Earthquake induced torsion in buildings: critical review and state of the art

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Abstract. The problem of earthquake induced torsion in buildings is quite old and although it has received a lot of attention in the past several decades, it is still open. This is evident not only from the variability of the pertinent provisions in various modern codes but also from conflicting results debated in the literature. Most of the conducted research on this problem has been based on very simplified, highly idealized models of eccentric one-story systems, with single or double eccentricity and with load bearing elements of the shear beam type, sized only for earthquake action. Initially, elastic models were used but were gradually replaced by inelastic models, since building response under design level earthquakes is expected to be inelastic. Code provisions till today have been based mostly on results from one-story inelastic models or on results from elastic multistory idealizations. In the past decade, however, more accurate multi story inelastic building response has been studied using the well-known and far more accurate plastic hinge model for flexural members. On the basis of such research some interesting conclusions have been drawn, revising older views about the inelastic response of buildings based on one-story simplified model results. The present paper traces these developments and presents new findings that can explain long lasting controversies in this area and at the same time may raise questions about the adequacy of code provisions based on results from questionable models. To organize this review better it was necessary to group the various publications into a number of subtopics and within each subtopic to separate them into smaller groups according to the basic assumptions and/or limitations used. Capacity assessment of irregular buildings and new technologies to control torsional motion have also been included.

Keywords: torsion; buildings; earthquake; review; eccentricity; codes; assessment; elastic; inelastic

1. Introduction

The torsional response of non-symmetric buildings under earthquake excitations makes their design for earthquake actions substantially more complicated than the design of symmetric buildings whose response is purely translational. And although this problem has been investigated for over 60 years since the emergence of earthquake engineering as a distinct field of engineering

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science, earthquake resistant design of asymmetric and irregular buildings is still an open area of research, while its treatment by modern codes varies significantly. The first study of this problem appeared in 1938 and continued in the late 50ties, with about 6 papers published from 1958 till 1970. Subsequently the number of pertinent publications started growing fast as indicated in the histogram of Fig. 1.

Currently, the total number of publications on this subject in refereed international Journals and in major Conferences probably exceeds 600. This large number must be attributed to the importance of torsion that adversely affects the vast majority of buildings with any type of eccentricity, to the new technologies applied for controlling torsional response and also to the many parameters affecting this problem. This last factor complicates the problem considerably, since it allows researchers to have different combinations of assumptions and different bases for comparisons. As a result, conclusions are often drawn, which although correct for the specific cases they were derived from, are unjustifiably generalized and appear conflicting to other conclusions based on different models or sets of assumptions. This generated debates and many papers supporting one or another view. It is only in the last 10 to 15 years that more realistic models have been introduced to study torsional problems in the inelastic range, allowing also assessments of results based on simplified one story models (Stathopoulos and Anagnostopoulos 2003, Kyrkos and Anagnostopoulos 2011). Such assessments showed that unless the one story models match closely the element stiffness and strength of the real buildings, as well as their three lowest periods, they may lead to erroneous conclusions and trends in behavior (Anagnostopoulos *et al*. 2008, 2009, 2010, see 8.3). This finding has raised questions and doubts about code provisions for torsion based on results from simplified, one-story models.

In order to make sense out of this huge volume of publications with the variety of models, parameters and assumptions, especially those using the simplified one-story inelastic model, it is necessary to group them in appropriate sub-categories following their basic topic and assumptions.

The grouping selected herein is as follows (numbers indicate the corresponding chapter or section and reference group in the paper):

5. Review papers
6. Torsion associated with non-uniform ground motion
7. Elastic torsional response
   7.1 One story simplified models
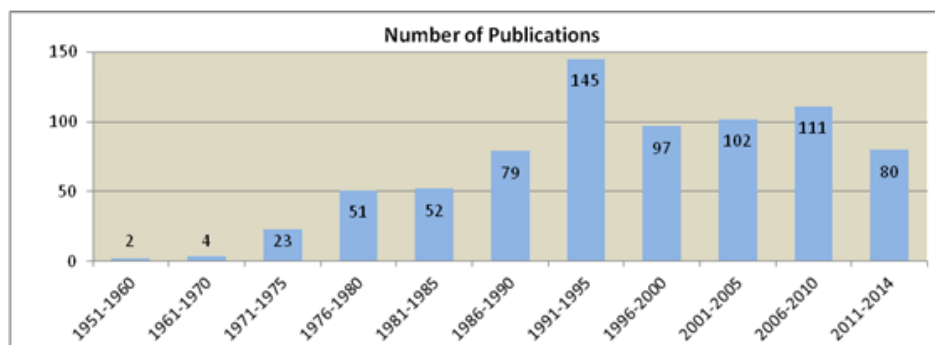   7.2 Multistory models (MST)
8. Inelastic torsional response

Fig. 1 Histogram of time distribution of publications on building torsion

8.1 One story inelastic shear beam models (1ST-INSB)

    8.1.1 e_x, K_y (unidirectional eccentricity, stiffness and ground motion)

    8.1.2 e_x, K_x+K_y (unidirectional eccentricity, bidirectional stiffness)

        8.1.1.1 One-component motions

        8.1.1.2 Two-component motions

    8.1.3 e_x+e_y, K_x+K_y (Bidirectional eccentricity, stiffness and motions)

8.2 Multistory models (MST)

    8.2.1 Approximate–simplified, usually shear beam type, models (MST-SIMP)

    8.2.2 Detailed, usually plastic hinge type, models (MST-PH)

8.3 One story shear beam (1ST-INSB) vs multistory plastic hinge (MST-PH) models

9. Accidental eccentricity

10. Design improvement for torsion

11. Experimental studies

12. Torsion with flexible diaphragms

13. Capacity assessment of asymmetric buildings

14. New technologies to control torsion

    14.1 Base isolation

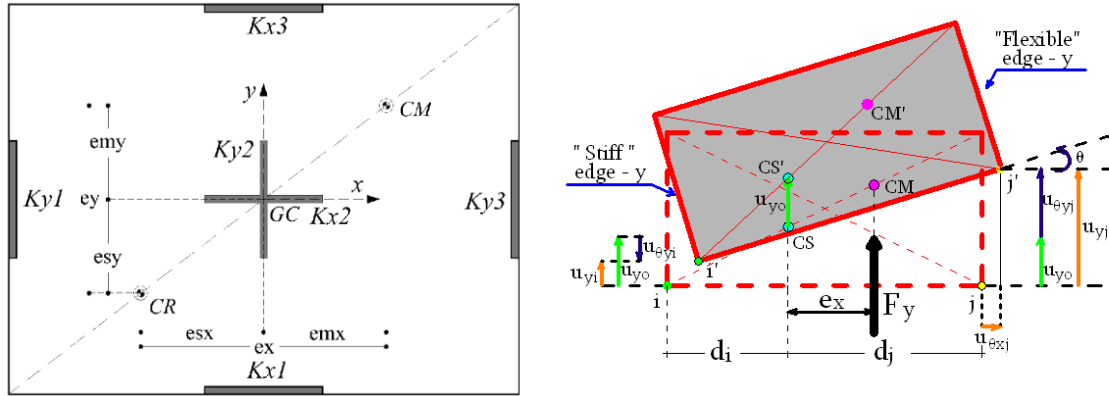   14.2 Energy dissipating devices

For easy reference, papers with assessments of code torsional provisions will be listed at the end of each chapter or subchapter of the above list. Note also that our reference list includes primarily publications in major refereed Journals and in the Proceedings of the World Conferences on Earthquake Engineering. Since some publications may address more than one of the above topics, they might be referenced in more than one place. Before going into each of the above items, we will give a brief set of definitions and terminology associated with torsional response to make this review easier to read. Moreover, conflicting results and controversies debated in the past will be also indicated along the way, while at the end of each chapter we will include brief comments on the pertinent progress made till today.

2. Causes of torsion in buildings

Earthquake induced torsion in buildings is due to (a) non-symmetric arrangement of the load resisting elements (stiffness and strength eccentricity) or non-symmetric distribution of masses, (b) torsional motion in the ground caused by seismic wave passage and by ground motion incoherency, (c) other reasons that are not explicitly accounted for in the design of the structure (stiffness of non-structural element such as brick infill walls, non-symmetric yielding of the load resisting elements, etc.). Since the causes of torsion listed under (b) and (c) cannot be explicitly addressed in design, building codes have introduced what is called accidental eccentricity to approximately account for them by requiring additional loading conditions generated by displacing the structural masses in both directions along the structure's *x* and *y* axes by a certain amount defined as accidental eccentricity. Chapter 9 below is dedicated to accidental eccentricity.

3. Definitions and terminology

For terminology and definitions, we will use the simplified 3-DOF model, shown in Fig. 2,

$$M_t = F_y \cdot e_x \qquad u_{yo} = F_y / K_y \qquad \vartheta_o = M_t / K_\vartheta \qquad u_{yi} = u_{yo} - \vartheta_o \cdot d_i \ < \ u_{yj} = u_{yo} + \vartheta_o \cdot d_j$$

Fig. 2 Symbols and terminology associated with building torsion

representing the layout of an eccentric, one-story building, having a horizontal slab, rigid in its plane, and supported by the indicated shear beam type vertical elements. It is assumed that the load bearing elements for lateral loads are oriented either along the *x* axis or along the *y* axis. An element having a different orientation is "broken" down to two equivalent elements along the *x* and *y* axes each. Most of the past research on earthquake torsional response of buildings has been based on this model, to which we will be referring as "simplified model". CM (or CG) represents the mass center and CS (or CR) the stiffness center, being the point on the slab through which a horizontal force causes only translation, no rotation, of the slab. CS is strictly defined for one story structures, while for multistory buildings an approximate CS may be defined for each floor separately (Stathopoulos and Anagnostopoulos 2005a, see 8.2.2) or an axis for minimal torsional effects (Makarios and Anastasiadis 1998a, 1998b, Marino and Rossi 2004, Georgousis 2010, see 7.2). For one story systems, CS coincides with the so called shear center, i.e., the point through which the shear resultant of the resisting elements passes. In the following, K_{xi} and K_{yi} are the stiffness of any element *i* along the *x* and *y* directions, respectively, y_i, x_i, their respective distances from axes *x* and *y* and K_x, K_y and K_θ the total stiffness along the axes *x*, *y*, *z* (K_θ torsional). For reasons of brevity, the following definitions (Eqs. (1)-(10)) are given only for quantities (eccentricities, radii) along the *x* direction. Replacing *x* by *y* and *y* by *x* we get the respective quantities along the y direction (Fig. 2). Moreover, if these quantities are divided by the pertinent lengths, indicating the maximum distances between edge elements in each direction, normalized values are obtained. In most of the publications dealing with torsion in buildings, the terms "flexible" and "stiff" edge or side are used. These characterize the sides where **<u>under a static eccentric lateral force</u>** through the CM, the displacement due to pure torsion is added or subtracted, respectively, to the common displacement due to pure translation (Fig. 2, right). This distinction is used also under dynamic excitations, but only for reference purposes, not implying the clear additions and subtractions of the definition based on a static force. The definitions below refer to the simplified model of Fig. 2.

Stiffness eccentricity, Stiffness or shear center (CS or CR)
$$e_{sx} = \frac{\sum_{i=1}^{n} K_{Yi} \cdot x_i}{K_Y} \qquad (1)$$

Strength eccentricity, Resistance center or
Plastic centroid (CP or CV)

$$e_{px} = \frac{\sum_{i=1}^{n} V_{PYi} \cdot x_i}{\sum V_{PYi}} \tag{2}$$

Mass eccentricity

$$e_{mx} = \frac{\int_{-L_x/2}^{L_x/2} x \cdot (\int_{-L_y/2}^{L_y/2} m(x,y)dy)dx}{M} \tag{3}$$

Physical eccentricity

$$e_x = \left| e_{mx} - e_{sx} \right| \tag{4}$$

Accidental eccentricity = Eccentricity from sources not accounted for in design

Torsional Stiffness(with respect to CM)

$$K_{\vartheta} = \sum_{i=1}^{\ell} K_{Xi} \cdot (y_i - e_{my})^2 + \sum_{i=1}^{n} K_{Yi} \cdot (x_i - e_{mx})^2 \tag{5}$$

Torsional stiffness radius

$$r_{kx} = \sqrt{\frac{K_{\vartheta}}{K_X}} \tag{6}$$

Radius of gyration

$$r_m = \sqrt{\frac{J_m}{M}} \tag{7}$$

Uncoupled natural periods

$$T_X = 2\pi\sqrt{\frac{M}{K_X}} \,, T_Y = 2\pi\sqrt{\frac{M}{K_Y}} \,, T_{\vartheta} = 2\pi\sqrt{\frac{J_m}{K_{\vartheta}}} \tag{8}$$

Torsional sensitivity (or flexibility) parameter

$$\Omega = \frac{r_{kx}}{r_m} \tag{9}$$

Torsionally balanced (TB) model

$$\text{CM}\equiv\text{CS}\equiv\text{CP} \tag{10}$$

4. Brief summary of modern code provisions for torsion

Applied research on building performance and response to various actions has as basic goal the production of safe buildings at reasonable costs. Since building design and construction is regulated by codes it is only natural that any pertinent progress be reflected in the continuously revised codes. Thus, in order to better appreciate the importance of the various contributions reviewed herein under a code perspective, a brief summary of current code torsional provisions is desirable. Table 1 below has been prepared just for this purpose and includes five of the best known modern codes (see References, Section 4). Papers including assessments of code

Table 1 Torsional provisions of five modern codes

Torsion Related Clauses	CODE				
	New Zealand 2004	Canada NBCC 2005	USA IBC 2012	Europe EC-8:2004	Mexico MOC-2008
Regularity Criteria	GEOMETRIC & STRUCTURAL	GEOMETRIC & STRUCTURAL	GEOMETRIC & STRUCTURAL	GEOMETRIC & STRUCTURAL	GEOMETRIC & STRUCTURAL
Torsional Sensitivity	Limits on ratio d_{max}/d_{avg}	Limits on ratio d_{max}/d_{avg}	Limits on ratio d_{max}/d_{avg}	NO	Limits on ratio e_s/L
Accidental Eccentricity	$\pm 0.10L$	$\pm 0.10L$	0.05L or $A(0.05L)$ $A=(\delta_{max}/1.2\delta_{avg})^2$	$\pm 0.05L$	$\pm 0.05L$
Amplification of static eccentricity	NO	NO	NO	NO	YES
Dynamic Analysis	ALL	ALL	ALL	ALL	ALL
Equivalent Static Analysis	Under conditions of regularity and lowest period	Under conditions of regularity and lowest period	Add to member forces (that include inherent torsional effects) the torque effects by $M_t=\pm e_{acc}F_i$	Under conditions of regularity in elevation and lowest period	Under conditions of regularity
Torsional effects (analysis and model dependent)	Move masses by $\pm e_{acc}$ or combine with static torque $M_t=\pm e_{acc}F_i$ or move static forces by $\pm e_{acc}$	Move masses by $\pm e_{acc}$ or combine with static torque $M_t=\pm e_{acc}F_i$ or move static forces by $\pm e_{acc}$	Same as in the static method above except that $e_{acc}=0.05L$ if included in the dynamic model	Move masses by $\pm e_{acc}$ or combine with static torque $M_t=\pm e_{acc}F_i$ or directly amplify element forces under conditions of symmetry etc.	Move static forces by $\pm e_{acc}$

torsional provisions are listed at the end of each chapter but we must note that some of those provisions have already been revised or replaced in later versions of the considered codes. The most important pertinent development is the replacement in most codes of the equivalent static method by the dynamic response spectrum method as the standard, generally applicable, procedure. This fact alone downgrades to a considerable degree the equivalent dynamic eccentricities that were a basic topic in many of the older publications.

5. Review papers

A number of review papers, about 10, will be found in the literature. The first one is by Rutenberg (1992) and is perhaps the most detailed and comprehensive review for the covered

period. Emphasizing that key findingsand results are based by far on the simplified, one story models (see below) Fig. 3, it summarizes them and reports: *"The picture emerging from the foregoing review is somewhat confusing, and the main confusion is that in addition to the linear properties of the system and element stiffness and location, the overall strength and its distribution among the elements are the most important parameters affecting the peak ductility demand of bilinear asymmetric systems. On the other hand, maximum displacements are easier to predict since they are less sensitive to the strength distribution".* The summary of conclusions starts as follows: *"Several discrepancies and inconsistencies among investigators have been reported in the preceding sections. Yet some general conclusions do emerge from the studies reviewed in this paper (and from unpublished investigations by the author).* Selectively, some interesting listed conclusions are: (1) The response (and conclusions) is affected by the model. (2) Peak ductility demands of asymmetric systems are larger than those of the corresponding symmetric systems. (3) Usually the most sensitive element (in terms of ductility demands) is the one near the stiff edge of the deck. (4) The strength eccentricity does not appear to be a useful parameter in allocating strength to the resisting elements. One of the reasons for the discrepancies and conflicting results, as often reported in various publications, has to do with the so called reference models used as the basis for comparison. This is addressed in a specialized review paper (Correnza *et al.* 1992) which evaluated the most commonly used reference models, namely the symmetric and the torsionally balanced (TB) model (TB models are those in which CS coincides with CM). The comparisons therein were all based on the simplified, one story deck model of Fig. 2. In their conclusions they point out (a) that the two models will give different results in the inelastic range, and (b) that accidental eccentricity must be considered in the design of the Reference model. With the goal of putting some order in the rather chaotic published results and conclusions, especially for inelastic torsion, Chandler *et al.* (1996) have identified and listed ten *"areas of concern where the use of differing definitions or the making of diverging assumptions has resulted in a basic lack of agreement between the results and conclusions of the research".* In the same paper
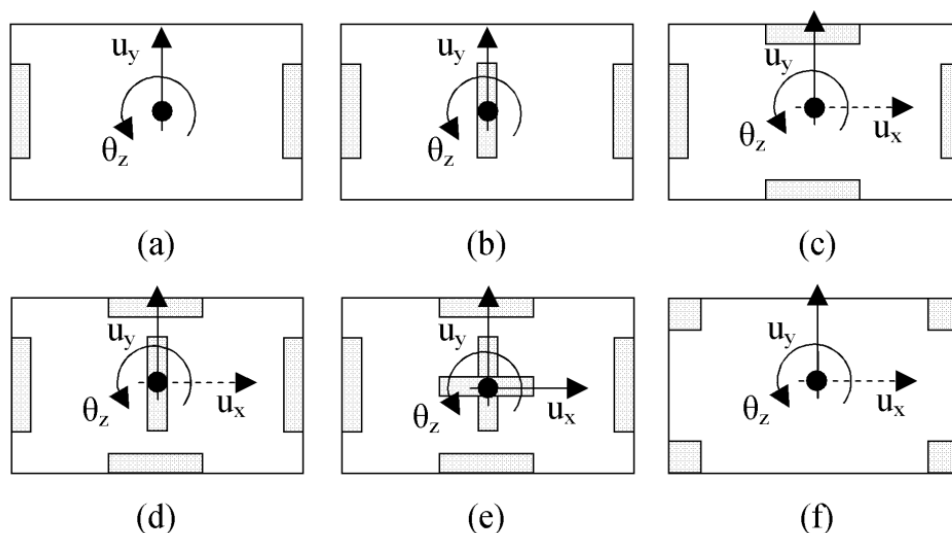

Fig. 3 Different types of 1 story, deck type, simplified models and DOF used (DOF u_x in dashed line may or may not be considered)

recommendations for developing code provisions are made and an example methodology is presented for deriving improved static design eccentricities.

The next review paper by Rutenberg and De Stefano (1997), addressed publications based on the simplified 1-story model, as well as a few newer publications based on multistory models, mostly of the shear beam types. Most of the listed conclusions focus on results from the multistory shear beam type models, whose shortcomings have already been discussed as far back as 1972 (Anagnostopoulos 1972, see 8.2.1). The next review paper, Rutenberg (1998) is essentially a repetition of the 1997 review. A limited review of results as background material for the EC8 Code can be found in Cosenza *et al.* (2000), while as follow up of his 1998 review paper, Rutenberg (2002) lists the progress since 1998 grouping the reviews to single story, simplified models and multistory models, some of them approximate and some more detailed, used either in dynamic or static pushover analyses. Again, the conclusions are varying, while many of them should be obvious without any analyses. Some experimental work reported in two papers is briefly reviewed and for the first time, publications on the use of energy dissipating devices (dampers) and base isolation to control torsional vibrations are also reported and briefly reviewed. The next review paper by De Stefano and Pintucchi (2008) covers the period since the 2002 review by Rutenberg, grouping the reviewed publications for (a) 1-story simple models. (b) Inelastic multistory models, both approximate and detailed (of the plastic hinge type), subjected to dynamic excitations as well as to static overloadings (pushover analyses) (c) Passive control methods including viscous as well as tuned mass dampers and base isolation, and (d) Vertically irregular multi story buildings having setbacks. Finally, the paper by Symans *et al.* (2008) (see 14.2) is an excellent review paper on the subject of energy dissipation systems for seismic applications in general.

6. Torsion associated with non-uniform ground motion

Non uniform ground motion may be caused (i) By *wave passage or travelling wave effects*, which occur when different points of the ground surface are excited by the same motion but with a phase lag (e.g., wave impinging the foundation at a finite angle). (ii) By *ground motion incoherence*, occurring when different points of the ground experience motions with different amplitudes and phase characteristics because of incoming waves from different locations of an extended earthquake source, waves impinging at different angles of incidence and at different times, wave reflections and refractions around the building foundation, or due to changes produced in the waves when travelling from the source to the structure through paths of different physical properties.

The very first paper on the subject of torsion due to seismic wave passage is the paper by Newmark (1969) who determined, using simple considerations, the torsional ground motion, torsional ground spectra and subsequently gave simple practical expressions for an equivalent eccentricity associated with this source of torsion. Newmark's work, applicable to one story symmetric systems, is simple, practical and has opened the way for more sophisticated solutions, such as the work by Luco (1976), who obtained the steady state solution for a simple elastic structure on a rigid circular disc sitting on an elastic halfspace and subjected to an obliquely incident plane SH wave. Nathan and Makenzie (1975) have also studied rotational ground motion, as did Tso and Hsu (1978), who gave solutions for the torsional ground motion and torsional spectra along the line of Newmark. Rutenberg and Heidebrecht (1985) provided similar solutions for a rigid base mat sitting on a Winkler type soil, Lee and Trifunac (1985) determined the surface

torsional motion based on analyses of available translational records and subsequently produced synthetic torsional accelerograms and finally, Castellani and Boffi (1986) estimated the rotational component of earthquake ground motions using data from the SMART array. More practical appears to be the work of Wu and Leyendecker (1984), who extended the investigation from a symmetric system to an eccentric system subjected to SH waves and determined that the rotational response of the system depends greatly on the physical (geometric) eccentricity, the dimensions of the foundation and the ratio Ω of the rotational frequency to the translational frequency.

A simplified derivation of torsional motion, calibrated to produce code specified eccentricities has been proposed by Vasquez and Ridell (1988), while Yeh *et al.* (1992) investigated the response of a 1-story biaxially eccentric system to torsional ground motions using data from the SMART array. The inelastic response of simple, one story system has been addressed by Inoue and Shima (1988) in a formulation accounting for travelling wave effects, and by Shakib and Datta (1993) for a biaxially eccentric inelastic system subjected to an ensemble of non-stationary random ground motions. A noteworthy study with interesting results was published by Hahn and Liu (1994), who investigated the response of both, symmetric and eccentric one story elastic systems, to random ground motions represented by a cross power spectral density function and an incoherence function. Three cases were examined: a symmetric structure with motion incoherence and an eccentric structure with and without motion incoherence, from which simple expressions were derived for effective eccentricities. Moreover, comparisons with the code value of 0.05L for accidental eccentricity were made and their dependence on the Ω ($=\omega_\theta/\omega_x$) was discussed.

De La Llera and Chopra (1994d) used one story elastic shear beam models to investigate the accidental torsion in buildings due to base rotational ground motion caused in 30 Buildings in California, for which base acceleration records from 3 earthquakes were available. They concluded that "*Accidental torsion has the effect of increasing the building displacements, in the mean, by less than 5 per cent for systems that are torsionally stiff or have lateral vibration periods longer than half a second. On the other hand, short period (less than half a second) and torsionally flexible systems may experience significant increases in response due to accidental torsion. .... two simplified methods are developed for conveniently estimating this effect of accidental torsion. They are the 'accidental eccentricity' and the 'response spectrum' method. The computed accidental eccentricities are much smaller than the typical code values, 0.05b or 0.10b, except for buildings with very long plan dimensions (b≥50 m)*". Publications on the same subject by Hao and Duan (1995, 1996), Hao (1996, 1997, 1998) presented results not in full agreement with those by Hahn and Liu. In fact Hao (1998), reports that for torsionally stiff systems (Ω>1), the physical eccentricity is more important than the non uniform motion induced eccentricity, while Hahn and Liu (1994) state exactly the opposite for Ω>~2, attributing that to the reduction of base shear caused by incoherency effects.

Shakib and Tohid (2002) provide results for a simplified elastic, one story and one way symmetric system based on random input motions and their main conclusion is that for torsionally stiff systems the 0.05b code value for accidental eccentricity is sufficient, but insufficient for overall stiff yet torsionally flexible systems. Moreover, the effect of rotational ground motion is much more significant for buildings on soft than stiff soil. Heredia-Zavoni and Levya (2003), use multi story torsionally stiff buildings on either stiff or soft soils and conclude that incoherence and wave passage effects did not induce significant torsional motion to the building but were important only for corner columns in the ground story. Moreover, the response was found to be more sensitive to wave passage effects than to loss of coherency and that for soft soils the code defined accidental eccentricity could underestimate shears in corner columns, especially in stiff buildings

with long dimensions. The effect of phase difference on the torsional response of simple asymmetric systems was statistically quantified in terms of energy by Alexander (2007), while Rigato and Medina (2007) examined the effects of earthquake incidence angle of two component motions on ductility demands of 1-story inelastic systems, symmetric as well as biaxially eccentric. Juarez and Aviles (2008) extended the work of Hahn and Liu (1994) by including foundation flexibility, along with wave passage effects, and developed a new simple equation for the total effective eccentricity, physical plus wave passage related (accidental). They examined these effects in relation to low and mid height buildings and noted that the greater importance of the foundation flexibility in increasing the effective eccentricity is a consequence of the reduction in base shear by interaction. Finally, the paper by Smerzini *et al*. (2009) dealing only with rotational ground motion as determined from instrumental data and available models, is of interest more to seismologists than to engineers.

In summary, there is little essential progress in this area of torsion in the past two and a half decades, as the main contributions date back to the 70ties and 80ties. Most of the newer papers could be viewed as refinements, in several ways, of Newmark's original work in 1969. These refinements are: modeling of the soil as an elastic subspace, introduction of ground motion variability as a combination of wave passage and loss of coherency effects, introduction of randomness in the ground motion and use of eccentric superstructures in addition to symmetric ones. Almost ALL of them make comparisons with code imposed accidental eccentricity for structural design, but it is almost always forgotten that the code required accidental eccentricity aims at capturing not only torsion induced by the ground motion but also from other sources such as mass and stiffness uncertainties.

7. Elastic torsional response

As expected, the first investigations of the earthquake induced torsional response were based on very simplified models, subject to many assumptions and limitations. Fig. 3 shows the typical one story rigid deck structure with two or three degrees of freedom (two translations and a rotation), supported on vertical, shear beam type elements. Initially the elements were only in one direction but later elements were placed in the perpendicular direction that allowed biaxial eccentricities and two component earthquake motions to be considered. In the years that followed, new knowledge accumulated while the computational power kept increasing due to rapid technological advances in hardware and software. As a consequence, more sophisticated models were used to study this problem and the assumptions made were less restrictive. So, starting with the simple one-story highly idealized, one way eccentric system responding elastically, the models were extended to idealized, elastic multistory special class buildings, to be followed by inelastic, one story simplified systems with uniaxial eccentricity, all subjected to one component ground motion. Subsequently, biaxial eccentricities were introduced and single, two component motions were initially used. When it was realized that conclusions based on single motions were shaky, groups of motions were applied and conclusions were based on average responses, thus becoming less dependent on the characteristics of specific single motions.

7.1 One story simplified models

It appears that the first dynamic investigation of torsion was carried out by Ayre (1938) who

made a theoretical and experimental investigation of one and two-story simple eccentric buildings aimed primarily at demonstrating the coupling between translational and torsional motion. Ayre's formulation of the equations of motion involves first the determination of the stiffness (rigidity) center for each floor and the corresponding principal axes of rigidity, i.e., the two orthogonal, horizontal axes through the stiffness center, along which a static horizontal force causes a parallel displacement of the slab. In those days, the stiffness center and the principal axes were essential instruments for approximate static solutions of eccentric buildings. Twenty years later, Housner and Outinen (1958), used a simplified 2 DOF asymmetric system subjected to ground acceleration and pointed out the dynamic amplification of the stresses at the flexible side, along with the pertinent shortcomings of the static solutions used till that time. Subsequently, based on the dynamic response of a simplified class of eccentric multistory buildings, Bustamante and Rosenblueth (1960), introduced the concept of dynamic, as opposed to static, eccentricity, and also concluded that *"A rough estimate of torsional dynamic effects in multi-story buildings can be obtained from the response of single-story structures with similar characteristics"*. This set the stage for using just the one story simplified model to investigate torsional problems and also for code torsional provisions through the introduction of *design eccentricities* e_d, aiming at the static approximations of the dynamic torsional effects at the flexible and stiff edges of the building as follows

$$\left.\begin{array}{l} e_d^+ = \alpha \cdot e_x + \beta \cdot L \\ e_d^- = \gamma \cdot e_x - \beta \cdot L \end{array}\right\} \qquad (11)$$

where e^+_d and e^-_d are the two design eccentricities to be considered for the lateral seismic force, if e_x is the natural or physical eccentricity along the x axis. Same expressions with y instead of x apply for the y axis. The coefficients α ($\alpha \geq 1$) and γ (≤ 1) increase and decrease the natural eccentricity aiming to approximate dynamic effects by static analyses. Note that this applies only to static analyses, while for dynamic analyses, included now in all modern codes, only the second term has been kept. This second term represents the so-called accidental eccentricity, i.e., eccentricity from a number of sources not foreseen in design, with coefficient β having values usually 0.05 or 0.10, producing accidental eccentricities 0.05L or 0.10L, respectively, where $L=$ length of the building perpendicular to the seismic motion direction. The first equation controls elements at the flexible side and the second equation contols elements at the stiff side. In the following years the majority of published papers dealt with torsion due to stiffness and/or mass eccentricities, while a smaller number, reviewed above, examined the problem of torsion induced in symmetric or asymmetric structures due to motion characteristics (e.g., travelling seismic waves). Torsion may also occur in symmetric or asymmetric structures as a result of lateral-torsional coupling caused by stiffness non linearities (Tso 1975, Antonelli *et al.* 1981). The formulation by Tso is for one story building while by Antonelli *et al.* is for multi-story buildings, with results presented for one and two story structures. Lateral torsional coupling can cause torsional instability under harmonic loading.

Practically all the papers reviewed in this section used one of the structural configurations of Fig. 3, which will be referred to as system type (a), (b) etc. Kan and Chopra (1977) used a type (e) layout and a flat-hyperbolic response spectrum for earthquake action and were among the first to carry out an extensive parametric study for elastic torsional response. They were followed by Dempsey and Irvine (1979), who used a type (a), 2 DOF system with one component motion, Kung and Pecknold (1984), with a type (c) system and bidirectional motion, Chandler and

Hutchinson (1986, 1988, 1992) who also used a type (c) system but with 2 DOF and unidirectional motion. In these papers results from parametric investigations are presented with emphasis placed on one or the other parameter. In all these papers the base was assumed as fixed. Soil flexibility modeled by a rigid base on 3 spring-dashpots, a translational, a rocking and a rotational, was included in Tsicnias and Hutchinson (1984), Chandler and Hutchinson (1987c) and Sikaroudi and Chandler (1992) for a simple, one story, 2 DOF model with uniaxial eccentricity. In the first of these publications the steady state harmonic response was studied, in the second the study was extended to include earthquake response to two small sets of real records, a narrow band and a broad band set, while the third paper is very similar to the second except that the set of motions used was different.

The influence of the eccentricity and of the uncoupled frequency ratio Ω was investigated and was found that the steady state effect of these two parameters on the coupling is not qualitatively affected by increasing soil flexibility. Somewhat similar conclusion was reached in the second and third papers. However, for buildings on very flexible foundations and with values of Ω close to 1.0 the torsional demands appeared to increase by 40% compared to those when the base is rigid. A more complete assessment of soil flexibility has been carried out in the frequency domain by Wu *et al.* (2001) who confirm well known behavioral trends, i.e., increasing height-to-base ratio generally amplifies translational and torsional response due to a more pronounced rocking motion and so do values of $\Omega\sim=1.0$. As a note here we could add that the effect of foundation flexibility may be viewed as a lengthening of the structural periods and this would be the primary mechanism for any changes in the torsional response and its effects.

A very detailed theoretical investigation of elastic torsion for type (a) or (b) systems with 2 DOF under unidirectional motion is presented in a 34 pages, text book type, paper by Anastasiadis *et al.* (1998), where application of the theoretically derived formulas can provide the influence of the parameters involved to the torsional response. Results and comparisons with inelastic response of type (e) systems subjected to two component motions have been reported by Stathopoulos and Anagnostopoulos (1998). Biaxial eccentricities and biaxial motions were considered by Hernandez and Lopez (2000), who concluded, based on results from type (d) systems, that bidirectional excitation could increase stiff edge displacements up to 50% and that bidirectional motion effects become more significant when the translational stiffnesses in the two directions are close to each other.

The next three papers, Gasparini *et al.* (2004, 2008), and Trombetti *et al.* (2008, 2012), use type (c) or (d) or (e) systems with biaxial eccentricity and introduce a new parameter, the α (alpha) parameter ($\alpha=\rho^*|u_{\theta,\max}/u_{y,\max}|$, ρ=gyration radius) found to be essential for torsional response. In the second of the two papers, the α method is used to introduce a new equation for the code specified design eccentricity and in the third paper it is used to derive a closed form expression for the maximum corner displacement under seismic excitation. Finally, Banerji and Barve (2008), using a 2 DOF, one way eccentric system, investigate the effect of uncertainties in ω_y, ω_θ and in the normalized eccentricity, on the torsional response. They found that the effects of these uncertainties increase for smaller eccentricities and for larger couplings and affect more the flexible edge than the stiff edge displacements.

Code assessment papers: Tso and Dempsey (1980), Tsicnias and Hutchinson (1981, 1982), Dempsey and Tso (1982), Chandler and Hutchinson (1987a, 1987b), Rutenberg and Pecau (1987, 1989), Rady and Hutchinson (1998), Shakib (2004).

7.2 Multistory models (MST)

MST models of eccentric buildings were used almost as early as one story simplified models. And while complete formulations for detailed models were possible, simplifications were introduced mainly to reduce the number of parameters involved and to make the problem computationally tractable so that parametric investigations would become more effective. The most frequently encountered simplifications are: (a) buildings with constant story geometry and mass over height and with constant or proportionally varying stiffness in all floors, which implies that all mass and stiffness centers of the floors are located on two vertical axes while the orientation of the principal axes of resistance remains constant with height. (b) shear type buildings (stiffness matrix tri-diagonal). We note here that in general, stiffness centers are not defined for a MST building, unless it satisfies the geometry and stiffness restrictions under assumption (a) above. Quite often, however, reference is made to the stiffness center of a story for any building in general, by loosely associating the story to one story structure.

One of the very first papers on torsion of MST buildings, using the simplification (a) above, by Shiga (1963) emphasized in its conclusions that torsional problems become important when the building is torsionally flexible ($\Omega < 1$). An interesting paper by Hart *et al.* (1975) presented measurements of periods and analyses of torsional response for several instrumented MST buildings due to ambient (primarily) and earthquake vibrations. In the same paper, a mean torsional response spectrum from four historical records was also derived and supported the conclusion that torsional response of the examined tall buildings was significant and strongly influenced by the torsional ground motion as well as by the building's asymmetry. In the papers that followed, approximate solutions were proposed for special classes of MST buildings, such as the method in Cardona and Esteva (1977), where an equivalent 2 DOF eccentric system was defined on the basis of the generalized mass and stiffness of the MST building, estimated from two approximate modes (e.g., story translations and rotations under a static load vector). Dynamic amplification factors for torque and shear are then computed for the 2 DOF systems and subsequently used to amplify story torques and shears of the MST building.

Kan and Chopra (1977a, 1977b, 1977c), have presented a simplified method for MST buildings subject to both limitations (a) and (b) above, using the modal properties and elastic forces of an associated uncoupled MST building as well as the dynamic response of an equivalent 3 DOF, one story building. Along the same line of approximation, Gluck *et al.* (1979) developed a similar method to that of Kan and Chopra, except that this appears to apply to MST buildings subject only to limitation (a). They also show that the one story torsional coupling analogy for MST asymmetric buildings holds (i) if the stiffness matrix of the building can be uncoupled in plan into 3 principal directions (or coordinates) and (ii) if the mode shapes in the 3 uncoupled directions are identical. In the meantime the response spectrum method of analysis started being applied to detailed 3-D elastic models of asymmetric buildings or structures in general, e.g., Rutenberg *et al.* (1978), while issues related to modal and spatial combinations of earthquake response were addressed in Rosenblueth and Elorduy (1969), Rosenblueth and Contreras (1977), Der Kiureghian (1979), Anagnostopoulos (1981).

In spite of the fact that "exact" dynamic analyses of detailed spatial models started to be more widely used, simplified approximate models kept appearing as in Hejal and Chopra (1989a, 1989b, 1989c, 1989d) because they were easier to use to investigate torsion. The above four publications by Hejal and Chopra are based on an extension of the earlier model by Kan and Chopra with the shear beam assumption removed, where the 1989d paper dealt with the same problem solved by

Rutenberg *et al*. in 1978. The new approximate model, formulated for one way eccentric systems under one component motion, is still subject to the limitations (a) above that make extrapolation of results to other buildings questionable. Note that its steps of application are almost the same as those of the earlier model based on the uncoupled MST variant of the real building and its one-story equivalent. In terms of results, the studies with this model confirm that lateral torsional coupling decrease the base shear, base overturning moment and top lateral displacement at the CM point. These effects increase with increasing eccentricity and for systems with close lateral and torsional frequencies. For the idealized class of MST buildings subject to assumptions (a) and (b), reduction methods to one story systems will be found in Wittrick and Horsington (1979), Zarate and Ayala (2004), and Adachi *et al*. (2011). In this last publication, a more general method is given, permitting reduced models with more than one story, e.g., 2 stories, leading to improved results.

Very interesting is the work of Lin and Tsai (2007a), applicable to one way eccentric multistory buildings under one component motion, for which they have derived an equivalent stick model with 2DOF corresponding to two parts of a modal vector: its translational component in the direction of motion and its rotational component. This model has been extended in Lin and Tsai (2009) to provide an equivalent inelastic 2DOF stick model, while in Lin and Tsai (2008a), it was extended to cover buildings with biaxial eccentricity under two component motions. The 2008a extension leads to an equivalent 3DOF stick model, where each DOF corresponds to one of the three directional components of the modal vector. This model has also been applied to 3D modal pushover analyses and has been used in Lin and Tsai (2012) to explain what the authors consider basic trends in inelastic torsional response. Discussion on the limits of the approximate methods will be found in Bosco *et al*. (2004).

Most of the codes for earthquake resistant design of buildings included till recently as main analysis method the so called equivalent static method and made reference to eccentricities defined in the form of Eq. (11). The definition of the physical eccentricities (Eq. (4)) requires the stiffness center, which, however, is strictly defined for one story buildings with a rigid floor diaphragm, as it was earlier explained, while for MST buildings it cannot be generally defined. This could create questions related to code applications. Answers to these questions were provided by Cheung and Tso (1986) who defined generalized centers of rigidity as the set of points at the floor levels such that when the equivalent static seismic loads are applied through these points no rotations of any of the floors will occur. They also showed by matrix methods how to locate these points. In a subsequent very useful and easy to follow paper by Tso (1990), the difference between floor and story eccentricities is explained as well as their dependence on the vertical distribution of the lateral seismic forces. This dependence makes clear that contrary to the one story building in which the eccentricity is an inherent property independent of loading, in MST buildings it depends also on the loading. It is rather interesting then to see the same topic covered in a very similar paper by Jiang *et al.* (1993) for shear type buildings only, when the more general case of buildings without the shear type restriction had already been covered by Cheung and Tso (1986) and Tso (1990).

Fresh, new interesting ideas in this area will be found in Makarios and Anastasiadis (1998a, 1998b) where the concept of "fictitious elastic axis" has been introduced and in Makarios (2005, 2008) with further applications. Marino and Rossi (2004) gave solutions for the "optimum torsion axis" and Georgousis (2009, 2010), introduced the modal center of rigidity. In these papers, axes for minimal torsion under lateral static loads are introduced and example applications are given. An improvement of this methodology can be found in Georgousis (2014). What makes such

definitions quite interesting is that they are applicable to any building without any restrictions on geometry, mass or stiffness distributions. An alternative method to locate floor and story centers of rigidity can be found in Basu and Jain (2007). It is interesting to note, however, that application of the static code provisions can be carried out using a method by Goel and Chopra (1993) that combines results by three additional static analyses without having to compute stiffness centers in the various floors. A very interesting theoretical investigation of a certain class of asymmetric multistory buildings, called isotropic buildings, and which are formed by planar load resisting elements with proportional lateral stiffness matrices, has been presented by Athanatopoulou *et al*. (2006). This study provides good insight for the seismic response of elastic asymmetric multistory buildings.

In the following years new publications appeared, though with very little new knowledge to offer. Hutchinson *et al*. (1993) used a 20 story shear beam building with corner columns under unidirectional excitation to conclude that the similarities between this special building class and the corresponding single story models are only qualitative while the degree of lateral torsional coupling is not uniform with height. Yoon and Smith (1995), assumed mass and stiffness centers located on two vertical axes and modeled MST buildings as continuous systems using an equivalent single cantilever beam for shear walls and an equivalent single shear cantilever beam for frames. The model was evaluated through 3-D analyses of detailed models and showed maximum differences in the coupled period ratios less than 6.0%. Obviously the method may be used for preliminary design, but again under the stated limitations for mass and stiffness distribution.

For so called regular buildings in plan and in height, the codes typically allow the static method of seismic analysis for design. A comparison of the equivalent static force procedure (ESFP) with the dynamic response spectrum method (DRSM) was carried out by Lam *et al*. (1997), using proportionately and non-proportionately framed MST buildings (in proportionately framed buildings, the stiffness matrices of the various lateral force resisting elements are proportional to each other and the translational and torsional mode shapes are identical). They concluded that the static solution gives good results for the former and differences up to 20% for the latter, whose effective eccentricities cannot be adequately predicted by the one story equivalent model. A similar but broader comparison was reported in Harasimowicz and Goel (1998), who used three, 9-story buildings with cantilever flexural walls and with plan layouts similar to plans (d) and (e) in Fig. 3 above. The buildings were not proportionately framed and the comparisons concerned the ESFP, as applied with different interpretations of the reference centers for torsional effects, with the DRSM. The basic conclusions were: (a) the ESFP gave very similar results for all the reference centers considered. (b) The ESFP gave good results only for the torsionally stiff building (one of the considered 3) and poor results for the torsionally flexible buildings (the two of the 3). An investigation of the torsional irregularity parameter proposed in the ASCE 7-05 (1995) document can be found in Ozhendekci and Polat (2008) while a story damage index has been developed by Wang J-F (2013), to characterize the damage condition of torsionally coupled buildings, based on the dominant natural frequencies and modal shapes. Tha latter are obtained by system identification techniques from a few floor acceleration records in the building. In this method, verified with real data and numerical calculations, the position variation of the center of rigidity may be used to identify the damaged area of the building.

Code assessment papers: Bustamante and Rosenblueth (1960), Hidalgo *et al*. (1992), Harasimowicz and Goel (1998)

8. Inelastic torsional response

8.1 One story inelastic shear beam models (1ST-INSB)

Most of the publications on torsion are in this group of papers. Any of the element configurations in Fig. 3, providing shear resistance to a rigid deck have been used, while no vertical loads were considered. The force-deformation relationships used were mostly bilinear, but in few cases stiffness or strength degradation were also considered. Stiffness and strength of the bearing elements were almost always selected independent of each other and in ways serving the purpose of the paper. The popularity of this model stems from its simplicity and the ease to carry out inelastic dynamic analyses with it. However, as will be explained later, these models suffer from many shortcomings so that the results obtained from them are strictly applicable to the models themselves. Unfortunately, this has been overlooked in most cases and unjustified generalizations and extrapolations of conclusions were often made. More on this issue will be found in subsequent sections.

Going over the 150 or so papers in this category one will certainly form the opinion that this model has been over-used and perhaps abused. As already explained, differences in the models and assumptions, often subtle, made by the various researchers, has led to significant differences in results, created controversies, heated discussions and generation of new papers to support claims or explain the differences. Thus, to make this review easier to follow, given the rather chaotic state of the literature on the subject, we have structured this subchapter into different sections. Models symmetric about the *x* axis with uniaxial eccentricity (e_x) constitute the overwhelming majority in the group of 1-story structures and will be found either as **mass eccentric** or **stiffness eccentric**, depending on whether the CM or CS, respectively, coincides with the geometric center of the deck. Since this differentiation does not change by itself results and conclusions to any significant degree, we did not use separate categories for each. And a last comment, the obvious reason to differentiate between single motions and ensemble of motions with averaging of results is because such averaging makes any conclusions less dependent on the characteristics of specific motions.

8.1.1 e_x, K_y (unidirectional eccentricity, stiffness and ground motion)

The simplest models to study torsion belong to this group and have 2 DOF, one translational perpendicular to the axis of symmetry (u_y) and one rotational (θ), with respective stiffnesses K_y and K_θ. They have been used with one component motions, either single or multiple. They correspond to buildings with one axis of symmetry, perpendicular to the direction of motion.

The first papers in this category are by Irvine and Kountouris (1980) and by Kan and Chopra (1981a, 1981b), all using 2DOF systems. The Kan-Chopra papers use an extension of the elastic model in Kan and Chopra (1977), to inelastic systems, by transforming again the multi-element system to an equivalent one element system. Three papers followed, Tso and Sadek (1985), Bozorgnia and Tso (1986), Tso and Bozorgnia (1986), where in the first of them, results are contrasted to those by Kan and Chopra (1977), Irvine and Kountouris (1980), who used a simpler two or one-element model instead of the 3 elements in the references by Tso and others. The 3-element model-type (b) in Fig. 3 has "*many characteristics common to many actual eccentric buildings so that the results obtained can provide guidelines in actual design*" and is more realistic than the 2-element model. The stated reasons for this is that the 3 element model in Tso *et al*. is statically indeterminate, while its periods vary by varying the stiffness, not the polar moment

of inertia as done with the 2-element model. In the first three of the above publications a small number of individual earthquake motions have been used, while in the fourth paper an ensemble of 6 motions was used and results were averaged. The findings relate ductility demands to the element strengths and eccentricity, indicating a smaller dependence of response on the ratio Ω, in comparisonn to elastic systems, and that an effective eccentricity may be defined for computing edge displacements in terms of the displacements of the corresponding elastic symmetric system.

A paper by Tso and Ying (1990) based on an ensemble of motions indicate the <u>element at the flexible side as the most critical</u> in terms of torsional effects, while Bruneau and Mahin (1988, 1990), Bruneau (1992), using an ensemble of motions investigate for bilinear and other nonlinear systems the influence of parameters such as e/r, Ω, ω_x and propose an equivalent SDOF system. The next paper by Chandler and Duan (1991) addresses the *"fundamentally contradictory conclusions"* by Tso and Ying (1990), concerning (a) the identity of the critical element (i.e., the most susceptible to the torsional effects), and (b) the *"adequacy of the design eccentricity formulae specified in some building codes for earthquake resistant design in terms of satisfactory ductility control ..... i.e., to provide consistent protection for symmetric and eccentric structures against structural damage"*. The paper concludes that *"When element strength is specified according to code design eccentricity expressions, <u>the element at the stiff edge is the critical element</u> which suffers significantly more severe damage than corresponding symmetric structures. The peak displacement ductility demand of the element at the flexible edge is always lower than that of corresponding symmetric structures"*. It also includes conclusions opposite to Tso and Ying (1990), concerning the Mexican codes of 76 and 87. We note that among other differences in the models used, the Tso and Ying results are based on an ensemble of 8 motions while results by Chandler and Duan are based on a single motion. Now this is a classic case of the contradictions noted before, like several others found in different publications. The interesting thing is that in both papers, the models used have little to do with actual eccentric buildings, the real behavior of which along with the answer to the above controversy was reported by Anagnostopoulos *et al*. (2010) (see 8.3).

The interest on this issue continues in Tso and Zhu (1992a, 1992b), Tso and Ying (1992), with new designs and new results, also based on averaging for an ensemble of motions. Chandler *et al*. (1991) return to the problem this time using the simple 2 DOF model but with Ramberg–Osgood properties and a design spectrum for input, while Mittal and Jain (1995), Annigeri *et al*. (1996), study the effects of structural eccentricity and of the Ω ratio and attribute the reported contradictions to differences in definitions and models. De Stefano and Rutenberg (1996) investigated torsional instability due to a negative slope in the post elastic branch of the bilinear relationships of the bearing elements, while Escobar (1996), Escobar and Ayala (1998) investigated the effects of random element properties on type (b) systems. Bugeja *et al*. (1999) after discussing differences and disagreements among Chandler and Duan (1990), Goel and Chopra (1990), Kan and Chopra (1981), Tso and Zhu (1992), and pertinent explanations in Correnza *et al*. (1994) (section 8.1.2.2 below), presented their own results based on one earthquake motion. Along similar lines are the papers by Gersi and Rossi (2000), Myslimaj and Tso (2002), Bensalah *et al*. (2012).

Code assessment papers: Rutenberg *et al*. (1992a, 1992b), Tso and Zhu (1992b), Chandler *et al*. (1994, 1995, 1996), Correnza *et al*. (1995), Chandler and Duan (1997), De Stefano and Rutenberg (1997), Moghadam and Tso (2000). In all these papers, the aforementioned contradictions and the torsional provisions of various codes are assessed and often criticized, based on the obtained results from the models used. We must point out once more that almost without

exception, in none of these papers is made clear that all conclusions are strictly applicable to the oversimplified models used and more important, such conclusions may or may not be applicable to actual buildings.

8.1.2 e_x, K_x+K_y (unidirectional eccentricity, bidirectional stiffness)

Introduction of shear elements in the perpendicular direction was the first step towards making the simplified 2 DOF system of the previous section a little more realistic. The new systems, in addition to the increased torsional stiffness coming from the elements parallel to the axis of symmetry, have 3 DOF instead of two and thus a third mode can influence the results. Since one may now use either one or two-component motions, it is helpful to separate the two groups of publications.

8.1.2.1 One component motions

A study of a 2 DOF nonlinear system under harmonic excitation was reported by Pecau and Syamal (1985) and by Syamal and Pecau (1985). Systems with bilinear and pinched behavior were examined and it was found that only the latter may, under certain conditions, be subject to torsional instability. Another conclusion is that structural elements at the stiff edge of eccentric buildings are only marginally affected by the magnitude of the eccentricity, thus indicating that seismic building codes which reduce design requirements for these elements underestimate actual behavior substantially. In our opinion, this is another example of extrapolation of results and conclusions to realistic buildings that may have little relation to reality. The following 4 publications by Goel and Chopra (1990, 1991a, 1991b, 1994) present parametric studies for a half cycle displacement pulse (the El Centro motion was also used in the 1991 papers) and in the last of the four papers a design procedure for asymmetric buildings is suggested. Several conclusions are presented about the influence of the various parameters, but one of them from the first of the three papers stands out*: "Thus the conclusions from earlier studies of systems without perpendicular elements are generally not applicable to most actual buildings which invariably include resisting elements in the two lateral directions to provide resistance to both horizontal components of ground motion".* This is a very serious though correct criticism to ALL the papers in section 8.1.1. above, but fails to recognize that the improved model suffers from the same "illness", i.e., although the addition of the perpendicular elements makes the new model more realistic, this model still remains a crude approximation of actual buildings, omitting many things that would greatly affect its response to real earthquake motions.

Sadek *et al.* (1992) went back to the equivalent single element of Kan and Chopra (1981a) (see 8.1.1), finding it easier to use, without any apparent concern as to how well it can represent realistic buildings. De Stefano *et al.* (1992, 1993) have used a 2 DOF system and two earthquake motions to study this problem and generated inelastic acceleration and overstrength factor spectra for different values of eccentricity. Using four different simple elements - types (b) and (d) and two variations of type (a), Fig. 3- a Ramberg-Osgood force-deformation relationship and the El Centro (one component) motion, Jiang *et al.* (1996) carried out parametric investigations that led to a number of conclusions concerning the influence of the transverse elements, the eccentricity *e*, the relative value of stiffness and strength eccentricities, the ratio Ω and the position of the critical element, found to be dependent on the phasing between translational and torsional motion. Rossi (2000) has looked into ductility and energy dissipation demands based on an ensemble of 30

artificial motions, and Pettinga *et al*. (2007) have investigated the effect of inelastic torsion on residual deformations.

One of the major simplifications in practically all of the studies with the simple models reviewed so far was the determination of element strength independent of its stiffness. Obviously this does not happen in real buildings and the inherent relation of these two parameters can significantly affect the results. This problem is addressed by Myslimaj and Tso (2004) who conclude that to minimize inelastic torsional effects a balanced location of the CV-CR (strength center-stiffness center) pair should be sought. The next paper by Lin and Tsai (2009) introduces the so called T-R (translational-rotational) response spectra for computation of the seismic demands of one way asymmetric buildings. The last paper in this section, Lucchini *et al*. (2009), investigates the inelastic response of a simple, 2 DOF system to groups of historical records and uses the Base Shear-Torque (BST) surface for its parametric studies. It makes also reference to the persisting conflicting conclusions on the subject. It is quite interesting to follow the discussion by Humar and Fazileh (2010), which refutes some of the findings and concludes: *"It is evident from the results presented here that the conclusions reached in the paper under discussion are applicable only to the model studied and cannot be generalized"* The authors response, Lucchini *et al*. (2010), correctly points out that the system selected by the discussers to make their point and refute the authors' conclusions suffers also from the same oversimplification "illness" their own model is accused for.

Code assessment papers: Chopra and Goel (1991), Zhu and Tso (1992), Chandler and Hutchinson (1992), Goel and Chopra (1992), Chandler and Duan (1994), De Stefano *et al*. (1993), Humar and Kumar (2000), Humar and Kumar (2004). All of the above papers assess the torsional provisions of various codes based on results from the simplified model.

8.1.2.2 Two component motions

Use of two-component motions to study torsional problems is somewhat more complicated but certainly more realistic. Most of the studies in this section have used type (e) or (f) elements of Fig. 3. Tso and Sadek (1984) have published results from a study of a simple system with four corner columns-type (f), Fig. 3- subjected to the two components of the El Centro record. Recognizing the limitation of their study, they report (i) substantially greater ductility demands than the symmetric structures but lower than those under unidirectional motion, where interaction effects are neglected and (ii) that a factor of 1.4 to simulate interaction effects in corner columns is reasonable. Sadek and Tso (1988, 1989) report that strength eccentricity correlates much better with inelastic torsional response than stiffness eccentricity. The significance and importance of: (i) the transverse elements, mainly through their contribution in increasing the torsional stiffness and consequently the value of the Ω parameter, and (ii) the two component excitation, is now recognized by Correnza *et al*. (1994) and five years later by Humar and Kumar (1999).

Wong and Tso (1994) showed the dependence of inelastic response on the ratio Ω of the system and how it affects ductility demands at the stiff and flexible edges and also the significance of accidental design eccentricity for strength distribution among the resisting elements that will limit the ductility demands to acceptable levels. Tso and Wong (1995b) looked into the flexible edge displacements for displacement based design applications and into the conservatism or non-conservatism of three different code procedures in the estimates of such displacements, while De La Llera and Chopra (1995) using both one and two component motions, carried out parametric studies and reached several conclusions on the basis of the Base Shear-Torque (BST) surfaces.

Goel (1997) investigated the problem with an energy based approach and an ensemble of 5, two-component real earthquake motions. As in most previous papers, there are conclusions for the flexible and stiff edge element response.

De Stefano and Rutenberg (1999) are interested in the effect of gravity loads and the force reduction factor, have included P-δ effects in their equations and have made comparisons of type (b), Fig. 3, systems under one component motion with type (e) systems under two component motions. Their conclusions focus on the influence of the design force reduction factor R on various aspects of the system's response. Detailed elastic and inelastic results for type (e) systems, designed in accordance with EC8 and subjected to 3 motion groups with different characteristics, will be found in Stathopoulos and Anagnostopoulos (1998). A period range 0.1 to 3.0 seconds and eccentricities e=0.0, .10, .20 and .30 have been covered. The cautionary statement in the beginning of the authors' conclusions is noteworthy: "*The results reported herein are strictly applicable only to the idealized systems from which they were produced. Any extrapolation to real buildings should be made with caution, given the highly idealized, simple model used for the investigation*".

Further investigations on the problem are reported by Riddel and Santa-Maria (1999), who compare results by one and two component motions, reaching pertinent conclusions, by De La Colina (1999) who has looked into the effects of the reduction factor *R*, the factors α and γ (see Eq. (11) above), the uncoupled period *T* and the stiffness eccentricity e_s. Additional papers are by Dusicka *et al*. (2000) who have looked into the element strength distribution, by Ghersi and Rossi (2001), whose key conclusion was that the presence of two motion components has only a minor effect on the response of the system. The last two papers, De Stefano and Pintucchi (2010) and Bosco *et al*. (2012) address the issue of inelastic edge displacements for extending pushover type of analyses to 3-D asymmetric buildings. In the second of the two papers, approximate equations are provided for the so called effective eccentricities, which define the points in plan where a static application of the seismic force will adequately predict the edge inelastic dynamic displacements needed for pushover analyses of eccentric buildings.

Code assessment papers: Tso and Wong (1993, 1995a, 1995b), Wong and Tso (1995), De Stefano *et al*. (1998), Tso and Smith (1999), Dutta and Das (2002), De Stefano and Pintucchi (2004), Dutta *et al*. (2005)

8.1.3 e_x+e_y, K_x+K_y (Bidirectional eccentricity, stiffness and motion)

One of the very first papers in this group is by Prasad and Jagadish (1989), where a type (f), Fig. 3, 3 DOF system with four corner columns was subjected first to one component of the El Centro record and then to both components. One of the main conclusions was that for small eccentricities, 2-component input did not affect much the results but for larger eccentricities ductility demands were reduced in some columns but the maxima increased up to 50 % over the demands of the symmetric system. This conclusion, however, is not supported from the data in that paper -see e.g., their Fig. 4- obtained for a system with period 0.25 s. Eight years later, Goel (1997) investigated the problem of biaxial eccentricity with a one-story, 3 DOF system with a type (e), Fig. 3 layout, subjected to a group of 5 pairs of real earthquake motions. He summarized findings in previous investigations (with more idealized models) as follows: "*These investigations generally concluded that elements on the stiff side (the same side of the center of mass as the center of rigidity)in code-designed, asymmetric-plan systems are likely to suffer more damage, whereas elements on the flexible side (the side opposite the stiff side) are expected to suffer less or similar damage compared to those in the reference system*". His (Goel's) conclusions indicated

the opposite, except for the stiff side of mid-period structures. More specifically, Goel (1997), states: *(i) The flexible-side elements undergo much larger hysteretic energy demands in an asymmetric-plan system than in the corresponding symmetric-plan systems. The stiff-side elements, on the other hand, do not necessarily experience any larger hysteretic demands in asymmetric-plan systems. (ii) The stiff-side element may experience larger ductility demands, in mid-period, asymmetric plan systems when compared to the same element in the corresponding symmetric-plan systems. The flexible-side element, on the other hand, undergoes much smaller ductility demands in asymmetric-plan systems".* Thus the conflicting conclusions kept accumulating.

The next three papers by Stathopoulos and Anagnostopoulos (2000,2003, 2007) are quite interesting because they include a comparison of the overused one story, 3 DOF shear beam type building model with a detailed plastic hinge model of the same building. To the best of our knowledge this is the first such reported comparison. An actual one story concrete building was designed and a detailed, member by member plastic hinge type model was prepared. The properties of the corresponding simplified model were determined as it was common practice in the past, with element strengths based only on earthquake actions as specified in Eurocode 8. The number of stories was first limited to 1 in an attempt to eliminate the multistory-multi mode effects as a sourse of difference between the two models, simplified and detailed (plastic hinge). The two models were subjected two 3 groups of 10 motion pairs (2 historical and one artificial) with different characteristics The investigation showed substantial qualitative differences between results from the two models, the most striking of which was that as far as the ongoing controversy on whether the stiff or the flexible side element is critical, (i.e., having higher ductility demands as a result of torsion) the detailed plastic hinge model showed the flexible side element to be the critical one, while the simplified model showed the opposite, i.e., the simplified model confirmed the prevailing view of higher ductility demands at the stiff edges. Figs. 4, 5, 6 and 7 based on the Ph. D. thesis of the third author show these results. Fig. 4 shows the layouts of one-story building, where the inelastic Plastic hinge (P-H) model is shown in (a) having three plane frames along each of the two directions x and y and the corresponding simplified model is shown in (b). The resisting elements in the simplified model (b) are indicated as EL1 to EL6, each corresponding to two-bay plane frames FR1 to FR6 in (a) (e.g., FR1 has columns C1, C4, C7 and FR3 has columns C3, C5,

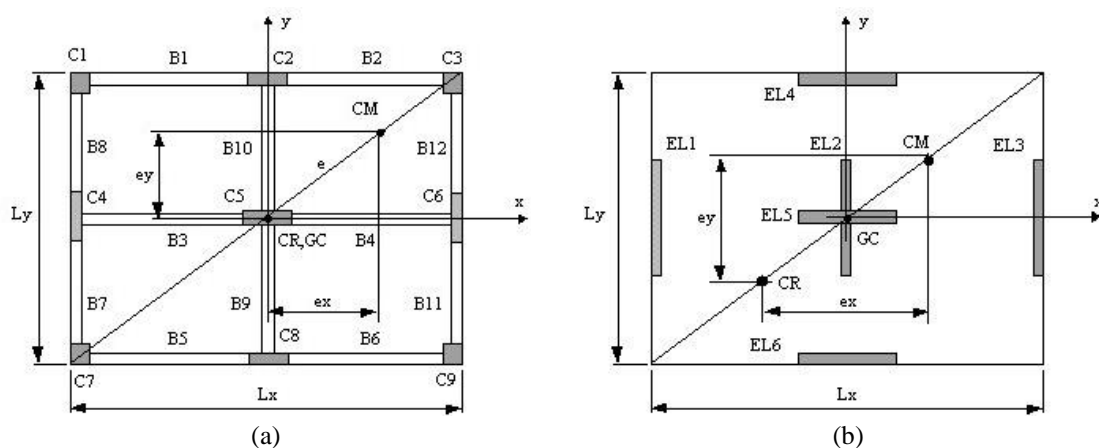

(a)   (b)

Fig. 4 Layouts of one-story buildings: (a) Plastic Hinge model, (b) Simplified one-story model

C9). Figs. 5 and 6 are for uniaxial mass eccentric systems, the first torsionally stiff and the second torsionally flexible. Fig. 7 is for a torsionally stiff, mass eccentric system, with biaxial eccentricity. The top graphs in each Figure give peak edge displacements normalized by the corresponding peak displacement of the symmetric system and the lower graphs give the corresponding ductility factors. The abscissa is the value of the eccentricity. Results in solid lines are for the detailed Plastic-Hinge (P-H) model and in dashed lines for the simplified model. While the displacements of the flexible sides are always higher than the displacements of the stiff sides, even for the torsionally flexible structure, and this is predicted by both models, the opposite happens with ductility demands but the two model predictions are opposite.

   Perus and Fajfar (2005) using an element layout type (e) of Fig. 3 presented an extensive parametric study for a group of 8, two-component historical earthquake records. The parametric investigation included mass and stiffness eccentric systems, the value of eccentricity, elastic vs inelastic response and the magnitude of the plastic deformations. The examined systems are torsionally stiff and the authors are careful in stating the limitations under which their results could be extrapolated to real multistory buildings. Additional parametric studies can be found in Ghersi and Rossi (2006) who used a simplified model similar to type (e) of Fig. 3 and investigated both, torsionally stiff and torsionally flexible systems for an ensemble of motions, and in Lucchini *et al.* (2011) who used a 3 DOF biaxially eccentric system with six columns on its perimeter, each with shear type resistance along the two main axes. They made use of the BST surfaces to study biaxial
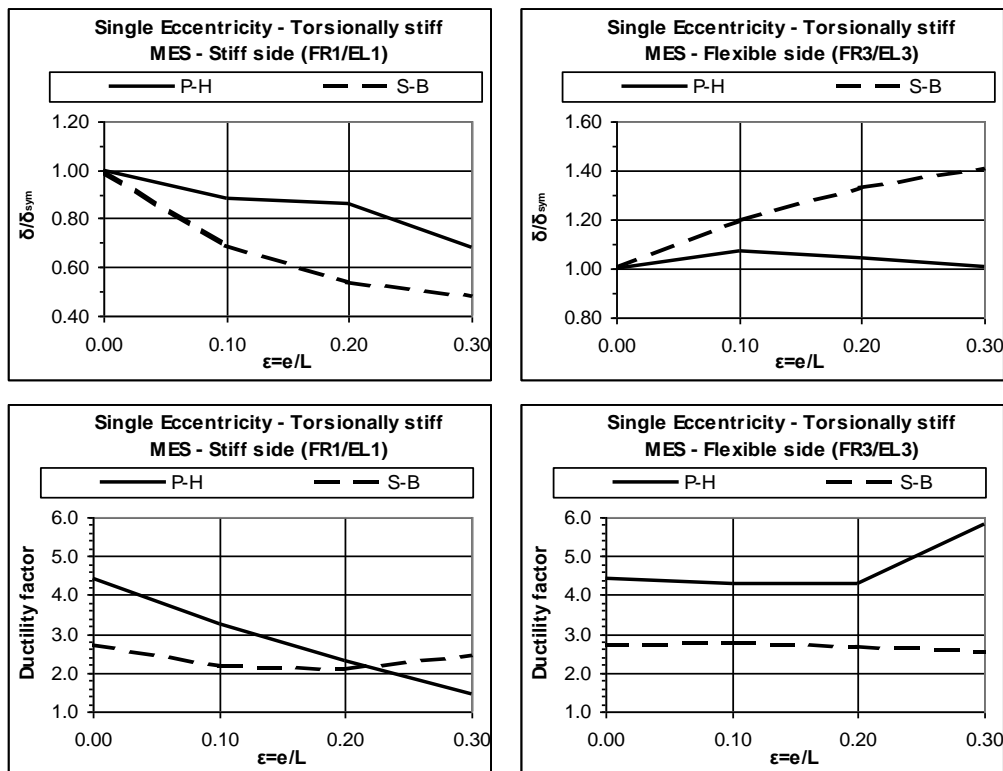

Fig. 5 Normalized displacements (top) and ductility factors (bottom) by P-H and simplified S-B models: Mass eccentric (MES), torsionally stiff systems with single eccentricity
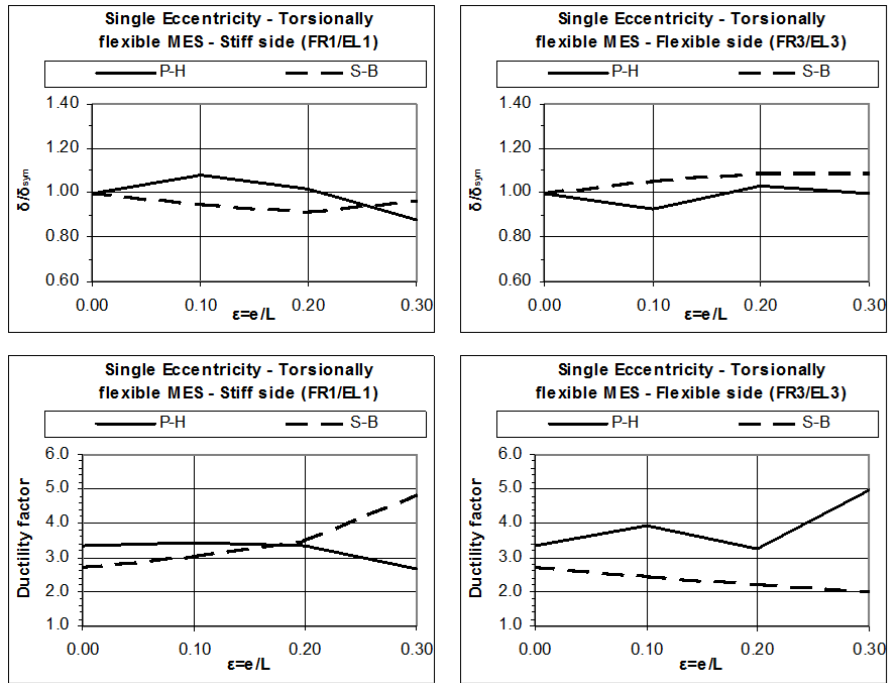
Fig. 6 Normalized displacements (top) and ductility factors (bottom) by P-H and simplified S-B models: Mass eccentric (MES), torsionally flexible systems with single eccentricity
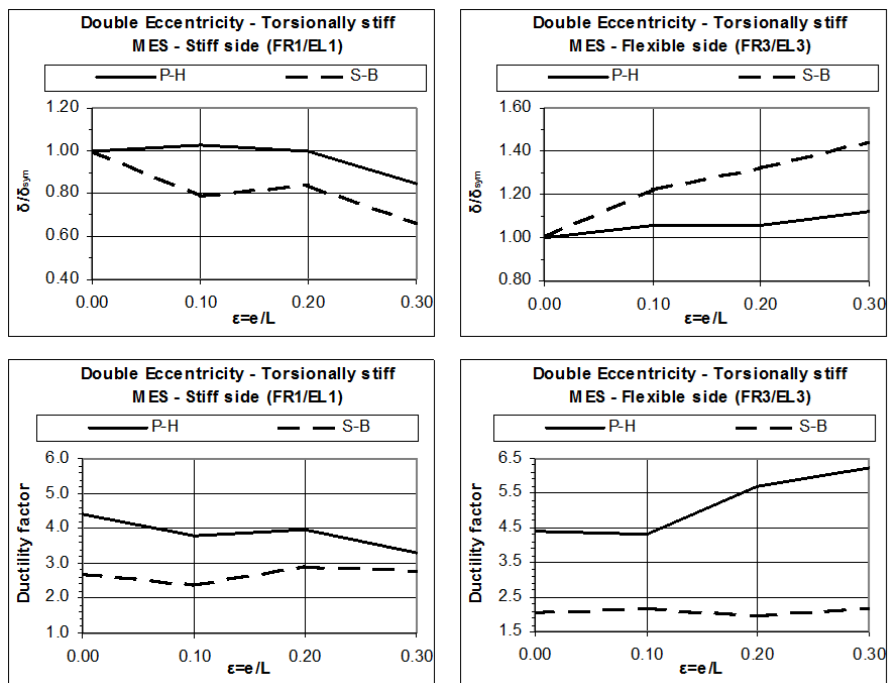

Fig. 7 Normalized displacements (top) and ductility factors (bottom) by P-H and simplified S-B models: Mass eccentric (MES), torsionally stiff systems with double eccentricity

effects, effects of motion intensity and of the incidence angle, and also how the results could be applied for pushover analyses of eccentric buildings. Finally, Aziminejad and Moghadam (2012), investigate the influence of near field and far field motion in type (f) systems with biaxial eccentricity.

Code assessment papers: Ayala *et al*. (1992), Dutta *et al*. (2005)

8.2 Multistory inelastic models (MST)

We distinguish the multistory models used in the investigation of torsion to approximate and detailed - advanced. The former come with different types of approximations (e.g., shear beam behavior) while the latter are based on realistic, 3-D detailed idealizations of a building, that include all its members (beams, columns, walls and sometimes partition walls), idealized with lumped plasticity (plastic hinge) models. Practically, this is the most detailed and advanced modeling with today's available software and hardware and it is only in the last 10 or 15 years that started been used in torsion research.

8.2.1 Approximate – simplified, usually shear beam type, models (MST-SIMP)

Although approximate inelastic multistory models constitute a step forward from the simplified, one story systems of the previous sections, they still miss basic characteristics of most real buildings. And although available since the early 70ties, such models did not "enter" the torsion investigation "arena" till quite later. One of the first, if not the first, approximate 3-D multistory models capable for nonlinear dynamic analyses under two-component earthquake motions was developed by Anagnostopoulos (1972), Anagnostopoulos *et al*. (1973). This is the program called STAVROS in the paper by Irvine and Kountouris (1980) (see 8.1.1), and can model a combination of frames and shear walls having any orientation in plan. The frames are modeled on a story by story basis as close coupled (shear beam) systems with different hysteretic rule possibilities (bilinear, trilinear, stiffness and/or strength degrading) while shear walls are modeled as far coupled systems with plastic hinges.

A very brief comparison of inelastic and elastic torsion effects for a 5 story building can be found in Anagnostopoulos and Roesset (1973), the very first for multistory inelastic torsional response. Twenty years later, Duan and Chandler (1993), Chandler and Duan (1993) presented results for the inelastic seismic response of code designed multistory buildings based on a highly approximate model with many limitations. More specifically the model consists of 3 shear beam type plane frames (totally rigid floors) all parallel to the *y* axis-type (b) plan of Fig. 3- with column properties uniform with height and with mass and stiffness centers located in two vertical axes. This is essentially a multilevel repetition of the one story simplified model of type (b) and still remains a very crude approximation of actual buildings for any realistic code assessment. Yet, without any mention of the great limitations of the model, results strictly applicable only to the highly idealized model are used for code assessment, criticism and recommendations for changes. This is the type of generalization seen in most of the one story models reviewed above. The model just described has also been applied in Duan and Chandler (1995) to investigate the inelastic torsional response of multistory buildings with setbacks and a modified static design procedure has been recommended.

Another simplified model has been reported in De La Llera and Chopra (1995) and was applied to investigate the inelastic response of asymmetric multistory buildings in De La Llera and Chopra

(1996). This is a refined extension of a previous model by Kan and Chopra (1981a) (see 8.1.1) and is based on the determination of one element per story for buildings with rigid floor diaphragms (shear beam behavior), symmetric about one axis (uniaxial eccentricity) and subjected to one component motion. For inelastic response assesments, use is made of the so called Story Shear Torque (SST) ultimate surfaces. The accuracy of the model is checked using a four story building with a setback, with steel columns and infinitely rigid floors, i.e., with a shear building. While this is certainly a refinement over the one story models, it still represents a crude approximation of real buildings and hence suffers from the same "illness", though less severe than the one story models. The authors' claim that the accuracy of their single element model is usually satisfactory for most design purposes, is, in our opinion, a long distance away from reality, unless the buildings to be designed are subject to the model's limitations (totally rigid floors- not just in their plane-, one way symmetric, one component motion, etc.). In the conclusions of the second paper, it is stated*: "The earthquake behavior of asymmetric single and multistory buildings of the class considered in this investigation shows similar trends and is affected by the following same building characteristics: the strength of resisting planes and intensity of ground motion in the orthogonal direction, the stiffness and strength asymmetry in the system, and the distribution of strength between the core and edges of the building".*

With 3-D inelastic dynamic analyses of several idealized frames, Tso and Moghadam (1998) investigated the adequacy of Eurocode8 to protect against excessive interstory displacements at the perimeter of torsionally flexible buildings and made recommendations for minimum torsional stiffness requirements that would render the equivalent static procedure applicable to MST buildings. De-la-Colina (2003) investigated the values of parameters α and γ of Eq. (11) by analyzing for the 2-component El Centro record, 7 variants of a 5-story shear beam frame building with the layout (e) of Fig. 3, each designed by a different static procedure, in accordance to some codes or to recommendations by other researchers. The study led to a proposal for the amplification of the static physical eccentricity, but without consideration of accidental eccentricities.

Another simplified model with one column element and 3 DOF per story has been proposed by Kosmopoulos and Fardis (2008) and validated with comparisons of results from 3-D analyses of detailed plastic hinge models. Without the limitations for vertical alignment of story CMs and CSs, this model has the flexibility of specifying the floor masses at their correct locations relative to either the CS, or CV or CT (center of twist), depending upon the user's choice as to which of these points should be the location point of the single vertical element. An apparent approximation of this model is that it brings all these points at the same vertical axis (that of the single element), but maintains in each floor the respective distance and relative location of the selected point to the CM of the floor. However, it is not limited to just one way symmetric buildings or to one component input motions. It is an interesting model that certainly needs further verifications. Another simplified MST model with type (e) element layout (see Fig. 3), rigid decks (shear beam type elements), constant masses and stiffness over height, was presented by Dutta and Roy (2012) with results provided for two and three story buildings subjected to a two component artificial motion, except that in one case the El Centro motion was also used. The system parameters were arbitrarily modified leading to models having little relation to realistic structures. As with so many papers reviewed before, the conclusions here also lack in generality and add very little, if anything, to the existing knowledge. In summary, one may conclude that simplified inelastic MST models can generally provide better approximations to the response of realistic buildings, compared to the simplified one-story models. However, they should be used only for preliminary studies that could

provide guidance for more detailed analyses using detailed plastic hinge models.

Code assessment papers: Duan and Chandler (1993)

8.2.2 Detailed, usually plastic hinge type, models (MST-PH)

This is the last, more detailed and advanced category of models, providing the most realistic idealization of buildings for inelastic analyses. Every structural element is included in the model with appropriate non- linear hysteretic rules describing its post elastic behavior. Inelastic behavior of columns, beams and flexural (shear) walls is approximated by plastic hinges normally at member ends, hence the term lumped plasticity model, where the plastification takes place. More advanced member idealizations are possible with the so called fiber model but this is too refined for dynamic analyses of realistic buildings. The authors are aware only of one study-Erduran and Ryan (2011)-(see below), where the fiber idealization has been used to model the columns of 3 story steel frames for studying inelastic torsional effects.

Among the first references in this group is the paper by Boroschek and Mahin (1992) who analyzed the response of an existing instrumented building to three earthquake motions, the largest of which recorded 0.11 g peak base acceleration and 0.36 g peak acceleration in the structure. Although the building did not suffer significant inelastic action, pertinent dynamic analyses were carried out using 5 other two component historical records scaled to various levels of intensity. The key finding here, which agrees with their analyses of simpler models, was: *"In general, the existence of torsional behavior in nearly regular space frames has the effect of increasing the stress or ductility demands in elements located far away from the center of rotation and changes the maximum translational displacements. These effects are more severe for elastic structures than inelastic structures and are highly dependent on the characteristics of the input ground motion"*. In other words the flexible edge was found to be the critical one. In the same conference where the previous paper appeared, Cruz and Cominetti (1992), reported results from the analysis of a 5-story building model with two, one bay, frames parallel to the x axis (axis of symmetry) and only one frame in the perpendicular direction parallel to the earthquake action. The reported conclusions have little application to the inelastic torsional response of realistic buildings.

De Stefano *et al*. (1995), reported results for three 4-story, eccentric reinforced concrete buildings with one axis of symmetry. The buildings were designed with an early version of Eurocode 8 and subjected to two, one component, motions. For the inelastic behavior a bilinear model and a stiffness degrading model (Clough's model) were used and results from them as well as from elastic analyses were compared. Their only conclusion relevant to torsion was that the buildings, designed as high ductility structures per EC8, were able to resist rather severe earthquakes but their degree of safety against collapse was lower than that of corresponding symmetric buildings.

One of the first systematic evaluations of torsion using detailed plastic hinge idealizations of 3-D buildings, symmetric about the x axis and formed by 12 plane frames, 8 in the y direction and 4 in the x direction, has been reported in Ghersi *et al*. (2000). It appears that for each frame there was no variation of beam and column sections, i.e., one section was used for all beams and another for all columns. They were varied proportionately from one frame to the other in order to obtain the desired eccentricities. Just this fact alone makes the buildings not so realistic and as a consequence it would be questionable to extrapolate the obtained results to many typical real buildings. The buildings were designed by standard modal analysis, accidental eccentricity was not considered at all and the investigation was carried out for an ensemble of 30 artificial, one

component motions. A new design procedure was also proposed that led to a better distribution of ductility demands among the various frames.

In Stathopoulos and Anagnostopoulos (2000), already referenced above, a comparison of simplified one-story models was carried out with a fully designed one story building, whose beams and columns were idealized with the plastic hinge model. Detailed results for sets of one, three and five story realistic, non-symmetric reinforced concrete buildings with biaxial eccentricity have been presented by Stathopoulos and Anagnostopoulos (2002, 2004, 2005a). These buildings were fully designed according to Eurocodes 2 (concrete) and 8 (earthquake resistant design), as if they were real buildings to be built. Analyses were carried out for 3 groups of ground motion: one with near field records, the other with far field records and the third with semi-artificial motions. For these analyses bilinear moment rotation relationships were used for all flexural members and biaxial moment-axial force interaction was included for columns. Moreover, two levels of section cracking were examined, one with EI values as defined by the code for designing new structures and another with lower values corresponding to the collapse prevention level of performance. The results showed a significant variation of ductility demands over the floor plan, with those at the flexible sides of the buildings up to 100 % higher than the reference symmetric building and those at the stiff sides at the same or reduced levels of the reference symmetric building. These contradict most of the findings in the past based on the simplified one-story models.

Extensive parametric studies with three, 5-story, moment resisting steel frame buildings have been carried out by Fajfar *et al.* (2004), Marusic and Fajfar (2005). Two of the buildings, the first symmetric and the second biaxially eccentric, were designed according to Eurocodes 3 and 8 (for steel and for earthquake resistant design, respectively) and both are torsionally stiff. A torsionally flexible third building was generated by moving into the interior of the plan the stiffer moment resistant bays that in the torsionally stiff building were located in the four corners. Asymmetry was introduced by assuming different mass eccentricities, but it appears that the buildings were not redesigned to account for them. The investigation was carried using planar beam and beam column elements with elastoplastic hinges assumed to form at their ends, whenever the corresponding yield moment was reached. Six different, two component, historical earthquake records were used in the parametric study and the basic conclusions were*: "The displacement in the mass centre of a plan-asymmetric building is roughly equal to that of the corresponding symmetric building. The amplification of displacements determined by elastic analysis can be used as a rough estimate also in the inelastic range. Any favourable torsional effect on the stiff side of torsionally stiff structures, i.e., any reduction of displacements compared to the counterpart symmetric building, which may arise from elastic analysis, may disappear in the inelastic range".*

An interesting investigation for 8 story buildings designed according to the Mexican code has been reported by Garcia *et al.* (2004). 25 models of biaxially eccentric buildings were subjected to the two components, 1985 Michoacan earthquake record at SCT and interesting results were presented, but limited to the first (ground?) story due to their large volume. These include rotational ductility factors for beams, the history of base shear vs base torque and the corresponding BST limiting surface and also the history of the instantaneous locations of the stiffness and shear centers. The main conclusion was that: *"providing in-plan strength distributions similar to the corresponding stiffness distributions leads to better behaviors, particularly in stiffness asymmetric models".*

Using many code designed multistory braced steel buildings with mass irregularities, Tremblay and Poncet (2006) make, without direct reference to torsion but only to mass irregularities, an interesting comparison of the pertinent IBC and Canadian code provisions, concerning primarily

the requirements for static and dynamic analyses. In another interesting paper by De Stefano *et al*. (2006), a 6-story, one way eccentric building was subjected to an ensemble of 30 one-component artificial motions, perpendicular to the axis of symmetry, to investigate the effect of member overstrength, typically not accounted for in the simplified 1-story models. Their results were to a large extent in agreement with those by Stathopoulos and Anagnostopoulos (2002, 2003, 2005a), concerning the often misleading conclusions coming from the crude, simplified, one-story models, and the pertinent consequences on code provisions. Ghersi *et al*. (2007), also recognizing the problem of code provisions based on the one-story model results, used the same 6-story buildings and the 30 artificial accelerograms as De Stefano *et al*. (2006), to recommend a design procedure for a more uniform distribution of ductility demands throughout the plan and elevation of multistory buildings.

Kosmopoulos and Fardis (2007) carried out 3-D elastic and inelastic, static and dynamic analyses to investigate the adequacy of predictions of member chord rotations by elastic analyses. Detailed 3-D models of 4 real asymmetric buildings, 3 to 6-stories high, were used for the study, subjected to an ensemble of 56, two component semi-artificial motions compatible with Eurocode 8 spectra. The basic conclusion of this study was that:…*"for multistorey RC buildings that typically have fundamental periods in the velocity-sensitive part of the spectrum, elastic modal response spectrum analysis with 5% damping gives on average unbiased and fairly accurate (within a few per cent) estimates of member inelastic chord rotations. If higher modes are insignificant, elastic static analysis in general overestimates inelastic chord rotations of such buildings, even when torsion is present"*. This is a very significant conclusion with potentially important practical implications that needs, however, further studies with more typical buildings to establish limits of and conditions for its application.

Rather similar to the goal of Kosmopoulos and Fardis (2007), appear to be the objectives in Fernandez-Davila and Cruz (2008), who used a simple 5-story frame building having a rectangular plan and three frames along each of the two main directions. The building was analyzed using the elastic response spectrum method and the results were compared with those from nonlinear dynamic analyses with an ensemble of 20, two component artificial motions. Based on such comparisons, where four spatial combination rules and 3 response reduction factors were considered, it was concluded that it is possible to estimate the maximum inelastic response to two component motions using the appropriate spatial combination rule of elastic modal analyses results for unidirectional excitation.

Another interesting study also comparing elastic with inelastic torsional response was presented in Erduran and Ryan (2011) who designed a 3-story braced steel building, whose earthquake resistance comes from four braced bays, one in each side of its perimeter. The remaining perimeter and interior bays were only designed for gravity loads. The excitation comprised an ensemble of 20, two component synthetic motions, scaled to four different hazard levels, with mean, 50 year, exceedance probabilities of: 50%, 10%, 2% and 1% that correspond to 72 year, 475 year, 2475 year, and 5000 year mean return periods, respectively. Obviously, the lower probabilities mean stronger motions and greater inelastic response or ductility demands. This is perhaps the only study of its kind where columns have been idealized using the fiber model, so axial and bi-directional moment interactions were accounted correctly, although none of the seismically designed bays includes a corner column in which such effects would be significant. The authors are also very careful (and modest) to caution in their conclusions that their results apply strictly to the investigated frame and that the behavior of braced frames with different brace arrangement might be significantly different from what has been observed in their study. Statements like this

are missing from the conclusions of the overwhelming majority of the papers reviewed herein which are based on the crude one-story models. One of the main conclusions is that: *"For the 1/50, 2/50 and 10/50 year events, the normalized story drifts from inelastic response analysis of the building are much higher than those for the same building responding elastically."* For the 50/50 event (the 72 year earthquake) the response was nearly elastic and hence the normalized story drifts were almost identical to the elastic ones. Moreover, for the same 3 events, the obtained median ratios of story drifts of the flexible edge to those of the mass center were 1.42, 1.35 and 1.25 respectively. The authors also make the following important observation: *"Our observations and conclusions are in contrast to those drawn by typical studies of asymmetric frame structures, that torsional amplifications in elastic systems exceed those in inelastic systems. For the braced frame system investigated here, the large rotational response is induced by a dynamic shift in the CR that results from substantial yielding/buckling of the braces on the flexible edge and near elastic response of the braces on the stiff edge"* This emphasizes even more the caution that must be used when generalizations of conclusions are attempted. Also important are the last two conclusions: (i) *"The normalized story drifts resulting from biaxial excitation are much larger than those resulting from uniaxial excitation",* and (ii) *"Two simplified analysis procedures, elastic RSA and pushover analysis, cannot capture the extent to which story drifts are amplified by the torsional component of response in braced frame buildings subjected to large ground motions"*.

As already mentioned earlier, detailed plastic hinge models were used in Stathopoulos and Anagnostopoulos (2002, 2004, 2005a, 2007) to investigate the shortcomings of the simplified one story models for reliable predictions of inelastic torsional response. Those studies were expanded in Anagnostopoulos *et al*. (2008, 2009, 2010) (see also 8.3), where it was shown that if the properties of the simplified one-story models are judiciously selected to properly reflect the corresponding properties of real buildings, then reasonable results may be expected from the one-story models, at least qualitatively. Additional studies in Stathopoulos and Anagnostopoulos (2005b, 2006, 2010) investigated the effects of code specified accidental design eccentricity on inelastic response of real buildings and provided indications that this provision of the code, introduced primarily on the basis of elastic response considerations, may have little effect on inelastic building response. More recently, Kyrkos and Anagnostopoulos (2011a, 2011b, 2011c, 2012a, 2012b, 2013a, 2013b), investigated the inelastic behavior of braced steel buildings designed according to Eurocodes 3 (steel) and 8 (earthquake resistant design). The study included 3 and 5-story high buildings, torsionally stiff and torsionally flexible, rectangular and L shaped, all with biaxial eccentricity. The excitation was an ensemble of ten, 2-component semi-artificial motions matching the EC8 design spectrum. For all these buildings, the mean ductility demands caused by the earthquake motions at the flexible edges were found to be substantially higher than the demands of the torsionally balanced reference building, while the demands at the stiff edges were either unaffected or slightly reduced. This prompted the proposal of a design modification leading to more uniform ductility demands. Finally, additional very recent studies with detailed models can be found in Karimiyan *et al*. (2023a, 2013b, 2014), who investigate the progressive collapse of 3 and 6 story symmetric and eccentric reinforced concrete buildings with various amounts of eccentricity and also in Adelka *et al.* (2014), who compare numerical predictions of the response of nonstructural components mounted on irregular reinforced concrete buildings, with the EC8 code provisions

Code assessment papers: Kyrkos and Anagnostopoulos (2011a, 2011b), Adelka *et al.* (2014).

8.3 One story shear beam (1ST-INSB) vs multistory plastic hinge (MST-PH) models

In the preceding sections, we often commented on the unjustified generalization of obtained results by different authors, especially results based on 1ST-INSB systems that were often used to make assessments and criticize torsional code provisions. Here we will expand on this issue by pointing the shortcomings of the 1ST-INSB model and at the same time suggest the conditions under which it could and should be used. These shortcomings have been pointed out in, among others, Ghersi *et al*. (1999), Stathopoulos and Anagnostopoulos (2002, 2003, 2004, 2005a) (see 8.2.2), Anagnostopoulos *et al*. (2010) and are repeated here:

(a) The stiffness and strength of the resisting elements of the 1ST-INSB model are usually specified and calculated independent of each other and only for seismic loads. In real buildings, member stiffness, strength and yield deformation are related to each other directly in a way that a change in one parameter entails changes in the other two. This problem has been addressed by Muslimaj and Tso (2001), Tso and Muslimaj (2003), but using again simplified one-story systems.

(b) In real buildings, a number of loading conditions and limitations are used for their design (vertical plus lateral loads, capacity design and drift limitations etc.) that are typically not considered with simplified models, whose strength and stiffness is almost always determined from the seismic loading alone. Hence the stiffness and strength (and the substantial overstrength) of the load bearing elements in real buildings, in absolute and relative values, are different from the corresponding quantities of the resisting elements of the 1ST-INSB models. Thus, the percentage changes of these quantities caused by applying Code provisions for torsion in real buildings are much smaller than the respective changes in the 1ST-INSB models and the same should be expected for the pertinent effects on the corresponding responses.

(c) Yielding of an end-element of the simplified model implies the practical elimination of the stiffness in that position (only the post-yield stiffness is left). A corresponding case in a real building would be the formation of a mechanism at the same side of the building, i.e., the simultaneous yield of all beam and column ends in all floors of the corresponding perimeter frame, which modern codes prevent through capacity design provisions. In real buildings, the post-elastic stiffness of any given frame is a significant fraction of its elastic stiffness, as it is controlled by the substantial number of members, typically columns that are elastic at any given instant. Thus, there are great differences in the post-elastic eccentricities between real buildings and the 1ST-INSB models.

(d) Higher mode effects are totally ignored by the 1ST-INSB models and thus the complex vibrational patterns of multistory buildings cannot be reproduced.

(e) In order to have at least qualitative agreement between results from 1ST-INSB and MST-PH models, key properties of the two models must be matched. Without such matching results will diverge not only quantitatively but qualitatively as well.

In practically every study of the past with the 1ST-INSB model element stiffnesses were selected more or less arbitrarily and corresponding element strengths were determined based only on the earthquake action. Mass and mass moment of inertia were then set to produce systems with the desired frequency and Ω ratios. When, however, a real building is designed, ALL these properties are interrelated and if one changes, the others change too. More important, most of the element stiffness and strength come from design for gravity loads and might be influenced by several other criteria, such as interstory drift limitations, capacity design provisions, minimum section and stability requirements etc. All these are absent from the 1ST-INSB. And of course there are also the equally great differences in number of stories, geometry and number of load

bearing elements. Therefore, unless condition (e) above is met, results from 1ST-INSB models should not be considered as directly (or indirectly and often without qualifications) applicable to real buildings.

An investigation as to what happens if the conditions under (e) above are met has been reported by Anagnostopoulos *et al.* (2008, 2009, 2010). In these publications two real biaxially eccentric R.Concrete buildings with 3 and 5 stories each were fully designed according to Eurocodes 2 (Reinforced Concrete) and 8 (Earthquake resistant design) for gravity and seismic loads. For these two buildings, whose typical floor plans are shown at the top of Fig. 8, equivalent 1ST-INSB models, shown in the lower part of Fig. 8, were generated as follows: A series of pushover analyses were carried out for each plane frame in both directions of each building and bilinear curves were fitted to them (Fig. 9). These provided the elastic stiffnesses of the elements forming the equivalent 1ST-INSB models. By applying a trial and error procedure, the mass and mass moment of inertia of the 1ST-INSB models were determined by appropriate reduction of the total mass and mass moment of inertia of the detailed multi-story models of the buildings, so that the 3 lowest periods of the simplified and MST-PH models were approximately matched. The element strengths of the 1ST-INSB models were determined from the strengths in the corresponding pushover bi-linear diagrams, but each was reduced by the ratio used to reduce the total building mass to arrive at the mass of the simplified models for matching the lowest 3 periods. In this
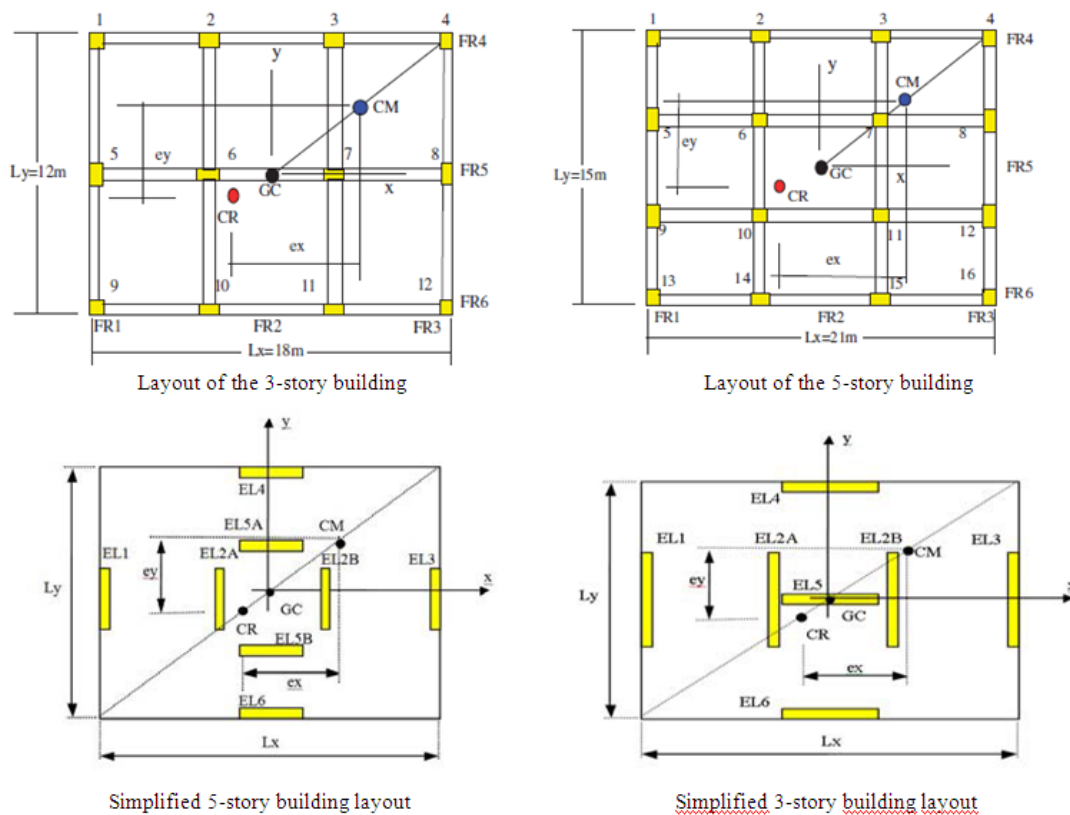

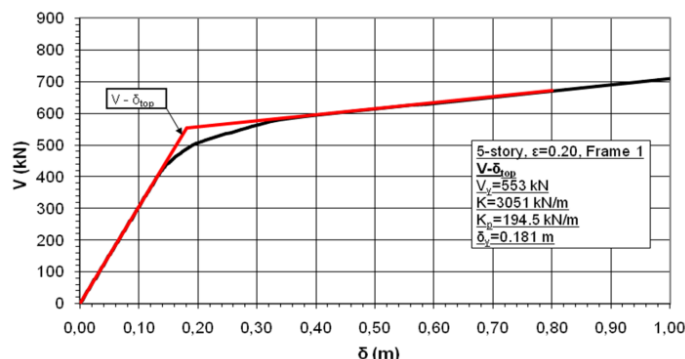Fig. 8 Layouts of 3 and 5-story buildings and 1ST-INSB their simplified models

Fig. 9 Fitting of bilinear push over curves

manner, the ratio of seismic forces to the element strengths was kept the same between the MST-PH and the 1ST-INSB models thus keeping strength as well as overstrength ratios between the elements the same for the MST-PH and the 1ST-INSB models. This was necessary because otherwise the simplified models would have substantially greater strength compared to the MST-PH models.

In addition to the 1ST-INSB so defined, which we call Plastic Hinge (PH) compatible models, a second 1ST-INSB was prepared for the 3 story building, differing from the PH compatible model ONLY in its element strengths, which in the second model were determined ONLY for earthquake loads as has been done traditionally by everyone in the past. In the results that follow the PH compatible simplified model is called SIMP1 and the traditional simplified model SIMP3. All models were subjected to an ensemble of 10 semi-artificial two component motions, compatible with the EC8 design spectrum. Due to space limitations, here we present results only for the 3 story building. Fig. 10 shows the variation with height of the maximum (mean from 10 motion pairs) rotational beam ductility factors for three values of physical eccentricity, 0, 0.10 and 0.20. The top 3 graphs, each for a different amount of eccentricity, are for frame 1 (stiff edge, solid lines) and frame 3 (flexible edge-dashed line) that are parallel to the *y* axis. Similarly, the bottom 3 graphs, are for frame 6 (stiff edge-solid line) and frame 4 (flexible edge-dashed line) parallel to the *x*-direction. It is clear that for both directions torsion penalizes more the flexible edge frames.

Peak top story displacements of both edges and in both directions *x* and *y* and correspondent ductility factors, based on the yield displacements of the fitted pushover curves of each edge frame (Fig. 9), are shown for the PH-model in Fig. 11 and for the PH compatible 1ST-INSB model in Fig. 12 as functions of the normalized eccentricity (abscissa). Top graphs are for displacements and lower graphs for ductilities. The graphs at left are for frames 1 and 3 parallel to the y direction and the graphs at right are for frames 6 and 4, parallel to the *x* direction. As in Fig. 10, stiff edge results are with solid lines and flexible edge results are with dashed lines.

We observe that in both directions, the flexible edge experiences not only the expected larger displacements but greater ductility demands (in agreement with the results of Fig. 10). Fig. 12 gives exactly the same data computed from the PH compatible, 1ST-INSB model (labeled SIMP1 model). We observe that although there are, as expected, quantitative differences, qualitatively the results show similar trends, especially in relation to the relative displacement and ductility demands between stiff and flexible edges in both *x* and *y* directions. If we go to Fig. 13, however,
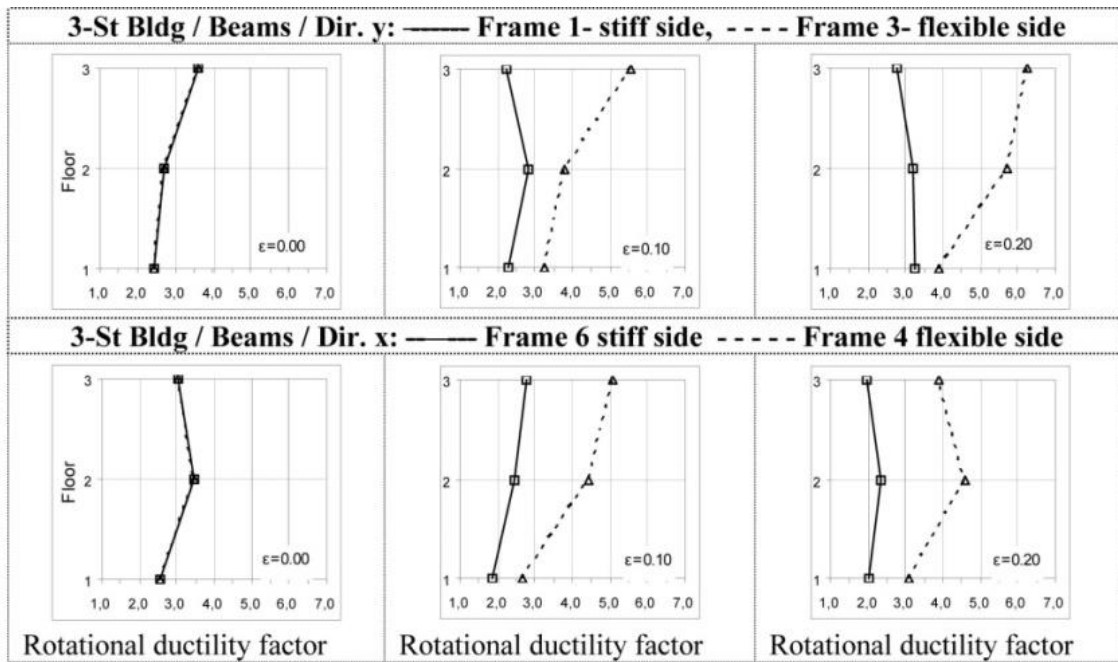
Fig. 10 Rotational ductility factors of the beams in frames of the 3-story buildings
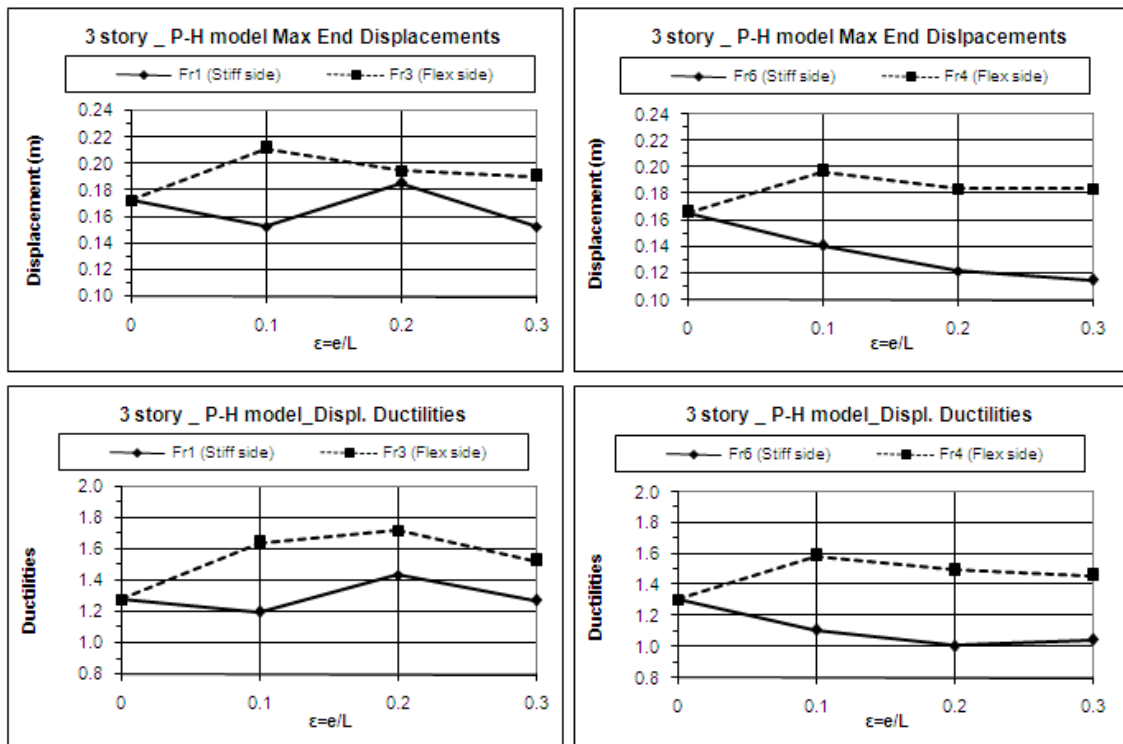

Fig. 11 Displacements and displacement Ductility factors of 3-story P-H model
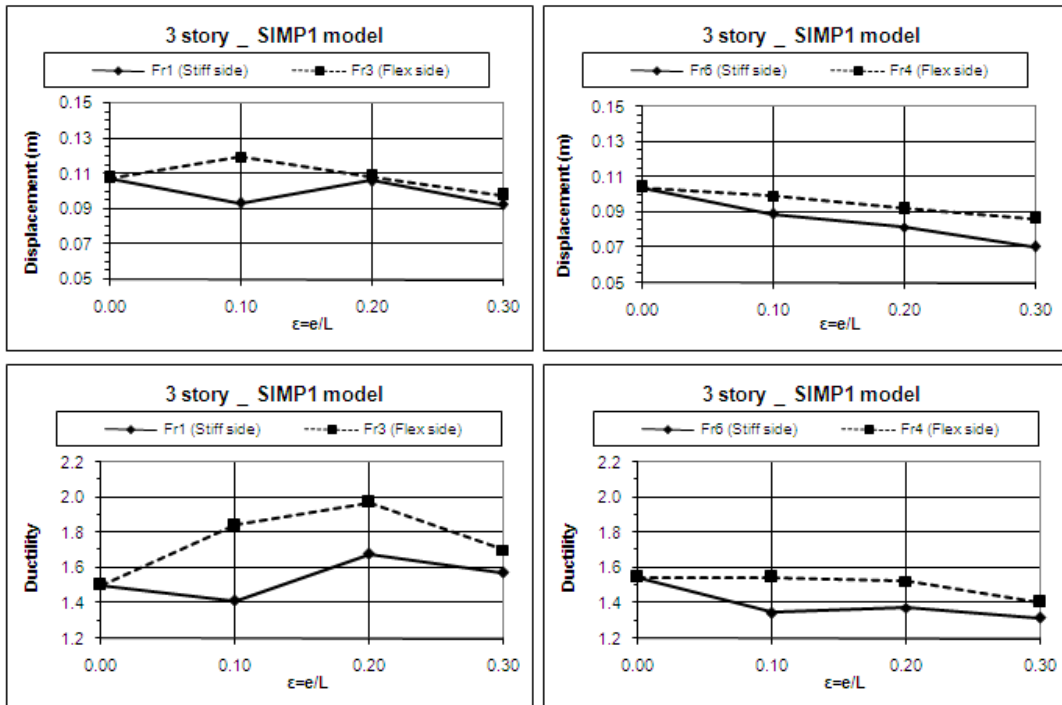
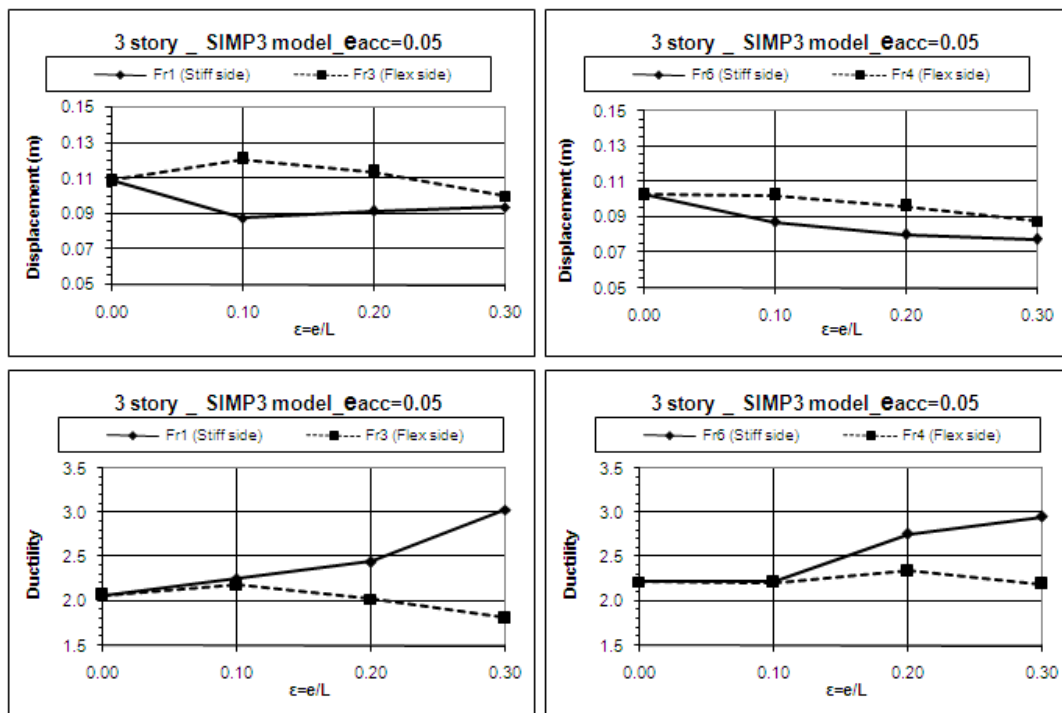Fig. 12 Displacements and displacement Ductility factors of 3-story SIMP1 model

Fig. 13 Displacements and displacement ductiliies of 3-ST- SIMP3 model

that presents the same results but for the traditional 1ST-INSB model (labeled SIMP3 model) we see that while peak displacements maintain the same trend, i.e., they are larger at the flexible than the stiff edges, the opposite happens with ductility demands shown in the lower two graphs. This confirms requirement (e) listed in the beginning of this section discussing the shortcomings of the 1ST-INSB models. We do believe that these results provide a strong support to our argument concerning the lack of reliability of the results from many earlier studies based on 1ST-INSB models, whenever such results were used to assess code provisions for actual buildings.

9. Accidental eccentricity

For the Dynamic method that modern codes specify as the basic method for earthquake response analyses of any building, only the second part in Eq. (11) is used to account for accidental eccentricity. The first part of these equations is to be used only with equivalent static analyses. As explained earlier in section 2, the reason for the design accidental eccentricity is to account for torsion by sources not explicitly included in design. Therefore even nominally symmetric buildings must be designed for this possibility. Publications on accidental torsion associated with non-uniform ground motion were examined separately in section 6 of the present document. Here we will review papers that address the problem of accidental torsion from any source from a design as well as building response perspective.

Among the first publications here are the two very similar papers by Pecau and Guimond(1988, 1990), who used a 2-element, type (a) in Fig. 3, 1ST-INSB model to investigate effects of unforeseen element strength variations and unbalanced hysteretic behavior. In their conclusions they state: *"(1) Compared with the symmetric response, both unforeseen variation of strength and unbalanced stiffness degradation can be expected to result in amplification factors of up to approximately 2. (2) For most structures, a code provision of 5% of the building width is adequate to account for plastic eccentricity ep*~<0.1, whereas 10% accidental eccentricity accounts for ep*~<0.25. (3) For torsionally flexible buildings (Ω<1.0), the above ranges reduce to ep*~<0.05 and 0.15, respectively. (4) When the results for different ground motions, frequency ratios Ω and periods of vibration T_o are combined, the 5% provision is found to be somewhat inadequate to account for the effect of unequal degradation in stiffness, whereas 10% of the building width becomes surprisingly accurate for the present model."* Although the authors are careful to indicate that their results are based on a two element simple system, they close by recommending a change in the Canadian code.

Substantially different are the conclusions in De La Llera and Chopra (1992, 1994a), where earthquake records from three nominally plan-symmetric buildings were used to assess the 5% code accidental eccentricity provision. Their conclusions state that for these 3 buildings, the 5% code value was more than sufficient, although in one of the buildings it caused increases in member forces as large as 30%, and further, this conclusion should apply to almost all nominally plan symmetric buildings. Accidental design eccentricity for two of the examined buildings and perhaps in most similar buildings need not have been considered. This would not be applicable to torsionally flexible buildings ($\Omega \leq 1.0$), elongated buildings, or buildings with uneven yielding. In the examined buildings, 25% to 45% of the recorded accidental torsion was due to rotational ground motion effects. Finally, it is concluded that *the code specified accidental eccentricity is probably a refinement inconsistent with other much higher uncertainties in the earthquake resistant design, especially in view of the extra work it requires to account for it in practical*

design.

A subsequent parametric study by De La Llera and Chopra (1994b) focused on accidental stiffness variations in a simplified one-story elastic model and led to essentially very similar conclusions. So did the study by De La Llera and Chopra (1994c), where in addition to the one story system, simplified MST buildings were also used. Wong and Tso (1994) investigated a 1ST-INSB system designed (a) without accidental eccentricity, (b) with accidental eccentricity applied by shifting the static load vector, and (c) with accidental eccentricity accounted by shifting the mass in a response spectrum analysis. Method (a) was found unconservative and method (b) preferable to (c). Towards the same goal for design applications, De La Llera and Chopra (1995) and Chopra and De La Llera (1996) recommend a new design method that avoids the extra analyses needed to account for accidental eccentricity from either the shifting of the static load vector or from the mass shifting in dynamic analyses. The new procedure consists of specifying an increase in edge displacements as a function of two building parameters: the ratio of the plan dimension *b* to the radius of gyration *r,* and the frequency ratio Ω. This method, evaluated in Lin *et al*. (2001) against recorded data of earthquake response of actual buildings, appears to have several advantages over the code methods accounting for accidental torsion.

Chandler *et al*. (1995) suggest a change in the three parameters of Eq. (11) by reducing the accidental eccentricity part to a minimum and increasing accordingly the dynamic eccentricity part reflected by the first terms of Eq. (11). Based on analytical and numerical investigation of one story simple elastic systems, Dimova and Alashki (2003) estimated that the code 5% accidental eccentricity leads to underestimation of torsional effects in symmetric systems up to 21 %. Simplified, multi-story shear beam models with a type (e), Fig. 3, typical floor plan were used in a probabilistic study of accidental eccentricity by De la Colina and Almeida (2004) and their main conclusion was that considering one random variable in the problem can lead to higher ductility demands than considering two or more. In addition they report that increasing eccentricities lead to lower ductility demands.

A practical assessment of the 0.05L accidental design eccentricities specified in most modern codes, including Eurocode 8 and IBC-2006, was carried out by Stathopoulos and Anagnostopoulos (2005b, 2006, 2010). They designed 3 sets of symmetric and biaxially eccentric buildings: with one, three and five stories. Besides the symmetric case, each set included biaxially eccentric variants with normalized physical eccentricities 0.10 and 0.20. A variant with ε=0.30 was included only in the one story set. Each of these variants was designed with 3 different accidental design eccentricities e_{acc}=0.0, 0.05L and *A** 0.05L, except that in the 5-story buildings the value of e_{acc}=0.10L was also included. The *A** 0.05L value is the amplified accidental eccentricity of the American IBC code. For each of the 3 and 5-story sets, designed with and without accidental eccentricity, two other sets were created to represent ACTUAL accidental eccentricity of ±0.05L. This was done by keeping the same structure but moving the masses on both sides by that amount. All these variants were modeled using the detailed plastic hinge idealization and were subjected to an ensemble of ten spectrum compatible, 2-component semi-artificial motions. Among the most interesting result to be quoted here was that accidental design eccentricity did not appear effective in reducing ductility demands in any of the examined buildings, nor in distributing such demands more evenly throughout the building. If this is confirmed by other studies for buildings of different types, then a revision of the codes would be desirable. With similar objectives, DeBock *et al*. (2014), investigated the influence of ASCE/SEI7 accidental torsion seismic design requirements on the performance of 230 archetypical buildings, designed with and without accidental torsion design provisions. Their conclusion, similar to those by Stathopoulos and Anagnostopoulos

(2005b, 2006, 2010), is that accidental torsion provisions are not necessary for seismic design of buildings without excessive torsional flexibility or asymmetry.

Aviles and Suarez (2006) used one story elastic, semi symmetric model on elastic half space to develop theoretical values of the dynamic and accidental eccentricity by matching the maximum displacement at the flexible edge from the static and dynamic solutions. They noted that since the parameters α and γ for dynamic eccentricity and β for accidental eccentricity depend on various combinations of the system's parameters, it is impractical to use constant values as the codes specify. Ramadan *et al.* (2008) used simplified multistory shear beam buildings with randomly distributed floor masses and carried out 300 time history analyses for a single, one component earthquake motion. Based on statistical evaluation of the results, they concluded that the accidental eccentricity is highly affected by the number of floors above the floor of interest and that such eccentricities are often lower than the 0.05L value specified by codes, especially for floors with large number of floors above. Similar conclusions will be found in De La Colina *et al.* (2011), who based their floor mass distributions on building surveys. Using then two different floor plans, a square and a rectangle, and different levels of slab self-weights, they made statistical analyses of vertical load distributions and determined probabilities for the code specified accidental eccentricities of 0.05L and 0.10L. It appears, however, that in both these studies, it is forgotten that the code specified accidental eccentricities reflect not only uncertainties in mass distribution but in other causes of torsional motion as well (e.g., non coherent ground motion).

Code assessment papers: Nearly all papers reviewed above include some form of code assessment

10. Design improvement for torsion

Most of the previously reviewed papers make assessment on torsional code provisions and often make recommendations for improvement. The overwhelming majority of such recommendations pertain to the equivalent static analysis based on the eccentricities in Eq. (11). Unfortunately, verification of the proposed modifications has also been carried out with the same simplified models and therefore their relevance to real structures remains highly questionable. In this category we will classify the proposals by Chandler and Duan (1993), Duan and Chandler (1997) based on previous studies in which a 3-element, 1ST-INSB model and two oversimplified 5 and 8 story shear beam models were used. In the 1997 paper, based only on the 1ST-INSB system, variable α and γ factors for Eq. (11) and a variable response reduction factor R were recommended for uniform ductility demands.

Along similar lines and based also on 1ST-INSB models, De Stefano *et al.* (1993), Mittal and Jain (1995) provide recommendations for the best location of the strength (or resistance) center CV, found to be near the midpoint between CS and CM for optimum strength distribution. Based also on a 2 DOF 1ST-INSB system, Goel and Chopra (1994) derived values for the coefficients α and γ in Eq. (11) as functions of the desired ductility factor and recommended them for a better, balanced, design covering two limit states: operational and ultimate. All these methods suffer from the same weakness: They were based on oversimplified crude models and were verified with the same. An interesting proposal came from Bertero (1995) who developed a static method based on limit analysis and the pertinent theorems. The method, good for preliminary design of buildings with aligned mass and stiffness centers in two vertical axes, is formulated having as objective the avoidance of torsional mechanism formation.

Another design method based on 1ST-INSB model and verified for buildings subject to a series of limitations was proposed by Ghersi and Rossi (1998), Ghersi *et al*. (2007). The limitations are: All CMs vertically aligned, all CSs vertically aligned and coincident with the CG, frame stiffness matrices proportional, columns not allowed to yield except at their base where axial-bending interaction is ignored, one beam and one column section used per frame and thus stiffness and strength in beams assumed independent. The method, requiring two modal analyses, a full 3D and one 2D with the torsional motion restrained, was verified using a detailed PH model subjected to a group of artificial one component motions. It was also shown that the method proposed by Duan and Chandler (1997) is over conservative for the flexible side and for some torsionally flexible systems, also for the stiff side. This supports our view about the weakness of such methods pointed out at the beginning of the chapter, a weakness also shared by the Ghersi *et al*. method, although to a lesser degree.

The most interesting and perhaps the most promising of the various design improvement proposals discussed in this subject, is a static procedure described in a series of papers by Paulay (1996, 1997a, 1997b, 1997c, 1998, 2000, 2001). It is a static method aimed at satisfying serviceability and ultimate limit state criteria, the former controlled by elastic structural response and the latter by addressing displacement ductilities of the elements and of the whole building. The method is based on determining equivalent bilinear strength curves for the various elements, defining a global maximum ductility factor of the building and deriving element strengths whose stiffness now become strength dependent. This requires the examination of possible translational mechanisms, which is easy under one component motion but becomes more complicated for two component motions. In the cited papers, no verification using inelastic dynamic analyses and detailed structural models has been presented.

A similar procedure with the one by Paulay has been presented by Crisafulli *et al*. (2004). Rather similar but much more interesting, complete and detailed is the design method presented by Sommer and Bachman (2005) for multistory shear wall buildings. Unfortunately, although they present a detailed step by step description of the proposed method, accompanied by an equally detailed example application, they have stopped short in verifying their results with an inelastic dynamic analysis of a detailed model.

In the two following papers, Tso and Myslimaj (2003), Myslimaj and Tso (2005), a procedure is presented for strength assignment to the lateral load resisting elements for the 1ST-INSB model and the same conclusion with De Stefano *et al*. (1993), Mittal and Jain (1995) is reached, i.e., that the optimum location for the CV (the strength center) is the midpoint between CM and CS. An attempt for providing an extension to a very limited class of multistory buildings may be found in Aziminejad *et al*. (2008). The aforementioned weakness exists in all these three publications as no verification with realistic buildings is provided.

Following an entirely different approach, Kyrkos and Anagnostopoulos (2011a, 2011b, 2011c, 2012a, 2012b, 2013a, 2013b) (see 8.2.2) have made seismic assessment of several steel braced buildings, all designed in full detail according to Eurocodes 3 and 8 for steel and earthquake resistant stuctures, respectively. Three and five story buildings were considered, with various degrees of biaxial eccentricity, torsionally stiff and torsionally flexible and with orthogonal and L shapes. 3-D analyses were carried out with detailed plastic hinge models for an ensemble of ten, two component, semiartificial motions compatible with the design spectrum. The results indicated a consistently uneven distribution of ductility demands in plan, with the flexible edges in either direction experiencing often substantially greater ductility demands, than the corresponding stiff edges. To alleviate this problem a simple modification of the design was applied based on strength

modification in the edge elements as a function of the top story computed edge displacements. In all cases, this modification reduced the ductility differences between flexible and stiff edges considerably, with each structure still meeting all the code requirements.

Displacement-based design of buildings with torsion has been presented and verified by Bahmani *et al.* (2014)

11. Experimental studies

Experimental studies on the problem of torsion, although few, are available and should provide some useful data for comparison with numerical studies. The first paper by Seki and Okada (1984) describes the static testing of four reinforced concrete, one story, one bay frames, with a deck supported on 4 columns and/or one shear wall at a scale of ¼. Four models were tested: one symmetric, two with uniaxial eccentricity and one with biaxial eccentricity. The tests were static cyclic with an imposed displacement at the center of mass. Hysteretic loops and numerical results are given. A shaking table test of ten reinforced concrete one story, one bay frames at 1:10 scale are reported by Kohyama *et al.* (1988). Four of them were symmetric with four corner columns while the rest were eccentric with one wall parallel to the direction of excitation. With three different percentages of hoop reinforcement considered, six specimens were subjected to sinusoidal motion and the rest to the 1940 El Centro EW record. The peak base accelerations varied from 210 cm/s^2 to 1380 cm/s^2 and test results were in good agreement with results from inelastic dynamic analyses.

Lin (1989), measured at various depths of foundation embedment the torsional response of a one-story, 1/11 scale model to ambient and steady-state incident wave motion. Modal frequencies and displacement ratios exhibited a qualitative dependence on embedment, consistent with the additional restraint of the foundation. Lumped parameter analysis, using a two DOF system that includes soil-foundation interaction effects, was used to predict the lower mode response. Shahrooz and Moehle (1989) presented shaking table test and numerical results for a 1/4 scale, 6-story ductile reinforced concrete frame building, 2 bay by 2 bay, with a 50% setback at midheight (Fig. 14). The model was subjected to 10 base excitations, (6 times the El Centro record, 3 times a Mexico City record and one time a Miyagi-Ken-Oki record), 4 parallel to the long direction and 6 along a 45 axis. Peak ground accelerations varied from 0.062 g to 0.634 g. Among the issues addressed were: (i) The influence of setbacks on dynamic response; (ii) the adequacy of current static and dynamic design requirements for setback buildings; and (iii) design methods to improve
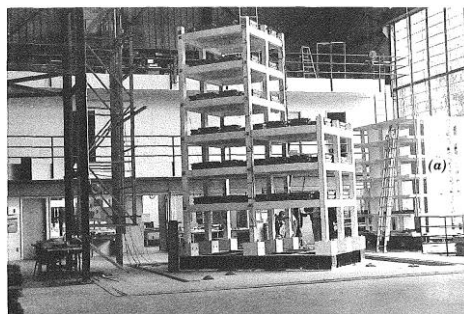

Fig. 14 Test structure in Shahrooz and Moehle (1989)

the response of setback buildings. The most important of the conclusions were that for this type of building the static analyses results were not notably different from modal-spectral analyses results. As a result suggestions were made that some code regularity requirements for permission of static analyses should be relaxed and further, a procedure was outlined to identify setback configurations for which excessive tower damage is likely. Finally, it was suggested that for setback structures identified as being irregular, and for which excessive tower damage is therefore deemed likely, the design should impose increased strength on the tower relative to the base. A static analysis method was proposed that amplifies design forces in the tower. This is a very useful paper with several practical implications.

Maheri *et al*. (1991) report a shaking table test of a 4-story, mass eccentric, small model (96 cm long) made of aluminum and their main conclusion was that *"the theory underestimates the significance of the fundamental torsional mode of vibration and overestimates the contribution of the first lateral mode. These effects compensate each other on the side of the structure which is most severely affected by torsional response, but produce large inaccuracies on the side of the building which is commonly assumed to be affected beneficially by torsional coupling"*. The next publication by Fardis *et al*. (1999) presents shaking table test and non-linear dynamic analyses results for the bidirectional response of a two-story RC frame structure with two adjacent sides infilled. Their main conclusion is that *"the peak displacement components of the corner column of the two open sides are about the same as (or slightly less than) those of the bare structure under the same bidirectional excitation, but take place simultaneously. This simultaneity of peak local demands from the two components of the motion seems to be the only effect of plan-eccentric infilling that needs to be taken into account in the design of the RC structure. Despite their very high slenderness (height-to-thickness ratio of about 30), infill panels survive out-of-plane peak accelerations of 0.6 g at the base of the structure or 1.3-1.75 g at their center"*.

Pseudo dynamic tests and numerical analyses under bidirectional excitation are given in Mola *et al*. (2004) for a 3 story reinforced concrete full scale model building, designed only for gravity loads with no earthquake provisions (Fig. 15). It was designed to be representative of older Greek buildings and the objective of the tests was to check the effectiveness of modern numerical methods for seismic capacity assessment. Several interesting conclusions are presented in the
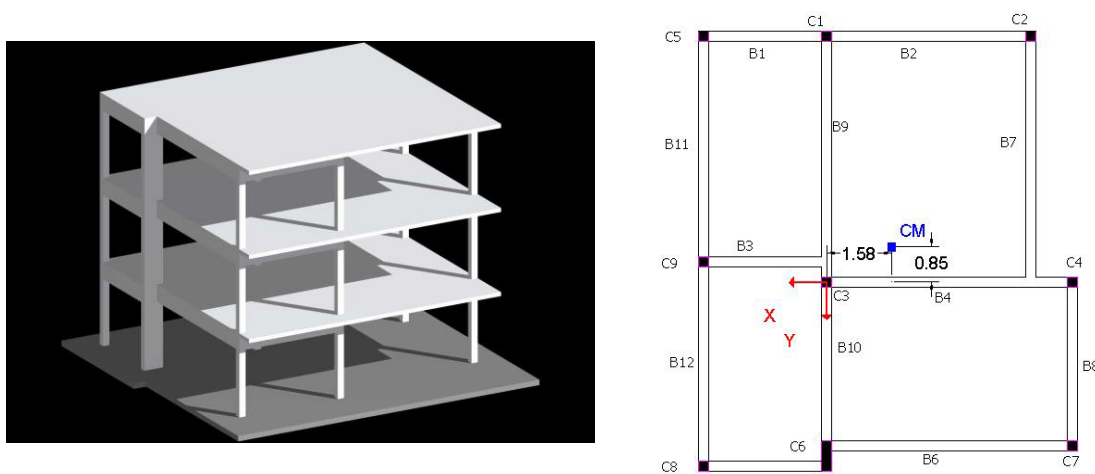

Fig. 15 Three-story SPEAR building (Mola *et al*. 2004)

paper, the most important of which states failure *"...in predicting the global failure mechanism of the structure; in fact, a first floor soft-story mechanism was predicted by all the pushover analyses, whereas in the test the second story was the most affected, with larger drifts and absorbed energy. This confirms that much care should be paid in applying simplified SDoF procedures to multi-story irregular buildings"*.

A set of shaking table tests of one story models with single and double eccentricities, Fig. 16, have been reported by Sfura *et al.* (2004). The model is 2.5 m long and was subjected to preliminary testing for determination of its dynamic properties and subsequently to a real one and two component earthquake motion, properly scaled to achieve ductilities of about 4.0~5.0. The purpose of the study was: *"to investigate the inelastic response of one-story, symmetric and asymmetric-plan steel moment-frame to biaxial lateral earthquake ground motions. ... The lateral-torsional response of the system was studied for eight different configurations of mass, strength, and stiffness eccentricity. The primary goals were to examine the adequacy of current building code torsional design assumptions and the ability of analytical software to predict inelastic response, for both a model tuned to the measured dynamic properties of the actual structure and a model based on common modeling assumptions"*. The basic conclusions were that results based on a "tuned" analytical model were similar with results using a more simplified model with "design" assumptions. For elastic response, "any *differences in the modal frequencies of the analytical model and actual structure can produce larger errors in the predicted response. When analyzing inelastic response, differences in the modal frequencies become less important...*" Also adding some strain hardening to an elastoplastic model led to significantly more accurate results.

Pseudo dynamic tests of a 1-bay, 2-story, one-way eccentric, reinforced concrete model building, Fig. 17, are reported by Bousias *et al.* (2007). The model design and detailing was representative of RC construction in Greece in the 1960s with little earthquake resistance. The tests were with one component excitations. Detailed numerical analyses were carried out and an overall good agreement with test results was reported, even for nonlinear behavior, provided that the proper member stiffness-secant to yield-was used. It is interesting to note that the conclusions herein are not similar, qualitatively, to conclusions by Maheri *et al.* (1991) and by Mola *et al.* (2004).
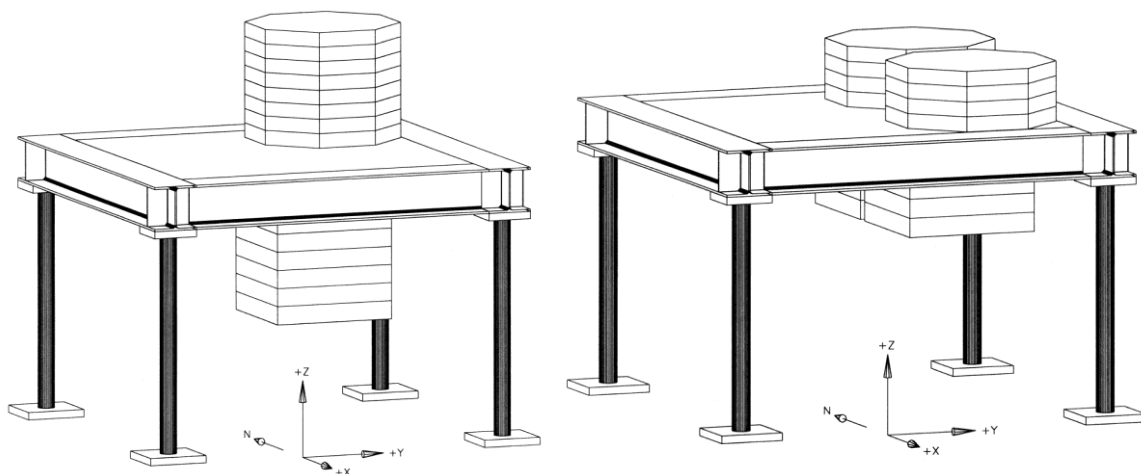

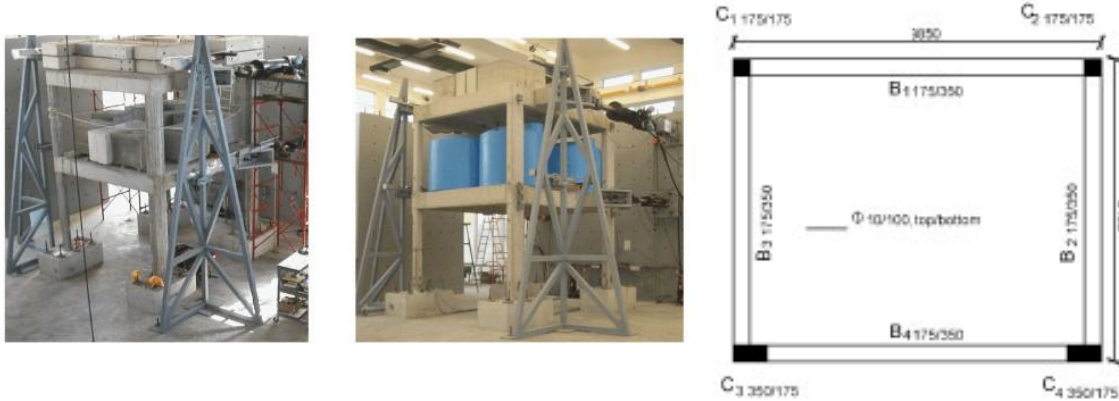Fig. 16 One-story tested structures (Sfura *et al.* 2004)

Fig. 17 1-bay, 2-story reinforced concrete building in Bousias *et al*. (2007)

De-la-Colina *et al*. (2007), used a one way eccentric, 2.0×0.88m steel deck supported by four steel columns, They considered two values of eccentricity, 0.05 and 0.15 and the test included both elastic and inelastic response. Their results indicated that torsion design factors (*α* and *γ*, Eq. (11)) depend on eccentricity and that for normalized eccentricities $\varepsilon \geq 0.025$, the amplification α can be between 2 and 3, while the *γ* factor can be between 0.0 and 1.6.

The last paper with experimental results, by McCrum, D.P. and Broderick, B.M. (2013),presents experimental and numerical comparisons of concentrically braced multistory buildings irregular in plan, and concludes that the EC8 code provisions are adequate in controlling ductility and interstory drifts but inadequate in controlling floor rotations.

12. Torsion with flexible diaphragms

The vast majority of buildings are built having floor diaphragms rigid in their plane, thus securing the so called "diaphragm action". This is beneficial for earthquake resistance. However, for various reasons mostly architectural, a floor might be flexible also in its plane and this flexibility must be accounted for in design. Reflecting the rare use of in plane flexible diaphragms, the number of pertinent publications is small. Snyder *et al*. (1996) have used a 2 DOF simple elastic model of a series of lumped masses interconnected with transverse springs simulating the in plane floor flexibility. The main conclusion is that this type of flexibility is important for torsional components and also that forces in roof members can exceed those in columns.

In a similar paper, De-La- Colina (2000), using a one story, one way eccentric system with one small frame at each perimeter side, concludes that: *"The peak displacement averages (PDAs) of lateral-resisting elements(frames) decrease for increasing in-plane floor flexibilities of systems with medium-to-large initial lateral periods (T>0.4 s). The PDAs of these elements increase (up to 50% higher) for systems with short initial periods (T<0.4 s). In all cases, the in-plane floor flexibility effect decreases for increasing values of the seismic force reduction factor R and the initial lateral period of vibration T"*. There is very little reference to torsion here. A paper by Murakami *et al*. (2000), use a modification of the procedure proposed by Paulay (1998) for one or
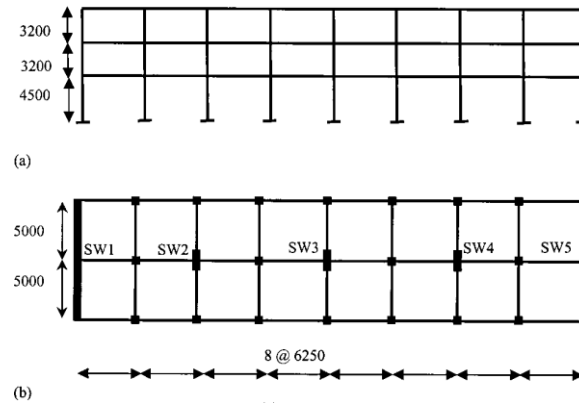
Fig. 18 Three-story building with flexible diaphragms in Basu and Jain (2004)

two story wooden houses with flexible diaphragms but in the conclusions it is stated that "*The accuracy of this simple procedure is not good enough but these predicted values may be utilized as a tentative criterion by introducing a safety factor after further numerical studies*".

Finally on the same topic, Basu and Jain (2004) recommend a procedure for buildings with flexible floor diagrams, extending the method by Goel and Chopra (1993) (see sect. 10), and computing a no-torsion condition restraining opposite sides to move in parallel. They say, however, very little about the diaphragm flexibility and how to model it into the proposed steps of analysis. They have also included an application for a 3-story elongated building (see Fig. 18) showing differences from the rigid deck assumption up to 36.4%. They conclude stating that "*the usual codal specification of accidental eccentricity as a fraction of the building dimension may be somewhat conservative for such buildings and this issue needs to be addressed in the future*".

13. Capacity assessment of asymmetric buildings

This is an area that has attracted considerable attention in the last 2 decades, due to the need for repair and strengthening of buildings after some catastrophic earthquakes. Since before any intervention the engineer must know the strength of the building, its capacity assesement is necessary. This requires knowledge of the behavior of the structure as a whole and in its details well into the inelastic regime and as a consequence it is necessary to use detailed models, either elastic with reduced loads or, preferably, inelastic. Since dynamic inelastic analyses of detailed structural models (PH models) are still considered too advanced for practical engineering applications, static limit analyses, popularly known as "pushover" analyses, soon became "the new game in town".

Used first with plane or symmetric structures, for which their application is straightforward, soon the need arose to expand it to 3-D non symmetric buildings subject to two component excitations. As no widely acceptable solution to this problem exists, it is at present an open area of research with torsion at the heart of the problem. It must be understood, however, that here the problem is different than the problem dealt before, in the sense that here we have an analysis problem while the problem dealt with in the papers already reviewed was essentially a design problem. Here, one tries to develop approximate methods to better approximate the response

predicted by the most accurate methods i.e., by nonlinear dynamic analyses of detailed structural models, while before, one was trying to find design solutions for irregular buildings that would exhibit optimum or satisfactory seismic performance. As happened before with the 1ST-INSB model, here too there is plenty of duplication, so we will be rather brief in our reviews.

Moghadam and Tso (1996, 2000), Tso and Moghadam (1997), were among the first to propose a procedure for monosymmetric systems subject to one component excitation. It uses an elastic spectrum analysis of the building to obtain target displacements and load distributions, subsequently needed for two-dimensional pushover analyses carried out for the lateral load resisting elements of interest. To investigate the efficiency of this method for different types of eccentric buildings, three simplified buildings were subjected to inelastic dynamic analyses: a ductile moment resisting frame building, a set-back building and a wall-frame structure. The analyses were performed for ten spectrum compatible motions. Comparisons of selected mean response results with those obtained by the proposed method demonstrated both the capabilities and limitations of the proposed procedure.

Rather similar are the procedures used in Kilar and Fajfar (1996, 1997), Faella and Kilar (1998), Fajfar *et al*. (2005) (N2 method), Dolsek and Fajfar (2007), Kreslin and Fajfar (2012), except that in the last two references the procedure is applicable to buildings with biaxial eccentricity under two component input. It is based on 2-D pushover analysis of the 3-D structure, combined with an elastic modal analysis to determine inluence factors for higher mode and torsional effects affecting the target displacement and the displacement variation in plan.

Evaluations of a simplified method where the static load vector was applied both at the CM and also displaced as dictated by the design eccentricities of UBC96, was carried out by De Stefano and Rutenberg (1998) and showed substantial differences from the nonlinear dynamic analysis results. On the other hand, much better agreement was reported in D'Ambrisi *et al*. (2009) for the method in Fajfar *et al*. (2005). More specifically they report: *"It is found that, even under such complex irregularity conditions, this 'modified' pushover analysis correlates well results from inelastic dynamic analysis almost up to failure, since, in most cases, its predictions of interstorey drifts and plastic rotations are conservatively close to values from inelastic dynamic analysis. Even failure mechanism, consisting of a floor mechanism at the third level, is correctly predicted, thus demonstrating adequacy of such method for actual framed structures".*

An interesting pushover method of analysis is the so called Modal Pushover Analysis (MPA) proposed originally by Chopra and Goel (2003), Goel (2004) for symmetric buildings subject to one component motions and subsequently extended by Reyes and Chopra (2011a, 2011b) to eccentric buildings subject to two component motions. The difference of this method from other pushover type methods is that it combines results from several pushover curves, each corresponding to a load pattern proportional to one of the elastic modes with results from corresponding bilinear SDOF systems, to obtain the final response values. Its accuracy is generally quite good for practical applications, although not always so with response quantities at the stiff and flexible edges. Moreover it is substantially more complex than other methods.

Fujii *et al*. (2004) have proposed a method for one way symmetric shear beam type buildings, with aligned CM and CS centers in two vertical axes, subject to one component motions. The method was extended in Fujii (2011) for use with asymmetric buildings subject to two component motions. It requires pushover analyses of plane elements and of two equivalent one-story systems, determination of the seismic demand of the two equivalent SDOF systems and from them evaluation of drift demands of each element in the equivalent one story systems and subsequently in each plane element of the actual building. The method appears to be rather complicated but the

presented results compare well with results from NLDA. However, more documentation is needed.

Lin and Tsai (2007) have proposed a version of modal pushover analysis, for one way symmetric buildings under one component motion and generalized it to fully asymmetric buildings under two component motions in Lin and Tsai (2008). In their first paper they derive an equivalent 2 DOF stick model and in the second paper a 3 DOF stick model (see also 7.2). The method is partially evaluated using two highly idealized 2 and 3-story buildings, but unfortunately the documentation and assessment with realistic buildings is lacking.

The next paper by Luccini *et al*. (2008) is applicable to one way eccentric regular buildings subject to one component motion normal to the axis of symmetry. It is very simple: it applies the load vector-first mode proportional- to the CS instead of the CM and appears to give good results while being much simpler than the N2 and MPA methods. In fact, according to the authors, while results by these two methods appear to deteriorate with increasing non linearity, the opposite appears to happen with the proposed method. Its major shortcoming, however, is its limited applicability to one way symmetric buildings subject only to one-component motions. Magliulo *et al*. (2008, 2012) have proposed an extension of the N2 method to account for accidental eccentricity in irregular buildings, but it is obviously quite cumbersome as it requires 4 modal response spectrum analyses and 8 nonlinear pushover analyses of the complete building.

Poursha *et al*. (2011) have proposed a multistage modal pushover analyses for one way eccentric buildings subject to one-component motions. Using 3 multistory buildings with 10, 15 and 20 stories, the authors applied the proposed method along with the MPA method and through comparisons with the NLDA results showed better predictions by their method. Of course the method is tedious as it requires pushover analyses in various stages, a fact that certainly defies the simplicity goal for practical applications. Very good agreement with results from NLDA has also been shown by using the so called adaptive pushover methods by Tabatabei and Saffari (2011), Shakeri *et al*. (2012) for one way eccentric buildings under one component motion. In adaptive methods, the load vector and/or its location is revised as the nonlinear analysis progresses, thus requiring several eigensolutions, which render such methods too cumbersome to use for practical applications. In fact, adaptive pushover methods are more complicated and more time consuming than NLDAs, i.e., the most accurate method that is generally used as a "yardstick" for all comparisons.

A variation of the MPA method was proposed in Manoukas *et al*. (2012) for one way symmetric systems subject to two componenet motions. Unfortunately, the accuracy of the method was cheked using only one story buildings. Another very recent proposal for a pushover analysis method, good for asymmetric buildings subject to two component motions, will be found in Bosco *et al*. (2012a, 2012b). Using various one story, one way eccentric systems and several artificial motions, two "corrective" eccentricities were derived as functions of the following four system parameters: stiffness eccentricity e_s, strength eccentricity e_p, ratio Ω and design response reduction factor R. The corrective eccentricities are subsequently utilized for application of the static load vector on either side of the CM to carry out two pushover analyses that envelope the plan distribution of maximum displacements. Application of the method to an L shaped, 5-story building gave results in very good agreement with results from NLDA.

Additional new procedures for extending pushover analyses to 3-D asymmetric buildings subjected to torsion under 2-component motions can be found in more recent papers by Bhatt and Bento (2014), Cimellaro *et al*. (2014), Putsha *et al*. (2014).

The last group of papers by Goel (2004), Anagnostopoulos and Baros (2008), Erduran (2008), Bento *et al*. (2010), Baht and Bento (2012), Bosco *et al*. (2013) include evaluations of pushover

methods, mainly of the N2, the MPA and the FEMA (2005) methods through applications to realistic buildings and comparisons with NLDA results.

14. New technologies to control torsion

Publications on new technologies for controlling earthquake induced torsion of irregular buildings appeared as early as 1980 and perhaps earlier, but only recently such technologies started to find their way into practice in increasing numbers. This resulted in an increase of pertinent publications.

14.1 Base isolation

We must note at the beginning that since base isolators behave just as shear beam elements, what differenciates the systems in this chapter from the idealized one story shear beam models of chapter 8 is primarily their distribution in plan.

Among the first publications addressing the problem of torsion with base isolated structures are Lee (1980), Rutenberg and Eisenberg (1984), Pan and Kelly (1984) whose main conclusion is that torsion is eliminated if the CS and CP centers of the isolation system are directly under the CM of the superstructure. Moreover they confirm that the properties of the isolators depend on the characteristics of the ground motion and show that the effect of base isolation reduces torsional coupling effects on the seismic structural response. Nakamura *et al.* (1988) reached the same conclusion based on experimental and theoretical investigations of a system with 4 bearings at the corners.

Nagarajaiah *et al.* (1993) have investigated the influence of various parameters on the response of an isolatated structure on elastomeric bearings to bidirectional ground motion and their results are used to explain: *"(1) The behavior of actual buildings;and (2) some inconsistencies in the conclusions of previous studies. It is shown that, although the total superstructure response is reduced significantly due to the effects of elastomeric base isolation, torsional amplification can be significant, depending on the isolation and superstructure eccentricity and the lateral and torsional flexibility"*.

Jangid and Kelly (2000) investigate analytically the response of a biaxially eccentric base isolated building and report that the effect of torsional coupling on the response is reduced when the ratio Ω is greater than one and also that the UBC static formula for the additional isolator displacements due to torsion is conservative. Shaking-table testing of an asymmetric base-isolated structure to triaxial base-excitation by Hwang and Hsu (2000) resulted in large rotations of the structure, which contributed significantly to the corner deformation. However, eccentricities in the system, imposed by an asymmetric arrangement of a small number of lead−rubber and natural-rubber bearings, were quite large (ε_x=0:20 and 0.39).

Using one and two component input and bilinear isolators, Tena-Colunga and Soberon (2002), Tena-Colunga and Escamilla-Cruz (2007, 2008) present parametric studies for base isolated eccentric structures, while Tena-Colunga and Zambrana-Rojas (2004, 2006) present results when eccentricities exist in the isolation system. Shakib and Fuladgar (2003) have investigated the effects of vertical earthquake components on the response of an idealized one story, one rectangular bay building supported on four pure-friction isolators at its corners. They concluded that the vertical motion component significantly affected the response of the coupled system. De

La Llera and Almazan (2003) presented results from an experimental study of a 3-story rectangular building model supported on four Friction Pendulum (FPS) isolators. Almazan and De La Llera (2003) have studied the effects of accidental torsion on the seismic response of a six story base isolated building on FPS isolators, caused by the variability in the vertical loads of the isolators due to the rocking motion of the building and its foundation.

Toyama *et al*. (2004) have presented a very interesting application of a conventional passive base isolation system, combined with a semi-active set of 20 dampers, half of which are passive oil dampers and the other half variable oil dampers. This system, the first semi-active base isolation system in Japan to be certified as a highly reliable system, was developed to minimize damage during small to medium level earthquakes and also to protect the building in a major event.

Ryan and Chopra (2004, 2006) have presented a procedure based on rigorous non-linear analysis that estimates the peak deformation among all isolators in an asymmetric building due to strong ground motion. It was shown in the first of the two papers that the peak isolator deformation was significantly underestimated by the U.S. building code procedures. The second of the two papers is an extension of the first to include, in addition to torsion, rocking motion. One of the conclusions therein was that accidental torsion in the isolation system from variation of the axial loads due to rocking of the structure was insignicant. Note, however, that this conclusion is applicable for the Lead Rubber bearings considered here and are different from the conclusions in Almazan and De La Llera (2003), where FPS type isolators had been used.

Seguin *et al*. (2008) investigate the linear earthquake response of seismically isolated structures with lateral–torsional coupling, with emphasis placed on developing simplified procedures for estimating the amplification of edge displacements of the superstructure and isolation base. An important conclusion, similar to the conclusion in Ryan and Chopra (2004), is that the UBC code formula, which is based on a static approximation, does not lead to accurate and conservative edge displacement predictions. In a newer paper, Seguin *et al*. (2013) present a design method for controlling the torsional response of seismically isolated, one way eccentric structures subjected to one component motions. Using probabilistic techniques, results are obtained suggesting that to counter-balance torsional effects in the base isolated asymmetric superstructure, it is necessary to introduce eccentricity in the isolation system. Moreover, the response of the superstructure may be substantially improved if the isolation system is torsionally flexible and if the center of stiffness of the isolated base lies in the vicinity of the (average) center of stiffness of the superstructure.

Kilar and Koren (2008), Koren and Kilar (2011) present and discuss the application of the N2 method to a one-way eccentric base isolated building. Comparisons of results with the average results of nonlinear dynamic analyses showed that the extended N2 method could, with certain limitations, provide a reasonable prediction of the torsional influences in minor to moderately asymmetric base-isolated structures. In one more publication, Kilar and Koren (2009) carry out a parametric study investigating the effect of various isolator arrangements on the seismic response of asymmetric buildings. To close this section, we will note the paper by Shimazaki (2012), where the effect of nonuniform base motion on the response of base isolated structures has been studied and was found that the maximum ratios of displacement increase due to this effect range in most cases between 1.2 and 1.3.

14.2 Energy dissipating devices

In the past, energy dissipating devices, mostly in the form of viscous dampers, had been used mainly in bridges and very rarely in some tall buildings, where they were applied in the form of

tuned mass dampers to control wind induced vibrations. Technological advances expanded the use of energy dissipating devices to buildings for reduction of earthquake caused motions. Such devices are used either in combination with seismic base isolators or alone and their function is to dissipate seismic energy, in addition to the energy dissipated by the various damping mechanisms inherent to the structure. In a building they are typically placed between floors, thus limiting interstory drifts, while in bridges they connect the deck with its abutments. There is a variety of such devices, the most common of which may be classified into the following broad categories: Friction devices (special joint mechanisms, slotted bolt connections etc.), yielding steel elements and viscous fluid dampers (see e.g., Constantinou 1994). More recently, dampers have been used as part of a control system, which may be active, semi active or hybrid control.

Structural control has been also a very active area of research, but it is only in the last two decades that applications to structures, mostly buildings and bridges, started to appear, primarily in Japan (Soong and Spencer 2000, Spencer and Nagarajaiah 2003, Casciati *et al*. 2011). An **active** structural control system consists of (a) sensors located in the structure to measure either external excitations, or structural response variables, or both; (b) devices to process the measured information and to compute the necessary control force needed, based on a given control algorithm, and (c) actuators, usually powered by external sources, to produce the required forces. A **hybrid** control system is a combination of passive and active control, and has the advantage that (a) it needs less power than an active control system and, (b) in case of power failure or insufficient battery power, the passive control will still work. A **semi active** control system is one that cannot inject mechanical energy directly into the controlled structural system (i.e., including the structure and control device) and hence it requires less energy supply, which can often come from batteries and thus the system will be unaffected by possible power failure. Examples of such devices are variable-orifice fluid dampers, variable-stiffness devices, controllable friction devices, smart tuned mass dampers and tuned liquid dampers, controllable fluid (magneto rheological) dampers, and controllable impact dampers. While this technology is rapidly developing, its wide application will require the development of a proper code or standard, as it has been done with base isolated structures.

One of the earliest published studies on the application of tuned mass dampers for active control of building response to strong earthquakes is the paper by Liu *et al*. (1984). They have considered a tuned mass damper in the top floor of a building controlled by two electrohydraulic servomechanisms along the *x* and *y* directions. Thus it is the translational motion that is directly controlled and indirectly through this the torsional motion. The effectiveness of the system is shown by a numerical example. Application of friction dampers to reduce torsional response has been investigated by Pecau and Guimond (1991), Pecau and Mastrangelo (1992), Pecau *et al*. (2000), while Lin *et al*. (1999) reported results of their study of seismic response reduction of irregular buildings by means of a passive tuned mass damper.

Goel (2000a, 2000b) has studied the harmonic response (2000a) and the elastic seismic response (2000b) of a simple, one-story, one way eccentric, type (f), Fig. 3 system and reported that best performance (highest motion reduction) is obtained if the dampers at the two sides are arranged so that the damping eccentricity is equal but opposite in sign to the structural (physical) eccentricity. The same conclusion is reported by Goel and Booker (2001), this time for a one story, type (e), Fig. 3 inelastic sytem. A simplified analysis of one way elastic eccentric systems with supplemental damping is proposed by Goel (2001). It is based on neglecting the off diagonal terms of the transformed damping matrix of a system with non-proportional damping and provides good results, at least for the one story eccentric system used as application. In Goel (2004), the seismic

response of a 1 story, one way eccentric, elastic and inelastic, type (f) Fig. 3 systems is compared for linear and nonlinear damping with an ensemble of 20 motions used as input. A similar investigation has been reported in Goel (2005) for the same systems and input but for non-linear fluid viscous dampers.

A detailed analytical investigation for the effects of supplemental viscous damping on the elastic seismic response of one-story, one way eccentric systems under one component motion may be found in Lin and Chopra (2001), where the most effective plan wise distribution of dampers for various response objectives was investigated. The next two papers, Lin and Chopra (2003a, 2003b), deal with the effects of non linear viscous damping on the elastic seismic response of one story, one way eccentric systems under one component earthquake motions. Comparisons with results using viscoelastic dampers, shows the higher effectiveness of the non linear dampers to better control the coupled building response. In the second of the two papers, an approximate method is suggested for systems with non linear dampers and a procedure recommended for designing non linear supplemental damping systems that satisfy given design criteria for a given design spectrum.

Using a genetic algorithm, Singh *et al*. (2002) have presented a method for optimal design of tuned mass dampers for controlling the seismic response of elastic, non symmetric MST buildings, with CM and CS centers vertically aligned, to two component excitations. Along similar lines is the paper by Ahlawat and Ramaswamy (2002), who have also used a genetic algorithm to determine optimal tuned mass damper solution for shear beam, multistory systems having a plan (f) -Fig. 3-layout with variable biaxial eccentricities in each floor and subjected to one component motion. In a study similar to those by Goel (2000a), Lin and Chopra (2001), Kim and Bang (2002), have looked into the optimum distribution of viscoelastic dampers using an elastic, one story, one way eccentric, type (f), Fig. 3, system under unidirectional excitation and applied their findings for a simplified, five story, shear beam, one-way eccentric building, with a type (a), Fig. 3 plan, with 2 DOF per floor.

Murnal and Sinha (2004) have presented results for one story, type (f) -Fig. 3-one way eccentric system supported on four pendulum type sliding isolators and subjected to one component motion. Huo and Li (2004) have also used a one story, one way eccentric, 2 DOF, type (a) -Fig. 3-system to investigate its earthquake response as controlled by Circular Tuned Liquid Column Dampers (CTLCD) and Palazzo *et al*. (2004) present their solution for best in plan arrangement of viscous dampers, based on the study of a one story, one way eccentric, 2 DOF, type (c) -Fig. 3-system. Single story asymmetric building models have also being used for displacement controlled response by Landi *et al.* (2013).

In the next series of papers: De La Llera *et al*. (2004), De La Llera *et al*. (2005), Vial *et al*. (2006), Garcia *et al*. (2007), Almazan and De La Llera (2009), the concept of Empirical Center of Balance (ECB) is introduced. Based on theoretical studies using one-story, type (c) -see Fig. 3-one way eccentric systems, and assuming that the earthquake responses are random ergodic processes with zero mean, the ECB is defined as the point in plan at which the translational and torsional responses have zero correlation. ECB is subsequently used for assessing the optimum locations of friction and/or viscous dampers in plan. The studies have been supported by experimental results carried out in a 6 story model building with a type (f) -Fig. 3-layout and in the last of this paper series, recommendations are made to use torsional balance as a design criterion.

The effectiveness of semi-active control with magnetorheological dampers to reduce torsional structural response has been investigated numerically by Yoshida and Dyke (2005), by means of genetic algorithms applied to two full scale buildings, the first with a 9-story plan irregular

building under one component motion and the second with an 8-story L shaped, steel braced building, under two component motions. The results of this study indicated that "*in general, a semiactive clipped-optimal controller in combination with MR dampers achieves similar performance as an ideal active control system in reducing the evaluated responses for several earthquakes. With a few exceptions, the ideal active controller performs slightly better than the clipped optimal controller, although the clipped-optimal controller achieves higher reductions in interstory drift responses in some cases. When comparing the semiactive controller using MR dampers and the passive-on controllers, the clipped-optimal controller offers significant performance gains in reducing acceleration responses*". Li and Qu (2005) have investigated by means of a simple, one story, one way eccentric system the application of multiple tuned mass dampers for "suppressing" coupled translational and rotational response, while Lavan and Levy (2006) solve the problem of best allocation of supplemental damping in the exterior frames of irregular, 3D multistory buildings by casting it as a multiobjective, non linear optimization problem.

Lin and Tsai (2007b, 2008b, 2008c) present an extension of a method, developed originally for proportional damping (See 7.2, Lin and Tsai 2007a, 2008a) to elastic multistory buildings, one and two-way eccentric, with non proportional damping. Here the same technique is used with a 2 DOF stick system for one component motions and with the 3DOF stick system for the two component motions, except that now non proportional damping can be accounted for. The accuracy of the method is evaluated by means of numerical integration for a one by one bay, 1-story and 3-story systems with non proportional damping. The same technique applied for proportional and non proportional damping by Lin and Tsai, is now applied by Lin *et al.* (2010a, 2011) to multistory one way and two way eccentric buildings with tuned mass dampers, subjet to one component and two compomnent motions respectively, to derive equivalent stick models with 2DOF and 3DOF. In the area of active control, Lin *et al.* (2010b) have presented results for soil–structure interaction (SSI) effects on vibration control effectiveness of active tendon systems for a two way eccentric building, subjected to two component earthquake excitations.

An approximate method to analyze eccentric buildings with linear viscous dampers was presented by Fujii (2008), as an extension of a method reported earlier (Fujii *et al.* 2004, Fujii 2011- see 13)) for reducing MST buildings to an equivalent one story eccentric system. Results for the effects of damper distribution in simple, one story 2DOF, one way eccentric system may be found in Mansoori and Moghadam (2008) while another method for seismic control of similar systems by optimizing the plan disrtribution of dampers is presented in Petti and Iuliis (2008). The same authors, Petti and Iuliis (2009), have also presented a method for designing a single tuned mass damper for controlling the response of a one story, one way eccentric simple system.

The next paper, by Shook *et al.* (2009), is an interesting presentation of experimental and numerical results for semi-active control using multiple magneto-rheological (MR) dampers for a 3 story, one way mass eccentric model structure. The system involves a fuzzy logic controller with genetic algorithms and works effectively in decoupling lateral and torsional modes and hence reducing substantially the torsional effects. Semi active control with variable dampers is presented by Mevada and Jangid (2012), for a simple, one story, one bay, one way eccentric system with four corner columns. It is found that "*the semi-active dampers reduce the lateral-torsional deformations significantly and effects of torsional coupling on effectiveness of control system are more sensitive to variation of eccentricity and torsional to lateral frequency ratio*".

The use of tuned mass dampers in three, one story systems, two with uniaxial eccentricity, and in one 15 story L shaped building, has been investigated by Almazan *et al.* (2012). The results

showed that *"the TMDs reduce the edge deformation in values varying from 20% to 50%. The highest reductions are obtained at the edges where deformation is greater. As a general rule, it has been found that the TMD should be located towards the corner where the uncontrolled response (without TMD) is greater…The findings for one or two TMDs are very similar, therefore, there is no significant improvement resulting from adding a second TMD. Preliminary results indicate that, when moderate, the inelasticity of the main structure does not significantly affect the optimized TMD frequency. However, for torsionally hybrid and flexible structures, the optimized TMD position is sensitive to inelasticity of the main structure"*.

The issue of optimal control of linear and nonlinear asymmetric structures by means of passive energy dampers is addressed by Aguirre *et al*. (2013). Three structures are used in the study: (a) a two story one bay plane frame (b) a one story, 2 DOF, one way eccentric, elongated rectangular building (5.0×15.0 m plan dimensions) and (c) a 19 story real asymmetric building with 2 basements. Using drift standard deviation as the performance index, it was found that optimal damper distribution achieves drift and torsional balance of the structure, which means that not only deformations along the peripheral edges of the structure at each story are reduced but also equalized solutions also achieved. Optimal damper distribution was similar for non linear metallic and viscous dampers. It was also found that the inelastic response of the frames was not only reduced, but also equalized. This is certainly an interesting study as it provides results also for application to a real building. Landi *et al.* (2013), have investigated the design of viscous dampers for the rehabilitation of plan-asymmetric buildings, while a study on optimization of viscous damper location and properties for torsional response reduction and seismic retrofitting is presented by Lavan and Amir (2014). On the same subject, Lin *et al.* (2014) use an energy based approach to optimize a set of viscous dampers in a building.

An excellent review paper on the subject of energy dissipation systems for seismic applications will be found in Symans *et al*. (2008). The paper has a wealth of information on the subject, describes the most frequently used energy dissipation systems, design principles for them and some characteristic applications to real buildings.

15. Conclusions

It is apparent from the reviewed literature that the accumulated knowledge over the past several decades in the field of earthquake induced torsion in buildings is in a rather chaotic state. Most of the people who have worked on this problem are aware of this, as they are aware of conflicting results and conclusions, a few of which were pointed out also in this paper. On the other hand, more and more people are becoming aware that the vast majority of the published work, including most of the papers with code assessment, was based until recently on crude oversimplifications and assumptions, leaving out essential properties and characteristics of actual buildings. As a consequence, even qualitative conclusions are now questionable and as it has been shown, erroneous trends were often predicted. What is interesting here is that when new studies were published with some of the earlier assumptions and model limitations removed, the validity of previous results was either questioned or rejected. For example, here is a quote from Goel and Chopra (1990), criticizing previous results by many authors based on systems with resisting elements in one direction (as in section 8.1.1 above): *"Thus the conclusions from earlier studies of systems without perpendicular elements are generally not applicable to most actual buildings which invariably include resisting elements in the two lateral directions to provide resistance for*

both horizontal components of ground motion".

Similarly, quite critical about results based on the simplified one-story shear beam model were Anagnostopoulos *et al.* (2010), who demonstrated the erroneous trends of behavior of the 1-story shear beam model, whose properties were determined in a very simplified manner as done in practically every past study. It is emphasized there that "*simplified models can approximate the inelastic response of real, asymmetric, multistory buildings, undergoing earthquake-induced torsion, **only if their three periods match closely the three lowest periods of the real building and the same happens with the element strengths, <u>which must reflect the total strengths of the building elements that have resulted from all loading conditions, including gravity, earthquake loads, drift limitations as well as capacity design effects</u>**. These strengths cannot be correctly determined if only the earthquake loading is considered, as was always done in the past. Moreover, use of a constant overstrength factor will not solve the problem, because the additional element strengths due to other loadings vary from one element to another. Only under the above stated conditions, simplified models could be used to provide useful insights about the influence of the many parameters affecting the inelastic earthquake response of irregular buildings and for understanding their behaviour. And only then they can provide guidance to code writers and perhaps preliminary checks for new code provisions.

The progress made in the past few decades is reflected also in modern codes which have introduced the dynamic method as the basic analysis method applicable to any building, having kept the static procedure only for special classes of buildings (e.g., for low rise regular buildings). As a result, the issue of dynamic amplification of the static eccentricity, as related to the values of parameters α and γ (Eq. (11)) of older codes and the related issue of "negative shear" i.e., whether design should account for shear force reduction at the stiff side due to torsion, two topics that have generated many papers in the past, have ceased to attract attention since modern codes address only the problem of accidental eccentricity. The basic building characteristics that influences code design provisions for torsion today and consequently the building response, are whether the building is regular or irregular and whether it is torsionally stiff or flexible. The latter is determined either by the ratio Ω of the uncoupled torsional period to the longest of the two translational periods or by the top story rotation expressed through a ratio of peak edge displacement to the average displacement of the two corners.

On the basis then of the presented review, the following topics appear to be among the active areas of research related to earthquake induced torsion in buildings:

(a) Design for torsion to secure uniform ductility demand distributions throughout the building plan. More specifically, designs that will lead to similar inelastic deformation demands along the flexible and stiff edges of an eccentric building, either torsionally stiff or torsioanally flexible.

(b) Code suggested values for accidental design eccentricity were mostly based on elastic response investigations. However, results based on measured building responses during earthquakes (De La Llera and Chopra 1994a, b, c, d) and on analyses of code designed buildings using detailed plastic hinge models (Stathopoulos and Anagnostopoulos 2005b, 2006, 2010) indicate that the 0.05L accidental eccentricity specified by several modern codes has little effect (a) on member sizing, and (b) on making ductility demand distribution more uniform in plan. It has thus been suggested that this design requirement should be reexamined and perhaps abolished, except, may be, for fully symmetric buildings, as it makes structural design more cumbersome by substantially increasing computational requirements.

(c) Capacity assessment of irregular (eccentric) buildings, which is affected by torsional building response. Extension of the well known pushover type of analysis, used primarily for plane

structures to 3D irregular building is a very active area of research in the past two decades.

(d) Finally, application of new technologies such as base isolation, passive, active or semi active energy dissipation devices, to control earthquake response in general and torsional response in particular is the other major area where intensive research activity will be found.

We have tried to present here a rather detailed literature review of earthquake induced torsion in buildings, including the progress made so far since the very first pertinent publication in 1938, to point some conflicting results, to discuss the current State of the Art and, finally, to indicate what we feel are the currently most active research areas in the field. It should be clear by now that because of the many parameters involved, great simplifications were introduced in the past that rendered the problem amenable to detailed numerical investigations. This allowed extensive parameter variations to be carried out and at the same time made it much easier to publish and generate volumes of papers with only small differences in some of the above parameters. Thus, while the obtained results and conclusions in each paper were **strictly applicable to the specific model used and subject to the underlying assumptions,** most authors made totally unjustified generalizations, having forgotten their simplifications and assumptions, often different than those of other researchers. Unfortunately, the cautionary statement we have just indicated in bold will rarely be found in any of these publications. If such a cautionary statement had accompanied the conclusions of the reviewed papers, it is most probable that the aforementioned controversies and conflicting results would have been greatly reduced.

It seems therefore appropriate when we try to solve a complicated problem, to remind ourselves the words of Albert Einstein: "We should try to make things as simple as possible but no simpler". And then, after we have obtained results that look interesting, to scrutinize them and give them a most critical look before publishing them, keeping in mind that "the easiest person to fool is ourselves" (R. Feynman).

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Elastic torsional response

One story elastic simplified models

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Inelastic torsional response

One story inelastic shear beam models (1ST-INSB)

e_x, K_y (unidirectional eccentricity, resistance and ground motion)

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e_x, K_x+K_y (unidirectional eccentricity, bidirectional resistance)

One-component motions

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In addition, some publications on 3-D pushover analyses of non-symmetric buildings may be found in:

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