Earthquakes and Structures, *Vol. 8, No. 5 (2015) 1215-1240*
DOI: http://dx.doi.org/10.12989/eas.2015.8.5.1215

Seismic design of chevron braces cupled with MRF fail safe systems

Alessandra Longo[*], Rosario Montuori[a] and Vincenzo Piluso[b]

Department of Civil Engineering, University of Salerno via Ponte Don Melillo, Fisciano, 84084, Italy

(Received April 17, 2014, Revised June 19, 2014, Accepted July 1, 2014)

Abstract. In this paper, the Theory of Plastic Mechanism Control (TPMC) is applied to the seismic design of dual systems composed by moment-resisting frames and Chevron braced frames. The application of TPMC is aimed at the design of dual systems able to guarantee, under seismic horizontal forces, the development of a collapse mechanism of global type. This design goal is of primary importance in seismic design of structures, because partial failure modes and soft-storey mechanisms have to be absolutely prevented due to the worsening of the energy dissipation capacity of structures and the resulting increase of the probability of failure during severe ground motions. With reference to the examined structural typology, diagonal and beam sections are assumed to be known quantities, because they are, respectively, designed to withstand the whole seismic actions and to withstand vertical loads and the net downward force resulting from the unbalanced axial forces acting in the diagonals. Conversely column sections are designed to assure the yielding of all the beam ends of moment-frames and the yielding and the buckling of tensile and compressed diagonals of the V-Braced part, respectively.

In this work, a detailed designed example dealing with the application of TPMC to moment frame-chevron brace dual systems is provided with reference to an eight storey scheme and the design procedure is validated by means of non-linear static analyses aimed to check the actual pattern of yielding. The results of push-over analyses are compared with those obtained for the dual system designed according to Eurocode 8 provisions.

Keywords: moment resisting frames; V-braced frames; global mechanism; design methodology; non linear static analyses

1. Introduction

Moment Resisting Frames (MRFs) and Concentrically Braced Frames (CBFs) are commonly used in the construction of steel buildings in seismic areas. MRFs are able to exhibit high ductility capacity compared to other structural typologies, because of the high number of dissipative zones subjected to cyclic bending, preferentially located at the beam ends. However, such structural system could be not able to provide sufficient lateral stiffness, as needed to fulfill serviceability

*Corresponding author, Ph.D., E-mail: alongo@unisa.it
[a]Ph.D.
[b]Professor, E-mail: v.piluso@unisa.it

http://www.techno-press.org/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

limit state requirements. Therefore, MRFs can require large member sizes to keep lateral drifts within mandatory limits provided by seismic codes.

Conversely, CBFs are a popular lateral load resisting system in high seismicity areas, because of their economy, easy construction and favorable stiffness. Because of the obstructions caused by cross-braces, chevron braces are often preferred to allow for door and windows openings. Conventional chevron frames (VBFs) consist of two braces forming an inverted V-shape and meeting the upper storey beam at mid-span. While the fulfillment of serviceability limit state requirements is easy to obtain by means of such structural typology, some uncertainties arise about its adequacy to resist strong seismic actions by undergoing severe excursions in the non-linear range. The energy dissipation capacity of CBFs is, in fact, almost completely related to non-linear hysteretic behaviour of diagonal braces under alternate tension and compression internal forces (Bruneau *et al*. 1998, Longo *et al*. 2008a). This behaviour is affected by a number of quite complex and not easily predictable aspects, such as the performance of end connections, the in-plane and out-of-plane overall buckling of compressed members and all the local damage phenomena, i.e., local buckling, low cycle fatigue, fracture propagation, related to the inelastic cyclic behaviour under axial and bending forces.

Because of the inherent drawbacks of both MRFs and CBFs, MRF-CBF dual systems are more and more attracting the interest of researchers and practitioners as they constitute a reliable structural scheme which allow to combine the advantages of both structural typologies, because of the exploitation of the local ductility supply of the beams of the moment resisting part and of the lateral stiffness provided by the diagonal members of the braced part. Therefore, dual systems constitute an effective structural solution able to satisfy both ultimate and serviceability limit state requirements.

Akiyama (1998) showed that dual systems composed by a flexible part and a rigid part acting in parallel are among the most efficient earthquake resistant structures. It is believed that the flexible subsystem will prevent drift concentration, whereas the rigid subsystem will dissipate seismic energy by plastic deformations. Iyama and Kuwamura (1998) investigated, form a probabilistic point of view, the advantage of combining CBF and MRF systems with different natural periods of vibration. They outlined that the resulting dual system can be termed “fail-safe”, because MRF provides an alternative load path for earthquake loading in the case of failure of the primary CBF system. Dubina *et al*. (2008, 2011), within the framework of a research devoted to removable links in MRF-EBF dual systems, have introduced to concept of “useful” ductility of the rigid subsystem as the limit value of the ductility demand of the rigid subsystem for which the flexible subsystem still remain in the elastic range. They pointed out that such useful ductility increases as far as the strength ratio between the flexible and the rigid subsystem increases and as far as the stiffness ratio between the rigid subsystem and the flexible subsystem increases. In addition, they showed that dual configuration results in smaller permanent drifts when compared to the permanent deformation of the rigid subsystem alone. This results has been pointed out by other researchers also in the case of dual systems using buckling restrained braces (Kiggings and Uang 2006).

Despite of MRF-CBF dual systems are able to provide an effective solution for both serviceability and ultimate limit state requirements and are able to reduce permanent drifts, they are not deeply dealt with in seismic codes. Therefore, they are usually designed by combining the design rules of MRFs and CBFs. In fact, code provisions are usually limited to a rough indication about the q-factor to be used in the definition of the design seismic forces and, sometime, to the suggestion of a minimum lateral resistance to be entrusted to the moment resisting part of the structural scheme. As an example, currently ASCE 7-05 design requirements (2006) are limited to state that moment frames must be capable of resisting 25% of the seismic forces while the moment frames and the braced frames of the whole structural system must be capable of resisting the entire seismic forces proportionally to their relative rigidities. Recent studies (Aukeman and Laursen, 2011) have shown that this design requirement at best is arbitrary.

In this paper, the use of the Theory of Plastic Mechanism Control is promoted for the seismic design of moment frame-chevron brace dual systems. In fact, it is well known that the energy dissipation capacity of structures is strongly influenced by the kinematic mechanism developed at collapse. Partial mechanisms undermine the global ductility supply and are responsible of low energy dissipation capacity. Therefore, the development of a collapse mechanism of global type becomes a relevant design goal in the plastic design of seismic-resistant structures. For this reason, the need to prevent collapse mechanisms having limited dissipation capacity and the need to promote the development of a global failure mode are universally recognized. The problem of the failure mode control is faced by modern seismic codes by means of recommendations which are based on simple hierarchy criteria which, generally, do not lead to structures failing in global mode and, in some cases, are not able to avoid the development of soft storey mechanisms. Therefore, aiming to the design of structures able to assure the development of a collapse mechanism of global type under destructive seismic actions, more sophisticated design procedures have to be defined (Mazzolani and Piluso 1997). Recently more sophisticated design procedures, aimed at designing structures failing in global mode by means of TPMC, have been proposed for moment resisting frames (Mazzolani and Piluso 1997, Montuori and Piluso 2000), concentrically braced frames (Longo *et al*. 2008a, 2008b), eccentrically braced frames (Montuori *et al*. 2013, 2014), Dissipative Truss Moment Frames (Longo *et al*. 2012a, 2012b) and MRF-CBF dual systems with *X* bracings (Giugliano *et al*. 2010, Longo *et al*. 2014). In the same fashion, TPMC is herein applied with reference to moment frame-Chevron brace dual systems and a comparison with Eurocode 8 design rules is presented.

The aim of the theory of plastic mechanism control is the development of an energy dissipation mechanism characterized by the involvement of all the dissipative zones, i.e., the diagonal members of the chevron braced bays and the beam ends corresponding to all the moment-resisting beam-to-column connections. Such global mechanism can be achieved by means of a design procedure whose robustness is based on the kinematic theorem of plastic collapse and, in order to account for second order effects also, on its extension to the concept of mechanism equilibrium curve. In this work, a detailed description of the proposed design methodology is reported by means of a worked example also. In addition, in order to show the accuracy of the design procedure, a push-over analysis has been carried out for the analysed structure by means of SAP 2000 computer program (1998). Furthermore, a comparison between the structure designed according to the proposed procedure and that obtained with reference to same structural scheme designed according to Eurocode 8 Provisions (CEN 2003) for MRFs and CBFs has been reported.

2. Eurocode 8 design criteria

The structural scheme commonly adopted for evaluating internal actions in beams, columns and diagonals of chevron braced frames subjected to seismic actions is based on the assumption that both tensile diagonals and compressed diagonals are active.

Dissipative zones of the chevron braced part are constituted by the diagonal members;

conversely, beam ends represent the dissipative zones of the moment-resisting part. Columns and connections to foundations are considered as non-dissipative zones so that, according to capacity design principles, they have to be designed to remain in elastic range. To this scope, according to Eurocode 8 CEN 2003), the design resistance of connections R_d of diagonals to any member has to satisfy the following hierarchy criterion

$$R_d \geq 1.1 \cdot \gamma_{ov} \cdot R_{fy} \tag{1}$$

where R_d is the design resistance of the connection, R_{fy} is the design plastic resistance of the connected dissipative member, the factor 1.1 is an amplification coefficient accounting for strain-hardening effects and γ_{ov} is an overstrength factor accounting for the random variability of material properties, varying from 1.0 to 1.25.

With reference to chevron braced frames, Eurocode 8 requires that the normalized slenderness braces $\bar{\lambda}(\bar{\lambda}=\lambda\,/\lambda_y$ with $\lambda_y = \pi\sqrt{E/f_y}$ where f_y is the yield stress E is the Young modulus) has to be properly limited assuring that $\bar{\lambda} \leq 2.0$. The aim of this limitation is the reduction of the plastic out of plane deformation of the gusset plates, due to brace buckling, which otherwise are prone to failure due to low cycle fatigue. Even though the fulfilment of such limitation to the brace slenderness is desirable due to the need to avoid the fracture of gusset plates, it cannot be forgotten that this slenderness limitation governs the overstrength of the bracing elements, especially at the top storey, thus affecting the size of beams and columns. In fact, the hierarchy criterion for beams and columns is based on an overstrength coefficient Ω of bracing elements defined as

$$\Omega = \min_{i=1}^{n}(\Omega_i) = \min_{i=1}^{n}\left(\frac{N_{br.Rdi}}{N_{br.Sdi}}\right) \tag{2}$$

where $N_{br,Sdi}$ is the design value of the brace axial force at *i th* storey and $N_{br,Rdi}$ is the corresponding design resistance and n is the number of storeys. Eurocode 8 (CEN 2003) requires the fulfilment of the following hierarchy criterion

$$N_{Rd}(M_{Sd}) \geq N_{Sd.G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot N_{Sd.E} \tag{3}$$

where $N_{Rd}(M_{Sd})$ is the axial resistance of the element (beam or column) according to Eurocode 3 (CEN 2005) reduced due to the contemporary action of the bending moment M_{Sd}, $N_{Sd,G}$ is the axial force due to the non seismic loads included in the seismic load combination, $N_{Sd,E}$ is the axial force due to the seismic loads, M_{Sd} is the bending moment due to the seismic load combination, 1.1 is again the amplification coefficient accounting for strain-hardening effects and, γ_{ov} is the already mentioned overstrength factor. The value of γ_{ov} factor ranges from 1.0 to 1.25 with the aim of including all the random effects of material properties. Therefore, considering the aim of such γ_{ov} factor, the accuracy of the design criterion suggested by Eurocode 8 should be investigated by means of Monte Carlo simulations. This work, conversely, is aimed to evaluate the nonlinear seismic response considering deterministic material properties; so that a γ_{ov} factor equal to 1.0 has been assumed.

It is useful to point out that the second member of Eq. (3) represents the axial force occurring in a non-dissipative member of the V-braced part (beam or column) when the first diagonal, corresponding to the storey where $\Omega_i = \Omega$, reaches its capacity, i.e., when the first diagonal is completely yielded and strain-hardened in tension up to its ultimate resistance. Furthermore, in order to promote the participation of all the storeys to the dissipative behaviour of the structure, the code prescribes that it should be checked that the maximum overstrength Ω_i does not differ from its minimum value by more than 25%. However, the main feature of chevron braced schemes when compared to *X*-braced schemes occurs because beams have to be checked against the vertical action resulting from the unbalanced brace axial forces. Such action is due to the fact that the compressed diagonal is buckled when the tensile one yields. According to Eurocode 8, this force can be approximately evaluated as follows

$$V_i = P_{t.i} \cdot sin\alpha_{i1} - \gamma_{pb} \cdot P_{c.Rd.i} \cdot sin\alpha_{i2} \tag{4}$$

where α_{i1} and α_{i2} are the angles between the diagonal axes and the beam axis at *i th* storey (typically $\alpha_{i1}=\alpha_{i2}$), $P_{t.i}$ is the design resistance of the diagonal in tension, $P_{c.Rd.i}$ is the buckling resistance of the compressed diagonal and γ_{pb} is a factor, whose suggested value is 0.3, used for estimating the post-buckling resistance of brace members.

It is important to underline that the design rule suggested for columns, i.e., Eq. (3), does not account for the column overloading deriving from the part of the above unbalanced vertical action transmitted by the beams to the columns.

Regarding the moment-resisting part of the scheme, the suggested hierarchy criterion is related to the desirable weak-beam strong-column behaviour. To this aim the typical design criterion requires at any joint

$$\sum M_{c.Rd} \geq \gamma_{Rd} \sum M_{b.Rd} \tag{5}$$

where $\sum M_{cRd}$ and $\sum M_{bRd}$ are, respectively, the sums of the design values of the bending resistance of the columns and beams converging in the joint and γ_{Rd} =1.3. To assure the above criterion, the following application rule is suggested

$$M_{cSd} = \left(M_{Ed.G} + 1.1 \cdot \gamma_{ov} \cdot \Omega \cdot M_{Ed.E}\right) \leq M_{cRd} \tag{6}$$

where $M_{Ed.G}$ and $M_{Ed.E}$ are the bending moments due to gravity loads and seismic forces, respectively, Ω is a beam over-strength factor determined as the minimum value of $\Omega_i=M_{pl.Rd}/M_{Ed.i}$, being $M_{Ed.i}$ the design moment of *i th* beam in the seismic design situation and $M_{pl.Rd.i}$ the corresponding design resistance.

Finally, a drift limitation is applied to assure both serviceability requirements and structural stability. Regarding interstorey drift limitation, according to Eurocode 8, the design interstorey displacement (d_r) is limited to a proportion of the storey height (h) such that

$$d_r \nu \leq \psi \cdot h \tag{7}$$

where ψ is a value depending on the typology of non-structural elements, given as 0.5%, 0.75% and 1.0% for brittle, ductile or non-interfering components, respectively, and ν is a reduction factor which accounts for the reduced intensity of the seismic action associated with damage limitation requirement; the value of ν is recommended as 0.4 or 0.5 according to the importance class of the structure.

3. Application of TPMC to moment frame-chevron brace dual systems

The Theory of Plastic Mechanism Control (TPMC) is based on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve so that it consists in

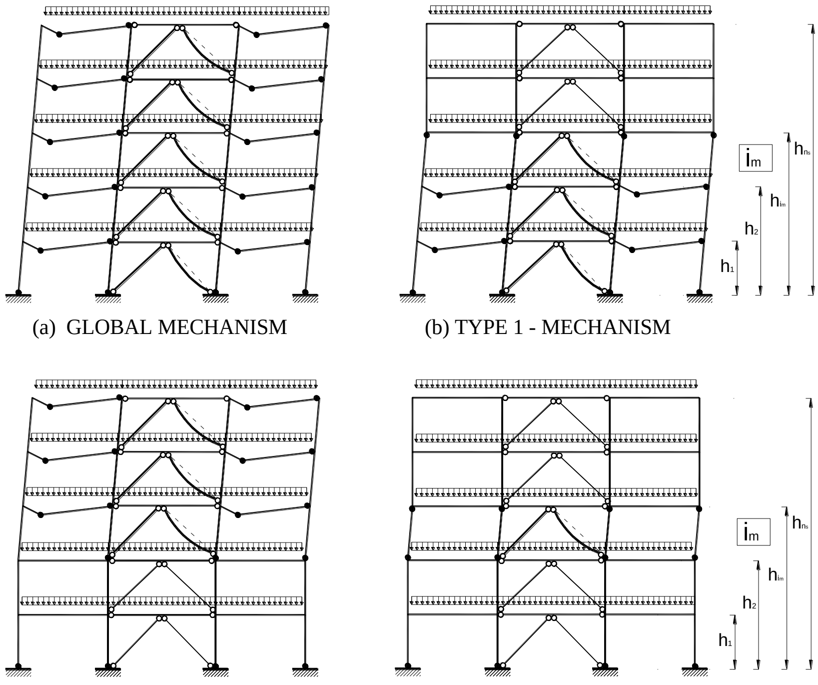

Fig. 1 Collapse mechanism typologies

imposing that the collapse mechanism equilibrium curve corresponding to the desired mechanism has to be located below the mechanism equilibrium curves corresponding to all the other undesired mechanisms up to a given ultimate displacement which is properly selected to be compatible with local ductility supply of dissipative elements. In case of moment frame-chevron brace dual systems, beams and diagonals are preliminarily designed to resist vertical and horizontal forces, respectively. The columns are the unknown of the design problem. TPMC starts from the observation that the collapse mechanism of dual systems under seismic horizontal forces can be considered as belonging to three main typologies (Fig. 1). The desired global mechanism is achieved when plastic hinges are developed at all the beam ends of the moment-resisting part and at the base of first storey columns and, in addition, with reference to the braced part, all the tensile diagonals are yielded and the compressed ones are buckled.

The linearized mechanism equilibrium curve is expressed as (Mazzolani and Piluso 1997)

$$\alpha = \alpha_0 - \gamma \cdot \delta \tag{8}$$

where α_0 and γ are, respectively, the kinematically admissible multiplier of horizontal forces, evaluated according to first-order rigid-plastic analysis, and the slope of the mechanism equilibrium curve. The kinematically admissible multiplier and the slope of the mechanism equilibrium curve can be evaluated for each one of the considered mechanisms. In the following, the notations $\alpha_0^{(g)}$, $\gamma^{(g)}$ and $\alpha_{im}^{(t)}$, $\gamma_{im}^{(t)}$ denote the kinematically admissible multiplier of horizontal forces and the slope of the softening branch of α-δ curve corresponding to the global mechanism and to the i_m *th* mechanism of *t th* type (with t=1 to 3), respectively.

With reference to the global mechanism the following relationship is obtained by means of the virtual work principle (Longo *et al*. 2014a, 2014b, 2015)

$$\alpha_0^{(g)} = \frac{\mathbf{M}_{\mathbf{c1}}^{\mathbf{T}}\mathbf{I} + W_d^{(g)} - tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{\mathbf{v}}^{(\mathbf{g})}\right)}{\mathbf{F}^{\mathbf{T}}\mathbf{s}^{(\mathbf{g})}} \tag{9}$$

where:

• $\mathbf{M_{c1}}^{\mathbf{T}}$ is the vector of plastic moments of first storey columns, reduced due to the influence of axial forces;

• $\mathbf{I}$ is the unit vector of order n_c;

• $\mathbf{F}$ is the vector of the design seismic horizontal forces equal to $\{F_1, F_2, \ldots\ldots, F_k, \ldots\ldots, F_{ns}\}$, where F_k is the horizontal force applied to the k*th* storey;

• $\mathbf{q}$ is the matrix of order $n_b \times n_s$ (number of bays × number of storeys) of uniform vertical loads q_{jk}, acting on the beams of *i th* bay of *k th* storey;

• $\mathbf{D_v}$ is a matrix of order $n_b \times n_s$ whose elements, $D_{v,jk}$, are coefficients related to the external work of the uniform load acting on *j th* beam of *k th* storey. In particular $D_{v,jk}=L_j\, x_{jk}/2$ when the second plastic hinge develops in section x_{jk} of the beam bay and $D_{v,jk}=0$ when the second plastic hinge develops at the first beam end (Mazzolani and Piluso 1997);

• $\mathbf{s}^{(g)}=\{h_1,h_2,\ldots,h_{ns}\}^T$ is the shape vector of horizontal virtual displacements when global mechanism occurs.

In addition, $W_d^{(g)}$ is the internal work of dissipative members, i.e., beams and diagonal braces, occurring in the global mechanism for a unit rotation of column plastic hinges.

Similarly, in case of i_m-mechanism of type-1, the kinematically admissible multiplier of horizontal forces is given by

$$\alpha_{0.i_m}^{(1)} = \frac{\mathbf{M}_{\mathbf{c1}}^{\mathbf{T}}\mathbf{I} + W_{d.i_m}^{(1)} + \mathbf{M}_{\mathbf{c}.i_m}^{\mathbf{T}}\mathbf{I} - tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{\mathbf{v}.i_m}^{(1)}\right)}{\mathbf{F}^{\mathbf{T}}\mathbf{s}_{\mathbf{i_m}}^{(1)}} \tag{10}$$

where $W_{d.i_m}^{(1)}$ is the internal work of dissipative members occurring in i_m-mechanism of type-1 for a unit rotation of column plastic hinges and $\mathbf{s}_{im}^{(1)}=\{h_1,h_2,\ldots,h_{im},h_{im},h_{im}\}^{\mathrm{T}}$ is the corresponding shape vector of virtual horizontal displacements, where the first element equal to h_{im} corresponds to the i_m *th* component.

Similarly, in case of i_m-mechanism of type-2

$$\alpha_{0.i_m}^{(2)} = \frac{W_{d.i_m}^{(2)} + \mathbf{M}_{\mathbf{c}.i_m}^{\mathbf{T}}\mathbf{I} - tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{\mathbf{v}.i_m}^{(2)}\right)}{\mathbf{F}^{\mathbf{T}}\mathbf{s}_{\mathbf{i_m}}^{(2)}} \tag{11}$$

where $W_{d.i_m}^{(2)}$ is the internal work of dissipative members occurring in i_m-mechanism of type-2 for a unit rotation of column plastic hinges and $\mathbf{s}_{im}^{(2)}=\{0,0,\ldots,h_{im},h_{im},h_{im}\}^{\mathrm{T}}$ is the corresponding shape vector of virtual horizontal displacements, where the first non-zero element is the h_{im} one.

Finally, with reference to the i_m *th* storey mechanism, it occurs

$$\alpha_{0.i_m}^{(3)} = \frac{2\mathbf{M}_{\mathbf{c}.i_m}^{\mathbf{T}}\mathbf{I} + W_{d.i_m}^{(3)}}{\mathbf{F}^{\mathbf{T}}\mathbf{s}_{\mathbf{i_m}}^{(3)}} \tag{12}$$

where $W_{d.i_m}^{(3)}$ is the internal work of diagonal members occurring in i_m *th* storey mechanism for a unit rotation of column plastic hinges and $\mathbf{s}_{im}^{(3)}$ =(0,0,…,h_{im}-h_{im-1},h_{im}-h_{im-1},h_{im}-h_{im-1}}^{T} is the corresponding shape vector of virtual horizontal displacements, where the first element different from zero is the i_m *th* one.

Regarding the slope γ of the mechanism equilibrium curve, it is related to the ratio between the second order work due to vertical loads and the work due to horizontal forces (Mazzolani and Piluso 1997)

$$\gamma = \frac{\mathbf{V}^{T}\mathbf{s}}{\mathbf{F}^{T}\mathbf{s}} \cdot \frac{1}{H_0} \tag{13}$$

where:

• $\mathbf{V}$ is the vector of the storey vertical loads $\{V_1,V_2,\dots,V_k,\dots,V_{ns}\}$, where V_k is the total load acting at k *th* storey ($V_k = \sum_{j=1}^{n_b} q_{j.k} L_j$);

• $\mathbf{s}\ \delta/H_0$ is the shape vector of the storey vertical virtual displacements, with $\mathbf{s}$ shape vector of the storey horizontal virtual displacements, δ top sway displacement of the structure and H_0 sum of the interstorey heights of all the storeys involved in the generic mechanism.

It is evident that the slopes of mechanism equilibrium curves $\gamma_{im}^{(1)}$, $\gamma_{im}^{(2)}$ and $\gamma_{im}^{(3)}$ for i_m *th* mechanism of t *th* type (t=1,2,3) and the slope $\gamma^{(g)}$ for the global mechanism can be immediately derived from Eq. (13) considering the appropriate shape vector of virtual horizontal displacements and taking into account that $H_0=h_{im}$, $H_0=h_{ns}-h_{im-1}$, $H_0=h_{im}-h_{im-1}$ for mechanism type-1, type-2 and type-3 respectively, and, finally, $H_0=h_{ns}$ for the global type mechanism.

The column sections required to guarantee the development of global mechanism can be obtained imposing that, in the range of displacements compatible with the local ductility of dissipative zones, defined by the design ultimate displacement δ_u, the mechanism equilibrium curve corresponding to the global mechanism has to be located below those corresponding to all the other undesired partial mechanisms. The resulting design conditions are expressed by the following relationships

$$\alpha^{(g)} - \gamma^{(g)} \cdot \delta_u \le \alpha_{0.i_m}^{(t)} - \gamma_{i_m}^{(t)} \cdot \delta_u \tag{14}$$

with i_m=1,2,3,...,n_s and t=1,2,3.

The fulfilment of the above design conditions can be assured by means of the following steps constituting the proposed design procedure:

a) Design of beam and diagonal sections to withstand vertical loads and seismic action, respectively.

b) Selection of the design ultimate top sway displacement δ_u to be compatible with the ductility supply of dissipative zones. To this scope, in the following, the plastic rotation capacity of beams is assumed equal to 0.04 rad so that $\delta_u = 0.04\, h_{ns}$ is assumed where h_{ns} is the height of the structure.

c) Computation of the axial load acting in the columns at collapse state, i.e., when the beam ends of the moment-resisting part and the tensile diagonals are yielded while the compressed ones are buckled (Fig. 1(a)).

d) Computation of the required sum of plastic moment of first storey columns, reduced due to the contemporary action of the axial load, $\mathbf{M}_{c.1}^{T}\mathbf{I}$, by means of the following relation

$$\mathbf{M}_{\mathbf{c.1}}^{\mathbf{T}}\mathbf{I} \geq \cdot \frac{\left[\mathrm{W}_{d}^{(g)} - tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{\mathbf{v}}^{(\mathbf{g})}\right) - \mathrm{W}_{d.1}^{(3)} \frac{\mathbf{F}^{\mathbf{T}}\mathbf{s}^{(\mathbf{g})}}{\mathbf{F}^{\mathbf{T}}\mathbf{s}_{\mathbf{1}}^{(\mathbf{3})}} + \left(\gamma_{1}^{(3)} - \gamma^{(g)}\right) \cdot \delta_{u} \cdot \mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}^{(\mathbf{g})} \right]}{\left(2 \cdot \frac{\mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}^{(\mathbf{g})}}{\mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}_{\mathbf{1}}^{(\mathbf{3})}} - 1\right)} \tag{15}$$

Eq. (15) is derived from design conditions () for $i_m=1$ and $t=1$ or $t=3$ (because for $i_m=1$ type-1 mechanism and type-3 mechanism are coincident). In addition, it has to be observed that for $i_m=1$ the internal work of dissipative zones, $W_{d.1}^{(1)} = W_{d.1}^{(3)}$, is due to the diagonal braces of first storey only.

Furthermore, it is important to underline that, for $i_m=1$, type-2 mechanism is coincident with the global mechanism, so that Eq. (14) becomes an identity. This observation is of paramount importance from the practical point of view, because it allows to design the first storey columns directly by means of Eq. (15) and to avoid any iterative procedure providing a closed form solution easy to be applied even by means of hand calculations.

e) The sum of the required plastic moments of first storey columns, $\mathrm{M}_{\mathrm{c.1}}^{\mathrm{T}}\mathrm{I}$, is distributed among the columns proportionally to the axial load $\mathrm{N}_{\mathrm{c.i1}}$ occurring in the *i* th column of first storey at collapse, thus obtaining the corresponding bending moment $\mathrm{M}_{\mathrm{c.i1}}$. Therefore, the first storey column sections can be designed by imposing that the point ($M_{c.i1}$, $N_{c.i1}$) is internal or belonging to the boundary line of the design plastic domain.

Because of the selection of columns from standard shapes, some overstrength occurs so that the actual sum of plastic moments of columns, accounting for the interaction with the axial force, generally is $M_{c.1}^{*T}I \geq M_{c.1}^{T}I$. As soon as the first storey column contribution to the internal work is known, the multiplier of seismic horizontal forces corresponding to the design ultimate top sway displacement can be computed as $\alpha^{(g)} = \alpha_0^{(g)} - \gamma^{(g)}\delta_u$, being now a known quantity.

f) Computation, at any storey i_m, of the required sum of plastic moment of columns, reduced due to the contemporary action of the axial load, $\mathbf{M}_{\mathrm{c.i_m}}^{\mathrm{T}}\mathbf{I}$, (for $i_m > 1$) by means of the following relations needed to avoid type-1, type-2 and type-3 mechanisms, respectively

$$\mathbf{M}_{\mathbf{c.1}}^{(1)T}\mathbf{I} \geq \left(\alpha^{(g)} + \gamma_{i_m}^{(1)}\delta_u\right) \cdot \mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}_{i_m}^{(1)} - \mathbf{M}_{\mathbf{c.1}}^{*\mathbf{T}}\mathbf{I} - \mathbf{W}_{d.i_m}^{(1)} + tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{v.i_m}^{(1)}\right) \tag{16}$$

$$\mathbf{M}_{\mathbf{c.1}}^{(2)T}\mathbf{I} \geq \left(\alpha^{(g)} + \gamma_{i_m}^{(2)}\delta_u\right) \cdot \mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}_{i_m}^{(2)} - \mathbf{W}_{d.i_m}^{(2)} + tr\left(\mathbf{q}^{\mathbf{T}}\mathbf{D}_{v.i_m}^{(2)}\right) \tag{17}$$

$$\mathbf{M}_{\mathbf{c.1}}^{(3)T}\mathbf{I} \geq \frac{1}{2}\left(\alpha^{(g)} + \gamma_{i_m}^{(3)}\delta_u\right) \cdot \mathbf{F}^{\mathbf{T}} \cdot \mathbf{s}_{i_m}^{(3)} - \mathbf{W}_{d.i_m}^{(3)} \tag{18}$$

Eqs. (16)÷(18) have been directly derived from Eq. (14) for $i_m>1$;

g) Computation of the required sum of the reduced plastic moments of columns for each storey as the maximum value among those coming from the above design conditions

$$\mathbf{M}_{\mathrm{c.i_m}}^{\mathrm{T}}\mathbf{I} = \max\left\{\mathbf{M}_{\mathrm{c.i_m}}^{(1)\mathrm{T}}\mathbf{I}, \mathbf{M}_{\mathrm{c.i_m}}^{(2)\mathrm{T}}\mathbf{I}, \mathbf{M}_{\mathrm{c.i_m}}^{(3)\mathrm{T}}\mathbf{I}\right\} \tag{19}$$

h) The sum of the reduced plastic moment of columns at each storey can be distributed among all the columns, at the same storey, proportionally to the axial load acting at the collapse state. The knowledge of these plastic moments, coupled with the axial force at the collapse state, allows the evaluation of the required column sections as already described in step e) with reference to the first storey columns.

i) If necessary, a technological condition is imposed, starting from the base, requiring that column sections cannot increase along the building height. If the application of this requirement leads to the revision of column sections at first storey (i.e., their increase) then a new value of $\mathbf{M}_{c.1}^{*T}\mathbf{I}$ is computed and the procedure has to be repeated from step f).

It has to be underlined that even in the case of structures designed to fail according to the global mechanism, the lateral stiffness needed to fulfil code requirements dealing with serviceability limit state could not be attained (CEN 2003). In order to design structures fulfilling also drift limitation requirements, different strategies requiring an iterative procedure can be implemented. In fact, the design procedure based on TPMC can be repeated by increasing the beam sections or by increasing the diagonal sections or by increasing the design ultimate displacement. However, past experience has shown that the most convenient solution is obtained by preliminarily designing the brace members for lateral stiffness requirements and by applying TPMC to design the columns sections aiming to guarantee the global mechanism (Giugliano *et al*. 2010).

Regarding Eqs. (15)-(18), it is useful to underline that the internal work of dissipative zones (i.e., beam ends and diagonal braces) due to a unit virtual rotation of column plastic hinges can be conveniently written in the following form

$$\mathrm{W}_{d.i_m}^{(t)} = 2tr\left(\mathbf{B}^{\mathbf{T}}\mathbf{R}_{\mathbf{b.i_m}}^{(\mathbf{t})}\right) + tr\left(\mathbf{N}_{\mathbf{t}}^{\mathbf{T}}\mathbf{E}_{\mathbf{t.i_m}}^{(\mathbf{t})}\right) + tr\left(\mathbf{N}_{\mathbf{c}}^{\mathbf{T}}\mathbf{E}_{\mathbf{c.i_m}}^{(\mathbf{t})}\right) \tag{20}$$

for i_m=1,2,..,n_s and t=1,2,3, where:

• $\mathbf{B}$ is a matrix of order $n_b \times n_s$ (number of bays × number of storeys) whose elements B_{jk} are equal to the plastic moments of beams ($B_{jk}=M_{b,jk}$);

• $\mathbf{R_b}$ is a matrix of order $n_b \times n_s$ whose elements, $R_{b,jk}$, are coefficients accounting for the participation of *j th* beam of *k th* storey to the collapse mechanism. In particular $R_{b,jk}=0$ when the beam does not participate to the collapse mechanism (and for beams of braced bays connected to the columns by means of connections transmitting shear forces only), otherwise $R_{b,jk}=L_j/(L_j-x_{jk})$, where L_j is the span of *j th* bay and x_{jk} is the abscissa of second plastic hinge of *j th* beam of *k th* storey. This abscissa is given by $x_{jk}=L_j-2\cdot(M_{b,jk}/q_{jk})^{1/2}$ when the uniform vertical load exceeds the limit value $q_{jk}=4\cdot M_{b,jk}/L_j^2$ and $x_{jk}=0$ in the opposite case (Mazzolani and Piluso 1997);

• $\mathbf{N_t}$ and $\mathbf{N_c}$ are matrixes of order $n_{br} \times n_s$ (number of braced bays × number of storeys) whose elements $N_{t,bk}$ and $N_{c,bk}$ are, respectively, equal to the yield axial forces in tensile diagonals and equal to the axial forces accounting for post-buckling behaviour in compressed diagonals of *b th* braced bay and *k th* storey, with reference to the collapse condition (Longo *et al*. 2008);

• $\mathbf{E_t}$ and $\mathbf{E_c}$ are matrixes of order $n_{br} \times n_s$ whose elements, $e_{t,bk}$ and $e_{c,bk}$, are coefficients representing, respectively, the elongation of the tensile yielded diagonal and the shortening of the buckled compressed one belonging to the *b th* braced bay and *k th* storey, due to a unit rotation of columns. They are given by $l_{bk}\cos\alpha_{bk}\sin\alpha_{bk}$ (l_{bk} is the brace length) when the diagonal participates to the collapse mechanism, conversely $e_{t,bk}=e_{c,bk}=0$.

It is important to underline that the internal work of compressed diagonals is evaluated accounting for the post-buckling behaviour. Therefore, in the collapse condition, for a given axial shortening or elongation δ_i of the generic diagonal, the axial force in the tensile diagonal of the *i-th* storey is equal to N_{ti}, while the axial force in the corresponding buckled compressed diagonal $N_{c.i}$ is evaluated as shown in Fig. 2.

The post buckling behaviour of the compressed diagonal braces can be predicted by combining the yielding condition of the midspan section of brace members with the relationship between the

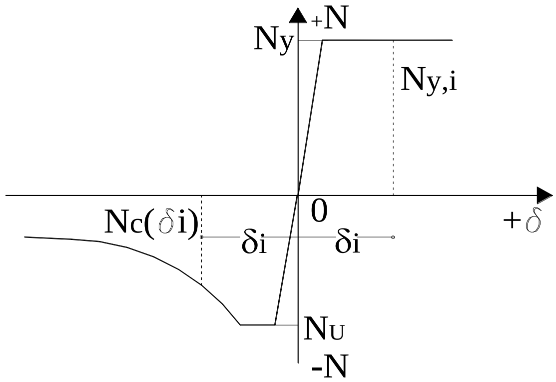

Fig. 2 Evaluation of compression and tension axial force depending on diagonal axial deformation

axial shortening and the midspan deflection (Longo *et al*. 2008). The yielding condition of the midspan section is simply given by

$$\mathrm{N} \cdot \mathrm{f_t} = M_{pl}(N) \tag{21}$$

where N is the axial force, f_t is the midspan deflection and $M_{pl}(N)$ is the plastic moment reduced due to the contemporary action of the axial force.

The relationship between the shortening of the brace member and the midspan deflection is given by (Longo *et al*. 2008)

$$\delta_a = \frac{N \cdot L_d}{EA} + \frac{\pi^2 \cdot f_t^{\,2}}{4 \cdot L_d} - \frac{\pi^2 \cdot f_0^{\,2}}{4 \cdot L_d} \tag{22}$$

where δ_a is the axial shortening, A and L_d are the cross-section area and the length, respectively, of the diagonal brace and f_0 is the initial midspan deflection due to equivalent geometrical imperfections.

In case of diagonal braces constituted by circular hollow sections, taking into account that *M*-*N* interaction plastic domain can be expressed as

$$M_{pl}(N) = 1.04 \cdot M_{pl}\left[1 - \left(\frac{N}{A \cdot f_y}\right)^{1.7}\right] \le M_{pl} \tag{23}$$

where $M_{pl}=Zf_y$ is the brace plastic moment, the equivalent geometrical imperfection can be obtained from

$$\frac{N_u \cdot f_0}{1 - \dfrac{N_u}{N_{cr}}} = M_{pl}(N_u) \tag{24}$$

where N_u is the buckling axial resistance of the brace member computed according to Eurocode 3 and N_{cr} is the elastic critical load.

Therefore, combining Eq. (23) and Eq. (24) the following relationship is obtained

$$f_0 = \Psi \cdot Z \cdot f_y \frac{1 - \dfrac{N_u}{N_{cr}}}{N_u} \tag{25}$$

with

$$\Psi = 1.04 \cdot \left[1 - \left(\frac{N_u}{A \cdot f_y} \right)^{1.7} \right] \le 1 \tag{26}$$

Finally, by combining Eqs. (21) and (22) and accounting for the plastic domain Eq. (23) the following relationship is obtained

$$\Psi = \frac{\sqrt{\dfrac{4 \cdot L_d}{\pi^2} \left(\delta_a - \dfrac{N \cdot L_d}{E \cdot A} \right) + f_0^{\,2}}}{Z \cdot f_y} \tag{27}$$

It is evident that Eq. (27) allows to compute the post-buckling resistance N corresponding to a given axial displacement δ_a. Eq. (27) can be easily solved by means of a trial and error procedure remembering that ψ is a function of N according to Eq. (26).

The axial shortening of compressed diagonal braces at any storey is related to the design ultimate top sway displacement by means of the following relationship

$$\delta_a = \frac{\delta_u}{h_{n_s}} \cdot h \cdot cos\,\alpha \tag{28}$$

where h is the interstorey height.

4. Design examples

Both the design procedure based on TPMC and the design based on Eurocode 8 provisions have been applied with reference to the eight storey structure depicted in Fig. 3, aiming to compare the corresponding seismic response.

It is assumed that the seismic resistant scheme depicted in Fig. 3 belongs to a building, 16×16 m in plane, where the seismic resistant system is constituted by two MRF-VBF dual systems both in x and y directions according to a perimeter structural system. As a consequence, neglecting the accidental torsion due to the variability of location of live loads, the distribution of the seismic horizontal forces among the vertical seismic resistant schemes is immediately derived. The storey height at each level is equal to 3.0 m. The dead load is equal to 3 kN/m^2 and the live load is equal to 2 kN/m^2. S275 steel grade has been adopted. The total seismic action has been evaluated by means of the design spectrum given by Eurocode 8 for soil type A and for high seismicity zone with a peak ground acceleration a_g equal to 0.35 g. All the beam-to-column connections of the chevron part are designed to transmit shear forces only whereas rigid connections are considered between beam and column members of the moment resisting part.

Aiming to fulfil the serviceability limit state requirements provided by Eurocode 8, diagonal

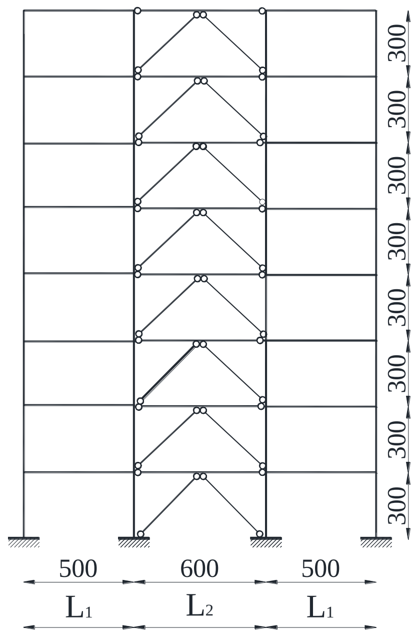

Fig. 3 Structural configuration of the analysed MRF-VBF dual systems

braces are preliminarily designed by assuming that all the seismic horizontal forces are withstood by the chevron braced part.

The seismic base shear is given by

$$F_b = \frac{S_e(T_1)}{q} \cdot \frac{W}{g} \cdot \lambda = S_d(T_1) \cdot \frac{W}{g} \cdot \lambda \tag{29}$$

where $S_e(T_1)$ is the value of the elastic spectral acceleration corresponding to the fundamental period T_1 of the building; a_g is the design peak ground acceleration; q is the behaviour factor provided by Eurocode 8 and taken equal to 4.8 for dual systems as suggested in (CEN 2003); $S_d(T_1)$=0.134 g is the design spectral pseudo-acceleration corresponding to a period of vibration equal to T_1=0.05·$H^{3/4}$=0.050·$24^{3/4}$=0.542 sec; λ is a coefficient depending on the vibration period and on the number of storeys of the building assumed equal to 0.85, W is the total seismic weight of the building equal to 7372.8 kN, being W_i=$(G_k$+0.3$Q_k)L_{tot}^2$=3.6·16^2=921.6 kN the seismic weight of the single storey.

The distribution of the seismic horizontal forces along the height of the building has been evaluated by means of the relationship

$$F_i = F_b \cdot \frac{z_i \cdot W_i}{\sum_{j=1}^{n} z_j \cdot W_j} \tag{30}$$

where F_i is the seismic horizontal force corresponding to the i-th storey; F_b is the total seismic base shear; z_i is the height of the i-th structural level with respect to the foundation level. Table 1 summarizes the results for the whole 8 storey building (F_i) and for the single seismic resistant structural scheme (F_i /2).

Table 1 Storey seismic forces

Storey	Seismic Force F_i [kN]	MRF-VBF seismic force $F_i/2$ [kN]
1	23.4	11.7
2	46.8	23.4
3	70.2	35.1
4	93.6	46.8
5	117.0	58.5
6	140.4	70.2
7	163.9	81.9
8	187.3	93.7

Table 2 Design of bracing members

Storey	N_{brace} [kN]	A_{min} [cm^2]	Diagonal section	$N_{pl.Rd}$ [kN]	$N_{br,Rd}$ [kN]	$\bar{\lambda}$	Ω_i	f_0 [cm]	$N_{c.k}(\delta_a)$ [kN]
1	298.0	11.4	CHS 127×6	627.22	350.08	0.795	1.07	1.235	56.59
2	289.7	11.1	CHS 127×6	627.22	350.08	0.795	1.10	1.235	56.59
3	273.2	10.4	CHS 121×6	596.11	310.35	0.841	1.03	1.226	51.18
4	248.3	9.5	CHS 121×6	596.11	310.35	0.891	1.14	1.226	51.18
5	215.2	8.2	CHS 114.3×6	561.38	267.78	0.931	1.13	1.214	45.45
6	173.8	6.6	CHS 114.3×5	472.14	228.17	1.071	1.12	1.216	38.56
7	124.2	4.7	CHS 108×4	359.39	160.90	1.181	1.18	1.205	27.96
8	66.2	2.5	CHS 108×2	183.15	84.38	1.316	1.16	1.209	14.51

The bracing diagonals have been dimensioned on the basis of the axial force due to the seismic horizontal forces and by considering the slenderness limitation required by Eurocode 8. Resulting from the above design requirements, the chosen profiles for the diagonal members are reported in Table 2, where also the overstrength coefficients Ω_i previously defined and the values of the non-dimensional slenderness $\bar{\lambda}$ are given.

The characteristic dead and live loads acting on the beams of the analysed structure are equal to 7.5 kN/m and 5.0 kN/m, respectively. The combination of actions under vertical loads only is $1.35G_k+1.5Q_k$, therefore the beams of the moment resisting part are designed to resist a uniform vertical load equal to 17.625 kN/m. The beam section of unbraced bays is IPE 180 are the same both for TPMC and Eurocode 8, whereas the beam sections of braced bays, whose connections are assumed to be designed to transmit shear forces only, are dependent on the considered design criterion. In fact, with reference to Eurocode 8 provisions, the beams of braced bays are designed for vertical loads without considering the intermediate support action given by the diagonals, in addition, for seismic conditions they are designed considering the unbalanced vertical seismic action (V) due to diagonals according to Eq. (4).

Conversely, regarding the structure designed according to the proposed procedure, beams of unbraced bays are designed to withstand the vertical loads deriving from non-seismic situation, whereas beams of braced bay are dimensioned considering a mid-span unbalanced vertical action

Table 3 Obtained sections for MRF-VBF designed according to Eurocode8

Storey	Beam MRF EC8 and TPMC	V_{EC8} [kN]	Beam VBF EC8	V_{TPMC} [kN]	Beam VBF TPMC	External Column EC8	Internal Column EC8
1	IPE 180	369.25	HE 340 B	403.50	HE 360 B	HE 140 B	HE 180 B
2	IPE 180	369.25	HE 340 B	403.50	HE 360 B	HE 140 B	HE 180 B
3	IPE 180	355.68	HE 320 B	385.32	HE 360 B	HE 140 B	HE 160 B
4	IPE 180	355.68	HE 320 B	385.32	HE 360 B	HE 140 B	HE 160 B
5	IPE 180	340.15	HE 320 B	364.82	HE 340 B	HE 140 B	HE 140 B
6	IPE 180	285.45	HE 300 B	306.59	HE 320 B	HE 140 B	HE 140 B
7	IPE 180	220.00	HE 280 B	234.36	HE 320 B	HE 140 B	HE 140 B
8	IPE 180	111.61	HE 220 B	119.25	HE 280 B	HE 140 B	HE 140 B

dependent on the design ultimate top sway displacement. In particular, the post-buckling axial force in the compressed diagonal is evaluated, according to Fig. 2, with reference to the axial brace deformation corresponding to the design ultimate displacement assumed in the proposed design algorithm. In the following, a detailed numerical example of the application of design procedure based on TPMC is reported and the obtained structure is compared with the one designed according to Eurocode 8 whose structural members are delivered in Table 3. In the same table also the values of unbalanced vertical action (V_{EC8} and V_{TPMC}) and the obtained section profiles for braced bay beams are reported.

The design procedure based on TPMC, described in Section 3, is herein presented in detail with reference to the structural scheme depicted in Fig. 3.

a) Design of beams and diagonals

As already stated, the first step of the proposed design procedure is the dimensioning of dissipative elements, i.e., beams and diagonal braces. The diagonal members are the same, both with reference to Eurocode 8 methodology and with reference to the proposed design methodology based on TPMC because, according to the first principle of capacity design, dissipative zones have to be designed considering the internal actions due to code specified seismic forces (Table 2).

However, it has been already described above, so that now reference is made to the resulting information needed for plastic design. In order to evaluate the internal work developed by beams in the global mechanism, the knowledge of the location of the second plastic hinge occurring in beams is needed. The beams of the internal braced bay are connected to the columns to transmit shear forces only. Therefore, their contribution to the internal work is equal to zero. Conversely, regarding the beams of unbraced bays, the design plastic resistance is equal to

$$M_{b,Rd}=W_{pl}\cdot f_y=166.4\cdot 10^{-6}\cdot 275\cdot 10^{3}=45.76 \text{ kNm} \tag{31}$$

The limit value of the uniform vertical load is given by

$$q_{\lim}=\frac{4\cdot M_{b,Rd}}{L^2}=\frac{4\cdot 43.58}{25}=6.97\ kN/m \tag{32}$$

With reference to the seismic design load combination, the uniform load $Q_d=G_k+\psi_2\cdot Q_k$ acting on the beams is equal to 9.0 kN/m (ψ_2=0.30 for residential buildings), which is greater than the limit value. Therefore, the abscissa of the second plastic hinge is given by (Mazzolani and Piluso 1997)

$$x = L - 2\sqrt{\frac{M_{b,Rd}}{q}} = 5 - 2\sqrt{\frac{45.76}{9}} = 0.49\,m \tag{33}$$

b) Selection of the design ultimate displacement

In order to define the design ultimate displacement to be compatible with local ductility supply of dissipative elements, a value equal to $\delta_u=\theta_{pu}\cdot h_{ns}$ is adopted, where θ_{pu} is the beam ultimate plastic rotation assumed equal to 0.04 *rad* and h_{ns} is the height of the structure. Such value can be attained provided that it is compatible with the maximum strain in diagonal members. In the examined case, the corresponding axial strain in diagonal braces is given by

$$\varepsilon_t = \frac{\delta_{pd}}{L_d} = \frac{\theta_{pu}\cdot h\cdot cos\,\alpha}{L_d} = \theta_{pu}\cdot sin\,\alpha\cdot cos\,\alpha = 0.057 \tag{34}$$

where h is the interstorey height, L_d is the brace length and α its angle of inclination. Therefore the design ultimate displacement can be assumed equal to 0.04·2400=96 cm.

c) Computation of column axial loads

Axial forces in the columns at collapse state can be determined starting from the knowledge of shear forces transmitted by the beam ends which are due to the distributed loads acting on the beams and to the beam end moments arising from the development of plastic hinges at the beam ends of unbraced bays. In addition, the contribution due to the vertical component of axial force in tensile and compressed diagonals needs also to be accounted for.

With reference to seismic forces acting from left to right, the shear forces transmitted by the beam ends of braced bays are given by

$$V_{jk}^{(left)} = \frac{q_{j.k}L_j}{2} + \frac{N_{t.j.k} - N_{c.j.k}}{2}\cdot sin\,\alpha_{j.k} - N_{t.j.k+1}\cdot sin\,\alpha_{j.k+1} \tag{29}$$

$$V_{jk}^{(right)} = \frac{q_{j.k}L_j}{2} + \frac{N_{t.j.k} - N_{c.j.k}}{2}\cdot sin\,\alpha_{j.k} + N_{c.j.k+1}\cdot sin\,\alpha_{j.k+1} \tag{30}$$

where reference is made to the j-th bay of k-th storey under the assumption of beam-to-column connections designed to transmit shear and axial forces only. Conversely, the shear forces transmitted by the beam ends of unbraced bays, whose beam-to-column connections are assumed to be rigid-full strength connections, are given by (Mazzolani and Piluso 1997)

$$V_{jk}^{(left)} = \frac{q_{j.k}L_j}{2} - \frac{M_{0.j.k} + M_{b.j.k}}{L_j} \tag{31}$$

$$V_{jk}^{(right)} = \frac{q_{j.k}L_j}{2} + \frac{M_{0.j.k} + M_{b.j.k}}{L_j} \tag{32}$$

where

$$M_{0.j.k} = M_{b.j.k} \text{ for } q_{j.k} \le \frac{4M_{b.j.k}}{L_j^2} \tag{33}$$

and

$$M_{0.j.k} = 2\sqrt{q_{j.k} \cdot L_j^2 \cdot M_{b.j.k}} - M_{b.j.k} - \frac{q_{j.k} \cdot L_j^2}{2} \tag{34}$$

in the opposite case.

Therefore, the load transmitted to the i-th column of k-th storey by the adjacent beams corresponding to the (i-1)*th* and i *th* bays is computed as

$$P_{ik} = V_{i-1.k}^{(right)} + V_{i.k}^{(left)} \tag{35}$$

so that the axial force occurring, at collapse state, in the i *th* column of k *th* storey is given by

$$N_{ik} = \sum_{j=k}^{n_s} P_{ik} \tag{36}$$

Similar formulations can be derived for seismic forces in the opposite direction. By means of this consideration, as compression loads provide the most severe design condition, the axial loads acting at collapse state have been evaluated as reported in Table 4.

d) Computation of the required sum of plastic moment of columns, reduced due to the contemporary action of the axial load, $M_{c.1}^{T}I$, for $i_m = 1$

As preliminarily stated, the sum of required plastic moment at first storey can be provided by means of Eq. (15) providing a value equal to 838.65 kNm. It has to be distributed among the columns proportionally to the total axial force acting at collapse state. The bending moment ($M_{pl,req}$) resulting from such distribution and the corresponding required plastic modulus ($W_{pl,req}$) are reported in Table 5 for internal and external columns. In the same table, the chosen standard section and the corresponding plastic bending moment ($M_{pl,obt}$) and plastic modulus ($W_{pl,obt}$) are given.

e) Selection of the column sections at first storey and computation of $M_{c,1}^{*}I$

As reported in Table 5 the selected profile of first storey columns are HE160B and HE320B for external and internal columns, respectively, so that, the sum of obtained column bending moments, reduced due to the contemporary action of axial load, at first storey, $M_{c,1}^{*}I$, is equal to 983.67 kNm.

Table 4 Axial forces acting at collapse state in the columns

Braced Bay		Unbraced Bay		Column Axial Force	
$V_{jk}^{(left)}$ [kN]	$V_{jk}^{(right)}$ [kN]	$V_{jk}^{(left)}$ [kN]	$V_{jk}^{(right)}$ [kN]	External [kN]	Internal [kN]
86.60	86.60	4.41	40.59	40.59	91.01
14.68	154.39	4.41	40.59	81.18	249.81
-73.09	200.70	4.41	40.59	121.77	454.92
-124.38	236.59	4.41	40.59	162.36	695.92
-177.21	251.71	4.41	40.59	202.95	952.04
-201.76	255.76	4.41	40.59	243.54	1212.21
-192.68	264.84	4.41	40.59	284.13	1481.47
-214.67	268.67	4.41	40.59	324.72	1754.55

f) Computation of the required sum of plastic moment of columns, reduced due to the contemporary action of the axial load, $M^{T}_{c.i_m} I$, for i_m>1 (Eqs. (16)-(18)), needed to avoid type-1, type-2 and type-3 mechanisms

In Table 6 reults of application of this step have been reported.

g) Computation of the required sum of plastic moment of columns (Eq. (19))

Also the results of this step are given in Table 6.

h) Design of column sections for the storeys above the first one by the distribution of the sum of the reduced plastic moment proportionally to the axial load.

The results of this step have been reported in Table 7.

i) Checking of the technological condition

From Table 7, it can be recognized that column sections of second storey of unbraced bays are

Table 5 Design of the column sections at first storey

Columns	N_q [kN]	$N_{Mb.Rd}$ [kN]	N_{diag} [kN]	N_{tot} [kN]	$M_{pl.req}$ [kNm]	$W_{pl.req}$ [cm^3]	$W_{pl.obt}$ [cm^3]	Profile	$M_{pl.obt}(N_{tot})$ [kNm]
1	180.00	-144.70	0.00	35.3	65.52	238.26	354.00	HE160B	86.27
2	396.00	144.70	-1098.9	-558.2	353.80	1286.57	2149.00	HE320B	405.56
3	396.00	-144.70	1502.0	1753.3	353.80	1286.57	2149.00	HE320B	405.56
4	180.00	144.70	0.00	324.7	65.52	238.26	354.00	HE160B	86.27

Table 6 Required reduced plastic moment of column at each storey

storey	$\boldsymbol{M_{c.im}^{(1)} I}$ (kN m)	$\boldsymbol{M_{c.im}^{(2)} I}$ (kN m)	$\boldsymbol{M_{c.im}^{(3)} I}$ (kN m)	$\boldsymbol{M_{c.im} I}$ (kN m)
2	963.03	623.94	755.25	963.03
3	1147.34	306.63	666.19	1147.34
4	1151.78	11.02	514.95	1151.78
5	1007.31	133.65	375.36	1007.31
6	781.07	156.17	266.58	781.07
7	479.80	118.10	156.98	479.80
8	194.11	17.17	88.47	194.11

Table 7 Design of column sections

Storey	**INTERNAL COLUMN**					**EXTERNAL COLUMN**				
	$\boldsymbol{N_{tot}}$ [kN]	$\boldsymbol{M_{pl.req}}$ [kNm]	$\boldsymbol{W_{pl.req}}$ [cm^3]	$\boldsymbol{M_{pl.obt}(N_t)}$ [kNm]	**Section**	$\boldsymbol{N_{tot}}$ [kN]	$\boldsymbol{M_{pl.req}}$ [kNm]	$\boldsymbol{W_{pl.req}}$ [cm^3]	$\boldsymbol{M_{pl.obt}(N_t)}$ [kNm]	**Section**
2	1480.31	404.00	1469.01	405.56	HE320B	284.11	77.53	1469.01	125.79	HE180B
3	1211.21	474.60	1736.87	446.84	HE320B	243.53	96.03	1736.87	129.17	HE180B
4	951.11	474.60	1725.89	487.53	HE320B	202.94	101.27	1725.89	132.39	HE180B
5	695.05	408.30	1484.68	526.85	HE300B	162.35	95.37	1484.68	97.35	HE160B
6	454.09	308.00	1119.84	483.72	HE260B	121.76	82.58	1119.84	97.35	HE160B
7	249.70	181.00	658.34	343.32	HE220B	81.18	58.85	658.34	67.49	HE140B
8	90.94	73.57	267.52	227.43	HE160B	40.59	23.49	267.52	28.66	HE100B

Table 8 Designed column sections - second and definitive iteration

Storey	INTERNAL COLUMN					EXTERNAL COLUMN				
	N_{tot} [kN]	$M_{pl.req}$ [kNm]	$W_{pl.req}$ [cm^3]	$M_{pl.obt}(N_t)$ [kNm]	**Section**	N_{tot} [kN]	$M_{pl.req}$ [kNm]	$W_{pl.req}$ [cm^3]	$M_{pl.obt}(N_t)$ [kNm]	**Section**
1	1753.31	353.81	1286.57	405.56	HE320B	324.70	65.52	238.26	122.41	HE180B
2	1480.30	384.21	1397.12	446.84	HE320B	284.11	73.74	268.15	125.79	HE180B
3	1211.22	462.89	1683.22	487.53	HE320 B	243.53	93.07	338.43	129.17	HE180B
4	951.11	464.40	1688.73	526.85	HE320 B	202.94	99.09	360.32	132.39	HE180B
5	695.05	401.97	1461.70	483.72	HE300 B	162.35	93.89	341.43	97.35	HE160B
6	454.09	304.74	1108.16	343.32	HE260 B	121.76	81.72	297.15	97.35	HE160B
7	249.70	179.97	654.45	227.43	HE220 B	81.18	58.51	212.75	67.49	HE140B
8	127.11	73.57	267.52	97.35	HE160 B	40.59	23.49	85.42	28.66	HE100B

greater than the corresponding ones at first storey. Therefore, because of technological requirements, the column sections at first storey are selected to be equal to the ones of second storey and the procedure restarts from step *f*) because a new value of $\boldsymbol{M}_{c.i_m}^{T}\boldsymbol{I}$ is obtained. The resulting column sections are reported in Table 8.

5. Non linear static analyses

In order to check the accuracy of the proposed design procedure aimed at the failure mode control of moment frame-chevron brace dual systems, a static non-linear analysis of the designed structure has been carried out by means of SAP 2000 computer program (1998). The resulting push over curve and the obtained pattern of yielding at collapse have been compared with those obtained with reference to the same structural scheme designed according to Eurocode 8 provisions. The analyses have been carried out under displacement control, taking into account both geometrical and mechanical non-linearities. In addition, out-of-plane stability checks of compressed members have been performed at each step of the non-linear analysis. Regarding the structural model, columns are assumed to be fixed at the base, whereas beams of braced bay and diagonals are pin-jointed to columns. Second order effects due to vertical loads acting on the beams have also been considered. Structural members have been modelled by means of nonlinear elements. In particular, beams and columns have been modelled using beam-column elements with the possibility of developing plastic hinges at their ends (or in an intermediate location when $q_{jk}>4M_{b,jk}/L_j^2$). Diagonals have been modelled accounting for the possibility of yielding of tensile members and for the occurrence of buckling of compressed ones. In particular, compressed diagonals have been modeled with two beam-column elements by means of an intermediate joint having an initial displacement selected to represent the effects of geometrical and mechanical imperfections. The magnitude of such equivalent geometrical imperfection has been evaluated according to the corresponding buckling curve of Eurocode 3 (2005). The study of influence of gusset plates on the actual restraining conditions of the diagonal braces at their lower ends is outside of the aim of this work. However, it can be useful to note that, when gusset plates are not directly modelled, their influence can be taken into account in the evaluation of the brace slenderness. In Fig. 4, the pushover curves (horizontal force multiplier versus top sway displacement) obtained from the analyses are depicted. In addition, both the straight line corresponding to the linear analysis and the global mechanism equilibrium curve are also plotted. The comparison between the linearized mechanism equilibrium curve and the softening branch resulting from the push-over analysis provides a first confirmation of the accuracy of the design methodology.

In order to show the accuracy of diagonal braces modelling by means of the fiber model of SAP 2000 computer program, in Fig. 4 the comparison between the predicted response and the experimental test results has been carried out with reference to a specimen tested by Kanvinde and Deierlein (2004), Fell *et al*. (2005, 2006).

Moreover, in Fig. 5 the pattern of yielding developed at the design ultimate displacement is depicted. Hinges in bracing members identify the yielding of the diagonals in tension and the occurrence of buckling for the compressed ones.

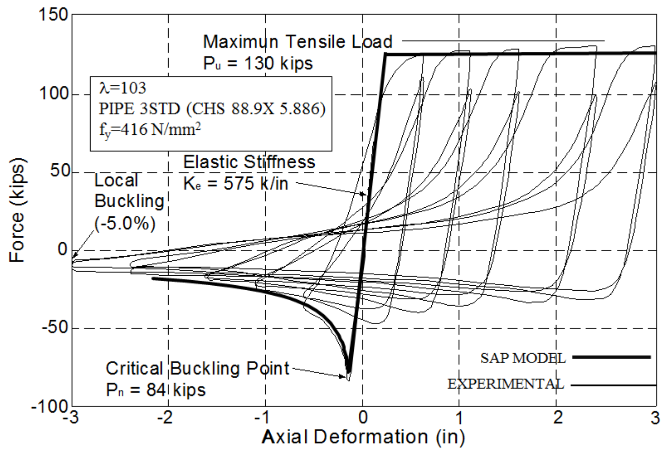

Fig. 4 Comparison between experimental and theoretical curve

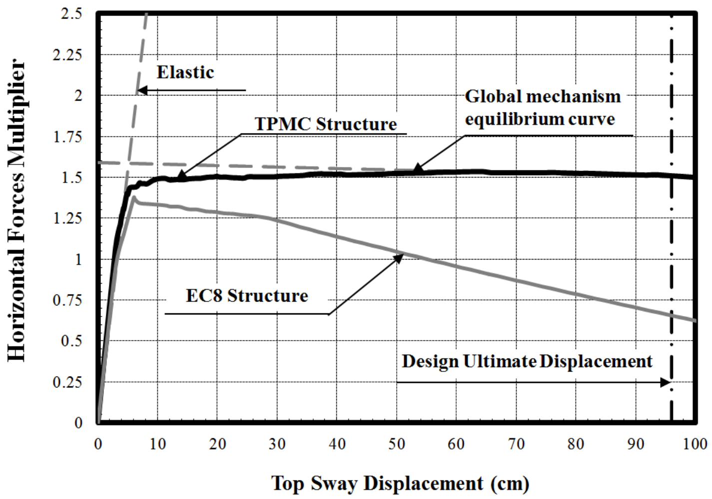

Fig. 5 Comparison between push-over curve and global mechanism equilibrium curve

The most important validation of the proposed design procedure regards the check of the fulfilment of the design goal, given by the yielding of all the dissipative elements, i.e., the development of the collapse mechanism of global type. In fact, in the case of the dual system designed by means of TPMC, all the tensile diagonals are yielded and all the compressed ones are in a buckled condition and, most all, plastic hinges develops at both ends of all the beams of all the storeys. Plastic hinges in the columns develop only at the base of first storey. In addition, Fig. 6 represents the pattern of yielding for a top sway displacement equal to the design one (96 cm). On the contrary, in the case of the dual system designed by Eurocode 8, even though tensile diagonals are still in elastic range, an undesired collapse mechanism of partial type is practically developed for a top storey displacement equal to 45 cm.

It is useful to not that the full control of the pattern of yielding can be achieved by means of a rigorous design approach like that based on TPMC. In fact, if the column sections are increased without any theoretical guiding rule, some improvement on the collapse mechanism typology can be obtained, but the global mechanism is still not assured. As an example, by increasing the column sections of the first two storeys with respect to those deriving from the application of Eurocode 8 (HEB160 instead of HEB140for external columns and HEB200 instead of HEB180 for internal columns) the partial mechanism typology does not improve as depicted in Fig. 7 involving the 2nd and the 3rd storey; an undesired failure mode is still obtained when compared to the global one.

It is evident that without any theoretical guiding rule, the improvement of the collapse mechanism typology up to the occurrence of the global mechanism, i.e., the design goal, could only be achieved by repeated push-over analyses of a new structural schemes where the column sections are progressively increased. However, the computational effort of such a procedure is quite cumbersome and, in addition, the convergence towards the structural solution assuring the global mechanism with the minimum structural weight is not assured.

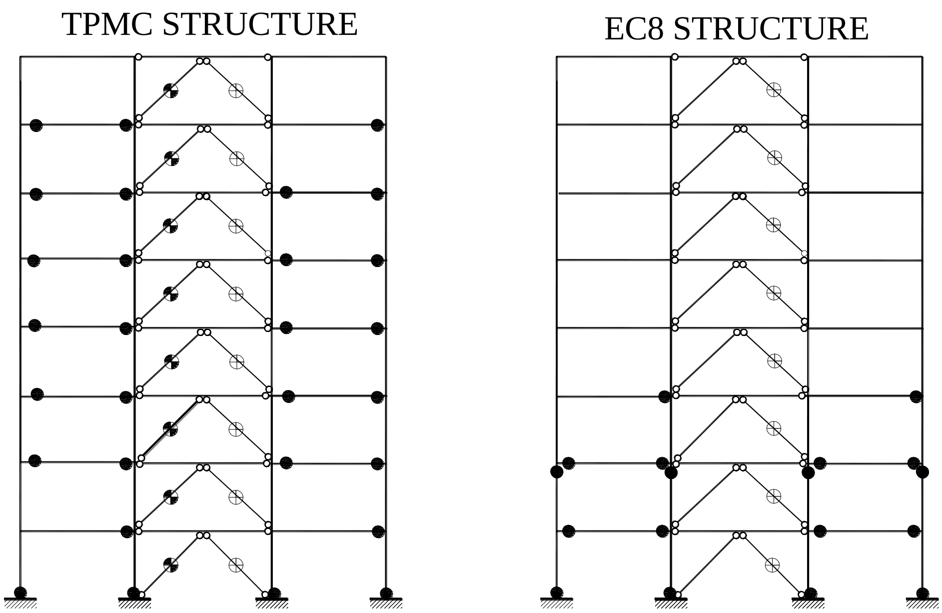

Fig. 6 Developed pattern of yielding for both structures

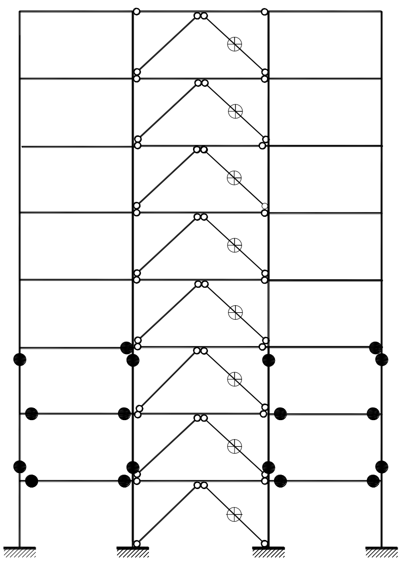

Fig. 7 Developed pattern of yielding for EC8 structure by increasing the column sections of the first two storeys

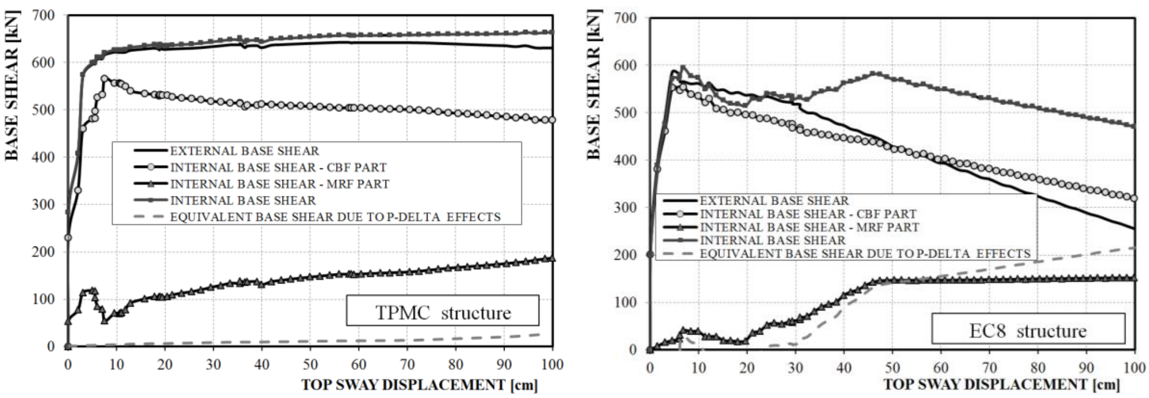

Fig. 8 Sharing of the base shear between the MRF part and the braced part

The above statement is better explained by means of Fig. 8 where the sharing of the base shear is examined according to the following equilibrium equation

$$V_{b.ext} = \alpha \sum_{i=1}^{n_s} F_i = V_{b.int.CBF} + V_{b.int.MRF} - \frac{\delta_1}{h_1} \sum_{i=1}^{n_c} N_{i.1} \tag{37}$$

where $V_{b.ext}$ is the external base shear computed as the sum of the seismic horizontal forces corresponding to the multiplier α, $V_{b.int.CBF}$ is the part of the base shear withstood by the CBF part of the structural system (internal action) computed from the axial forces in bracing members,

$V_{b.int.MRF}$ is the part of the base shear withstood by the MRF part of the structural system (internal action) computed from the shear forces in the columns and the last term is the equivalent base shear due to second order effects. This last term is computed from the axial forces in the columns, being δ_1 the first storey horizontal displacement and h_1 is the corresponding interstorey height.

It is evident that, in the case of the dual system designed according to TPMC, the moment resisting part provides a substantially increasing contribution which, at the design displacement, attains a value equal to 185 kN. At this displacement level, the total internal base shear, given by $V_{b.int.CBF}+V_{b.int.MRF}$, is equal to 664 kN. Therefore, the base shear withstood by the MRF part is equal to about 28% of the total internal base shear. The influence of second order effects is less than 5%. Conversely, in the case of the dual system designed according to Eurocode 8, the base shear withstood by the MRF part attains its maximum value for a displacement level equal to about 50 cm corresponding to the development of a kinematic mechanism for the MRF part. This contribution is equal to about 152 kN. At the same displacement level the total internal base shear is about 570 kN. Therefore, the contribution of the MRF part is about 27%. Despite of this contribution is similar to the one obtained for the structure designed according to TPMC, the resulting behaviour of the structure designed according to Eurocode 8 is adversely affected by second order effects which, due to the poor collapse mechanism typology, practically eliminate the benefits of the MRF part. Therefore, it can be concluded that the suggestion of a minimum lateral resistance to be entrusted to the moment resisting part of the structural scheme, as given in ASCE 7-05 design requirements, cannot be considered sufficient to assure that the MRF part really behaves as a survival structural system, as required for an effective dual system.

In addition, in Figs. 9(a) and (b), for different levels of the top sway displacement (ranging between 10 cm to 96 cm, i.e., up to the value of design displacement adopted in TPMC application), the interstorey drift values along the building height are reported both for TPMC and EC8 structure, respectively. From these figures, it is possible to observe that the distribution of damage in terms of interstorey drift is more uniform in the case of the proposed design procedure, compared to the dual system designed according to Eurocode 8 which, conversely, exhibits an important damage concentration in first and second storey. It is useful to note that this result confirms that the use of push-over analysis as a tool to estimate the seismic response of structures is more appropriate in case of structures failing according to the global mechanism, rather than in case of structures, like those designed according to Eurocode 8, exhibiting partial mechanisms.

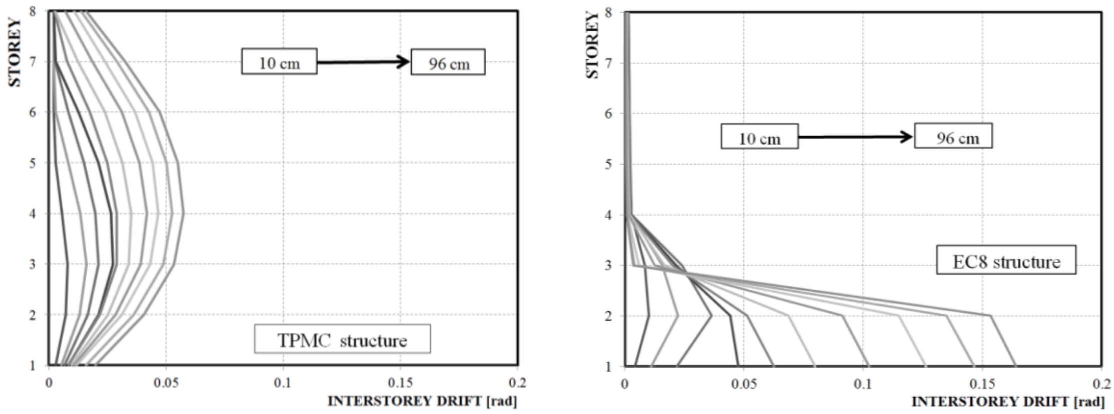

Fig. 9 Interstorey drift ratios of all the storeys for increasing values of top sway displacement

Even though the sole application of TPMC has led, with reference to the worked example herein presented, also to the fulfilment of serviceability limit state requirements, it is useful to underline that, despite of a collapse mechanism of global type, if the maximum interstorey drift exceeds the limit value specified by the code then the design procedure can be easily repeated by increasing the beam or the diagonal sections or by increasing the design ultimate displacement, thus reducing the maximum drift. However, the most convenient solution is obtained by increasing the size of diagonal braces (Giugliano *et al*. 2010). Obviously, in such a case, the design of column sections by means of TPMC has to be repeated to assure the desired collapse mechanism.

6. Conclusions

In this paper, the theory of plastic mechanism control has been implemented for moment frame-braced frame dual systems. In particular, a numerical example of the application of the proposed design methodology has been reported in detail in this work ,with reference to a three-bay eight-storey dual system, aiming to demonstrate the simplicity of the method. In order to check the accuracy of the design methodology, a push over analysis has also been carried out. The obtained pattern of yielding has shown that all the tensile diagonals are yielded, all the compressed ones are in a buckled condition and plastic hinges take place at all the beam ends and at the base of first storey columns. Therefore, analysis results have shown the accuracy of the design procedure which is able to prevent the development of undesired partial mechanisms assuring the global mechanism. The same structural scheme has also been designed according to Eurocode 8 design methodology with the aim to compare the structural behaviour of the structures. The static push over analysis has shown the inability of Eurocode 8 design provisions for guaranteeing the development of dissipative type of collapse mechanisms, because partial mechanisms usually develop.

Finally, the detailed analysis of the sharing of the base shear has pointed out that the suggestion of a minimum lateral resistance to be entrusted to the moment resisting part of the structural scheme cannot be considered sufficient to assure that the MRF part really behaves as a survival structural system, as required for an effective dual system. Conversely, the control of the failure mode is able to guarantee a fail safe behaviour.

Even though the results herein presented are based on push-over analyses only, it is important to underline that they have been confirmed by incremental dynamic analyses repeated for different earthquake records. For sake of shortness, such results are not herein reported, but they will constitute to the main content of a forthcoming paper on the achievements of the research activity.

References

Akiyama, H. (1998), "Behaviour of connections under seismic loads. control of the semi-rigid behaviour of civil engineering structural connections. COST-C1", *Proceedings of the International Conference*, Liège.

Aukeman, L.J. and Laursen, P. (2011), "Evaluation of the ASCE 7-05 standard for dual systems: Response history analysis of a tall buckling-restrained braced frame dual system", *Struct. Congress*, ASCE, 2707-2717.

Bruneau, M., Uang, C.M. and Whittaker, A. (1998), *Ductile design of steel structures*, McGraw-Hill, Network.

CSI (1998), *SAP 2000: Integrated Finite Element Analysis and Design of Structures*, Analysis Reference,

Computers and Structures Inc., University of California, Berkeley.
CEN (2003), *PrEN 1998-1, Eurocode 8: Design of Structures for Earthquake Resistance, Part 1: General Rules*, Seismic Actions and Rules for Buildings.
CEN (2005), *EN 1993-1-1, Eurocode 3: design of steel structures, Part 1: general rules and rules for buildings*, Comité Européen de Normalisation.
Dubina, D., Stratan, A. and Dinu, F. (2011), "Re-centering capacity of dual-steel frames", *Steel Constr.*, **4**(2), 73-84.
Dubina, D., Stratan, A. and Dinu, F. (2008), "Dual high-strength steel eccentrically braced frames with removable links", *Earthq. Eng. Struct. Dyn.*, **37**(15), 1703-1720.
Fell, B.V., Myers, A.T., Deierlein, G.G. and Kanvinde, A.M. (2006), "Testing and simulation of ultra-low cycle fatigue and fracture in steel braces", *Proceedings of the 8th National Conference on Earthquake Engineering 2006*, San Francisco, USA.
Fell, B.V., Myers, A.T., Deierlein, G.G. and Kanvinde, A.M. (2006), "Testing and simulation in steel braces", *Proceedings of the conference 8th U.S. National Conference on Earthquake Engineering*, San Francisco.
Giugliano, M.T., Longo, A., Montuori, R. and Piluso, V. (2010), "Failure mode and drift control of MRF-CBF dual systems", *Open Constr. Build. Technol. J.*, **4**, 121-133.
Iyama, J. and Kuwamura, H. (1999), "Probabilistic advantage of vibrational redundancy in earthquake-resistant steel frames", *J. Constr. Steel Res.*, **52**(1), 33-46.
Kanvinde, A.M. and Deierlein, G.G. (2004), *Micromechanical simulation of earthquake induced fracture in steel structures*, Blume Center TR145.
Kiggings, S. and Uang, C. (2006), "Reducing residual drift of buckling-restrained braced frames as dual system", *Eng. Struct.*, **28**(11), 1525-1532.
American Society of Civil Engineers (ASCE) (2006), *ASCE 7-05 Minimum Design Loads for Building and Other Structures.*
Longo, A., Montuori, R. and Piluso, V. (2008), "Failure mode control of X-braced frames under seismic actions", *J. Earthq. Eng.*, **12**(5), 728-759.
Longo, A., Montuori, R. and Piluso, V. (2008), "Plastic design of seismic resistant V-Braced frames", *J. Earthq. Eng.*, **12**(8), 1246-1266.
Longo, A., Montuori, R. and Piluso, V. (2012), "Theory of plastic mechanism control of dissipative truss moment frames", *Eng. Struct.*, **37**, 63-75.
Longo, A., Montuori, R. and Piluso, V. (2014), "Theory of plastic mechanism control for MRF-CBF dual systems and its validation", *Bull. Earthq. Eng.*, **12**(6), 2745-2775.
Longo, A., Montuori, R. and Piluso, V. (2012), "Failure mode control and seismic response of dissipative truss moment frames", *J. Struct. Eng.*, **138**(11), 1388-1397.
Longo, A., Montuori, R., Nastri, E. and Piluso, V. (2014), "On the use of HSS in seismic-resistant structures", *J. Constr. Steel Res.*, **103**, 1-12.
Longo, A., Montuori, R. and Piluso, V. (2015), "Moment frames - concentrically braced frames dual systems: analysis of different design criteria", *J. Struct. Infrastruct. Eng.*, Taylor & Francis.
Mazzolani, F.M. and Piluso, V. (1997), "Plastic design of seismic resistant steel frames", *Earthq. Eng. Struct. Dyn.*, **26**(2), 167-191.
Montuori, R. and Piluso, V. (2000), "Plastic design of steel frames with dog-bone beam-to-column joints", *Third International Conference on Behaviour of Steel Structures in Seismic Areas, STESSA 2000*, Montreal, Canada.
Montuori, R., Nastri, E. and Piluso, V. (2013), "Theory of Plastic Mechanism Control for Eccentrically Braced Frames with Inverted Y-scheme", *J. Constr. Steel Res.*, **92**, 122-135.
Montuori, R., Nastri, E. and Piluso, V. (2014), "Rigid-plastic analysis and moment-shear interaction for hierarchy criteria of inverted Y EB-Frames" *J. Constr. Steel Res.*, **95**, 71-80.