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An iterative hybrid random-interval structural reliability analysis

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Abstract. An iterative hybrid structural dynamic reliability prediction model has been developed under multiple-time interval loads with and without consideration of stochastic structural strength degradation. Firstly, multiple-time interval loads have been substituted by the equivalent interval load. The equivalent interval load and structural strength are assumed as random variables. For structural reliability problem with random and interval variables, the interval variables can be converted to uniformly distributed random variables. Secondly, structural reliability with interval and stochastic variables is computed iteratively using the first order second moment method according to the stress-strength interference theory. Finally, the proposed method is verified by three examples which show that the method is practicable, rational and gives accurate prediction.

Keywords: structure; interval load; random strength; strength degradation; hybrid model; dynamic reliability

1. Introduction

Structural reliability is an important indicator in structural performance evaluation. One of the challenges in reliability analysis is that loads and structural strength are uncertain. Random methods (Ditlevsen and Madsen 1996, Madsen 1985), Madsen *et al*. 1986, Tee *et al*. 2013, Tee *et al*. 2011) and fuzzy analysis (Ayyub and Lai 1992, Jiang and Chen 2003) are among the main solutions to cope with the problems in structural engineering. However, all the developed methods are strongly relied on known information. Probability distribution function and fuzzy membership function are not easy to determine because sufficient data is difficult to obtain. The reliability prediction results are very sensitive to the accuracy of the estimated distribution parameters. The results may contain large error due to inaccurate distribution parameters (Ellishkoff 1995). Therefore, the random methods and fuzzy analysis have been rarely applied in engineering practice because of this limitation.

On the other hand, the interval or convex model can be used to solve structural reliability

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problems when the information of the distribution parameters is little known (Ben-Haim 1994, Elishakoff 1995) Besides that, the structural non-probabilistic reliability model based on interval analysis has been developed, and the structural reliability under interval load has been estimated (Qiu 2005, Guo *et al*. 2001). In real-world engineering applications, both random and interval uncertain elements should be considered together in reliability analysis of structures during their whole life services. Thus, the hybrid model which takes into account both elements (random-interval) is more credible than the model considers only one element (Zhu *et al*. 2008). The Sequential Single-Loop Method (SSLM) has been employed to solve random-interval reliability problems, but only static force is considered in the analysis (Du *et al*. 2005). In addition, static force has been researched as an interval variable and structural reliability is estimated numerically (Jiang *et al*. 2008), Gao *et al*. 2007), Gao *et al*. 2007). Nevertheless, structural reliability based on time response is also influenced by other external and internal uncertain effects, such as vibration, shock, fatigue, corrosion, etc (Jiang *et al*. 2014)). The probability density evolution method has been proposed to evaluate structural dynamic reliability under repeated random loads (Fang *et al*. 2013). Two cases with and without structural strength degeneration are considered in the analysis. The structural non-probabilistic reliability model using time response has been investigated and the model can precisely predict structural reliability under interval loads with structural strength degeneration (Fang *et al*. 2012).

This study is based on structural dynamic reliability theory with little information on the applied loads. The interval loads which are applied on the structure several times are analyzed using the random-interval theory. The hybrid structural dynamic reliability prediction model under repeated interval loads is obtained with and without stochastic structural strength degradation. The interval parameters are converted to the random parameters and the structural dynamic reliability index is computed by using the first order second moment method (FOSM). Finally, the proposed method is verified by three examples which show that the method is practicable, rational and gives accurate prediction.

2. Analysis of applied loads and structural strength based on time response

2.1 Applied loads

The loads applied on the structure are uncertain in terms of their amplitude and interval variables. The characteristics and assumptions on interval load analysis for structural reliability are given as follows.

(1) The structural service period T is divided into n equal-time sections $\tau = T/n$. The interval upper bound $\overline{s_i(t)}$ and interval lower bound $\underline{s_i(t)}$ of the stress of the maximum interval load at time τ can be determined by using statistical analysis. Thus, $\overline{s_i(t)}$ and $\underline{s_i(t)}$ are considered as the interval stress where $s_i^I(t) = [\underline{s_i(t)}, \overline{s_i(t)}], t = i\tau, i = 1, 2, \cdots, n.$ The mean and standard deviation of interval stress are $s_i^c(t), s_i^r(t)$, respectively and the diameter of the interval is $\Phi s_i^I(t) = \overline{s_i(t)} - \underline{s_i(t)}$.

(2) It is assumed that $s_i^I(t),\ t = i\tau$ is independent with each other.

If the structure does not fail under the maximum interval stress, then the structure will also not fail under the multiple-time repeated interval loads. Thus, the structural reliability analysis under the multiple-time repeated interval loads is equivalent to the analysis under the maximum interval load. This can be expressed by $\Phi s^I(t) = \max\{\Phi s_i^I(t)\}$. $\Phi s^I(t)$ is denoted as $s^I(t)$ where $s^I(t)$ is the maximum interval stress under the multiple-time repeated loads.

Suppose

$$s^I(t) = [\underline{s(t)}, \overline{s(t)}] \tag{1}$$

where $\underline{s(t)}, \overline{s(t)}$ is the lower bound and the upper bound of the maximum interval stress, respectively.

The mean and standard deviation of $s^I(t)$ is given as follows, respectively

$$s^c(t) = \frac{\underline{s(t)} + \overline{s(t)}}{2} \tag{2}$$

$$s^r(t) = \frac{\overline{s(t)} - \underline{s(t)}}{2} \tag{3}$$

2.2 Structural strength

Structural strength is considered as a random variable during its service period because it is influenced by external and internal uncertain effects, such as vibration, shock, fatigue, corrosion, etc. The remaining structural strength at time *t* can be described using Weibull distribution (Schaff, J.R., Davidson, B.D.(1997)) and is calculated as follows.

$$r(t) = r(0) - (r(0) - s(t))(\frac{t}{T})^e \tag{4}$$

where $r(0)$ is the initial strength, $s(t)$ is the time history of applied stress on the structure, T is the structural service period and e is the degradation index of the material.

Based on the property of the Weibull distribution and Eq. (4), the mean of the remaining structural strength at time *t* is given as follows.

$$\mu(r(t)) = \mu(r(0)) - (\mu(r(0)) - \mu(r(1)))(\frac{t}{T})^e \tag{5}$$

where $\mu(r(0))$ is the mean of the initial structural strength and $\mu(r(1)) = \mu(s(t))$ is the mean of the structural strength at the end of service period.

The variance of the remaining structural strength can be calculated as follows.

$$\sigma(r(t)) = (\frac{\sigma(r(0))}{\mu(r(0))} - (\frac{\sigma(r(0))}{\mu(r(0))} - \frac{\sigma(r(1))}{\mu(r(1))})(\frac{t}{T})^e)\mu(r(t)) \tag{6}$$

where $\sigma(r(0))$ is the variance of the initial structural strength and $\sigma(r(1))$ is the variance of the structural strength at the end of service period.

3. Computation of random-interval structural reliability

For structural reliability problem with random and interval variables, the interval variables can be converted to uniformly distributed random variables. Thus, the random-interval problem is reduced to traditional random reliability problem and the reliability can be computed using FOSM.

Based on the above concept, Eq. (1) can be rewritten as follows

$$s^I(t) \sim U(\underline{s(t)}, \overline{s(t)}) = F(s(t)),\ s(t) \in [\underline{s(t)}, \overline{s(t)}] \tag{7}$$

where $U(\underline{s(t)},\ \overline{s(t)})$ is uniformly distributed interval $(\underline{s(t)},\ \overline{s(t)})$, its probability distribution and probability density function are $F(s(t))$ and $f(s(t))$, respectively. For simplicity, $\Phi s^I(t)$ is denoted as $\Phi s(t)$.

$$f(s(t)) = \frac{1}{\overline{s(t)} - \underline{s(t)}} = \frac{1}{\Phi s(t)},\quad \underline{s(t)} < s(t) < \overline{s(t)} \tag{8}$$

The mean and variance of Eq. (8) are given as follows

$$\mu(s(t)) = \frac{\underline{s(t)} + \overline{s(t)}}{2} \tag{9}$$

$$\sigma^2(s(t)) = \frac{(\Phi s(t))^2}{12} \tag{10}$$

The limit state function under multiple interval loads with structural strength degradation can be obtained from Eq. (11) based on the stress-strength interference theory.

$$g(r(t), s(t)) = r(t) - s(t) \tag{11}$$

For structural reliability estimated by FOSM, $s(t)$ and $r(t)$ at any time *t* can be transformed to the equivalent standard normal distribution $(X(t),\ Y(t))$. Then, the structural reliability index can be obtained as follows

$$\beta(t) = \sqrt{X^2(t) + Y^2(t)} \tag{12}$$

The initial condition can be determined by Eq. (10) and structural reliability index at any time *t* can be obtained by computing Eq. (12) iteratively as follows

$$\beta^*(t) = \sqrt{X^{*2}(t) + Y^{*2}(t)} \tag{13}$$

where $X^*(t)$, $Y^*(t)$ corresponds to $(r(t),\ s(t))$ which is the maximum failure point in Eq. (11).

According to the iterative checking point method for accelerating convergence, the $(k+1)$th step of load calculation can be determined from Eq. (14)

$$s^{(k+1)}(t) = s^{(k)}(t) + (\mu(s(t)) \pm a \cdot \sqrt{\sigma(s(t))} - s^{(k)}(t))A^{(k)} \tag{14}$$

where a can be assigned to 2.25 in the first iteration and in the next subsequent iterations it is assigned to 0.75(Allaix, D.L., Carbone, V.I.(2011)). The initial value of Eq. (14) is given as $s^{(0)}(t) = \overline{s(t)}$. A in Eq. (14) can be obtained as follows

$$A^{(k)} = \frac{g(r^{(k)}(t), s^{(k)}(t))}{g(r^{(k)}(t), s^{(k)}(t)) - g(\mu(r(t)) \pm b \cdot \sqrt{\sigma(r(t))}, \mu(s(t)) \pm a \cdot \sqrt{\sigma(s(t))})}$$

On the other hand, for structural strength $r(t)$ calculation, if it is normal distribution, Eq. (15) can be neglected, or else Eq. (15) will be used.

$$r^{(k+1)}(t) = (r^{(k)}(t) + (\mu(r(t)) \pm b \cdot \sqrt{\sigma(r(t))}) - r^{(k)}(t))B^{(k)} \tag{15}$$

where b is assigned to 3 in the first iteration and in the next subsequent iterations it is assigned to 1 (Kang, S.C., Koh, H.M, Choo, J.F.(2010)). The initial value of Eq. (15) is given as $r^{(0)}(t) = \mu(r(t))$. B in Eq. (15) can be obtained as follows

$$B^{(k)} = \frac{g(r^{(k)}(t), s^{(k)}(t))}{g(r^{(k)}(t), s^{(k)}(t)) - g(\mu(r(t)) \pm b \cdot \sqrt{\sigma(r(t))}, \mu(s(t)) \pm a \cdot \sqrt{\sigma(s(t))})}$$

The iteration will be stopped when the circumstance of Eq. (16) is met.

$$\left| \beta^{(k+1)}(t) - \beta^{(k)}(t) \right| < \varepsilon \tag{16}$$

In summary, the step-by-step procedure for random-interval structural reliability analysis based on time response under multiple-time repeated interval loads with structural strength degradation is given as follows.

Step 1. The equivalent maximum interval stress under the interval loads can be determined as shown in Section 2.1.

Step 2. The interval stress will be converted to random stress which obeys normal distribution using Eq. (7).

Step 3. The random stress and random strength can be transformed to the equivalent standard normal distribution.

Step 4. The initial structural reliability index $\beta^{(1)}(t)$ can be computed using FOSM.

Step 5. The subsequent structural reliability index $\beta^{(k+1)}(t)$ can be determined iteratively using Eqs. (14-15).

Step 6. Stopping criterion for iterative reliability analysis is computed using Eq. (16).

Step 7. If the circumstance of the stopping criteria has been met, the iteration will be stopped, or else, go to Step 5.

Finally, structural reliability can be determined using the converged value of $\beta^{(*)}(t)$ and the standard normal distribution function table.

Table 1 The comparison of the computed results for Example 1

Methods	No. of Iterations	Reliability
Proposed Method	5	0.99995
SSLM	9	0.99994
MCS	10^6	0.99996

Table 2 The comparison of the computed results for Example 2

Time (*h*)	No. of Iterations	Proposed Method	MCS
0	5	0.99995	0.99996
500	6	0.99968	0.99970
1500	5	0.99871	0.99871
5000	10	0.98886	0.98889
9000	8	0.92601	0.92621

4. Examples

Example 1: Automobile Real Axle without Strength Degradation

An automobile rear axle is used to validate the proposed method for the case without strength degradation. Its initial strength $r(0)$ obeys normal distribution with mean of 160MPa and variance of 15MPa. The maximum interval stress under the applied multiple-time repeated loads has been computed as shown in Section 2.1. The lower and upper bounds of the maximum interval stress are $s(t) \in [45,\ 115]\text{Mpa}$. The reliability of the automobile real axle based on time response is calculated using the proposed method, SSLM and Monte-Carlo simulation (MCS). The results are compared in Table 1. The estimated reliabilities of the automobile real axle are almost identical among the three methods. However, there is a huge difference in the number of iterations. MCS is simulated with 10^6 times of iteration whereas SSLM and the proposed method require only 9 and 5 iterations, respectively to achieve a suitable accuracy. Therefore, the proposed method is the fastest in terms of computational time for convergence.

Example 2: Automobile Base with Strength Degradation

Next, an automobile base with strength degradation is used as a worked example where its strength $r(t)$ decreases with service time. Its initial strength $r(0)$ obeys normal distribution with mean of 150MPa and variance of 15MPa. The service period for the automobile base is given as 10000 hours and the degradation index of the material is 4.092. The lower and upper bounds of the maximum interval stress are $s(t) \in [45,\ 115]\text{Mpa}$ based on Section 2.1. The reliability of the automobile base under multiple-time repeated loads is calculated using the proposed method and MCS. The estimated reliabilities of the automobile base are shown in Table 2 at the time t = 0, 500, 1500, 5000 and 9000 hours. Similarly, the results obtained from the two methods are almost identical with number of iterations for the MCS is 10^6 and the proposed method is within 5 to 10 as shown in Table 2. As expected, the estimated reliability is decreased with the service time due to structural strength degradation. The results can be used not only to examine the structural life in its whole service period, but also as a reference basis for structural design.

Example 3: Cantilever Tube with Strength Degradation

Finally, a cantilever tube as shown in Figure 1 is used to further validate the proposed method for the case with strength degradation. A more complicated applied force is used in this example where the external forces $F_1(t)$, $F_2(t)$, $P(t)$ and torque $T(t)$ are applied together on the cantilever tube. The service period for the cantilever tube is given as 10000 hours and the degradation index of the material is 4.092. As shown in Table 3, there are 11 uncertain parameters where the parameter 1 and parameter 2 are represented by the mean and variance, respectively for random variables whereas the upper bound and lower bound, respectively for interval variables. If the cantilever tube is reliable, the equivalent stress applied on the cantilever tube $s_{\max}(t)$ should be less than its strength $r(t)$. Thus, the limit state function is given as follows

$$g(r(t),s(t)) = r(t) - s_{\max}(t)$$

where

$$s_{\max}(t) = \sqrt{s_x^2(t) - 3\kappa_{zx}^2(t)}$$

$$s_x(t) = \frac{P(t) + F_1(t)\sin\theta_1 + F_2(t)\sin\theta_2}{A} + \frac{M(t)}{I}$$

$$\kappa_{zx}(t) = \frac{T(t)d}{4I}$$

A is the cross section of the cantilever tube which can be calculated as follows

$$A = \frac{\pi}{4}[d^2 - (d-2\iota)^2]$$

M(*t*) is bending moment which can be calculated as follows

$$M(t) = F_1(t)L_1\cos\theta_1 + F_2(t)L_2\cos\theta_2$$

The reliability of the cantilever tube under multiple-time repeated loads is calculated using the proposed method and MCS (10^6 iterations). As shown in Table 4, at initial time $t = 0$ when there is no degradation in structural strength, both approaches provide an identical result (0.9716).

Table 3 Statistical properties of random and interval variables for the cantilever tube

Variables	Parameter 1	Parameter 2	Distribution
ι (mm)	5	0.1	Normal
d(mm)	42	0.5	Normal
$F_1(t)$(N)	2450	3600	Interval
F_2(i)	2450	3600	Interval
L_1 (mm)	110	130	Interval
L_2 (mm)	50	70	Interval
θ_1 θ_1 (°)	0	10	Uniform
	5	15	Uniform
θ_2 (°)	12000	1200	Normal
P(t)（N）	90	9	Normal
T(t)（Nm）	220	20	Normal
r(0)（MPa）			

Table 4 The comparison of the computed results for Example 3

Time (*h*)	No. of Iterations	Proposed Method	MCS
0	6	0.9716	0.9716
500	11	0.9668	0.9667
1500	9	0.9533	0.9533
5000	10	0.8802	0.8819
9000	12	0.8169	0.8188

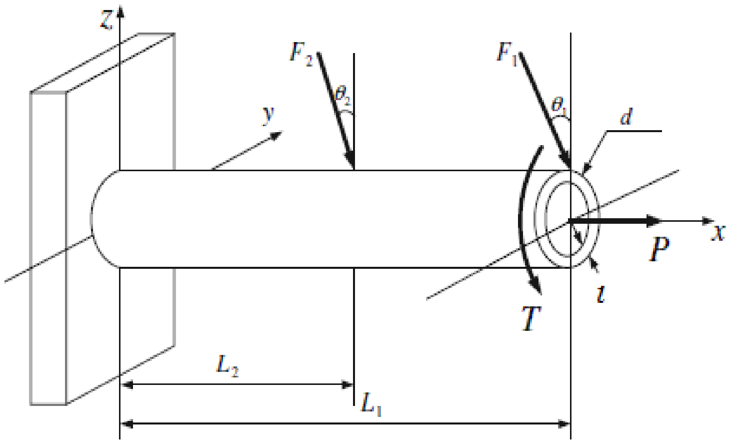

Fig. 1 A cantilever tube

In addition, the same result is also obtained from SSLM with 16 iterations [15]. However, the proposed method only requires 6 iterations to achieve the accuracy. The computed results from the proposed method and MCS are reasonably close during strength degradation at time t = 500, 1500, 5000, 9000 hours. Similarly, the estimated reliability of the cantilever tube is decreased with service time while the number of iterations for the proposed approach is increased.

5. Conclusions

An iterative hybrid structural dynamic reliability prediction model has been developed based on time response under multiple-time interval loads with and without consideration of stochastic structural strength degradation. This study is based on little statistical information on the applied loads. The interval loads which are applied on the structure several times are analyzed using the random-interval theory. The random-interval problem is reduced to traditional random reliability problem and the reliability can be computed using FOSM. The proposed method is the fastest in terms of computational time for convergence compared to MSC and SSLM.

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