

## Compressibility of Changi sand in $K_0$ consolidation

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**Abstract.** The one-dimensional compressibility of sand is an important property for the estimation of settlement or deformation of sand deposits. The  $K_0$  value of sand is also an important design parameter. Experimental results are presented in this paper to study the compressibility of sand in  $K_0$  consolidation tests. The  $K_0$  consolidation tests were carried out using a triaxial cell and a plane-strain apparatus. Specimens prepared using both the moist tamping and the water sedimentation methods were tested. The testing data demonstrate that the type of testing apparatus does not affect the  $K_0$  measurement if proper boundary conditions are imposed in the tests. The data also show that the compressibility and the  $K_0$  value of loose sand specimens prepared using the moist tamping method are very sensitive to the variation of void ratio. The  $K_0$  values measured from these tests do not agree with the  $K_0$  values calculated from Jaky's equation. The compressibility and  $K_0$  values of sand obtained from tests on specimens prepared using different preparation methods are different which may reflect the influence of soil fabrics or structures on the one dimensional compression behavior of sand.

**Keywords:** earth pressure; compressibility; consolidation; laboratory tests; sand fabric; triaxial; plane-strain.

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### 1. Introduction

The coefficient of lateral pressure at rest,  $K_0$ , is an important design parameter for geotechnical engineering.  $K_0$  is defined as the ratio of horizontal effective stress to vertical effective stress

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measured under a zero lateral strain condition, *i.e.*,

$$K_0 = \frac{\sigma'_{ho}}{\sigma'_{vo}} \quad (1)$$

where  $\sigma'_{ho}$  – the horizontal effective stress and  $\sigma'_{vo}$  – the vertical effective stress in situ.

The value of  $K_0$  cannot be determined using routine laboratory tests. Jaky's equation (Jaky 1944) is often used to estimate the  $K_0$  value of normally consolidated clay or loose sand:

$$K_0 = 1 - \sin \phi' \quad (2)$$

where  $\phi'$  is the effective friction angle of soil. However, the friction angle of soil depends on several factors, *e.g.*, void ratio (or relative density), confining pressure, stress conditions and shearing rate. For example, the friction angle of soil increases when the void ratio decreases. As the friction angle of soil is void ratio dependent, Eq. (2) implies that the  $K_0$  will vary with the void ratio of soil. It should be pointed out that Eq. (2) is not a precise theoretical prediction of “the the earth pressure at rest” and the horizontal-to-vertical stress ratio depends on the history of the deposition of granular material, as discussed in detail by Michalowski (2005). For loose sand, the study of Chu and Gan (2004) has shown that Eq. (2) can overestimate the  $K_0$  value by up to 36% using the friction angle obtained in triaxial tests.

It should also be mentioned that for linear isotropic materials the value of  $K_0$  can be obtained based on elasticity theory as:

$$K_0 = \frac{\nu}{1 - \nu} \quad (3)$$

where  $\nu$  is Poisson's ratio. However, the true linear elastic range is very small for sand (Jardine 1992). Therefore, in practice,  $K_0$  is normally measured in a range beyond linear elastic.

A  $K_0$  condition can also be viewed as a special case of a plane-strain condition. Therefore, a  $K_0$  test can also be conducted using a plane-strain apparatus. In this case, it is arguable whether the friction angle obtained under axisymmetric or plane-strain conditions may be used. It is well known that the friction angle obtained under axisymmetric conditions is generally lower than that under plane-strain conditions (*e.g.* Cornforth 1964, Bishop 1966, Alshibli *et al.* 2003, Wanatowski and Chu 2006). Therefore, the  $K_0$  value calculated from Eq. (2) using the friction angle for plane-strain conditions will be lower than that using the friction angle for axisymmetric conditions.

In this paper, results of  $K_0$  tests on sand conducted using triaxial and plane-strain apparatuses are presented. The compressibility characteristics of loose and medium loose sand were studied. Specimens prepared using both the moist tamping (MT) and the water sedimentation (WS) methods were tested. The effect of specimen preparation methods on the measurement of  $K_0$  is also discussed. The  $K_0$  values measured are compared with that predicted using Jaky's equation.

## 2. Testing arrangement

$K_0$  consolidation tests presented in this study were carried out either in a triaxial cell or in a plane-strain apparatus.

The triaxial experiments were carried out using a fully automated triaxial testing system as described by Chu and Leong (2001) and Wanatowski and Chu (2007b). The testing system

comprised a computer, a triaxial machine, a hydraulic actuator, three digital pressure/volume controllers (DPVCs) and an eight-channel data-logger. The hydraulic actuator was used to control the load and the data-logger was used to convert analogue data to digital format. A submersible load cell was also used to measure the vertical load. DPVC #1 was used to control the vertical load together with the hydraulic actuator. DPVC #2 was used to control the back pressure and to measure volume change of the specimen at the same time. DPVC #3 was used to control the cell pressure. In addition, the pore water pressure at the top and the bottom of the specimen was measured by two pressure transducers. Two submersible linear variable differential transformers (LVDTs) were attached onto the specimen, for the measurement of axial strain up to approximately 4%. An external LVDT of 50 mm travel was mounted directly on top of the triaxial chamber for the purpose of measuring large strains. The specimen used was 200 mm in height and 100 mm in diameter. To minimize the bedding errors and achieve as uniform strain distribution as possible during the test, free-ends with enlarged platens were used (Rowe and Barden 1964). The diameter of the enlarged top and base platens was 115 mm and each platen had a 38 mm diameter porous stone at the centre.

The plane-strain test system developed by Wanatowski and Chu (2006) was also used in this study. A prismatic soil specimen 120 mm in height and 60 by 60 mm in cross section was tested. Two 35 mm thick by 74 mm wide by 120 mm high rigid vertical platens were fixed in position by two pairs of horizontal tie rods to impose a plane-strain condition. The lateral stress in this direction ( $\sigma_2$ ) was measured by four submersible total pressure transducers. Two transducers were used for each platen. The total lateral pressure was evaluated as an average value obtained from the four individual transducers. All rigid platens were properly enlarged and lubricated using a free-end technique (Rowe and Barden 1964) to reduce the boundary frictions and to delay the occurrence of non-homogeneous deformations. For the top and base platens, latex disks were used, whereas for the two vertical platens, Teflon<sup>®</sup> sheets were adopted. A pair of miniature submersible linear variable differential transformers (LVDTs) was used to measure the vertical displacement. An external LVDT was also used to measure the axial strain when the internal LVDTs ran out of travel. A digital hydraulic force actuator was mounted at the bottom of a loading frame to apply axial load. The actuator was controlled by a computer via a digital load/displacement control box. A 10 kN submersible load cell was used to measure the vertical load. The cell pressure was applied through a digital pressure/volume controller (DPVC). Another DPVC was used to control the back pressure from the bottom of the specimen while measuring the volumetric change at the same time. A pore pressure transducer with a capacity of 1000 kPa was also used to record the pore water pressure at the top of the specimen. For details of the plane-strain apparatus, see Wanatowski and Chu (2006).

### 3. Testing methods

Numerous laboratory  $K_0$  tests have been conducted in the past (Mayne and Kulhawy 1982) and various testing methods have been used for conducting  $K_0$  tests (El-Sohby 1969, Abdelhamid and Krizek 1976, Daramola 1980, Okochi and Tatsuoka 1984, Fukugawa and Ohta 1988, Lo and Chu 1991, Mesri and Hayat 1993, Chu and Gan 2004). As reported by Lo and Chu (1991), most test methods attempt to simulate a condition of zero lateral strain. For example, El-Sohby (1969) conducted anisotropic consolidation tests along a range of principle stress ratio paths and deduced  $K_0$  by interpolation. This method inherently assumes that the volumetric to axial strain increment

ratio,  $d\varepsilon_v/d\varepsilon_1$ , remains constant during an anisotropic consolidation test. The most popular method of measuring  $K_0$  appears to be based on loading a sample in a triaxial cell under the condition of no lateral strain ( $d\varepsilon_3 = 0$ ) by continuously adjusting the cell pressure. A range of lateral strain detection devices have been used to provide necessary feedback (Okochi and Tatsuoka 1984).

An alternative method of determining  $K_0$  is based on strain path testing method developed by Chu and Lo (1991) and Chu *et al.* (1992) in which the strain increment ratio  $d\varepsilon_v/d\varepsilon_1$  of a triaxial specimen is precisely controlled. In  $K_0$  consolidation tests, the condition of zero lateral strain is controlled by maintaining the ratio of volumetric strain increment to axial strain increment to be the same, that is,  $d\varepsilon_v/d\varepsilon_1 = 1$  (Lo and Chu 1991). This method was adopted in the present study.

Under plane-strain conditions,  $d\varepsilon_2 = 0$ . The control of  $d\varepsilon_v/d\varepsilon_1 = 1$  leads to  $d\varepsilon_3 = 0$ , i.e., the condition of zero lateral strain. A  $K_0$  test can thus be conducted by regulating the volume change of the specimen in accordance with the axial strain to achieve  $d\varepsilon_v/d\varepsilon_1 = 1$ . The same control method of  $d\varepsilon_v/d\varepsilon_1$  can therefore be used in either triaxial cell or plane-strain apparatus.

Studies by Lo and Chu (1991), Chu *et al.* (1992, 1993), have shown that a strain path can be specified by strain-increment ratio,  $d\varepsilon_v/d\varepsilon_1$ . Such a strain path can be imposed on a triaxial soil sample by controlling the volume change of the sample via the DPVC in accordance with the measured axial deformation. A DPVC, developed by Menzies (1988) is now commonly available and can be either purchased 'off the shelf' or be fabricated in a research laboratory. Apart from the DPVC, a strain path can be controlled precisely without the use of sophisticated instruments, as shown by Lo and Chu (1991) and Chu *et al.* (1992).

In general, during a test, the strain increment ratio can be either maintained constant or varied in accordance with a certain predetermined relationship. This enables the multi-linear or non-linear path to be implemented. If an overconsolidated condition is needed, it can be simulated by loading the specimen to a preconsolidation axial stress and then unloading (and reloading if appropriate) along the same strain path to the current effective axial stress.

In this study all specimens were  $K_0$  consolidated from an initial isotropic stress state of 20 kPa. This is because a  $K_0$  consolidation test on sand cannot start from a free stress state as a saturated clean sand specimen cannot stand without an initial stress to hold it during the specimen preparation and saturation stage. However, this initial stress does not affect the resulting  $K_0$  path when the consolidation stress is beyond 4 times the initial stress, as established by Lo and Chu (1991). All experiments were carried out under a deformation-controlled loading mode using a strain rate of 0.05%/min. The cell pressure was maintained constant (within 0.5 kPa) whereas the expulsion of sample fluid was controlled by a computer via the DPVC to satisfy the condition of  $d\varepsilon_v/d\varepsilon_1 = 1$ . This led to a change in pore water pressure and thus the effective horizontal stress  $\sigma'_3$ . The control loop consisted of the following steps:

- (1) compute the axial strain from the pair of internal LVDTs,
- (2) compute  $\varepsilon_v$  and hence the volume change to be imposed by the DPVC from  $d\varepsilon_v/d\varepsilon_1 = 1$ ,
- (3) send signal to DPVC to target the required volume change,
- (4) read all transducers, plot the strain path followed and the effective stress state, record stress-strain parameters and return to step.

The continuous plotting of the actual strain path and the effective stress state followed at step (4) enabled the detection of any out of control. Satisfactory control was achieved for all the tests. A summary of the  $K_0$  tests conducted is given in Table 1.

Table 1 Summary of  $K_0$  consolidation tests conducted

Test No. <sup>a</sup>	Initial State <sup>b</sup>			$K_0$ -consolidated State								Figure No.
	$e_0$	$Dr_0$ (%)	$e_c$	$Dr_c$ (%)	$p'_c$ (kPa)	$q_c$ (kPa)	$\eta_c$	$\varepsilon_{1c}$ (%)	$\sigma'_3$ (kPa)	$\sigma'_1$ (kPa)	$K_0$	
MT05tc	0.846	18.3	0.830	22.5	199.1	247.9	1.24	0.72	116.5	364.4	0.32	3,4,12
MT06tc	0.852	16.7	0.834	21.4	198.6	234.2	1.18	0.87	120.5	354.7	0.34	12
MT10tc	0.858	15.1	0.840	19.8	200.2	234.7	1.17	0.93	122.0	356.7	0.34	3,4,12
MT08tc	0.886	7.8	0.857	15.4	198.5	205.4	1.03	1.32	130.0	335.4	0.39	3,4,12
MT07tc	0.895	5.5	0.863	13.8	200.4	197.8	0.99	1.47	134.5	332.3	0.40	12
MT04tc	0.897	5.0	0.871	11.7	197.4	193.9	0.98	1.50	133.5	325.2	0.41	12
MT03tc	0.910	1.6	0.873	11.2	199.0	192.5	0.97	1.67	135.5	331.6	0.41	3,4,12,14
MT01tc	0.928	-3.1	0.888	7.3	201.4	158.8	0.79	1.79	148.5	307.3	0.48	3,4,12
WS19tc	0.744	44.9	0.734	47.5	201.7	215.1	1.07	0.81	130.0	345.1	0.38	5,6,12
WS12tc	0.789	33.2	0.768	38.6	199.8	224.3	1.12	0.95	125.0	349.3	0.36	5,6,12
WS11tc	0.802	29.8	0.780	35.5	198.8	233.5	1.17	0.96	121.0	354.5	0.34	5,6,12
WS10tc	0.818	25.6	0.793	32.1	200.9	228.7	1.14	1.10	124.7	353.4	0.35	5,6,12
MT01psc	0.765	39.4	0.750	43.3	200.2	232.7	1.16	0.86	86.0	344.0	0.25	8,9,12,13
MT02psc	0.788	33.4	0.769	38.4	202.2	226.2	1.12	0.93	90.5	335.2	0.27	12
MT03psc	0.813	26.9	0.795	31.6	199.4	219.6	1.10	1.07	97.0	334.5	0.29	8,9,12
MT04psc	0.888	7.3	0.868	12.5	198.3	205.4	1.04	1.23	94.5	262.5	0.36	8,9,12
MT05psc	0.910	1.6	0.884	8.4	198.8	190.8	0.96	1.55	111.0	284.6	0.39	8,9,12,14
MT06psc	0.946	-7.8	0.902	3.7	198.4	168.4	0.85	2.45	120.0	272.7	0.44	8,9,12
MT07psc	0.978	-16.2	0.915	0.3	198.0	157.2	0.79	3.29	135.0	281.3	0.48	7,8,9,12
WS01psc	0.676	62.7	0.665	65.5	200.6	230.0	1.15	0.68	106.0	294.4	0.36	10,11,12
WS02psc	0.689	59.3	0.679	61.9	200.0	222.4	1.11	0.69	107.5	298.6	0.36	10,11,12
WS03psc	0.707	54.6	0.694	58.0	202.9	218.1	1.07	0.80	122.0	348.6	0.35	7,10,11,12
WS04psc	0.768	38.6	0.756	41.8	202.9	203.5	1.00	0.93	130.0	333.3	0.39	10,11,12,13
WS05psc	0.802	29.8	0.784	34.5	200.4	190.1	0.95	0.98	120.5	325.7	0.37	10,11,12

<sup>a</sup> MT = moist tamping, WS = water sedimentation, tc = triaxial compression, psc = plane-strain compression.<sup>b</sup>  $\sigma_{30} = 420$  kPa,  $u_0 = 400$  kPa,  $p'_0 = 20$  kPa,  $q_0 = 0$  kPa

#### 4. Material and specimens tested

The granular soil tested in this study was a marine dredged silica sand, the so-called Changi sand, used for the Changi land reclamation project in Singapore (Leong *et al.* 2000). The Changi sand has the specific gravity ( $G_s$ ) of 2.60, the mean grain size ( $D_{50}$ ) of 0.30 mm, the coefficient of uniformity ( $C_u$ ) of 2.0, and the coefficient of curvature ( $C_c$ ) of 0.8. The fines' content is approximately 0.4%. According to the Unified Soil Classification System (ASTM D2487) it is medium grained, poorly graded, clean sand. The individual particles of the sand are mainly subangular in shape. The minimum and maximum void ratios are 0.533 and 0.916, respectively. The minimum void ratio ( $e_{\min}$ ) was determined according to ASTM D4253 and the maximum void ratio ( $e_{\max}$ ) according to ASTM D4254. Since the Changi sand is dredged from the seabed, it contains shells of various sizes ranging from 0.2 to 10 mm. The shell content of the Changi sand is approximately 12%. Scanning electron microscopy (SEM) of Changi sand revealed that the shells are flaky and generally much

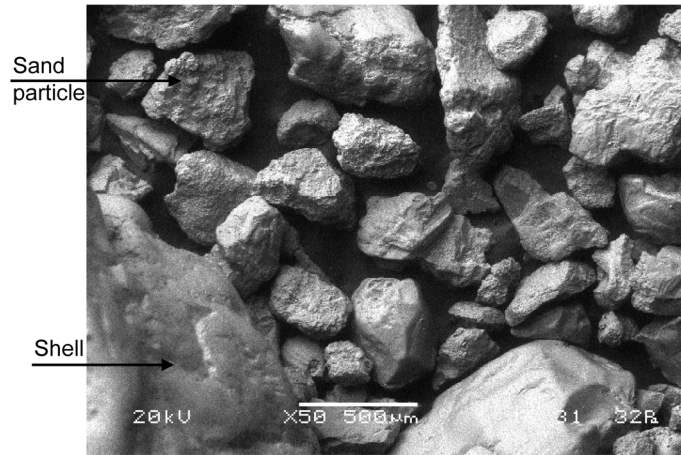


Fig. 1 SEM photograph of Changi sand

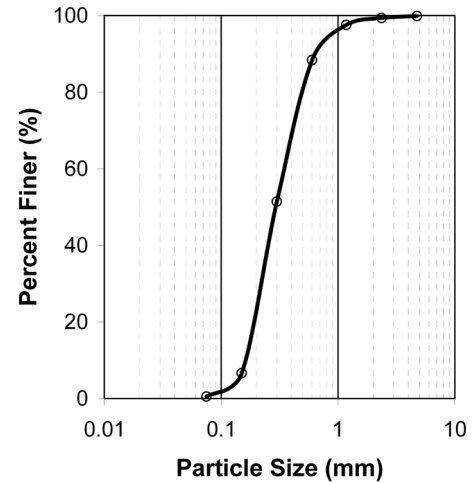


Fig. 2 Grain size distribution curve of the Changi sand

larger than the sand particles, as shown in Fig. 1. The grain size distribution curve of the Changi sand is given in Fig. 2. For more details on the material parameters, see Leong *et al.* (2000), and Wanatowski and Chu (2006).

Laboratory reconstituted specimens were used in this study. Two different specimen preparation methods, the water sedimentation (WS) and the moist tamping (MT) were employed. A four-part (plane-strain apparatus) or three-part (triaxial cell) split mould was used for the preparation of all the specimens. A latex membrane was fitted into the mould. A vacuum pressure of 10 kPa was used to achieve a tight fit between the mould and the membrane. In the WS method, sand was pluviated into the mould which was half-filled with de-aired water. Deposition of sand was done by moving the tip of the funnel in a circular motion 1-2 cm above the water surface. In the MT method, the oven-dried sand was first mixed with 5% of de-aired water. After mixing, the moist sand was deposited into the mould in five layers and each layer was compacted using a small tamper. The number of blows applied for each layer was carefully controlled. To achieve a greater uniformity of specimens, the undercompaction method, proposed by Ladd (1978), was used. For each layer, the compactive effort was increased towards the top with the undercompaction ratio of 2.5%. For saturation, the specimen was flushed with de-aired water from the bottom to the top for 60 min under a water head of about 0.5 m. After that a back pressure of at least 400 kPa was applied to ensure complete saturation of the specimens. The Skempton's pore water pressure parameter (*B*-value) greater than 0.96 was obtained for all the specimens. A liquid rubber technique (Lo *et al.* 1989) was adopted to reduce the bedding and membrane penetration errors. For more details of the sample preparation procedures, see Leong *et al.* (2000) and Wanatowski and Chu (2006).

## 5. Results

### 5.1 Triaxial tests

The effective stress paths and stress-strain curves of five  $K_0$  consolidation tests carried out in the

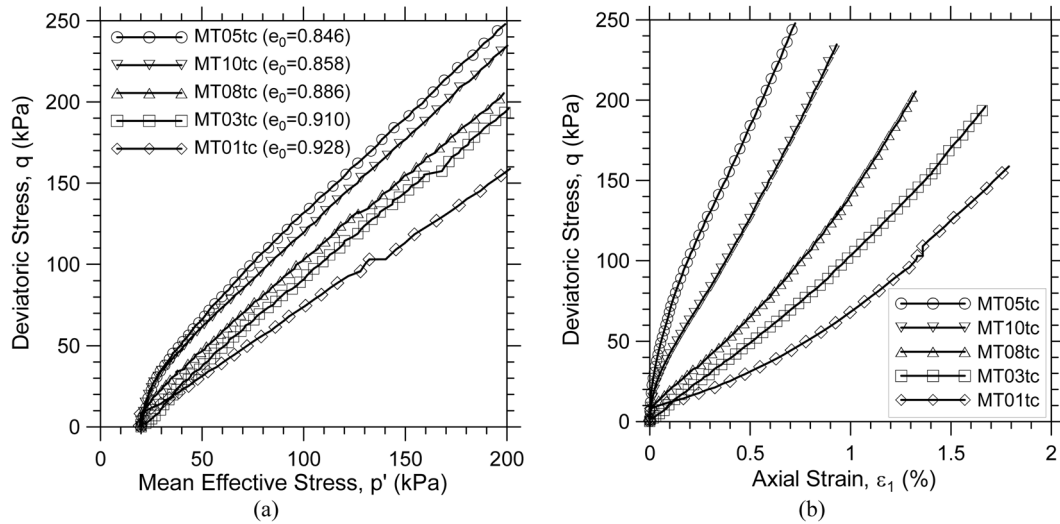


Fig. 3 Results of  $K_0$  consolidation tests conducted on MT specimens using triaxial cell: (a) effective stress paths; (b) stress-strain curves

triaxial cell on very loose and loose specimens are presented in Fig. 3. The specimens were prepared using the MT method. The initial void ratios of the specimens tested in this series vary from 0.846 to 0.928. All the tests were terminated when the mean effective stress,  $p'$ , reached 200 kPa. As shown in Figs. 3(a) and 3(b), the differences in the resulting effective stress paths and the stress-strain curves are obvious. The data shown in Figs. 3(a) and 3(b) indicate a consistent trend that the denser the soil, the higher the gradient of the effective stress path and the stiffer stress-strain response.

The variations of  $K_0$  with the effective vertical stress,  $\sigma'_1$ , and axial strain,  $\varepsilon_1$ , in the five tests are plotted in Figs. 4(a) and 4(b), respectively. It can be seen from Fig. 4(a), that as the tests started from an isotropic state, there is an initial transition from the isotropic state to the  $K_0$  state. This explains the sharp reduction in  $K_0$  at the beginning of the test. After this transition period,  $K_0$  becomes more or less constant after the mean effective stress exceeds 80 kPa which corresponds to an axial strain of 0.4-0.5% (Gan 2002). This observation is consistent with what has been established by Lo and Chu (1991). It can be further observed from Fig. 4 that the looser the specimen, the higher the  $K_0$  value and the larger the axial strain developed during  $K_0$  consolidation.

Fig. 4 also shows that two different trends of  $K_0$  behavior were observed for specimens tested in the triaxial cell. For specimens with  $e_0$  looser or equal to 0.886 (*i.e.* tests MT01tc, MT03tc, and MT08tc), the  $K_0$  value reduces gradually with the effective vertical stress and the axial strain. For specimens with  $e_0$  denser than 0.858 (*i.e.* test MT05tc and MT10tc), the  $K_0$  reduces quickly to a minimum value and then increases slightly afterward. Such differences in the  $K_0$  behavior may be due to the differences in the soil structures. In preparing specimens with void ratios smaller than 0.858 using the MT method, each layer has to be densified with more than 20 blows. As a result, a more distinct soil structure may have developed in the specimen. According to Leroueil and Vaughan (1990), a temporary higher yielding stress is required to break the soil structure, which results in a temporary higher stress ratio at the yielding point. This may explain why for soil specimens with  $e_0$  less than 0.858 (test MT05tc), a distinct yielding point can be observed in the stress-strain curve (see Fig. 3b). In Fig. 4, the minimum  $K_0$  value point observed in test MT05tc

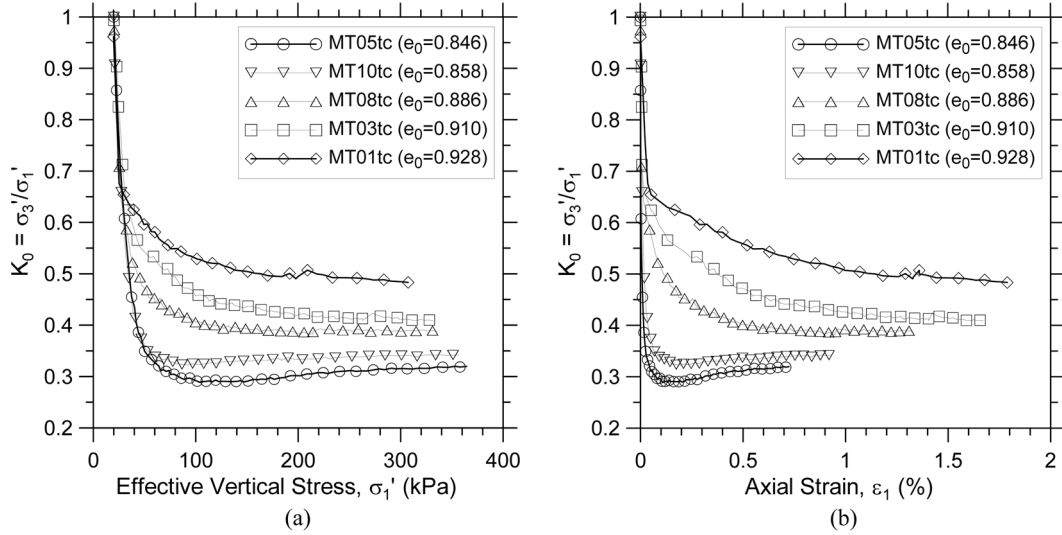


Fig. 4 Curves of  $K_0$  for MT specimens tested in triaxial cell: (a)  $K_0 - \sigma'_1$ ; (b)  $K_0 - \epsilon_1$

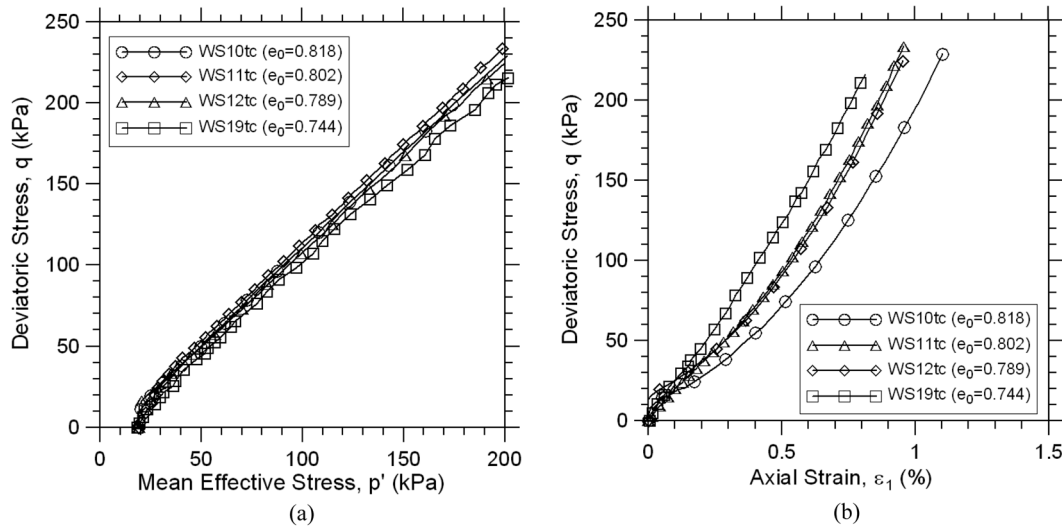


Fig. 5 Results of  $K_0$  consolidation tests conducted on WS specimens using triaxial cell: (a) effective stress paths; (b) stress-strain curves

may reflect the temporary higher stress ratio point. The subsequent slight increase in  $K_0$  may represent the process of a gradual degradation in soil structure. Similar behaviors for other soils have also been reported by other researchers (Leroueil and Vaughan 1990, Coop and Atkinson 1993, Vatsala *et al.* 2001). Tests MT01tc and MT08tc show a different behavior because the specimens were looser and the soil structures were relatively weaker.

The effective stress paths and the stress-strain curves of four  $K_0$  consolidation tests obtained from triaxial specimens prepared using the WS method are shown in Figs. 5(a) and 5(b). The variations of  $K_0$  with effective vertical stress and axial strain for those three tests are also shown in Figs. 6(a) and 6(b). The initial void ratios of the specimens tested in this series vary from 0.744 to 0.818.



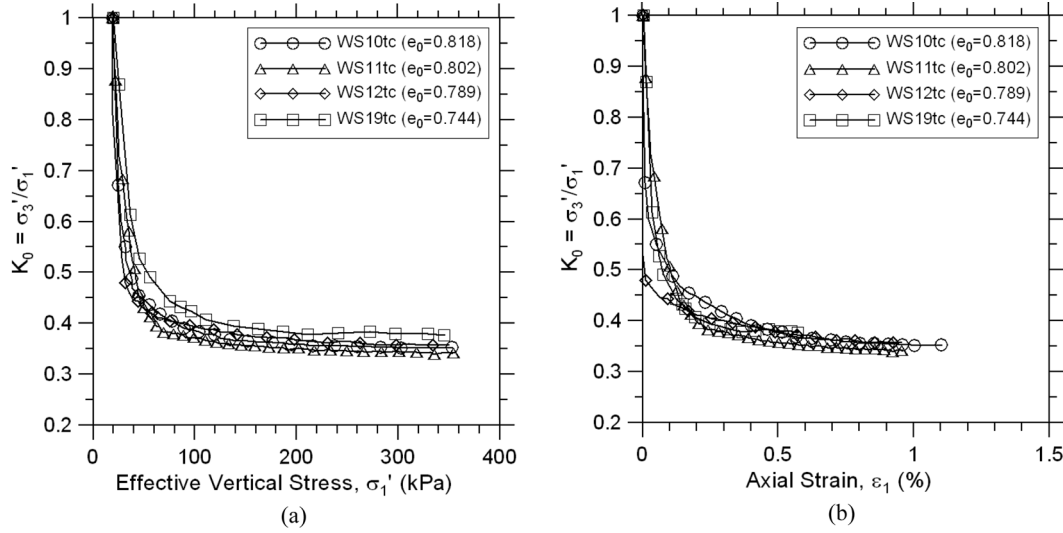


Fig. 6 Curves of  $K_0$  for WS specimens tested in triaxial cell: (a)  $K_0 - \sigma'_1$ ; (b)  $K_0 - \epsilon_1$

It can be seen from Figs. 5(a) and 5(b) that within the range of void ratio, effective stress paths and stress-strain curves do not vary much with void ratio. The data presented in Figs. 6(a) and 6(b) indicate that within the range of void ratio tested, the  $K_0$  values of the WS specimens fall within a narrow range. The variation of  $K_0$  seen in Fig. 6 is similar to that observed in tests MT01tc, MT03tc, and MT08tc shown in Fig. 4 (*i.e.* the  $K_0$  value reduces gradually with the effective vertical stress and the axial strain). This may be indicative that the WS method produces a less structured triaxial soil specimen as compared with specimens prepared using the MT method.

## 5.2 Plane-strain tests

As mentioned earlier, the plane-strain apparatus used in this study allows independent measurement of two lateral pressures  $\sigma'_2$  and  $\sigma'_3$ . The values of  $\sigma'_2$  and  $\sigma'_3$  measured from two typical  $K_0$  consolidation tests carried out in the plane-strain tests are compared in Fig. 7. The  $\sigma'_2$  and  $\sigma'_3$  versus axial strain curves obtained from tests MT07psc and WS03psc are shown in Figs. 7(a) and 7(b), respectively. It can be seen that the two curves are almost identical for both specimens, that is,  $\sigma'_2 = \sigma'_3$  is obtained under the  $d\epsilon_2 = d\epsilon_3 = 0$  condition. Similar observations were made from all the other  $K_0$  consolidation tests conducted on Changi sand in the plane-strain apparatus. This suggests that there is no strong anisotropy in the  $\sigma'_2$  and  $\sigma'_3$  directions, that is, the specimen is essentially cross-anisotropic. Therefore, in terms of  $K_0$  value there is no difference whether  $\sigma'_2$  or  $\sigma'_3$  is used as  $\sigma'_{ho}$  in Eq. (1). Furthermore, this also serves as a verification of the reliability of the plane-strain apparatus used in this study.

The results of six  $K_0$  consolidation tests carried out on very loose to medium loose sand in the plane-strain apparatus are presented in Fig. 8. All the specimens were prepared using the MT method. The initial void ratios,  $e_0$ , of the specimens were in the range between 0.765 and 0.978. The specimens were consolidated to a mean effective stress of 200 kPa. Similar to the  $K_0$  test in the triaxial cell, a  $K_0$  test on sand carried out in the plane-strain cell has to be started from an isotropic stress state as explained earlier; hence there is an initial transition from the isotropic state to the  $K_0$

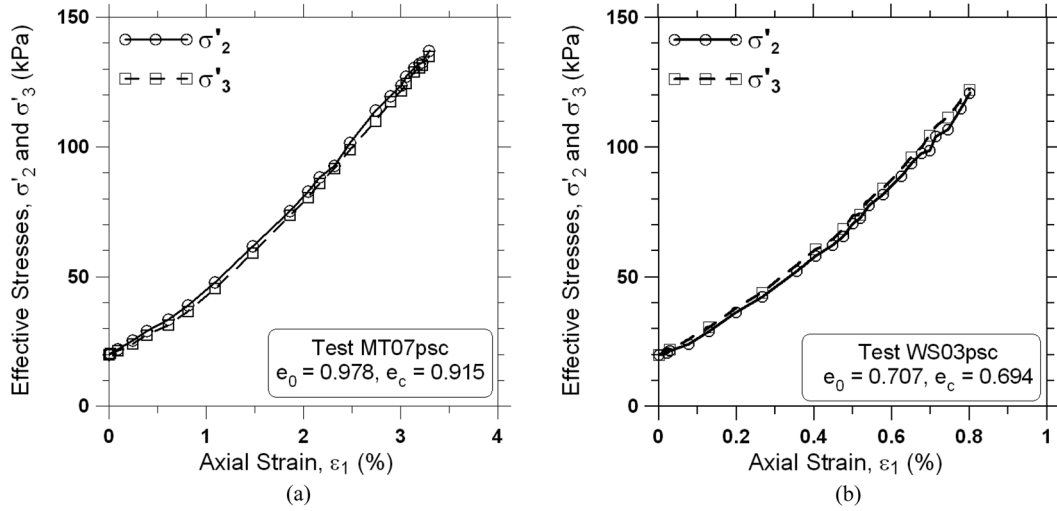


Fig. 7 The lateral stress response obtained from plane-strain tests: (a) MT07psc; (b) WS03psc

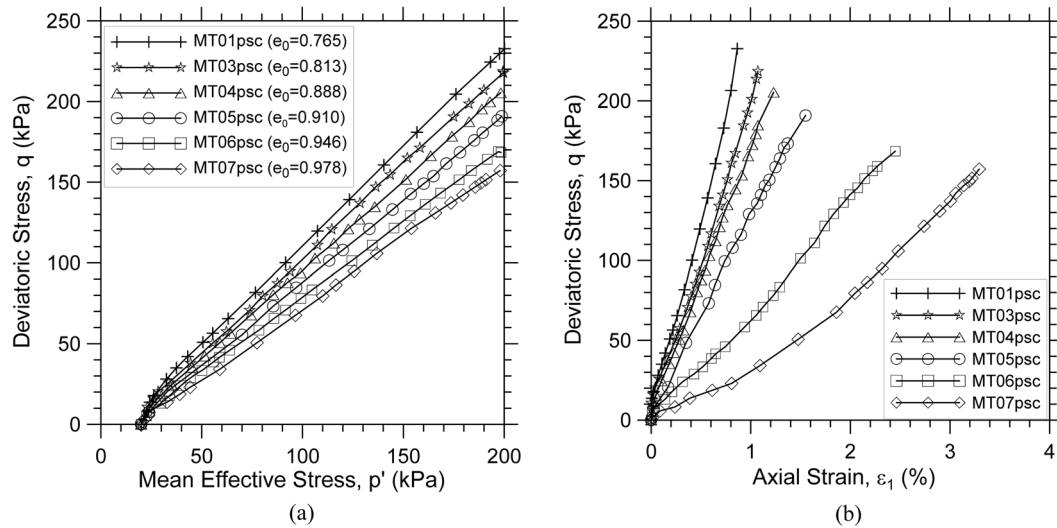


Fig. 8 Results of  $K_0$  consolidation tests conducted on MT specimens using plane-strain apparatus: (a) effective stress paths; (b) stress-strain curves

state. However, this transition only affects the  $K_0$  value at the initial period. The  $K_0$  value approaches more or less a constant value after the mean effective stress exceeds 80 kPa which corresponds to an axial strain of 0.35-0.40% (Wanatowski 2005). This observation is consistent with that made for the triaxial tests.

As shown in Fig. 8(a), the effective  $K_0$  paths obtained from the six tests are affected by the variation of void ratio. The looser the soil the smaller the effective stress ratio  $\eta_c$ , which is calculated as the slope of the effective stress path  $q/p'$ . Stress-strain curves are affected by the void ratio too. As shown in Fig. 8(b), the denser the specimen the stiffer stress-strain response during  $K_0$  consolidation.

The variation of  $K_0$  with the effective vertical stress and the axial strain in the six plane-strain

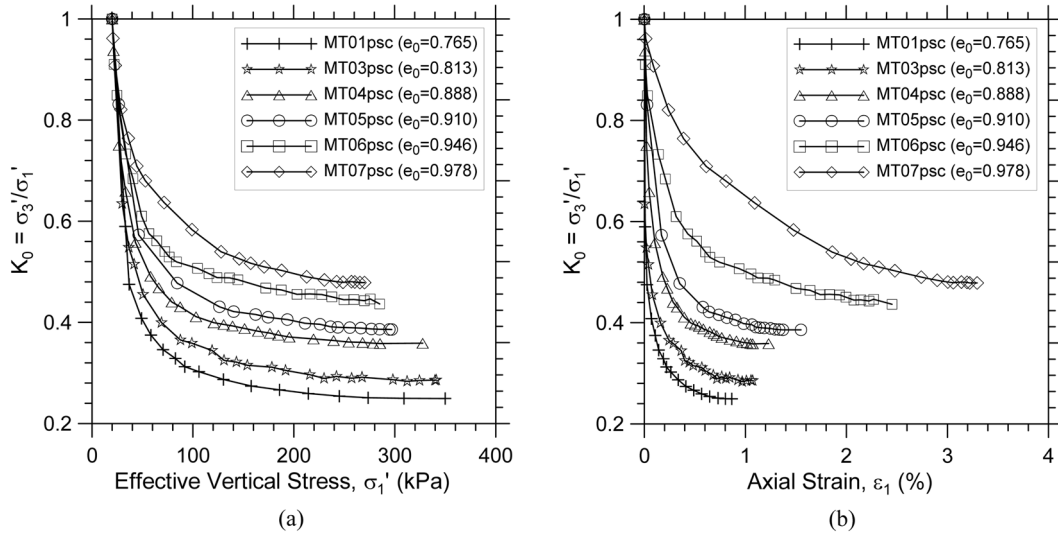


Fig. 9 Curves of  $K_0$  for MT specimens tested in plane-strain apparatus (a)  $K_0 - \sigma'_1$ ; (b)  $K_0 - \epsilon_1$

tests is plotted in Figs. 9(a) and 9(b), respectively. Similar to the  $K_0$  behavior of loose sand obtained from triaxial cell (see Fig. 4), there is an initial transition from the isotropic state to the  $K_0$  state in Figs. 9(a) and 9(b). After that the  $K_0$  value reaches more or less a constant value at the end of consolidation in each test. As shown in Fig. 9(a), the looser the specimen, the lower effective vertical stress is required to consolidate the specimen to  $p'_c = 200$  kPa. It can be further observed from Fig. 9(b) that the looser the specimen, the higher the  $K_0$  value and the larger the axial strain developed during  $K_0$  consolidation.

The results of five  $K_0$  consolidation tests obtained from medium loose to medium dense

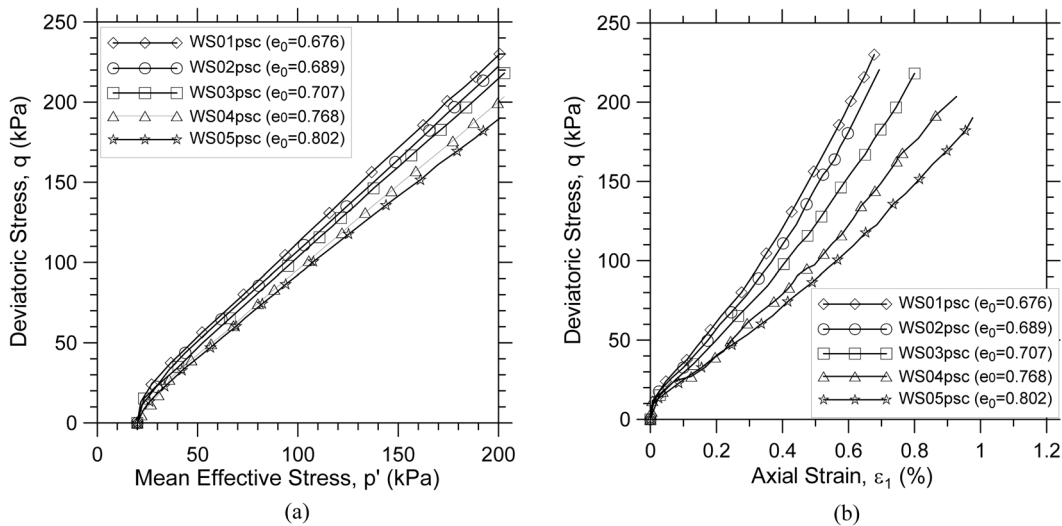


Fig. 10 Results of  $K_0$  consolidation tests conducted on WS specimens in plane-strain apparatus: (a) effective stress paths; (b) stress-strain curves

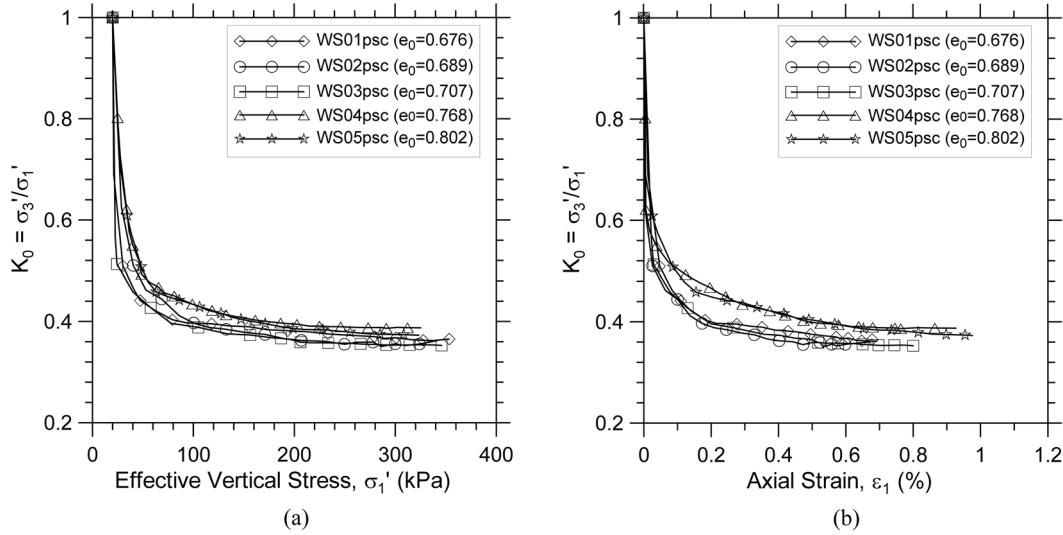


Fig. 11 Curves of  $K_0$  for WS specimens tested in plane-strain apparatus (a)  $K_0 - \sigma'_1$ ; (b)  $K_0 - \epsilon_1$

specimens prepared using the WS method are given in Fig. 10. The initial void ratios of the specimens varied from 0.676 to 0.802. All the specimens were consolidated to a mean effective stress of 200 kPa. The effective stress paths and the stress-strain curves are plotted in Figs. 10(a) and 10(b), respectively. Similar to  $K_0$  consolidation tests performed on the MT specimens in the plane-strain apparatus, the effective stress paths and the stress-strain curves are affected by the void ratio. The larger the initial void ratio (or the looser the soil), the smaller the effective stress ratio  $\eta_c$  and the less stiff stress-strain behavior, as shown in Figs. 10(a) and 10(b). However, the variation in the  $K_0$  behavior is much smaller compared with the results shown in Fig. 8 for MT sand.

The variation of  $K_0$  with effective vertical stress and the axial strain is presented in Figs. 11(a) and 11(b), respectively. Despite void ratio dependent  $K_0$  behavior of WS specimens (Fig. 10), the data presented in Fig. 11 indicate that within the range of void ratio tested, the  $K_0$  values of the WS specimens fall within a narrow range. Similar behavior for the same sand has been observed in triaxial tests (see Fig. 6).

## 6. Discussion

The  $K_0$  values for Changi sand obtained from this study and the study by Wanatowski and Chu (2008) carried out in both triaxial and plane-strain apparatuses are compared in Fig. 12. All the  $K_0$  values were calculated at the end of consolidation stage that is at a mean effective stress of 200 kPa. It can be seen from Fig. 12 that within the range of void ratio tested the  $K_0$  values of the WS specimens fall within a relatively narrow range. In other words, the  $K_0$  values obtained from the tests on WS specimens show little dependence on the initial void ratio. On the other hand, the  $K_0$  values obtained from the tests on MT specimens form a relationship with the initial void ratio. As shown in Fig. 12 the looser the MT specimen, the higher the  $K_0$  value. Chu and Gan (2004) and Wanatowski and Chu (2007a) have also reported that  $K_0$  values obtained from the tests on Changi sand do not agree well with Jaky's equation (see Eq. 2). To illustrate this further, the variations of

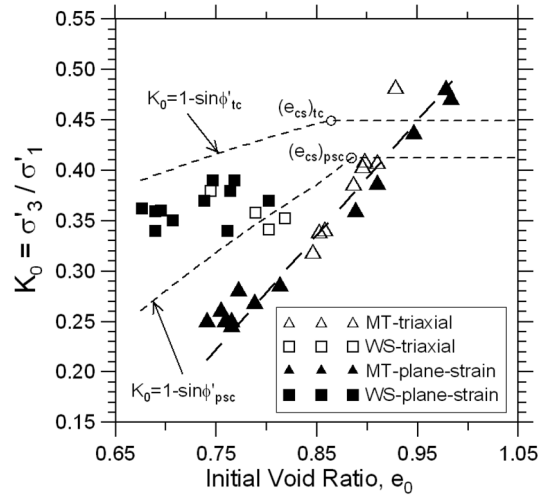


Fig. 12 Summary of  $K_0$  consolidation tests for Changi sand

$K_0$  with void ratio established for both triaxial and plane-strain tests were plotted in Fig. 12. The  $K_0$  values were calculated using Eq. (2) and the peak friction angle and void ratio relationships determined for Changi sand by Wanatowski and Chu (2006).

Within the void ratio range of the specimens tested in this study, the peak friction angle obtained from triaxial tests,  $\phi'_{tc}$ , varies between  $33.4^\circ$  and  $39.1^\circ$ , and from plane-strain tests,  $\phi'_{psc}$ , between  $36.0^\circ$  and  $43.4^\circ$  (Wanatowski and Chu 2006, 2007b). The lower values  $33.4^\circ$  and  $36.0^\circ$  are the critical state friction angles ( $\phi'_{cs}$ ) under axisymmetric,  $(\phi'_{cs})_{tc}$ , and plane-strain conditions,  $(\phi'_{cs})_{psc}$ , respectively. The  $K_0$  versus void ratio relationships using both  $\phi'_{tc}$  and  $\phi'_{psc}$  are plotted in Fig. 12 as two curves in which the curve calculated using  $\phi'_{tc}$  gives the upper bound and the curve using  $\phi'_{psc}$  the lower bound.

It can be seen from Fig. 12 that the  $K_0$  values obtained from the tests on WS specimens show little dependence on the initial void ratio, thus do not agree with Jaky's equation well. Nevertheless, most of the data points fall within the zone bound by the two curves. The  $K_0$  values obtained from the tests on MT specimens form a linear relationship with void ratio. However, this relationship does not agree with Jaky's equation either. Therefore, Jaky's equation does not seem to give a good prediction of the  $K_0$  value of sand as far as this study is concerned.

It can also be seen from Fig. 12 that the  $K_0$  values obtained from tests on the MT specimens are different from those on the WS specimens at the same void ratio. The  $K_0$  values obtained from the MT specimens are generally lower than those from the WS specimens. This is illustrated further in Fig. 13(a) where the  $K_0$ – $\varepsilon_1$  curves obtained from two plane-strain tests conducted on WS and MT specimens with similar void ratios are compared. It can be observed from Fig. 13(a) that the higher  $K_0$  value and the lower axial strain were measured during  $K_0$  consolidation of the WS sand.

The effective stress paths obtained from the two  $K_0$  tests are plotted in Fig. 13(b). It can be seen that the effective  $K_0$  paths obtained from the WS and MT specimens are different. This shows that the specimen preparation method affects not only the  $K_0$  value of sand but also the effective stress ratio  $\eta_c$  determined at the end of  $K_0$  consolidation, as discussed earlier. As shown in Fig. 13(b), the higher effective stress ratio  $\eta_c$  was obtained for the MT sand. Similar behavior was observed from other  $K_0$  consolidation tests conducted in the plane-strain apparatus (Wanatowski and Chu 2008).

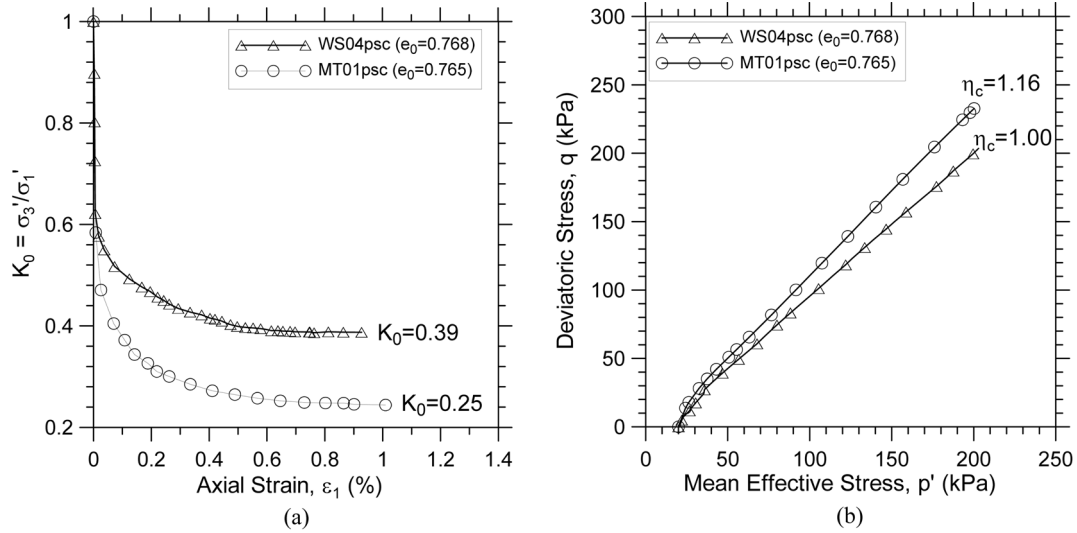


Fig. 13 Comparison of  $K_0$  tests conducted on WS and MT specimens using plane-strain apparatus: (a)  $K_0 - \varepsilon_1$  curves; (b) effective stress paths

The differences in the stress-strain behaviors of MT and WS sand are due to different soil fabrics and structures produced by each method (Ladd 1974, Kuo and Frost 1996, Vaid *et al.* 1999, Frost and Jang 2000, Frost and Park 2003, Chu and Gan 2004, Yamamuro and Wood 2004, Wanatowski and Chu 2008). Such differences in the fabric and structures of granular soils are related to different deposition and densification processes involved in different preparation methods. In this study, the MT method involved placing moist sand in several layers whereas the WS method involved raining dry sand through water. Furthermore, the vertical stresses applied during preparation of the MT and WS specimens were different. In the case of loose and medium loose specimens, tamping had to be used for the MT specimens, but not for the WS specimens. In the case of medium dense specimens, tamping was used for both MT and WS specimens. However, for the MT specimens the tamping effort was much greater than for the WS specimens. These differences, in turn, affected the global uniformity and the grain contact structure of specimens. As shown by Kuo and Frost (1996), Vaid *et al.* (1999), and Frost and Park (2003), the MT specimens are always more nonuniform in void ratio distribution than the WS specimens. Studies by Yamamuro and Wood (2004) and Wood *et al.* (2008) also suggest that MT specimens might retain more unstable grain contacts than WS specimens. In other words, an unstable and highly compressible particle microstructure is produced by MT method. Therefore, it can be concluded that the differences in the stress-strain behaviors of the MT and WS specimens during  $K_0$  consolidation observed in this study may be related to the differences in soil fabrics and structures resulting from different specimen preparation methods.

Fig. 12 also shows that the plane-strain test data agree well with that from triaxial tests on MT specimens. This observation supports the fact that the type of testing apparatus does not affect the  $K_0$  measurement if proper boundary conditions are imposed in the tests. To illustrate this point further, the  $K_0$  results obtained from two tests MT03tc and MT05psc conducted on two MT specimens with the same void ratios, are compared in Fig. 14. Both tests were conducted in a similar way. However, test MT03tc was carried out in the triaxial cell and test MT05psc in the plane-strain apparatus. It can be seen from Fig. 14 that a good consistency in  $K_0 - \varepsilon_1$  curve (Fig. 14a)

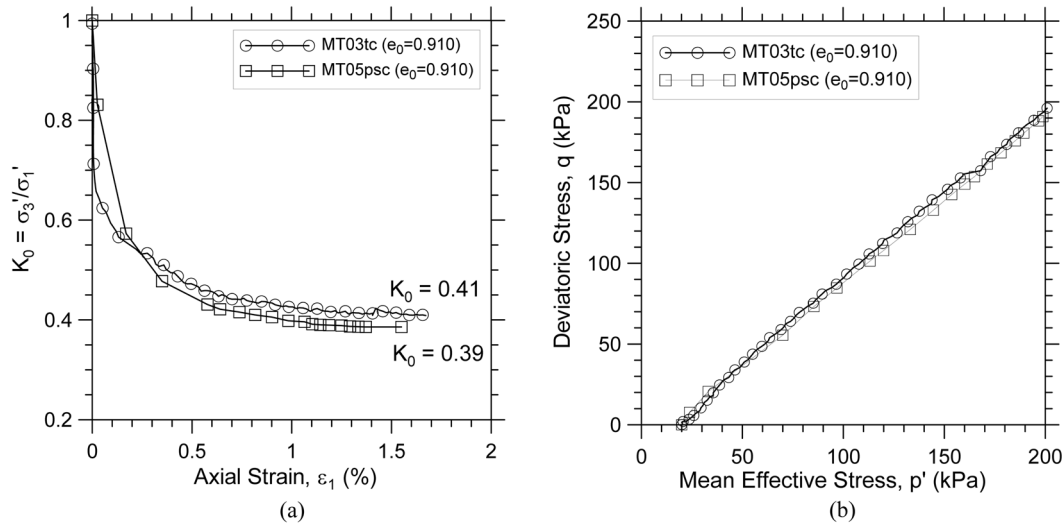


Fig. 14 Comparison of  $K_0$  tests results obtained from triaxial cell and plane-strain apparatus (a)  $K_0 - \varepsilon_1$  curves; (b) effective stress paths

and the effective stress paths (Fig. 14b) was obtained.

The good agreement between the triaxial and plane-strain data also indicates that both the triaxial and plane-strain testing systems adopted in this study are reliable in measuring the stress-strain behavior of sand under  $K_0$  conditions.

## 7. Conclusions

The results of  $K_0$  tests using a triaxial cell and a plane-strain apparatus are presented and compared. The  $K_0$  condition was imposed by controlling  $d\varepsilon_v/d\varepsilon_1 = 1$ . Very loose to loose sand specimens prepared using the MT method and medium loose to medium dense specimens prepared using the WS method were tested. The data show that the  $K_0$  values determined by plane-strain tests are in good agreement with those by triaxial tests. Therefore, the type of testing apparatus does not affect the  $K_0$  measurement if proper boundary conditions are imposed in the tests.

The test results show that for sand specimens prepared using the MT method, the  $K_0$  value and the stress-strain behavior of Changi sand are strongly void ratio dependent. Generally, the denser the sand, the lower the  $K_0$  value, and the stiffer the stress-strain behavior. However, the effect of void ratio on  $K_0$  behavior is less significant for specimens prepared using the WS method.

The  $K_0$  values obtained from the tests on WS specimens show little dependence on the initial void ratio, thus do not agree with  $K_0$  values calculated from the Jaky equation using the peak friction angles obtained from either triaxial or plane-strain tests. The  $K_0$  values obtained from the tests on MT specimens form a linear relationship with void ratio. However, this relationship does not agree with the Jaky equation either. The difference in the  $K_0$  values measured for WS and MT specimens appears to be related to the differences in the soil fabrics and structures resulting from different specimen preparation methods.

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