted by the GA and G–N optimization algorithm are almost the same. The advantages of the GA for solving the inverse problem cannot be found. Perhaps local minimization of the objective function for the inverse problem approaches global minimization. This may be because the radial strain is not sensitive to model parameters. The larger changes of model parameters will only produce a smaller difference of radial strain.

5. Conclusions

Triaxial compression experiments were conducted on rockfill materials obtained from a dam construction site. The stress–strain change relationships of the rockfill materials are presented and discussed. The inversion procedure used for estimating model parameters is presented based on a GA. Some comparisons were performed for predicting the deformation behaviors of the rockfill materials by using the GA and a gradient-based optimization search algorithm, respectively. The investigations revealed that the proposed inversion procedure based on the GA provides more satisfactory prediction accuracy than a traditional gradient-based optimization algorithm.

Acknowledgments

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