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Effective stiffness in regular R/C frames subjected to seismic loads

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Abstract. Current design codes and technical recommendations often provide rough indications on how to assess effective stiffness of Reinforced Concrete (R/C) frames subjected to seismic loads, which is a key factor when a linear analysis is performed. The Italian design code (NTC-2008), Eurocode 8 and ACI 318 do not take into account all the structural parameters affecting the effective stiffness and this may not be on the safe side when second-order P-Δ effects may occur.

This paper presents a study on the factors influencing the effective stiffness of R/C beams, columns and walls under seismic forces. Five different approaches are adopted and analyzed in order to evaluate the effective stiffness of R/C members, in accordance with the scientific literature and the international design codes. Furthermore, the paper discusses the outcomes of a parametric analysis performed on an actual R/C building and analyses the main variables, namely reinforcement ratio, axial load ratio, concrete compressive strength, and type of shallow beams. The second-order effects are investigated and the resulting displacements related to the Damage Limit State (DLS) under seismic loads are discussed. Although the effective stiffness increases with steel ratio, the analytical results show that the limit of 50% of the initial stiffness turns out to be the upper bound for small values of axial-load ratio, rather than a lower bound as indicated by both Italian NTC-2008 and EC8. As a result, in some cases the current Italian and European provisions tend to underestimate second-order P-Δ effects, when the DLS is investigated under seismic loading.

Keywords: effective stiffness (in R/C); R/C frames; columns; beams; seismic design

1. Introduction

Seismic design of ordinary R/C frames commonly relies on linear analysis, whose results are strongly influenced by the assumed values of effective stiffness. However, such procedure neglects progressive cracking and time-dependent phenomena such as shrinkage or creep.

Despite scientific studies suggest a more accurate evaluation of the effective stiffness of R/C

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elements under seismic loading by means of non-linear analyses as perfomed by Borzi *et al*. (2013) or compared by Carvalho *et al*. (2013), design codes such as Italian NTC-2008, EC-8 (2005) and ACI 318 (2002) provide indications that may appear too simplified. In fact, NTC-2008 allows designers to reduce the effective stiffness by 50% compared to the initial one, depending on the entity of the normal force; while EC8 suggests to assume a cracked stiffness equal to 0.50 times the gross stiffness, unless a more accurate analysis (nonlinear) is performed. According to the authors of this paper, instead, such an approach may be improved since the above mentioned codes do not specify how to assign different stiffness values according to the different limit states (Seismic Ultimate Limit State - SULS - and Seismic Damage Limit State SDLS). Furthermore, the cracked stiffness of the beams is introduced in the same way as that of the columns, and main structural parameters influencing the stiffness of members appear to be omitted within the provisions.

Other international design codes provide more specific indications about the effective stiffness of members to be adopted in the design of R/C frames under seismic conditions. The New Zealand code has gone a gradual evolution as analyzed by Fenwick and MacRae (2009). The current design code, NZS 3101 (2006), provides values for the effective stiffness at the ultimate and serviceability limit state for different R/C members. American ACI 318 (2002) contains design recommendations for both serviceability and failure limit states. FEMA 356 (2000) dedicates the whole Chapter 6 - namely “Concrete” - to provide design recommendations. Finally the Canadian Standard Association (2005) design code assigns specific values for the reduced stiffness of structural members. Table 1 reports an overview of the provisions recommended by the cited design codes.

NZS 3101 (2006) considers specific provisions for columns which have a high level of protection against hinge formation at ULS but it offers no provisions on shear stiffness reduction. The values included in Table 1 refers mainly to beams with low to moderate reinforcement content, but those values may be on the low side for beams with moderate to high reinforcement content. In that case the use of the following formula is recommended: $I_e = \big(0.32 + 40(\rho - 0.0075)\big)I_g$, being ρ the reinforcement content. FEMA 356 gives instead provisions on the axial stiffness of members as well as on the flexural stiffness. Finally all of these codes deal with coupling beams as well, mainly suggesting reduced values for both flexural and especially shear stiffness.

As regards the stiffness of walls, the distinction between the cracked and uncracked conditions is addressed in ACI 318; while Wallace and Orackal (2002) and Petrini *et al.* (2004) report a few comments on the stiffness of cracked or uncracked walls along their length, Chen and Scawthorn (2003) have discussed the importance of considering a reduced stiffness in cracked walls as well.

Despite the significance of this topic, there is unfortunately only a limited number of scientific works dealing with it. Table 2 reports a synthetic view of the most relevant literature contribution. Branson (1965) was the first to deal with the problem of computing the stiffness in cracked regions, and his work is still a reference (ACI-318). Sugano (1970) then proposed a reduced stiffness for beams and columns, based on some experimental results and on the ratio between the secant stiffness at yielding and the stiffness of the gross uncracked section.

Grossman (1981) proposed to compute the stiffness of cracked beams subjected to bending on the basis of the ratio between the first-cracking moment M_{cr} and the applied moment M_a. He also introduced a parametric coefficient *Ce* for high moment values, which is dependent on the mechanical properties of concrete and steel. Mirza (1990) introduced an empirical expression for the effective stiffness of R/C columns.

Paulay and Priestley (1992) then indicated the values reported in Table 3 for the effective stiffness of both columns and beams, based on a range of possible values for the moment of inertia of the gross section. These recommendations were taken as a reference by ACI guidelines. Later Priestley (2003) correlated the effective stiffness with the nominal bending resisting moment of the section and the curvature at yielding of the rebars φ_y. This last depends on the geometry of the member. For circular and rectangular columns φ_y is taken respectively as 2,25 ε_y/D and as 2,10 ε_y/h_c, while for rectangular cantilever walls the value are lowers at 2,00 ε_y/l_w and for T-beams at 1,70 ε_y/h_b. D, h_c, l_w, and h_b, are respectively the diameter of the column, the height of the column section, the width of the wall section and the height of the beam section.

Table 1 Overview of effective stiffness in structural members according to recent seismic design codes

CODE	***MEMBER***	***CONDITION***	***Stiffness***				
			Ultimate Limit State		***Serviceability Limit State***		
			Flexural	**Shear**	**Flexural**		
			f_y=400 MPa (f_y=500 MPa)	-	**μ=1.25**	**μ=3**	**μ=6**
NZS 3101	R/C beam	Rectangular	$0.4\ E_{40\mathrm{MPa}}\cdot I_g$ $(0.32\ E_{40\mathrm{MPa}}\cdot I_g)$	-	Ig	$0.7\cdot I_g$	$0.4\ E_{40\mathrm{MPa}}\cdot I_g$
		T/L	$0.35\ E_{40\mathrm{MPa}}\cdot I_g$ $(0.27\ E_{40\mathrm{MPa}}\cdot I_g)$	-	Ig	$0.6\cdot I_g$	$0.35\ E_{40\mathrm{MPa}}\cdot I_g$
	column	$N^*/f_c'\ A_g$>0.5	$0.80\cdot I_g\ (0.80\cdot I_g)$	-	Ig	$1.0\cdot I_g$	$1.0\cdot I_g$
		$N^*/f_c'\ A_g$=0.2	$0.55\cdot I_g\ (0.50\cdot I_g)$	-	Ig	$0.8\cdot I_g$	$0.66\cdot I_g$
		$N^*/f_c'\ A_g$=0.0	$0.40\cdot I_g\ (0.30\cdot I_g)$	-	Ig	$0.7\cdot I_g$	$0.45(0.35)\cdot I_g$
	wall	$N^*/f_c'\ A_g$=0.2	$0.48\cdot I_g\ (0.42\cdot I_g)$	-	Ig	$0.7\cdot I_g$	A_s at ULS
		$N^*/f_c'\ A_g$=0.1	$0.40\cdot I_g\ (0.33\cdot I_g)$	-	Ig	$0.6\cdot I_g$	
		$N^*/f_c'\ A_g$=0.0	$0.32\cdot I_g\ (0.25\cdot I_g)$	-	Ig	$0.5\cdot I_g$	
Canadian Standard Association	beam		$0.4\ I_g$	-	-		
	column		$\alpha c\ I_g$	-	-		
	wall		$\alpha w\ I_g$	$\alpha_w A_g$	-		
NTC2008	all		$0.5\ E\cdot I_g$	$0.5E\cdot I_g$	-		
FEMA 356	R/C Beam		$0.5\ E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
	P/C Beam		$E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
	Column	$N^*/f_c'Ag\geq0.5$	$0.7\ E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
		$N^*/f_c'Ag\leq0.3$	$0.5\ E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
	Uncracked wall		$0.8\ E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
	Cracked wall		$0.5\ E_c\cdot I_g$	$0.4\ E_c\cdot A_w$	-		
ACI 318	R/C Beam		$0.35\cdot I_g$	-	$0.5\cdot I_g$		
	Column		$0.70\cdot I_g$	-	I_g		
	Uncracked wall		$0.70\cdot I_g$	-	I_g		
	Cracked wall		$0.35\cdot I_g$	-	$0.5\ I_g$		

Note: I_g is the moment of inertia of the gross uncracked section with respect to the central axis;

h, l_c are the height of the cross section and the span of the beam respectively;
A_g is the area of the gross section, disregarding the area of the reinforcing steel;
f'_c is the compressive strength of concrete;
N^* is the normal force acting at the ultimate limit state, that is taken positive in compression mode;
E_c is the elastic modulus of concrete;
E_{40MPa} is the elastic modulus for concrete with a strength of 40 MPa (taken as a reference regardless of the actual concrete strength);
A_w is the area of the shear resisting section;
α_c , α_w are parameters including the sum of a fixed value and the ratio between the applied normal stress and compression strength for column and wall respectively. They have to be minor or equal to 1;
μ is the ductility factor

Table 2 Effective stiffness of structural members resulting from scientific literature

Authors Reference	Experimental Findings	Limitations
Branson (1965)	$I_{eff} = (M_{cr}/M_a)^3 I_g + [1-(M_{cr}/M_a)^3] I_{cr}$	
Sugano (1970)	$\mathrm{K_r/K_i} = (0.043 + 1.64\mathrm{n}\rho_\mathrm{t} + 0.043\,\mathrm{a/h} + 0.33\eta)(\mathrm{d}/h)^2$	$0.4 < \rho_\mathrm{t} < 2.8\%$ $2 < a/h < 5$ $0 < \mathrm{P/A_g f'_c} < 0.55$
Grossman (1981)	$I_{eff}=(M_{cr}/M_a)^4 I_g$ for $M_a/M_{cr} \leq 1.6$ $I_{eff}=0.1c_e(M_{cr}/M_a)^4 I_g$ for $1.6<M_a/M_{cr}\leq 10$	
Mirza (1990)	$EI_{eff} = [(0{,}27 + 0{,}003\,l/h - 0{,}3\,e/h)E_c I_g + E_s I_{se}]$	
Priestley (1998)	$EI_{eff} = M_n/\varphi_y$	
Wang (2001)	$I_{eff} = (2M_{cr}^2/M_a^2 + M_s^2)I_g + [1-(2M_{cr}^2/M_a^2 + M_s^2)]I_{cr}$ $I_{eff} = (M_a + M_s/M_a + M_s)I_g + [1-(2M_{cr}/M_a + M_s)]I_{cr}$ $I_{eff} = I_g$	$for\ M_{cr} \leq M_s$ $for\ M_s \leq M_{cr} \leq M_a$ $for\ M_{cr} \geq M_a$
Mehanny (2001)	$EI_{eff}/EI_{g,tr} = (0{,}4 + P/2{,}4P_b) \leq 0{,}9$	
Khuntia Ghosh (2004)	$\mathrm{EI_{eff}} = \mathrm{E_c I_g}(0.80 + 0.25\rho_\mathrm{c}) \times \left(0.30 + 0.5\frac{P}{\mathrm{P_u}}\right) \leq \mathrm{E_c I_c}$, $\geq \mathrm{E_c I_{beam}}$ $\mathrm{EI_e} = \mathrm{E_c I_g}(0.10 + 25\rho_\mathrm{b})\left(1.2 - 0.2\frac{\mathrm{b}}{\mathrm{h}}\right) \leq 0.6\mathrm{E_c I_g}$ $\mathrm{EI_e} = \mathrm{E_c I_g}(0.10 + 25\rho_\mathrm{b})\left(1.2 - 0.2\frac{\mathrm{b}}{\mathrm{h}}\right) \times (1.15 - 4\times 10^{-5}\mathrm{f'_c}) \leq 0.6\mathrm{E_c I_g}$	$(1.2 - \frac{0.2b}{h}) \leq 1$ $\mathrm{P_u} = 0.85\mathrm{f'_c}(\mathrm{A_g} - \mathrm{A_{st}}) + fyAst$
Elwood Eberhard (2006, 2009)	$\mathrm{E_c I_{eff}/EI_g} = 0.2$ if $\eta \leq 0.2$ $\mathrm{E_c}I_{eff}/EI_g = 5/3\eta - 4/30$ if $0.2 < \eta \leq 0.5$ $\mathrm{E_c}I_{eff}/EI_g = 0{,}7$ if $\eta > 0.5$	For rectangular columns
	$\frac{EI_{effcalc}}{EI_g} = \frac{0.45 + 2.5\eta}{[1 + 110(\phi_r/D)(D/a)]} \leq 1.0\ \&\ \geq 0.2$	For rectangular and circular columns
Kumar Singh (2010)	$\frac{\mathrm{E_c}I_{eff}}{E_c I_g} = \begin{pmatrix} 0.35\ for\ \eta \leq 0.2 \\ 0.175 + 0.875\,\eta\ for\ 0.2 \leq \eta \leq 0.6 \\ 0.7\ for\ \eta \geq 0.6 \end{pmatrix}$	For normal strength concrete
	$\frac{\mathrm{E_c}I_{eff}}{E_c I_g} = \begin{pmatrix} 0.35\ for\ \eta \leq 0.1 \\ 0.24 + 1.1\,\eta\ for\ 0.1 \leq \eta \leq 0.6 \\ 0.9\ for\ \eta \geq 0.6 \end{pmatrix}$	For high strength concrete

Note: I_g , f'_c , E_c, h as per Table 1.

E_s is the elastic modulus of steel and $n=E_s/E_c$; EI_{eff} is the effective stiffness;

I_{eff} , I_{cr} are the effective and first cracking moments of inertia of the gross section;

I_{se} is the moment of inertia of steel reinforcements with respect to the central axis of gross section;

M_{cr},M_a are the cracking and SLS applied moments; M_s is the maximum bending moment due to short-term loads;

M_n is the nominal bending strength of the section;

ρ_t is the ratio of reinforcement area in tension over the total concrete area; A_{st} is the reinforcement area;

ρ_c,ρ_b are the reinforcement ratios in columns and beams respectively;

K_r/K_i is the ratio between the secant stiffness at yielding and the initial stiffness of the member;

c_e is a parameter related to concrete and steel; l and e are the buckling length and eccentricity of the normal load;

P, P_u are respectively the design normal load and the ultimate load in compression;

P_b is the normal action for balanced failure; $\eta = P/A_g f'_c$ is the normalized axial stress (compression is positive);

φ_y is the curvature at yielding of rebars; ϕ_r is the rebar diameter;

D is the diameter of a circular columns or the height of a rectangular column;

b is the width of the rectangular section; a is the shear span; d is the effective height of the section

Table 3 Paulay and Priestley (1992) recommendations, taken as reference by ACI318

R/C member	**Range**	**Recommended Stiffness**
Rectangular beams	0.30-0.50 I_g	0.40 I_g
T/L beams	0.25-0.45 I_g	0.35 I_g
Columns for η >0.5	0.70-0.90 I_g	0.80 I_g
Columns for η =0.2	0.50-0.70 I_g	0.60 I_g
Columns for η = -0.05	0.30-0.50 I_g	0.40 I_g
Coupling beams	-	$0.20I_g/[1+3(h/l_n)^2]$
Walls	-	$I_e = (100/f_y + \eta)I_g$

Note: η and I_g as per Table 2;

h, l_n are respectively width and clear span of the coupling beam;

f_y is the yielding stress of reinforcement bars.

In 2001 Wang proposed a revisited Branson's approach for beams in bending; in the same year Mehanny (2001) introduced a value for the effective stiffness based also on the area of the reinforcement as well. Crowley (2003) confirmed then the validity of the earlier experimental approach proposed by Sugano (1970).

Recently Khuntia and Ghosh (2004) focused on the evaluation of the effective stiffness of R/C frames subjected to lateral loads, by introducing a new parameter: namely the steel ratio. The effective stiffness EI_{eff} of a R/C column under seismic or wind loads was proposed as well, together with a simplified expression for R/C beams casted with normal-strength concrete and another expression valid for high-performance/high-strength concrete.

Based on the results of their test, Elwood and Eberhard (2009) proposed the values reported in Table 2 to reduce the stiffness for R/C columns with rectangular cross sections in R/C frames under seismic loading. A general equation was proposed for rectangular and circular columns, disregarding the reinforcement ratio in the cross section. In this last, tentative values for ϕ_r/D may be 1/25 for bridge piers and 1/18 for columns in ordinary buildings.

Recent research by Kumar and Singh (2010) has provided design values for the effective stiffness of cracked R/C frames, for both normal-strength or high-strength concrete. Table 2 summarizes the formulations derived from the cited scientific studies.

Table 3 reports a detailed list of effective stiffness values for R/C structural members according to the recommendations by Paulay and Priestley (1992), taken as reference by ACI318.

According to the scientific literature and to international design codes, five different approaches were used in the parametric analysis presented herein for an ordinary multi-storey building. The values of the effective stiffness of R/C elements were computed according to the following codes or models: NTC-2008 and EC8 (based on the same approach); NZS-3101 code; Khuntia and Ghosh (2004), Elwood and Eberhard (2009), Kumar and Singh (2010).

2. Structural analysis

A response spectrum analysis was performed on a six-storey building placed in a seismic area having a rectangular plan section (size 31 m×22.5 m) and two vertical symmetry planes. A very limited asymmetry occurs with respect to the vertical mean plane aligned with the long side, due to the presence of the stairwell and of the lift. The long side is aligned with the horizontal axis (*x* axis) of the global reference system and the short side is aligned with the vertical axis (*y* axis), as illustrated in Fig. 1. Such building has been designed according to the modern seismic provisions of EC8 and Italian code. Its structural typology is very representative of residential building that were built over the last decades in many European countries.

According to both Eurocode 8 and the Italian design code, with reference to the gross inertia of structural members the building is to be considered as a dual system since vertical and lateral loads are mainly carried by both frame elements and walls. The concrete slabs consist of unidirectional T-beams oriented in the *y* direction as indicated in Fig. 1; the building (Fig. 2) has an inter-storey height of 3.80 m on the ground floor and 3.0 m at the remaining levels. The stairway and lift well have R/C walls (Fig. 1).

The building is an earthquake resistant R/C frame designed in low ductility class (CDB) according to NTC-2008 and EC8 (2005). In the seismic analysis the floors were considered as in-plane rigid diaphragms. The walls and the columns are 3 and 44, respectively, with axis-to-axis spacing equal to 4-5 m in the *x* direction, and to 4.5 m in the y direction (Fig. 1). The section of the columns is 500×500 mm for the first three floors, 400×400 mm for the fourth floor and 300×300 mm for the last two floors (Fig. 2). According to these schemes different parameters are chosen to compare the different approaches for the evaluation of the effective stiffness under seismic loads.

The response spectra were used in conformity with Italian NTC-2008 and modal responses were combined by means of the CQC (Complete Quadratic Combination) rule, which is more effective for closely spaced modes. The elastic spectrum at the ultimate limit state (10% probability) in the site of the building (residential/office use), for damping equal to 5%, regular topography and category of soil classified as type B (very dense sand and gravel or consistent clays) is shown in Fig. 3, which implies an amplification factor equal to 1.2 in the spectra value. The design spectrum was obtained from the elastic spectrum by dividing for the dissipation factor q (computed according to NTC-2008, and EC8): $q=q_0K_R=3.0\alpha_u/\alpha_1K_R=3.0\times1.3\times1.0=3.9$. The design spectra for the ultimate and damage limit states are shown in Fig. 3.

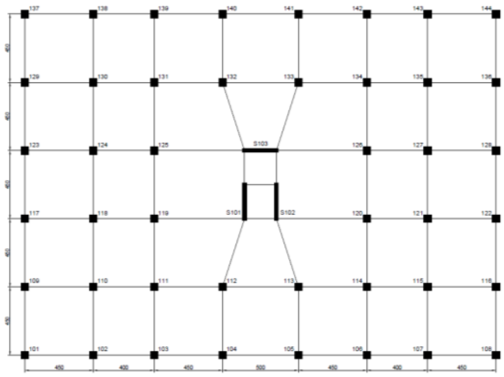
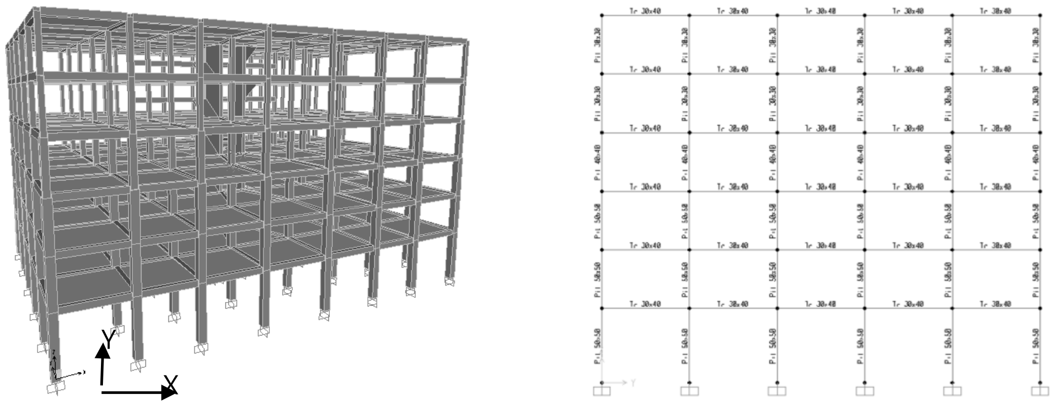
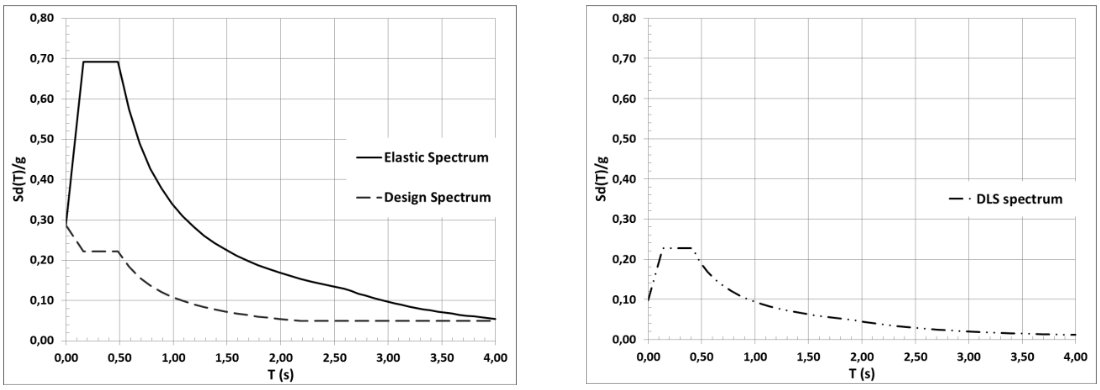


Fig. 1 Plan section of the R/C frame



(a) (b)

Fig. 2 Geometry and size of the R/C frame investigated in this study



(a) (b)

Fig. 3 Acceleration spectra for the Ultimate Limit State (a); and for the Damage Limit State (b)

According to the above-mentioned design assumptions, a parametric study was performed in order to test the response of the modal analysis according to the five different approaches adopted for the evaluation of the effective stiffness. The variables chosen in the parametric study are: type/geometry of shallow R/C beams; compressive strength of concrete; reinforcement ratio; axial-load ratio. The influence of beam cross section height in the seismic behavior of R/C has been investigated by Xing (2013) in terms of shear stress and failure mode of beam-column joints. Beam types are distinguished in two classes, namely *ordinary* and *wide beams*. Herein beams are defined *ordinary* or *deep* if they have section height greater than slabs height (250 mm), otherwise when the height of the beam is equal to that of the slabs, they are considered *wide* or *flat*.

Ordinary beams with section size 300×400 mm were used as an alternative to wide beams having a section size of 700×250 mm. Two concrete grades were taken under examination: C20/25 and C45/55 (compressive strength on cylinders $f_c^{'}$=20 MPa and 45 MPa, respectively). Concrete cover was set to 30 mm for all cases. Two steel ratios were considered for the longitudinal reinforcement in the columns (1% and 2%). This last parameter affects only the Khuntia and Ghosh model in the evaluation of the reduced stiffness, since the other recommendation do not take account of this variable. Beam-column panel zones have not been considered in the analysis.

The evaluation of the reduced stiffness shall take into consideration the distinction between SULS and SDLS, in order to maximize the displacements in the evaluation of second-order effects (parameter θ in EC-8) at the ultimate limit state. However, in the case under examination there were no significant differences between the outcomes of the analyses performed with reduced stiffness at the SULS and SDLS. At both limit states the spectral design accelerations corresponding to the first modes periods, are almost the same. Indeed in both cases the mass of the structure and the axial load ratios $P/A_g f_c'$ in vertical members, which affects the global lateral stiffness, are the same.

2.1 Effect of concrete compressive strength

In the parametric study two values of $f_c^{'}$ (20 and 45 MPa) have been considered. For the columns, the axial-load ratio $P/A_g f_c'$ ranges from 0.05 to 0.25 (ordinary/deep beams or wide/flat beams); the results obtained with the various stiffness models show that in the selected range the influence of concrete is quite small and the effective stiffness ratio $EI_{eff}/E_c I_g$ decreases slightly at increasing values of $f_c^{'}$, as illustrated in Fig. 4.

This result agrees with Kumar and Singh's findings (2010), namely that the concrete compressive strength has a nonlinear complex dependence on the effective stiffness. In the range of the axial-load ratio 0.4-0.7, the effective stiffness ratio increases significantly with $f_c^{'}$. This fact was also observed by Khuntia and Ghosh (2004), and by Elwood and Eberhard (2009). For smaller values of the axial-load ratio, however, the effect of $f_c^{'}$ is much smaller and of opposite sign (that is, the effective stiffness-ratio decreases at increasing values of $f_c^{'}$). For low values of the axial-load ratio, the crisis of the section is governed by the yielding of the steel; therefore, the effective stiffness is hardly sensitive to the concrete strength. Furthermore, at very high values of the axial-load ratio a similar trend can be observed but this range of the axial-load ratio has little practical significance. NTC-2008 brings in a limitation for the axial load in columns; when the axial-load ratio does not exceed 0.65 the influence of concrete compressive strength is low and the effective stiffness ratio decreases slightly or remains almost constant with any increase of $f_c^{'}$.

In the present case, the model less sensitive to the compressive strength was found to be that of Kumar and Singh which is the only model that provides different equations for normal-strength and high-strength concretes in columns. With reference to beams, all the stiffness models considered in this study are devoid of any dependence on concrete compressive strength, except Khuntia and Ghosh's model (2004). In this case, the greater is the compressive strength $f_c^{'}$ the smaller the effective stiffness-ratio, as shown in Fig. 5.

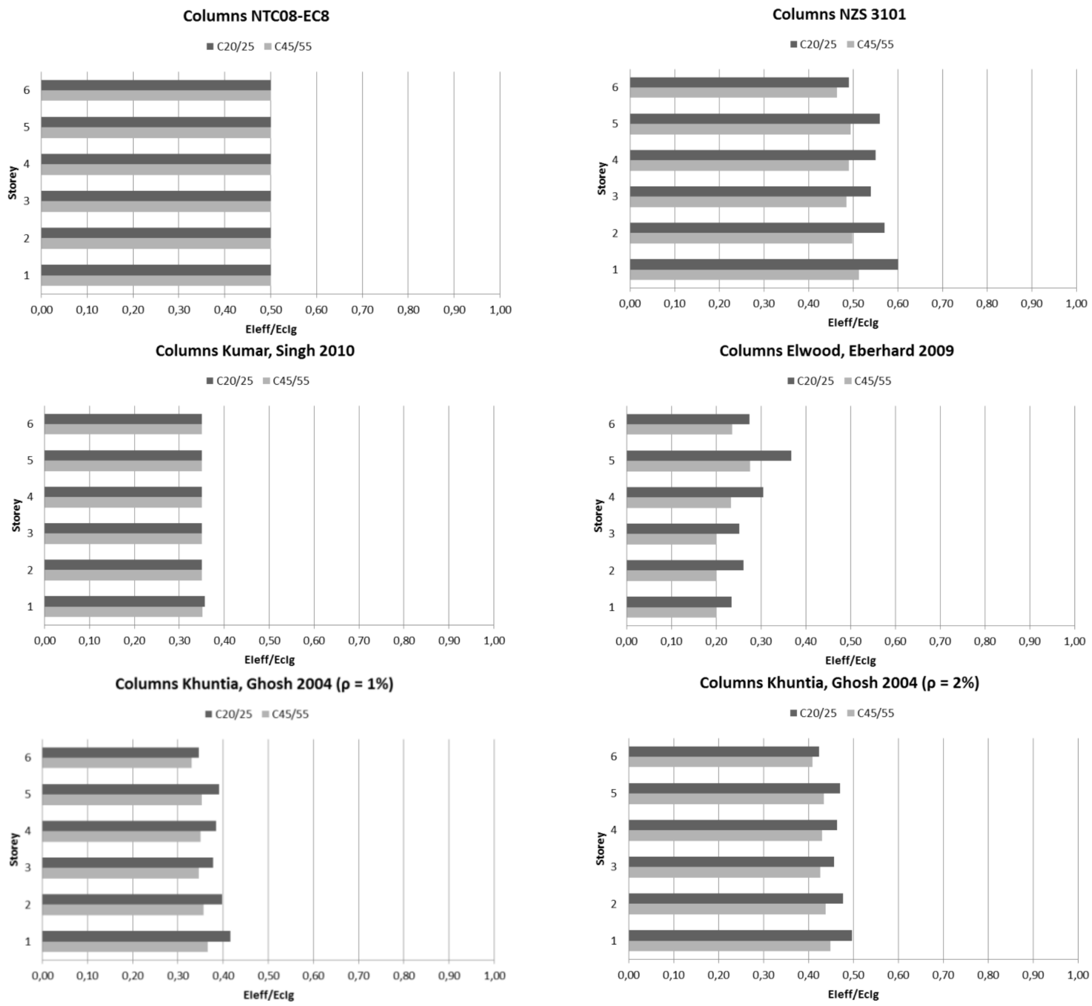


Fig. 4 Effective stiffness-ratio in the columns for different values of concrete strength f_c'

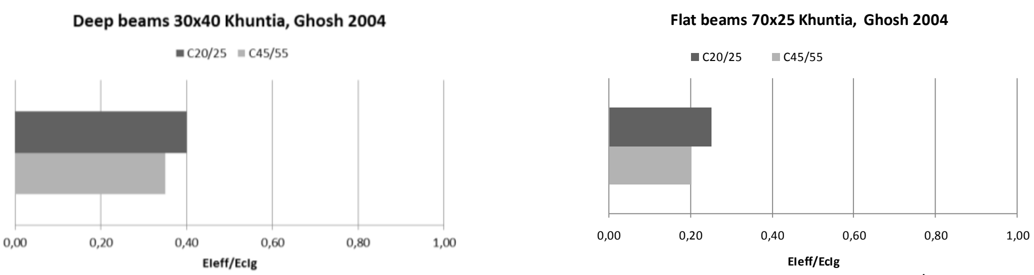


Fig. 5 Effective stiffness-ratio in beams for different values of concrete strength $f_c^{'}$ according to Khuntia & Ghosh's model

2.2 Effect of the beam type

To study the influence of the beam type on the effective stiffness, shallow beams, either ordinary (or deep with section $b \times h$=300×400 mm) or wide (or flat with section $b \times h$=700×250

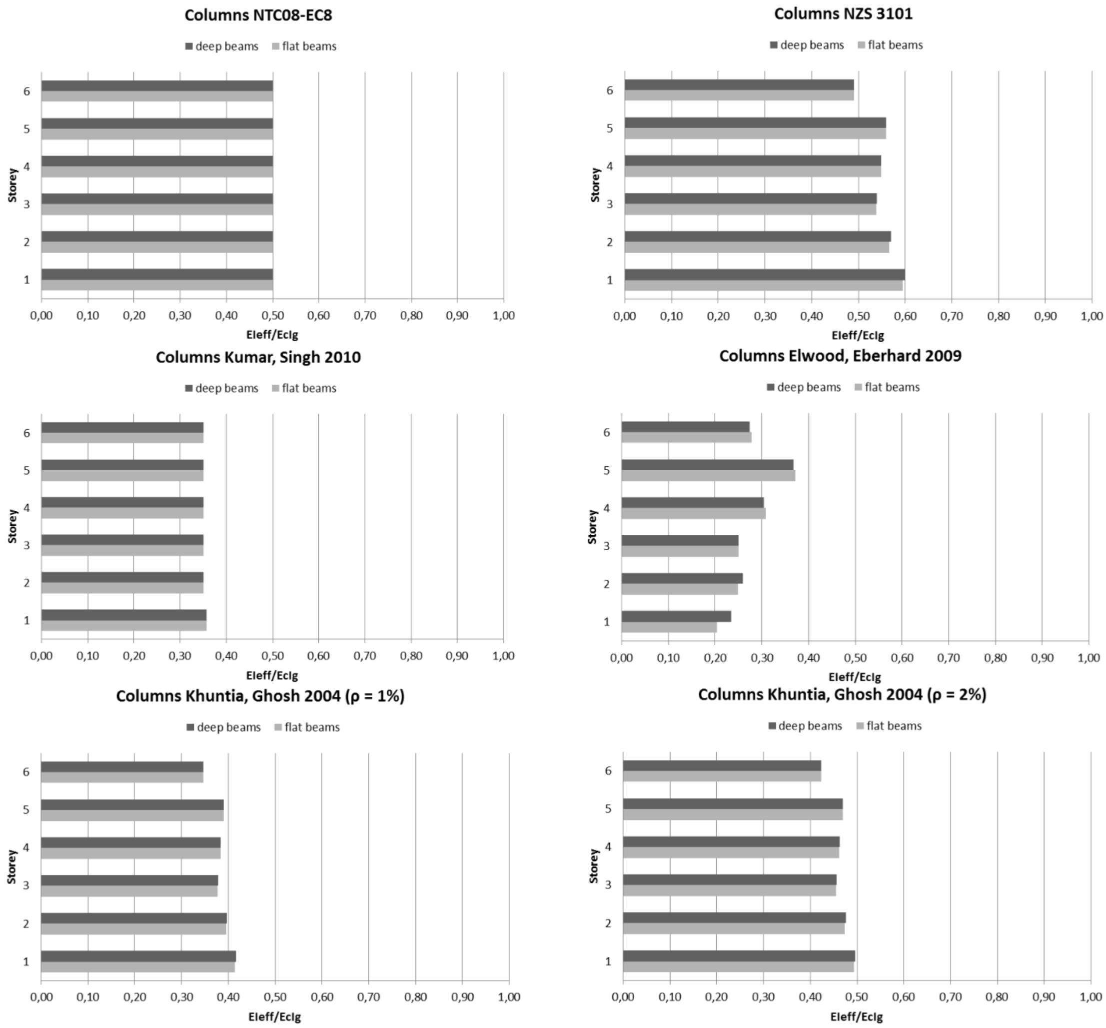


Fig. 6 Effective-stiffness ratio in the columns for different beam types

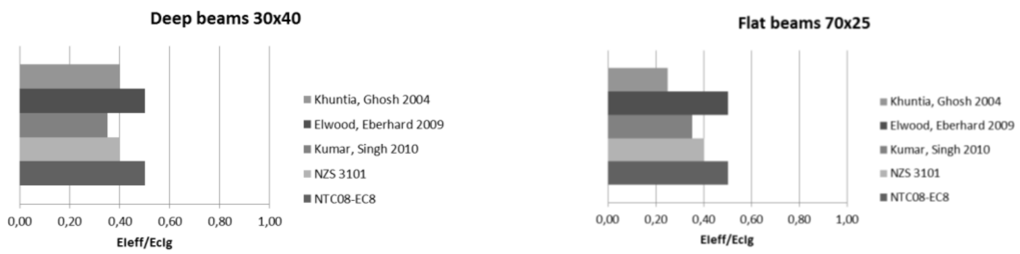


Fig. 7 Effective-stiffness ratio in the beams for different beam types (deep/flat - ordinary/wide beams)

mm), were considered as slab-bearing members; all other coupling beams had a rectangular section (400×250 mm).

For both low-grade and high-strength concretes, the results obtained with the various stiffness models show that the effective-stiffness ratio for the columns is little affected by the beam type, as illustrated in Fig. 6, because in neither case (deep beams or flat beams) the axial load P (and the corresponding internal force) changes. Only the model of Elwood and Eberhard (2009) showed an influence of the beam type on the stiffness of the columns. The effect is very small and depends on the D/a ratio between the depth of column section in the loading direction and the shear span. Thus in the case of flat beams, especially in the lower floors, there is a reduction of the shear span of the columns with respect to the case with deep beams, leading to increasing values for the ratio D/a and consequently to decreasing values for the effective stiffness. As for the beams, all stiffness models provide identical values for the deep beams and for the flat beams, except Khuntia and Ghosh's model (2004). That model includes the b/h ratio (between the width of the section and its effective depth). In this case, any increase of the b/h ratio is accompanied by a reduction of the effective-stiffness ratio, which means that the stiffness of flat beams is reduced more than that of deep beams (by almost 50%, see Fig. 7).

2.3 Effect of steel ratio

NTC-2008 provides limitations on the area of the longitudinal reinforcement in columns. It has to be comprised between the lower limit 0.01 A_g and the upper limit 0.04 A_g. Design practice, however, commonly limits the steel ratio to 2-3% to avoid any reinforcement congestion. Therefore, two values of reinforcement ratios (ρ=1% and 2%) are considered in the present paper.

The effect of the steel ratio in columns was evaluated exploiting the stiffness model proposed by Khuntia and Ghosh (2004). The other models do not include that parameter. In either ordinary or wide beams, made of either low-grade or high-grade concrete, any increase of the steel ratio leads to a significant increase of the effective stiffness of the columns. Fig. 8 reports the effective stiffness ratio for deep beams. The same appreciable increase of stiffness due to a small increment in steel ratio occurs in the case of shallow beams as well.

It can be seen that the effect of the steel ratio is not relevant for all possible values of the axial-load ratio. For $P/A_g f_c'$<0.4, as required by NTC-2008, the crisis of the section is governed by the yielding of the reinforcement; therefore, the influence of the steel ratio is greater than for the values of the axial-load ratio comprised between 0.4 and 0.7. At very high axial-load ratios (≥0.7), steel ratio plays a major role but the related values of axial force are of no practical significance.

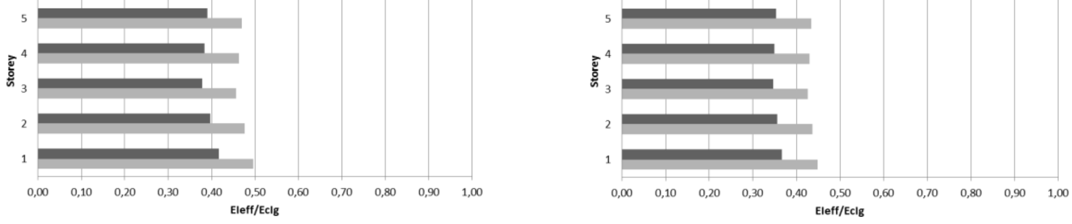


Fig. 8 Effective stiffness ratio in columns for different reinforcement ratios (1% and 2%) - ordinary beams

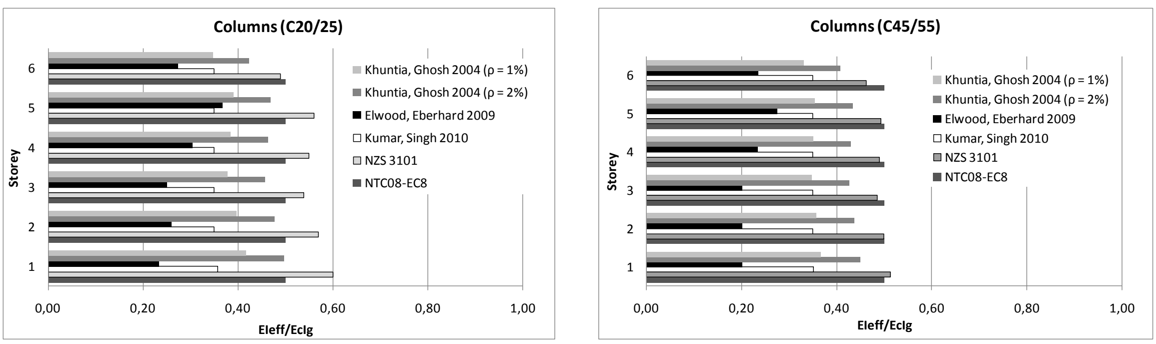
3. Discussion of the analytical results

The analytical results obtained with the different stiffness models for columns are shown in Fig. 9(a) and (b), respectively for low-grade (C20/25) and high-grade (C45/55) classes of concrete. In the case of low-grade concrete the effective stiffness prescribed for columns by NZS 3101 is greater - on average - by 10% in the highest floors ($0.55E_cI_g$) and by 20% at the first level ($0.60\ E_cI_g$) compared to the lower limit of $0.50E_cI_g$ provided by NTC-2008 and EC-8. This does not hold true for the last level, where a similar stiffness ratio was obtained.

For high-grade concrete the NZS 3101 guidelines are aligned with NTC-2008 and EC-8 in terms of effective stiffness, except for a slightly lower value for the last level because of its low axial load. Similarly, in the case of low-grade concrete, Khuntia and Ghosh's model provides the same stiffness values as EC-8 only in the first floor for ρ=2%. Elsewhere, at the contrary, appear lower values, especially when a 1% steel ratio is adopted. In this case an effective stiffness lower by 10÷15% was found if compared with EC-8 and NTC-2008. The effective-stiffness values were in fact equal to $0.40E_cI_g$ and $0.35E_cI_g$ for C20/25 and C45/55, respectively. A residual stiffness value of $0.35E_cI_g$ was obtained for the columns by using the equations recommended by Kumar and Singh (both classes of concrete).

With reference to columns, Elwood and Eberhard's model turned out to be the most conservative. For low-grade concretes the effective stiffness of $0.25E_cI_g$ (valid for the first three levels) tends to increase at higher floors (up to approximately $0.35E_cI_g$) and to decrease again at the top floor. That variation along the height of the building is due - as already mentioned - to the effect of the shear span of the columns - on the stiffness relation by Elwood and Eberhard, beside the effect of the axial-load ratio P/A_gf_c'. With a slight numerical difference, the same considerations apply to the building made of high-grade concrete. The New Zealand recommendations and scientific literature, however, agree in attributing lower stiffness values to the columns placed along the sides and in the corners. That is not taken into account in the Italian and European codes, which provide constant values to the reduced stiffness.

Concerning the effective stiffness ratio (EI_{eff}/E_cI_g) in the columns, Fig. 10 presents a comparison among different models as a function of the axial-load ratio (P/A_gf_c'), for both the low-grade and high-grade classes of concrete. It should be observed from the figures that - for low axial loads, such as those imposed by NTC-2008 (P/A_gf_c'≤0.31 for high-ductility class and P/A_gf_c'≤0.37 for low-ductility class) - the models under investigation display, for the cracked



(a) (b)

Fig. 9 Reduced stiffness in R/C columns for (a) f_c'=25 MPa (b) f_c'=45 MPa

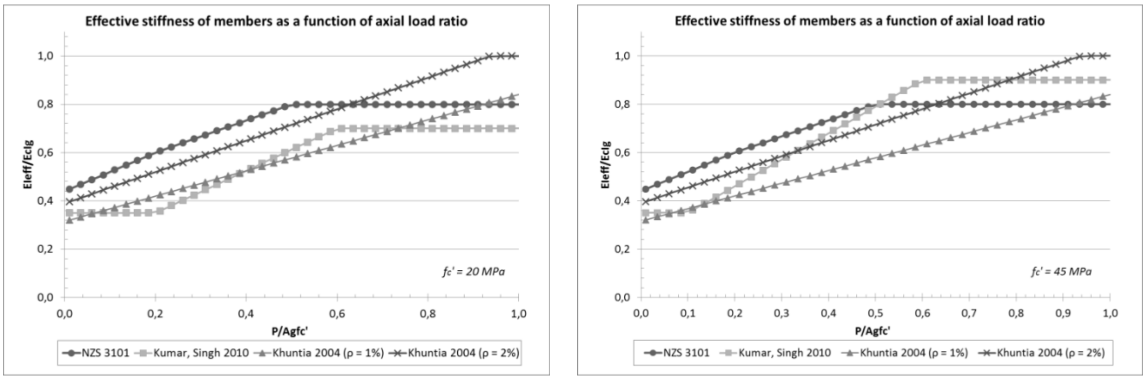


Fig. 10 Effective stiffness ratio of the columns as a function of the axial-load ratio

sections, lower stiffness values than the lower limit required by both NTC-2008 and EC-8 design codes. With reference to beams, all the models provide lower stiffness values than those suggested by both NTC-2008 and EC-8. NZS 3101 assumes for the beams a constant value (=$0.40E_cI_g$), while according to Kumar and Singh's model the value of the effective stiffness is equal to $0.35E_cI_g$ for both ordinary and wide beams, and for both concrete grades. Instead, Khuntia and Ghosh's model introduces a reduction in the effective stiffness with an increase of the b/h ratio. Thus wide beams exhibit a greater stiffness reduction compared to ordinary beams. Moreover, as in the case of the columns, an increase of f_c' provokes a reduction of the effective-stiffness ratio. In particular, for wide beams and high-grade concrete there is a minimum value of $0.20E_cI_g$, while for the ordinary beams and the low-grade concrete there is a maximum of $0.40E_cI_g$.

The results of the analyses carried out were instrumental in quantifying second order P-Δ effects (if any) because of the reduced stiffness. NTC-2008 and EC8 (2005) neglect geometric nonlinearities when - at each floor - the following ratio does not exceed 10%

$$\theta = \frac{P \cdot d_r}{V \cdot h_f} \leq 0.10 \tag{1}$$

where P is the total vertical load applied above the level of the structure under consideration in the seismic design; d_r is the average lateral drift between two subsequent floors; V is the total horizontal shear force at the floor in question; h_f is the vertical spacing between the floor under investigation and the floor underneath.

The displacements d_E under seismic action at the ultimate limit state (ULS) are obtained by multiplying the values d_{Ee} (obtained from linear analysis, dynamic or static) by the factor μ_d, according to the following expression

$$d_r = d_E = \pm\mu_d \cdot d_{Ee} \tag{2}$$

where d_r is the displacement under the design seismic action at the ULS; d_{Ee} is the displacement determined by linear analysis; and μ_d depends on the natural frequency of the structure and seismic spectrum, easy to be computed according to NTC-2008 and to EC8 (2005). When θ ranges from 0.1 to 0.2, the effects of geometric nonlinearities may be taken into account by increasing the effect of the horizontal seismic action by a factor equal to $1/(1-\theta)$, while for θ between 0.2 and 0.3 it is necessary to perform a nonlinear analysis; however, θ can never exceed 0.3.

In this paper the factor θ is investigated through numerical analyses run according to the structural parameters already introduced, and according to the proposed effective stiffness models for beams and columns. The displacement d_r has been monitored with the purpose to determine whether and to what extent NTC-2008 and EC-8 may underestimate geometric nonlinearities, if compared to other design recommendations. The results of such analyses are presented first in Fig. 11, with reference to the most severe cases of low-grade concrete, for seismic forces acting along the *x* and *y* axes.

In the case of high-grade concrete (Fig. 12), the above-mentioned results were then confirmed. The analysis gives rather useful outcomes, since NZS 3101, Khuntia-Ghosh's and Elwood-Eberhard's models bring in values of θ greater than 0.1 in both *x* and *y* directions. Among these, only Elwood-Eberhard's model leads to some second-order effects in the first floor (direction *y*). The highest *P*-Δ effects are obtained for Kumar and Singh's model; in that case, the effective stiffness leads to values of θ that are close to 0.10-0.13 in the first and second floor in *x* direction, and values of θ that are close to 0.19 in the second and third floor in *y* direction, for the case of shallow beams.

Furthermore, it is interesting to observe that Khuntia-Ghosh's model brought in values of θ in the *x* direction very similar to those obtained with Kumar-Singh's model, while in the *y* direction θ is lower, as predicted also by Elwood and Eberhard's model. This is due to a factor of effective stiffness reduction - for beams, - accounted for through the b/h ratio. The results seem interesting since the stiffness reduction of the wide beams in the *x* direction (*b*=700 mm) is greater than the reduction of the wide coupling beams (*b*=400 mm) in *y* direction.

The results of the analysis for the Seismic Damage Limit State were compared with the limitation introduced by NTC-2008

$$d_r < 0.005\ h_r \tag{3}$$

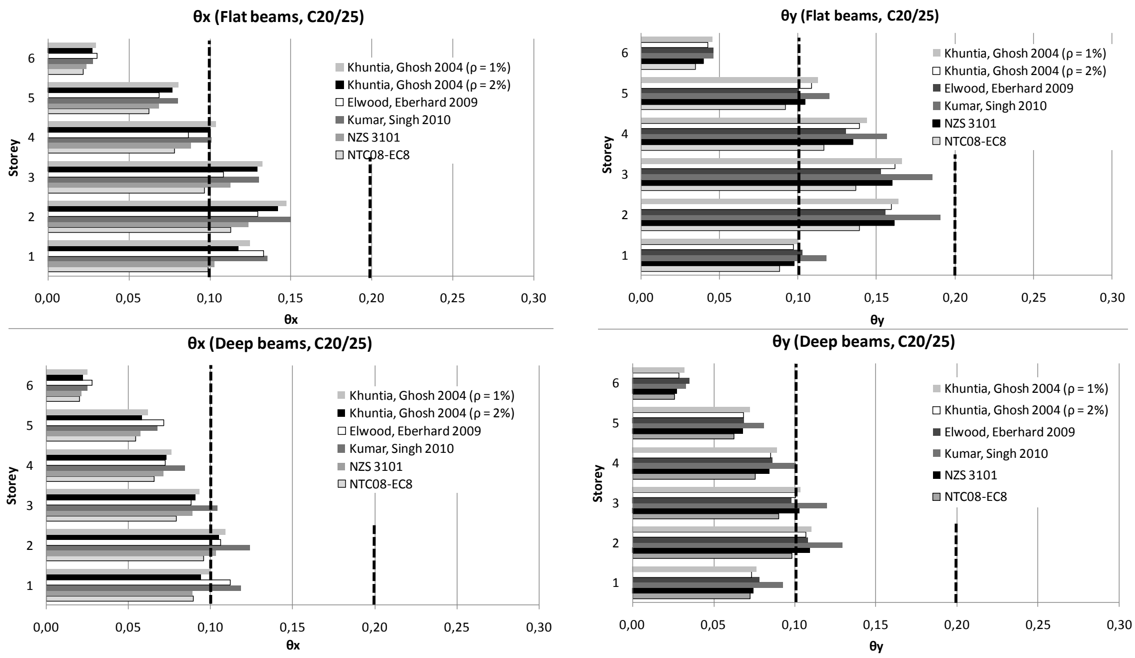


Fig. 11 *P*-Δ effect (f_c'=20 MPa); deep/flat beam=ordinary/wide beam

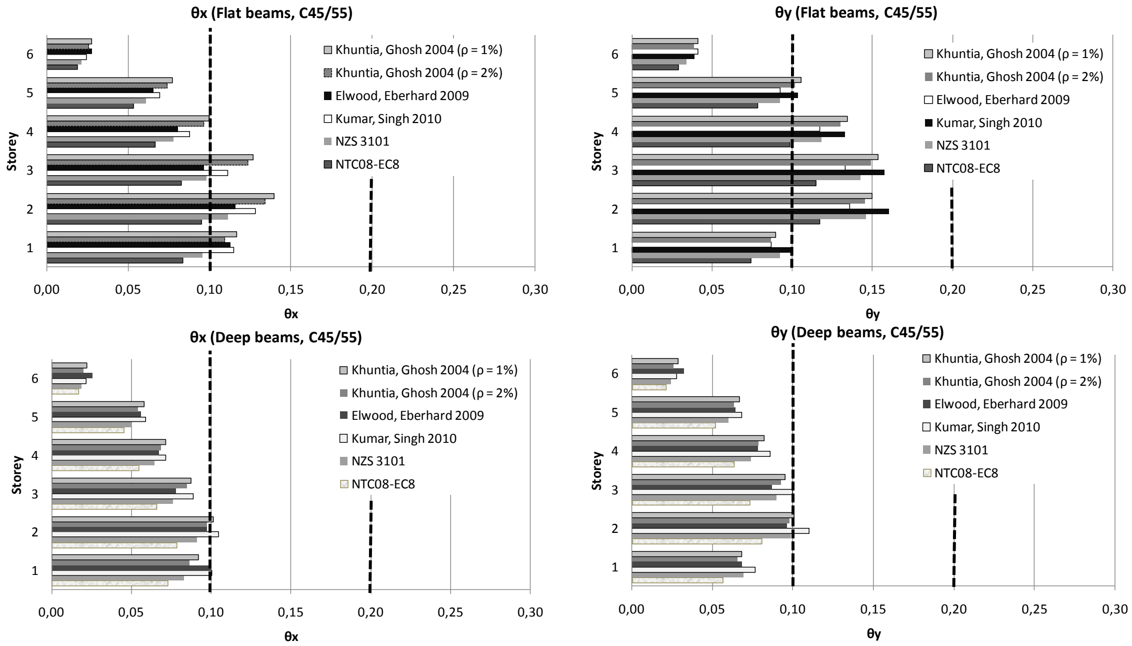


Fig. 12 P-Δ effect ($f_c^{'}$=45 MPa); deep/flat beam=ordinary/wide beam

that means a 0.5% limit ratio between the lateral drift and the inter-storey height.

In this case, the model was assumed with reduced stiffness for both columns and beams. Note that in general the values of the internal forces at the DLS may result different from those at the ULS. This is why the internal forces are computed according to different load combinations based on a different ground acceleration spectrum. In such a case, NTC-2008 and EC8 (2005) amplify the drift according to the values of θ. This analysis was aimed to check whether the results obtained by using NTC-2008 or EC-8 may underestimate the drift compared to the values computed with different reduced-stiffness models. The results shown in Fig. 13, related to the low-grade concrete case, show sizeable differences among the different approaches. According to New Zealand Design Code, the building with ordinary beams exhibits lateral-drift values which are by 4% higher than those computed according to EC-8 and NTC-2008. Furthermore, such difference increases by applying the equations provided by Khuntia-Ghosh's model (+7% and +5% for ρ=1% and ρ=2%, respectively).

Comparing the results obtained with EC-8 with those obtained with Elwood and Eberhard's model, the latter exhibits a reduced stiffness greater by 15% at the first level, which decreases by 8-9% at the top level. The largest difference occurs when comparing the displacements computed with EC-8 and those computed according to Kumar-Singh's reduced-stiffness model. The difference in this case is about 15% along the total height of the structure. It is worth noting that in all cases the limitation provided by Eq. (3) is always verified (the largest values - 0.004 - were found at the second floor using Kumar-Singh's model).

In the case of wide beams, the differences between EC-8 and the other approaches are more evident. NZS 3101 provides drift values 7% higher, while Elwood and Eberhard's model brings in an increment of 20% at the first level, decreasing until 10% at the top level. The reduced stiffness recommended by Kumar and Singh leads to lateral-drift values that are 17-19% higher than those obtained with both NTC-2008 and EC-8, for all the floors of the building. In the last case, Eq. (3) is verified when adopting the stiffness provided by the Italian Code, while the ratio exceeds the value 0.005 at the levels 2-3-4 and 5 when the other models are used.

For high-grade concrete classes (Fig. 13), it was found that in all cases, both for deep and flat beams, Eq. (3) is satisfied and the highest values of the drift in direction *y* were equal to 0.0035 and 0.0046 h_r respectively. Applying different approaches shows how NZS 3101, Elwood-Eberhard's model and Khuntia-Ghosh's model reduce the effective stiffness when the value of $f_c^{'}$ increases, leading to large lateral drifts in both *x* and *y* directions, which are closer to those computed by using Kumar and Singh's model.

Summing up, the stiffness reduction proposed by both NTC-2008 and EC-8 in many cases may underestimate the lateral drift at the Damage Limit State, compared to the predictions provided by other non-European code provisions and results of research studies in the field, presented herein.

Finally, the adoption of effective stiffness of structural members for linear analysis requires an in-depth reflection due to its remarkable influence on the estimation of lateral force demand. These last results to be reduced because of an induced increment of the first natural periods. In such a way, the spectral accelerations and thus the global lateral shears computed in a response spectra analysis are reduced. In a force-based design, it is of major interest the estimation of how much the base shear demands decrease. For the case under examination, Table 4 reports the first mode period and the ordinate value of the SULS design spectrum in the *y* direction.

For all cases the first mode period of the structure in the y direction, computed for uncracked sections, is in the descending branch of the design spectrum and ranges between 0.67÷0.74 s for deep beams and 0.82÷0.91 s for flat beams. For deep beams the first natural period increment ranges between 39÷73%, leading to a decrease in the global lateral actions of about 29÷38%. For flat beams the effect of the application of effective stiffness leads to similar results: the lateral force decrements range between 19-38%. For shorter or stiffer buildings such decrement of lateral seismic forces is expected to be even greater since the first mode periods may be closer to the plateau of the design spectrum and the derivative of the descending branch is greater. The most conservative results are provided by NTC-2008 and EC-8 which lead to a lateral forces reduction included in the range of 28÷39%, while the maximum decrease is recorded for Kumar (2010) formula (decrement of about -39%).

The stiffness reduction takes into account the fact that after the first shakes the structural members can be considered mostly cracked. Anyway shall not be forgotten that the design spectrum is already derived from the elastic one by considering the ductile capacity of the building itself. Therefore further studies should address the correct estimation of force demands on the structure at ULS contemporarily considering cracking and time-dependent phenomena and ductility by means of the strength reduction factor. Underestimation of force demand shall be prevented.

4. Conclusions and recommendations

Response spectrum analysis is the most common method used in seismic design of ordinary R/C frames, even if linear analysis is unable to take into account nonlinear phenomena provoked by concrete cracking and section reduction, which engender continuous variation of the structural stiffness. Thus, reduced-stiffness values are recommended to introduce these phenomena through the adoption of secant stiffness values. Comparing different approaches for the evaluation of reduced stiffness is, at the same time, necessary yet complicated, as shown in this study for R/C columns and beams belonging to a R/C frame building subjected to seismic loadings.

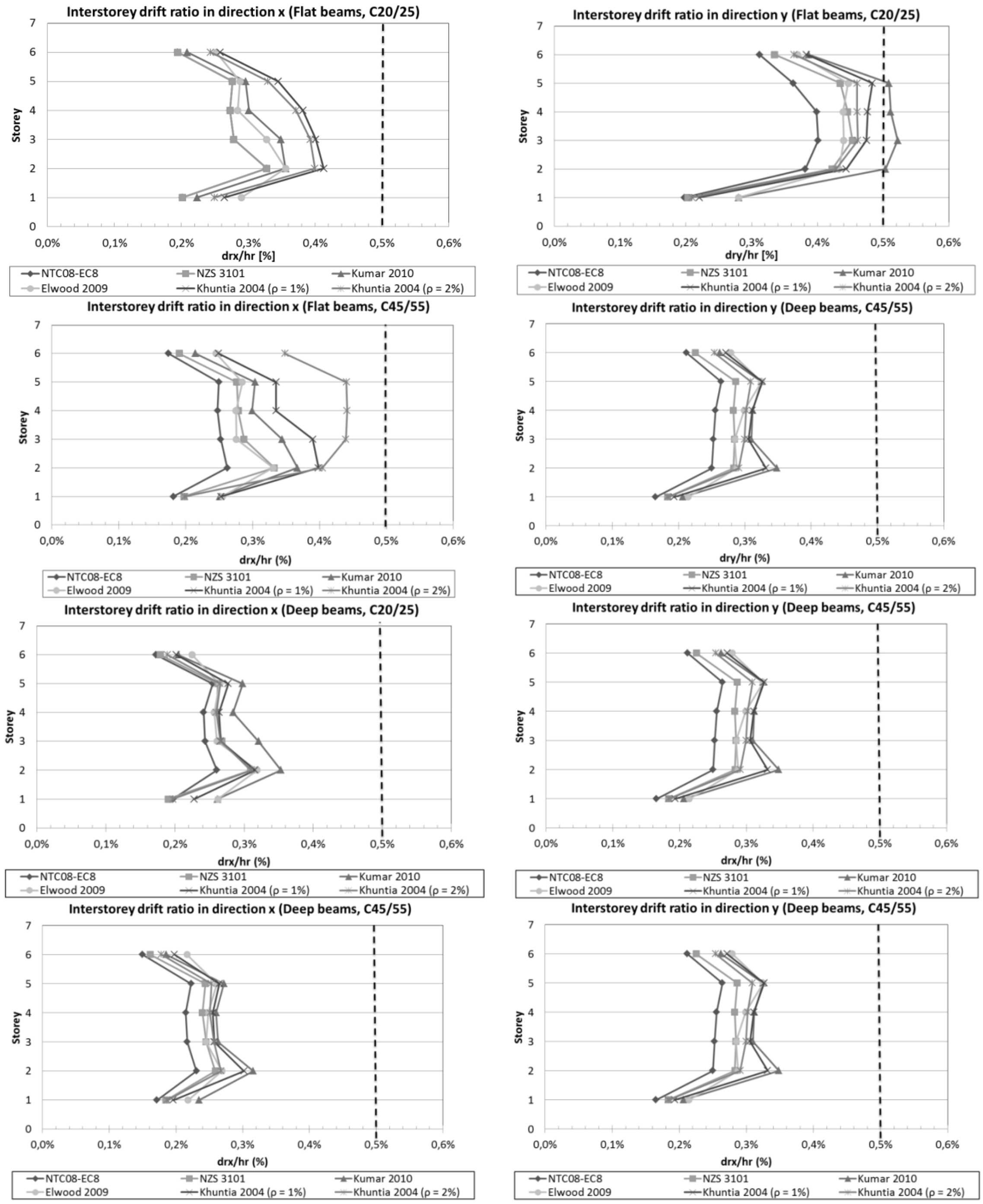


Fig. 13 Lateral inter-storey drift at the DLS in directions *x* and *y* (C20/25-C45/55, flat/deep beams)

Table 4 First natural periods and corresponding seismic action reduction at the ULS for seismic loading

ULS - Deep Beams (30×40) - C20/25					***ULS - Deep Beams (30×40) - C45/55***				
STIFFNESS MODEL	***T_{1y}***	***ΔT(%)***	***S_d(T)***	***ΔS_d(T) (%)***	***STIFFNESS MODEL***	***T_{1y}***	***ΔT(%)***	***S_d(T)***	***ΔS_d(T) (%)***
Uncracked Members	0,74	-	0,118	-	Uncracked Members	0,67	-	0,130	-
Khuntia 2004 - 1%	1,09	49%	0,079	-33%	Khuntia 2004 - 1%	1,04	56%	0,082	-37%
Khuntia 2004 - 2%	1,12	53%	0,080	-32%	Khuntia 2004 - 2%	1,07	61%	0,085	-35%
Elwood 2009	1,14	55%	0,080	-32%	Elwood 2009	1,08	61%	0,085	-35%
Kumar 2010	1,27	73%	0,073	-38%	Kumar 2010	1,15	73%	0,080	-38%
NZS3101	1,13	54%	0,080	-32%	NZS3101	1,05	58%	0,085	-35%
NTC-2008/EC08	1,10	49%	0,083	-30%	NTC-2008/EC08	0,93	39%	0,092	-29%
ULS - Flat Beams (70×25) - C20/25					***ULS - Flat Beams (70×25) - C45/55***				
STIFFNESS MODEL	***T_{1y}***	***ΔT(%)***	***S_d(T)***	***ΔS_d(T) (%)***	***STIFFNESS MODEL***	***T_{1y}***	***ΔT(%)***	***S_d(T)***	***ΔS_d(T) (%)***
Uncracked Members	0,91	-	0,094	-	Uncracked members	0,82	-	0,105	-
Khuntia 2004 - 1%	1,38	52%	0,062	-34%	Khuntia 2004 - 1%	1,31	60%	0,065	-38%
Khuntia 2004 - 2%	1,36	50%	0,064	-32%	Khuntia 2004 - 2%	1,29	57%	0,067	-36%
Elwood 2009	1,33	46%	0,065	-31%	Elwood 2009	1,23	49%	0,07	-33%
Kumar 2010	1,47	62%	0,058	-38%	Kumar 2010	1,34	62%	0,064	-39%
NZS3101	1,34	48%	0,063	-33%	NZS3101	1,26	53%	0,068	-35%
NTC-2008/EC08	1,25	38%	0,069	-27%	NTC-2008/EC08	1,13	38%	0,076	-28%

Note: T_{1y} is the first mode period (s) of the frames in the y direction
$\Delta T(\%)$ is the increment of the first mode period for cracked frames
$S_d(T)$ is the design spectra acceleration at the ULS
$\Delta S_d(T)$ is the variation in the design spectra acceleration at the ULS

NTC-2008 and EC8 provide the simple recommendation to account for cracked stiffness reduced by 50% with respect to the stiffness of the gross section, for the verification at ULS. But in some cases, that does not seem to be on the safe side, as the effects of a number of relevant variables are neglected (such as steel ratio, beam stiffness, concrete properties). This study performed a parametric analysis focusing on R/C frame buildings designed according to EC-8 and Italian NTC-2008. The structural analysis was carried out by comparing the European recommendations with equations provided by New Zealand Design Code, which includes further structural parameters in the computation of the effective stiffness.

Three different theoretical models available in the scientific literature, each one based on experimental evidence and on different theoretical assumptions, but suitable to the design of structural members, were adopted and compared, to predict the behavior of a dual system building which is very representative of Italian and European residential buildings designed over the last decades. The parameters investigated in this study were: concrete compressive strength, beam type (wide beams as an alternative to ordinary beams to support the floors slabs), steel ratio and axial-load ratio (that is variable along the height of the frame). Two concrete grades were considered, in order to represent older existing buildings (made of low-grade concrete, C20/25) and more recent earthquake-resistant buildings (made of high-grade concrete, C45/55).

It was found out that, when the axial load is within the limitations imposed by NTC-2008, there is no significant effect of concrete strength in the computation of the reduced stiffness, while the steel ratio plays a sizeable role. Hence, it is evident that an accurate evaluation of the effective stiffness should include at least the steel ratio.

In the design at the Ultimate Limit State, the presence of wide beams or rather ordinary beams does not influence the effective stiffness of the columns, since the axial load remains almost the same in both cases.

Also, it should be noted that New Zealand's code and the analytical models considered in this study unanimously consider a reduced stiffness for the columns, placed along the sides - or even at the corners - of the building, while this is not mentioned in EC-8 and in NTC-2008. In more details, the reduction of the effective stiffness computed by using the above-mentioned models may be remarkable indeed (more than -50% of the stiffness of the gross section), even within the range of axial loads allowed by the Italian Code. In particular, in the heavily-loaded columns of lower floors, the reduced stiffness is limited to $0.50E_cI_g$ by EC-8 and NTC-2008, while in the upper floors - especially in tapered columns - a value of P/A_gf_c'>0.2 is hardly possible. Hence, values for the reduced stiffness as low as 0.35-$0.40E_cI_g$ are often found. With regard to this point, values contained in the range of 0.45-$0.50E_cI_g$ are admissible according to the New Zealand's Design Code, which is rather close to what is assumed in EC8 and NTC-2008. Therefore, current indications specified by EC8 and by other European codes should be revised, in order to consider the value of $0.50E_cI_g$ at the ULS as an upper limit - rather than lower - for low axial loads.

As for the reduced stiffness of cracked beams, the European approach is less conservative if compared with the other four methods of analysis adopted in this study. Only Khuntia-Ghosh's model takes care of the aspect ratio b/h of the cross section, by making reference to wide beams ($b>h$) and ordinary beams ($b<h$).

Another topic investigated in this study is the P-Δ effect, which requires second-order analysis. It was found that geometrical nonlinearities may occur, depending on the way the reduced stiffness is computed. As a matter of fact, using the analytical models found in the scientific literature leads to an amplification of the seismic action, something that is not contemplated in the European and in the Italian design codes. In this type of analysis, sizeable differences ensue from the beam type, since ordinary beams strongly contribute to the reduction of second-order effects, for both of the concrete classes taken into examination. The same conclusions hold true at the Damage Limit State (Serviceability Limit State under seismic loading), since the values of the lateral drift significantly change depending on the model adopted in the analysis. For instance, when wide beams are used, differences between the Italian Code and other models or codes are more evident. NZS 3101 leads to higher drift values (+7%); Elwood and Eberhard's model leads to even greater values (+20% at the first level and 10% at the top level). The same occurs with the reduced stiffness recommended by Kumar and Singh (+17÷19%) for each floor of the building. Hence, if a more accurate evaluation of the effective stiffness were performed, then the maximum lateral drift admitted by the Italian code would not be respected.

To conclude, it is necessary to focus on two final remarks. Firstly, in the analysis at the ultimate limit state configurations with both cracked and uncracked columns should be considered. The subsequent analysis should be based on cracked sections to maximize the inter-storey drift and the possible non-linear P-Δ effects. For the damage limit state, instead, a cracked model should be used for the computation of reliable lateral deformations.

Secondly, the present study strongly suggests to update the design indications on how to compute the effective stiffness of R/C frames under seismic loads in both EC8 and the Italian Design Code considering a larger number of structural parameters, as found in scientific literature.

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