

A probabilistic analytical seismic vulnerability assessment framework for substandard structures in developing countries

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Abstract. This paper presents a framework for analytical seismic vulnerability assessment of substandard reinforced concrete (RC) structures in developing countries. A modified capacity-demand diagram method is used to predict the response of RC structures with degrading behaviour. A damage index based on period change is used to quantify the evolution of damage. To demonstrate the framework, a class of substandard RC buildings is examined. Abrupt accumulation of damage is observed due to the brittle failure modes and this is reflected in the developed vulnerability curves, which differ substantially from the curves of ductile structures.

Keywords: seismic vulnerability; reinforced concrete; sub-standard construction; non-linear static, damage index; analytical; probabilistic, capacity-demand diagram methods

1. Introduction

Recent worldwide experience indicates that even though new design codes have been introduced in most seismic regions, they have not yet contributed as much to the minimization of earthquake losses, primarily as a result of the fact that the majority of the existing building stock pre-dates modern codes (i.e., sub-standard construction). An unfortunate verification of the above is given in the 2011 annual report of the Centre for Disaster Management and Risk Reduction in Germany CEDIM-Annual-Report-2011 (2012), which concluded that economic losses from earthquakes and their consequences have peaked in 2011 estimated at a staggering \$365 billion U.S. dollars. The quantification of the damage potential of the existing building stock can be achieved through vulnerability curves which represent the mathematical relationship between

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damage and seismic hazard. In recent years, analytical procedures are used to derive vulnerability curves, which in general use Finite Element Analysis (FEA) of structural models to simulate the buildings response and determine the damage distribution through an appropriate response-based damage index. In order to produce such curves for non-seismically designed buildings (defined here as sub-standard buildings) it is important to consider their deficiencies both in design and detailing and accommodate their brittle failure modes in the structural modelling.

The aim of the current study is to develop a framework for conducting probabilistic analytical vulnerability assessment of low and mid-rise substandard (RC) structures. This framework is based on improved modelling assumptions, use of new capacity models for sub-standard structures, improved performance evaluation method for sub-standard structures with low ductility and the probabilistic assessment of capacity related uncertainties. The effects of predominant failure modes, such as bond and shear, are addressed through static non-linear analysis. For simplicity and efficiency of the framework, the improved equivalent linearization procedure (modification of the capacity-spectrum method used in ATC-40 1996, which is included in FEMA440 2005) is used to predict the nonlinear response. The brittle failure modes were successfully captured for the case study structures using a damage index having secant period as a response parameter. Vulnerability curves were derived as a function of peak ground acceleration (PGA).

This paper initially presents the background on analytical vulnerability assessment where the research needs are highlighted. This is followed by the description of the framework and its applications for 2 case-studies.

2. Background

Predicting the seismic damage potential of specific categories of old RC buildings, which have not been designed and constructed according to modern seismic provisions, is still a challenge for the earthquake engineering community. One of the first comprehensive attempts to quantify the expected damage potential for different intensity levels was made by Whitman *et al.* (1973), based on observed damage data from the 1971 San Fernando earthquake. This empirical method was used by introducing for the first time the concept of Mean Damage Ratio (MDR) as the mean ratio between repair and replacement cost which is still the most widely used economic damage indicator. Since then, various methods of vulnerability assessment have been developed (e.g. Mosalam *et al.* 1997, Lang 2002, Gardoni *et al.* 2003, Franchin *et al.* 2003, Crowley *et al.* 2004, Rossetto and Elnashai 2005, Erberik and Elnashai 2005, Erberik 2008, Celik and Ellingwood 2008) differing in level of detail and precision. In general most of these methods utilize static or dynamic analysis for the determination of the structural response and are referred to as analytical vulnerability assessment methods.

Analytical vulnerability assessment has become very popular in recent years especially due to the evolution of capacity-demand diagram methods (Chopra and Goel 1999), for the prediction of the seismic response. Both simple and detailed analytical methods exist in the literature, which differ in the determination of the non-linear capacity of the structure and the procedure used for comparison with the seismic demand. Simple methods do not require the analysis of the structure but rely on simple equations to derive its capacity.

The most widely used simple analytical procedure was proposed by Calvi (1999), and it is

based on the ratio between the displacement capacity of a structure corresponding to several limit states and the displacement demand from an earthquake event as obtained from the corresponding displacement spectrum. The displacement capacity of a structure is based on simplified formulae using their mean geometrical and material properties. The procedure is based on the elastic properties of the building and utilises the principles of the direct displacement-based design. More recently, Borzi *et al.* (2008) have proposed another simplified assessment method that builds upon Calvi (1999) but obtains the details of the structures (in terms of beam and column sections and detailing) from a simulated design, after which a simplified pushover curve is produced. The accuracy in the prediction of these two methods is discussed in Crowley *et al.* (2008).

Detailed analytical procedures are more thorough and demanding, and are intended to be used when the resulted response of a structure is obtained from static or dynamic non-linear analysis. In general these methods rely on:

- structural modelling and analysis for the determination of the structural response,
- correlation of the estimated structural response to damage levels.

2.1 Modelling

As far as modelling is concerned and especially in the case of substandard RC buildings, a number of failure modes (such as flexure, shear, local buckling and debonding of reinforcement) should be considered. It is noted that most analytical vulnerability studies concentrate primarily on the flexural and, to a smaller extent, shear failures at the member level (Rossetto and Elnashai 2005, Dymiotis 2000, Ahmed 2006) whereas buckling and debonding failures at the local level are disregarded completely. Slip of reinforcement in the joint region due to inadequate anchorage length or lack of confinement in the anchorage region, can cause bar debonding and result in large cracks in the region around the joint as observed in relatively recent earthquakes in the Mediterranean region (Elnashai 1999, Sezen and Moehle 2003). Besides the joint region, inadequate shear link spacing (especially in sub-standard constructions) can lead to local buckling of the column reinforcement bars and subsequently to severe loss in column capacity.

It can be argued that, in vulnerability studies, the modelling issue is not addressed at a satisfactory level in general in the published literature, since the sophisticated analytical tools that exist in modern finite element software are not fully exploited. A number of sophisticated local interface models exist in the literature that can be used to simulate the above mentioned failures. The simulation of such failures requires the use of local force versus displacement models (fiber element based models) with different compressive to tensile strength characteristics for each failure mode rather than the global moment versus curvature models (plastic hinge models) commonly used in vulnerability studies, which cluster the effect from a number of failures and are very demanding in model calibration.

2.2 Structural response

Most detailed analytical methods use capacity-demand diagram methods to estimate the structural response. In brief these methods compare the static non-linear response of the structure with the response spectrum of a specific seismic acceleration representative of the area under consideration, to compute the expected response. In ATC-40 (1996) highly damped spectra are used to characterise the seismic demand (Capacity-Spectrum Method); whereas in FEMA356 (2000), the displacement coefficient method is used as a simple alternative. Improvements of both

methods are included in FEMA440 (2005). In particular for the case of the ATC-40 (1996) method, an improved linearisation procedure is proposed which uses a Modified Acceleration-Displacement Response Spectrum (MADRS) to represent the seismic demand. This increases the sophistication of the method, since it reduces the elastic response spectrum based on both the increase in damping and ductility.

In contrast, Singhai and Kiremidjian (1997) derived analytical curves using nonlinear time history analysis, applied to model frames of low, medium and high-rise RC bare frames in California, USA. The ground motion was represented by the average spectral acceleration ordinate and the shape of the curve was described by a lognormal distribution. The curves derived for low-rise frames were verified against actual damage observations. Similarly Mosalam *et al.* (1997), computed analytical curves for the typical buildings in Memphis, USA (low-rise reinforced concrete frames with and without infill walls) using artificial earthquake records of PGA to compute time-history analysis on an equivalent SDOF system.

The increase in computer power and the availability of robust and sophisticated software packages in the past 20 years has led to an increase in the application of detailed analytical vulnerability assessment methods a selection of which is given in the previous paragraphs. The aim of this paper is to take advantage of the benefits provided by the detail analytical modelling.

2.3 Structural damage

Further to the prediction of the structural response, another important step for the derivation of analytical vulnerability curves is the correlation of this response to structural damage. For this purpose, a number of structural damage indicators can be found in the literature that correlate the structural response to the expected damage potential. Timchenko (2002) clustered the variety of damage indicators (DI) in the following three categories based on the structural parameters required for their calculation.

- Dynamic parameters of the structure
- Displacement parameters
- Displacement and cumulative damage

A commonly used dynamic damage indicator was proposed by Dipasquale and Cakmak (1988) and is based on the concept of final softening. The change in the fundamental period of the structure is used as a measure of the change in the stiffness caused by the earthquake. The advantage of the final softening method is that it can be evaluated from the initial natural period and the final period determined from vibration field-testing after the earthquake. On the other hand, it does not provide any information about local and storey damage. In addition, the period calculation at the final time step of the excitation may be affected by the randomness of the instantaneous tangent stiffness at the end of the dynamic load (Ghobarah *et al.* 1999).

The proposition that damage is related to an increase in period (or decrease in frequency) was verified by using experimental data. Calvi *et al.* (2006) concluded from results of experimental tests on RC frames that *a significant period elongation occurs during strong ground motion and this can be attributed to the accumulation of damage in the structure*. Zembaty *et al.* (2006) moved a step forward by producing an experimental damage scale relating observed damage to the decrease in frequency and suggested that a 70% decrease can be regarded as structural failure.

Displacement parameters are the most commonly used for vulnerability assessment purposes, since they can easily be obtained analytically. In addition, it is generally accepted (e.g., Ambraseys

2002, Priestley 1997), that displacement parameters (such as drift and ductility) simulate better the structural response in the inelastic range, and that damage is related to maximum response displacements rather than accelerations (Priestley 2003).

Damage models were also developed to account for both energy dissipation and peak displacement. The most popular DI of this category was derived by Park and Ang (1985). The ductility level at each displacement increment is superimposed on the hysteretic energy dissipated in the structure up to the specific displacement. The calibration of this DI is rather demanding and requires laboratory or field data and, as with most cumulative damage indices, depends strongly on the hysteretic model of the elements. Strength based damage indicators are used to denote the failure (collapse) of a structure. Most frequently in vulnerability studies of existing non-seismically designed buildings, a threshold in the loss of strength is defined beyond which the building is characterised as unstable and is expected to actually collapse. In several studies, such as Kappos (2006), a strength loss of 20% in the capacity curve (or push-over curve) is regarded as an indication of collapse.

The increase in computer power and the availability of robust and sophisticated software packages in the past 20 years has led to an increase in the application of detailed analytical vulnerability assessment methods a selection of which is given in the previous sections. The degrading behaviour of substandard RC structures and the variety of failure modes appears to be the main issue not addressed fully by previous vulnerability studies, both as far as the prediction of the structural response and level of damage are concerned. The aim of this paper is to take advantage of the benefits provided by the explicit detailed analytical modelling for various failure modes, and produce a probabilistic vulnerability assessment framework that is suited to the special conditions of degrading structures considering the randomness of the main basic variables. The lack of ductility and the degrading nature of the structural response necessitate the use of alternative approaches for the determination of response and damage. The description of the procedures involved in this framework is given in the following sections along with an example case study.

3. Description of the framework

3.1 Introduction

The flowchart diagram for the proposed probabilistic analytical vulnerability framework is presented in Fig. 1. Unlike similar work, based on capacity-demand diagram methods for assessment purposes, this framework uses a reverse application of the concept treating each point (SA_i , SD_i) on the capacity curve as a performance point (PP) and estimates the PGA corresponding to each PP (see procedures 10-15, Fig. 1). The estimations are achieved using the improved

equivalent linearisation procedure in FEMA440 (2005). The ratio of the period $\frac{T_{sec}}{T_{initial}}$ at each PP

is evaluated and is linked to damage estimates using a damage indicator (DI). Based on multiple performance criteria, such as inter-storey drift and strength loss, the secant period at collapse is defined and is assumed to correspond to a value of DI=100%. The values of DI corresponding to the each point on the capacity curve are then found by interpolation. The corresponding calculated PGA value at each point is linked to the interpolated values of DI to form the vulnerability curve.

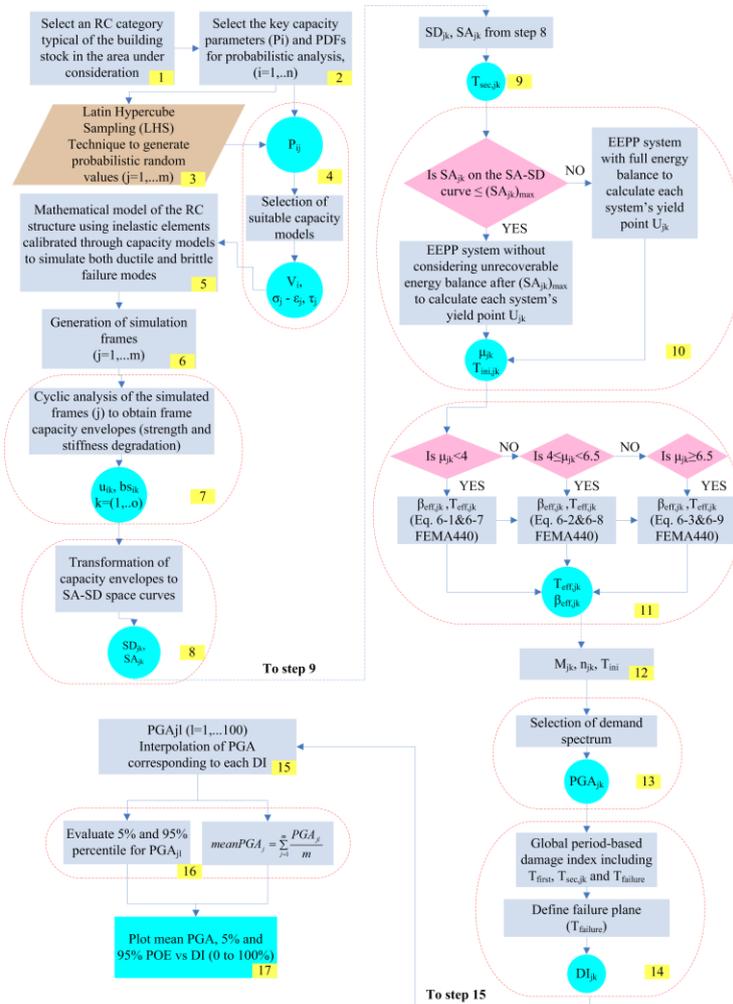
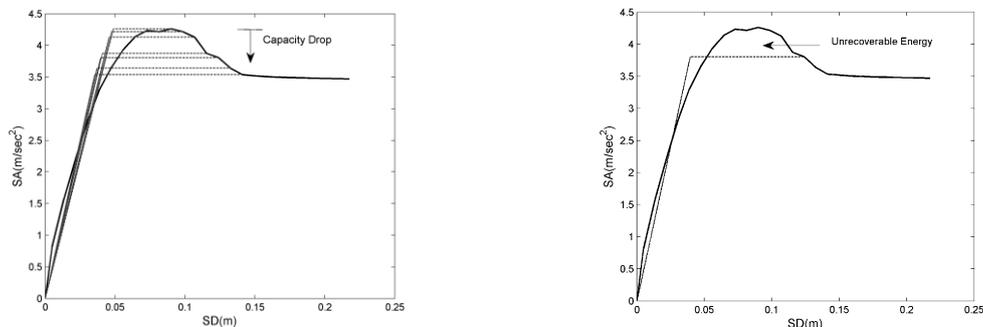


Fig. 1 Proposed framework for the probabilistic analytical vulnerability assessment of RC structures



(a) Series of equivalent EPP systems at each PP (b) Unrecoverable energy at higher displacement

Fig. 2 Energy balance calculation

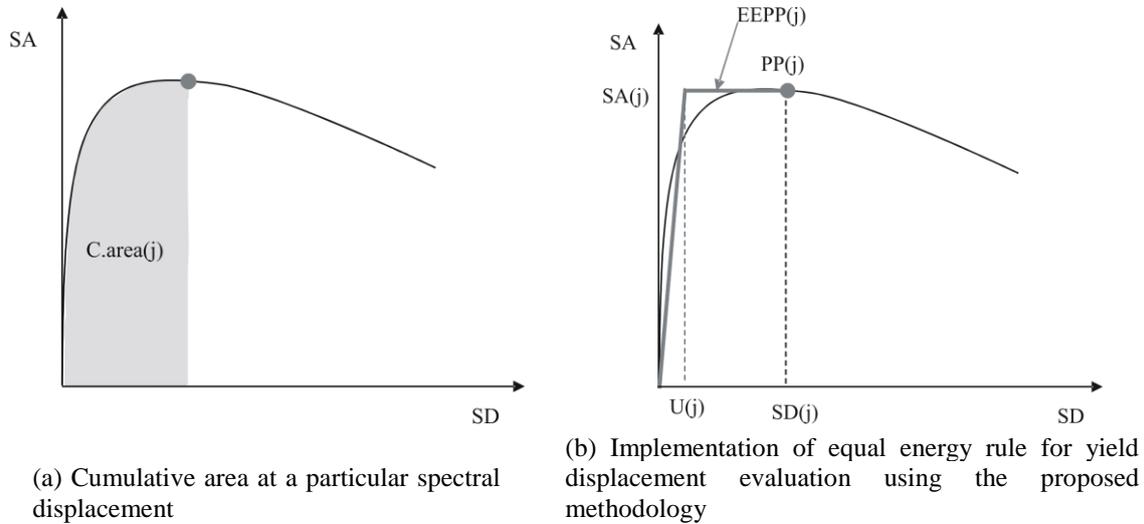


Fig. 3 Evaluation of yield displacement for EEPP

Table 1 Mean and standard deviation of seismic demand prediction error

No	Building	Error (MADRS)	Error (MADRS)	Error (EEPP)	Error (EEPP)
		% (μ)	% (σ)	% (μ)	% (σ)
1	2 storey 1bay	18.6	5.3	8.3	5.7
2	2storey 2bay	10.8	5.4	5.0	2.7
3	3storey 3bay	19.7	5.4	3.2	3.1
4	5storey 4bay	15.2	4.1	3.7	2.3

3.2 Modelling complex degradation behaviour of sub-standard equivalent systems

The prediction of the seismic demand using non-linear static methods can be achieved through the use of capacity-demand diagram methods. A number of such methods exist in the literature, which arrive at the reduced earthquake demand calculations using either the increased damping (β) due to damage or the increase in ductility (μ). Regardless of the procedure used, these methods compare the reduced earthquake demand (reduced spectrum) to the non-linear capacity curve of the structure in order to arrive at the performance point (PP) on the capacity curve. This is regarded as the expected displacement of the structure for the specific earthquake (response spectrum).

For degrading systems, which is the focus of this study, the capacity curve is expected to follow a degrading behaviour which cannot be idealised using an elastic-perfectly plastic approximation. In an attempt to address this issue, the improved equivalent linearisation procedure, proposed in FEMA440 (2005), includes a strength and stiffness degradation elasto-plastic (E-P) system idealisation alternative, which allows for a linear strength degradation at the post-elastic part of the curve. The maximum allowable negative post-elastic slope is 5%. Although this is a step in the right direction, it still fails to capture the characteristics of a degrading curve due to strength loss, since the energy of the capacity curve differs considerably from the energy of the E-P system at all displacements. In addition, the actual strength loss of the structure can be considerably higher than

the modelling capabilities given in the procedure and may follow a steeper or non-linear path.

In the context of this work, the use of a single E-P approximation is considered insufficient to model the more complex degrading behaviour encountered in sub-standard structures. Therefore, in order to maintain the special characteristics of the capacity curve, it is proposed that the shape of the curve is approximated by a number of different elastic-perfectly plastic systems with zero post-yield stiffness. Each SA_i - SD_i coordinate on the capacity curve is treated as the strength and ultimate displacement of an equivalent elastic-perfectly plastic system (EEPP), defined using the equal energy rule. However, after degradation, energy dissipated above the current force level is considered unrecoverable and is excluded from the energy balance calculation (Fig. 2a and b). Hence, the proposition for idealisation of the capacity curve is based on its discretization into a number of PP's, each corresponding to a single EEPP system. The idealised bilinear curve of each system is based on equal energy rule up to the maximum response, and the exclusion of the unrecoverable energy for the degraded part of the capacity curve. Kyriakides (2008) and Kyriakides *et al.* (2012) show detailed information and examples of the idealisation procedure.

Fig. 3a shows the cumulative area under the capacity curve at $SD(j)$ corresponding to the maximum capacity point. EEPP corresponding to this point is shown in Fig. 3b. The equal area rule (Equations 1 and 2) is applied to evaluate the yield displacement $U(j)$ using Equation 3.

$$C.area(j) = \frac{U(j)SA(j)}{2} + (SD(j) - U(j))SA(j) \quad (1)$$

$$C.area(j) = \frac{U(j)SA(j) + 2SD(j)SA(j) - 2U(j)SA(j)}{2} \quad (2)$$

$$U(j) = - \frac{2(C.area(j) - (SD(j)SA(j)))}{SA(j)} \quad (3)$$

The performance of the proposed idealisation EEPP procedure was assessed numerically in predicting the seismic demand of a range of sub-standard structures. A small building population (10 simulation buildings from each building type) was derived from probabilistic data, based on key capacity parameters (concrete strength, steel yield strength, cover and development length) as described in Ahmad (2011). The seismic demand (i.e. performance point) for each simulation building was evaluated using the proposed idealisation procedure and the one included in FEMA440 (2005). The spectrum used in Eurocode 8 (2004) for soil type A was used. The

calculated demand values were compared to demand predictions (exact predictions) obtained from time history analysis using a scaled acceleration record derived from the same spectrum. The error in the predictions was defined as the percentage difference to the exact prediction. Table 1 outlines (for each building category) the error obtained in seismic demand predictions by using the idealisation procedure included in FEMA440 (2005) and the proposed EEPP procedure. Based on these results, it is concluded that for deficient structures experiencing strength degradation and brittle failure modes, the unrecoverable energy should be removed from the energy balance calculation.

3.3 Calculation of PGA

Since each point (SA_i , SD_i) on the capacity curve is treated as a PP, the capacity-demand diagram methods can be implemented in a reverse manner (back analysis) to estimate the

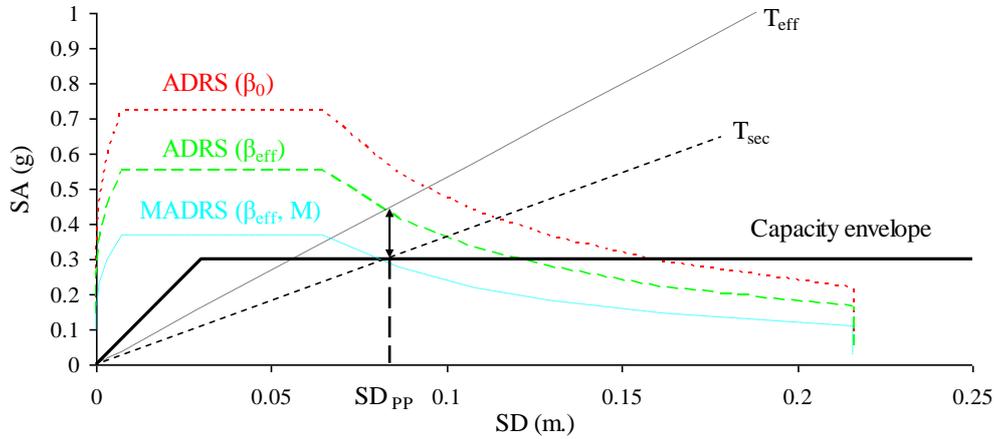


Fig. 4 MADRS in FEMA 440 (2005) for use in CSM

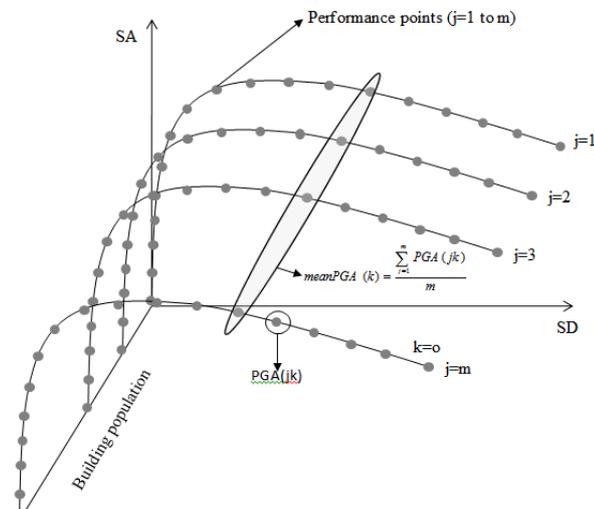


Fig. 5 Schematic representation of the PGA calculation from a PP on capacity spectrum

corresponding PGA. For that purpose, the improved equivalent linearization procedure (i.e. MADRS) proposed in FEMA440 (2005) is used; the main reason for this choice is that the MADRS was proven by Kyriakides (2008) and Kyriakides *et al.* (2012) to provide more accurate estimates of the response of the sub-standard structures than the widely used N2 method (Fajfar and Gašperšič 1996). The MADRS method is based on a 2-stage reduction of the elastic spectral acceleration-displacement response spectrum (ADRS). At the first stage, the reduction is computed using an effective damping (β_{eff}) value due to damage; whereas, at the second stage, the ADRS is further reduced to account for the ductility of the structure.

The assumption of multiple performance points (SA_i, SD_i) and equivalent systems on the capacity curve is used to evaluate T_{eff} and β_{eff} corresponding to each PP (step 11 in Fig. 1).

Subsequently, β_{eff} is substituted in the elastic spectrum equation to calculate the reduced acceleration-displacement response spectrum (ADRS) with increased damping (β_{eff}). To finalise the reduction of the spectrum, each ordinate of SA_i on the ADRS is multiplied by a factor M to generate the modified ADRS (MADRS) spectrum. Factor M corresponds to the difference in ductility between the nonlinear (T_{sec}) and “equivalent” linear (T_{eff}) SDOF systems. The derivation of the MADRS spectrum is shown graphically in Fig. 4.

For the Eurocode 8 (EC8 2004) response spectrum, the MADRS spectrum is used to estimate the PGA at each PP (Fig. 5) using the elastic response spectrum equation in the code, depending on the spectrum branch which intersects the PP. For a PP that intersects the third branch of the spectrum ($T_C < T < T_D$) and for a reduction factor M to account for the transformation to MADRS, equation 4 is used to calculate the PGA at a specific PP. In Figure 5 a number of capacity curves j for different buildings are shown with the corresponding PP's on each curve ranging from $k = 1$ to O .

$$PGA_{jk} = \frac{SA_{jk} \cdot T_{\text{initial}_k}}{\alpha \cdot S \cdot M_{jk} \cdot \eta_{jk} \cdot T_c} \quad (4)$$

where;

- j = building population
- k = a performance point on capacity curve
- SA = spectral acceleration
- T_{ini} = initial period of the equivalent system
- T_c = site characteristic period
- α = spectral amplification coefficient
- S = soil factor
- η = reduction factor
- M = modification factor

The advantage of using back analysis for PGA calculation is that the demand PGA can be defined for each SD step without the need of repeating the static analysis. This is quicker and much simpler and requires fewer assumptions to be made than using time history analysis, where a lot of artificial or natural ground motion record sets (corresponding to different PGA levels) are required to cover the entire spectrum of SDs. It should be noted though that using a static method, the uncertainty in demand is not properly addressed when compared to the 'exact' solution from time-history analysis, since the demand spectrum used (ADRS) represents the mean spectrum of an area. The variability in the spectrum should also be accounted and added to the general variability arising from the probabilistic study. The authors are currently investigating ways to address the uncertainty in demand, such as with the use of derived mean spectra from a wide range of acceleration records.

3.4 Damage Index

The proposed framework uses a newly damage index developed by Kyriakides (2008), based on the increase in period due to increased displacement demand (increased damage); and it assumes the fundamental mode to be the predominant mode for structures having small inelastic deformation. In mathematical terms, the definition of damage (D) is based on the ratio between T_{initial} and the secant period corresponding to each point on the capacity curve (T_{sec}) as shown in Eq. 5.

$$D = \frac{T_{sec}}{T_{initial}} - 1 = \frac{T_{sec} - T_{initial}}{T_{initial}} \quad (5)$$

where; ' $T_{initial}$ ' is the period of the structure with no damage and is regarded as the initial threshold value. Whereas ' T_{sec} ' is the secant period at each PP, which can be evaluated from the capacity curve using Eq. 6.

$$T_{sec} = 2\pi \sqrt{\frac{SD_i}{SA_i}} \quad (6)$$

where;

SD_i = spectral displacement at PP 'i' on capacity curve

SA_i = spectral acceleration at PP 'i' on capacity curve

An additional threshold value is considered which corresponds to the collapse of the structure and is referred to as $T_{failure}$ (DI=100%). Equation 7 gives the final damage index (DI) including both threshold values.

$$DI = \frac{T_{sec} - T_{initial}}{T_{failure} - T_{initial}} \quad (7)$$

The DI in Eq. 7 is evaluated at each PP and is able to capture the severe increase in damage in degrading systems through the abrupt change in period.

The threshold value for period at the assumed collapse ($T_{failure}$) of the structure can be related to inter-storey drift (ISD) or spectral displacement (SD) or even loss of strength values. Displacement-based collapse values were proposed in a number of vulnerability studies, such as in HAZUS (1999) and Rossetto and Elnashai (2003). The failure line corresponding to $T_{failure}$ is defined by the radial line corresponding to the limit value of SD (Fig. 6). Any building with capacity envelope crossing this failure line is regarded as collapsed. The threshold values for $T_{failure}$ used in the framework are discussed in section 4.3.1.

In order to use the adopted DI for the scope of this work, its predictions should be correlated to the Mean Damage Ratio's (MDR), since this is the most widely used economic damage indicator. MDR represents the ratio of the repair to the replacement cost, i.e. the fraction of cost that has to be invested for repair in comparison the cost to replace a structure. Thus, the shape of the function relating DI with MDR needs to be defined using Equation 8.

$$MDR = f(DI) \quad (8)$$

For that purpose, empirical results of damage at various PGA levels, observed in Cyprus and Pakistan, are used as reference (Kyriakides 2008, A.N.ASSOCIATES 2006a,b). A database was created from the recording of recent earthquakes in these areas, including the post-earthquake repair cost, the area of the buildings and the level of damage based on the judgment of the local engineers conducting the post-earthquake evaluation for the local authorities. Guidelines for the definition of the level of damage were provided and were based on the European Macroseismic Scale (EMS 1998). The ratio of repair to replacement was determined representing the MDR and was compared to the anticipated damage grade. It was observed that a linear increase in the level of damage caused an exponential increase in the MDR. Since the adopted DI increases exponentially with an increase in the level of damage (due to the exponential increase in period caused by the nonlinear behaviour of the structures), it is decided to assume that the DI is linearly correlated to the MDR with a correlation coefficient equal to 1. This assumption requires further justification using more empirical data, which is currently understudy.

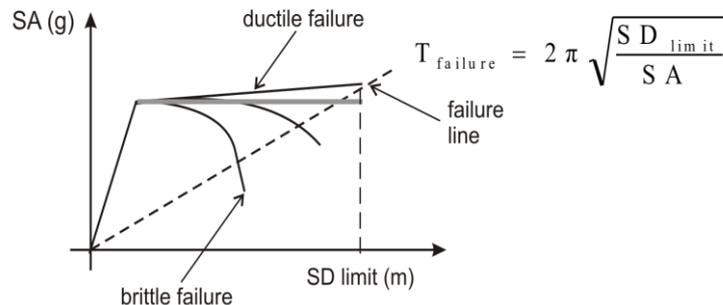


Fig. 6 Definition of the failure line

3.5 Variability of capacity parameters for analytical vulnerability curves

To study the uncertainty of the analytical vulnerability curves, the randomness of the various key parameters (involved in the calibration of capacity models) needs to be accounted for. Therefore, it is essential to account for variations in design and detailing of buildings of the same type and construction period, since the randomness of these parameters can cause significant variability in the vulnerability curves.

The main source of variability comes from parameters involved in the calibration of the capacity models of the main RC structural members (beams and columns); the main capacity models influencing the structural response of the members are the flexural, shear and bond models. The key parameters, involved in the calibration of the capacity models, can be divided into three broad categories, as outlined in Table 2: strength-related, geometrical and design parameters. The strength-related parameters are regarded as the key parameters for the generation of probabilistic vulnerability curves.

The variability of strength-related parameters can be evaluated using either expert judgement, code provisions or by using available statistics to define the probability density function (PDF) of each parameter. In this framework, the statistical approach is followed; the PDFs used for the strength of materials, the development length and the transverse reinforcement spacing are given in Table 3.

In this study, the uncertainty of the vulnerability curves was accounted by generating simulation frames, where the values of the key parameters were pseudo-randomly generated (i.e. steps 1 to 6 in Fig. 1). The variance reduction technique Latin Hypercube Sampling (LHS) (McKay *et al.* 1979) was utilised for the generation of the random values, since it uses a stratified technique and this can help reduce the number of required simulations. To find the optimum number of simulations, a study was conducted and 25, 50 and 75 simulations were found to lead to convergence, whereas, 10 simulations showed significant variation. Rossetto and Elnashai (2005), Kyriakides (2008) and Kyriakides *et al.* (2012) used successfully 25 simulations in their studies, but due to the inclusion of additional uncertainty parameters in the current study (such as the spacing of shear reinforcement and lap length), 50 simulations are recommended for each building.

Table 2 Calibration parameters for capacity models

Capacity model	Key parameters	Deterministic parameters	Design parameters
Flexure:	f'_c, f_y	$b, d, k=f_{ult}/f_y, \epsilon_{su}$	ρ
Shear:	f'_c, s	b, d, f_{yw}	A_{sw}, s
Bond:	f_{ct}, s, l, c		s, l, d_b

Where:

f'_c = concrete compressive strength f_y = steel yield strength f_{ult} = steel ultimate strength s = shear link spacing f_{ct} = concrete tensile strength l = anchorage length c = concrete cover	d_b = longitudinal bar diameter d_{bw} = shear link bar diameter f_{yw} = shear link yield strength b, d = section dimensions ρ = longitudinal reinforcement ratio ϵ_{su} = strain in steel at ultimate steel stress
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Table 3 Probabilistic data for key parameters used in the framework

Parameter	Probability density function	Design value	Mean (μ)	S.deviation (σ)	Minimum value	Maximum value
Concrete compressive strength f'_c (MPa)	Log-normal	21	8	11	$\mu - 0.3\sigma$	$\mu + 1.5\sigma$
	Log-normal	21	18	7	$\mu - \sigma$	$\mu + 3\sigma$
	Log-normal	28	20	10	$\mu - \sigma$	$\mu + \sigma$
	Normal	34	28	9	$\mu - \sigma$	$\mu + \sigma$
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	Log-normal	28	20	10	$\mu - \sigma$	$\mu + \sigma$
	Normal	34	28	9	$\mu - \sigma$	$\mu + \sigma$

4. Application of the framework

The remaining of the paper focuses on the application of the framework on 2 case-study RC building frames. The selected frames along with the material strengths and capacity models for the probabilistic analysis were chosen to be typical of the construction and design practise found in developing countries at the last part of the previous century before modern seismic design codes were enforced legally. Further details on the building types are given in section 4.1.

To demonstrate the framework, DRAIN 3DX (Prakash *et al.* 1994) was selected as the analytical tool due to the availability of suitable inelastic elements for brittle structures in its library. Appropriate material models are used, as described in section 4.2, enabling the more accurate prediction of the degrading behaviour of some of the selected buildings.

4.1 Building selection for vulnerability assessment

A very large array of low rise (LR) RC frames over different construction and design periods (CDP) can be examined. To illustrate the proposed vulnerability framework, seismic vulnerability curves are generated for two CDP (pre-seismic and 1st generation design codes). For the LR category, a typical 2 storey 2 bay structure typically used for commercial or residential purposes in both urban and rural areas is chosen, as shown in Fig. 7. There can be a number of different design sub-categories and configurations, but three typical design sub-categories and a bare frame with a regular configuration are chosen as an example. These design sub-categories were chosen to reflect the variation in section geometry of beams and columns, which may lead to the weak column and strong beams effect.

A random building population was generated using the PDF parameters for the key (strength-related) capacity parameters through LHS. Due to the random assignment of the capacity parameters, each sub design category may exhibit either non-ductile (e.g. bond or shear failure) or ductile failure mode and, thus, the derived vulnerability curves cover the whole spectrum of such buildings in many developing countries.

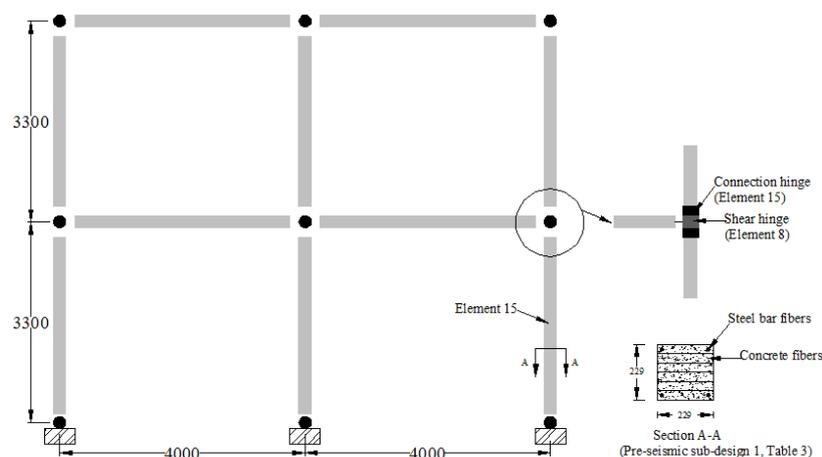


Fig. 7 LR building for vulnerability assessment (two storey two bay regular bare frame)

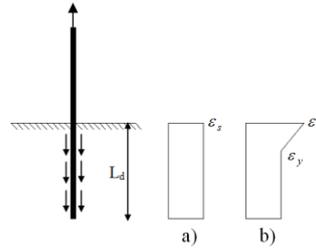


Fig. 8 Strain distributions for different conditions a) up to yield b) post yield case

4.2 Material and capacity model for analysis

Modelling of frame members in DRAIN 3DX was carried out using the fiber element (element 15). The calibration of the material model for concrete was conducted using a low strength concrete (LSC) $\sigma - \epsilon$ material model developed by Ahmad (2011) by defining five $\sigma - \epsilon$ points. To incorporate the effect of bond-slip ($\tau - s$) behaviour in the analysis, connection hinges (element 15) were also used at the joints. The hinges comprising of fibres were used to model both the pullout and gap effects. These hinges were located at the element ends, where the steel is replaced by pullout fibres and the concrete by gap fibres. To model the characteristics of a pullout hinge, a monotonic tri-linear envelope is used and a stress-displacement envelope should be defined both in tension and compression. More details on the modelling aspects of this work can be found in Kyriakides *et al.* (2012) and Kyriakides (2008).

The LSC bond strength model developed by Ahmad (2011) and a suitable assumption for the strain distribution were used to model the $\tau - s$ behaviour. To evaluate the initial bond stiffness, a uniform strain distribution (Fig. 8) was assumed over the embedment length and Equation 9 was used to define the elastic slip. For post yield, an initial linear strain distribution is assumed as shown in Fig. 8b and Equation 10 was used to evaluate the plastic slip.

$$\text{When } f_s \leq f_y, \quad Slip = \frac{f_s^2 d_b}{8E_s \tau_e} \quad (9)$$

$$\text{When } f_s > f_y, \quad Slip = \frac{f_y^2 d_b}{8E_s \tau_e} + \frac{(f_s - f_y) f_y d_b}{4\tau_y E_s} + \frac{(f_s - f_y)^2 d_b}{8\tau_y E_h} \quad (10)$$

where: τ_e = elastic bond strength, τ_y = yielded bond strength, d_b = steel bar diameter, f_s = bar stress, f_y = yield strength of bar, E_h = steel hardening modulus, E_s = steel modulus of elasticity

The choice of appropriate shear capacity model is very important for the assessment of such RC frames. The Sezen and Moehle (2004) model with full concrete contribution and without the k factor was used (equation 11). Element 8 of DRAIN 3DX was used to model the shear behaviour of the joint.

$$V_n = V_s + V_c = k \left(\frac{A_v}{s} f_y d \right) + k \left(\frac{0.5\sqrt{f_c}}{a/d} \sqrt{1 + \frac{P}{0.5\sqrt{f_c} A_g}} \right) 0.8A_g \quad (11)$$

where;

P = axial force

k = ductility related strength degradation value

a/d = aspect ratio

A_g = cross-sectional gross area

A_v = shear link area

f_y = yield strength of steel

f_c = concrete compressive strength

The value of ' k ' between 1 and 0.7 is related linearly to ductility levels of 2 to 6. The ductility factor is applied to both the steel and concrete contribution, since the contribution of concrete or steel to shear degradation is assumed to be equally significant.

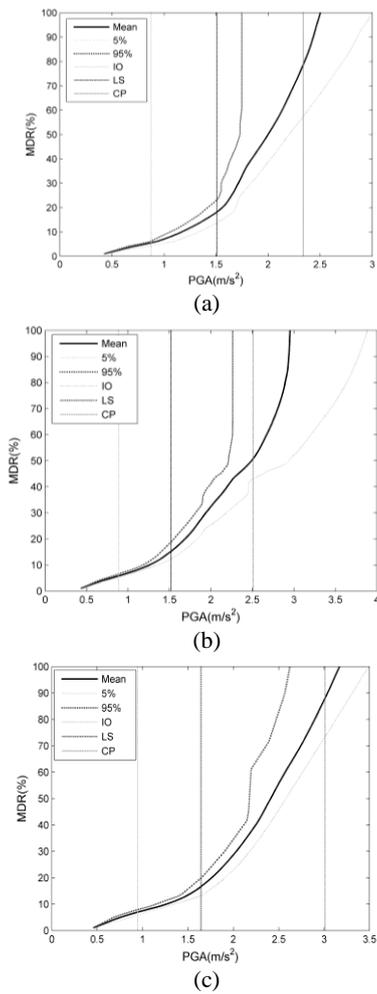


Fig. 9 Pre-seismic LR RC structure vulnerability curves: (a) sub-design 1, (b) sub-design 2, (c) sub-design 3

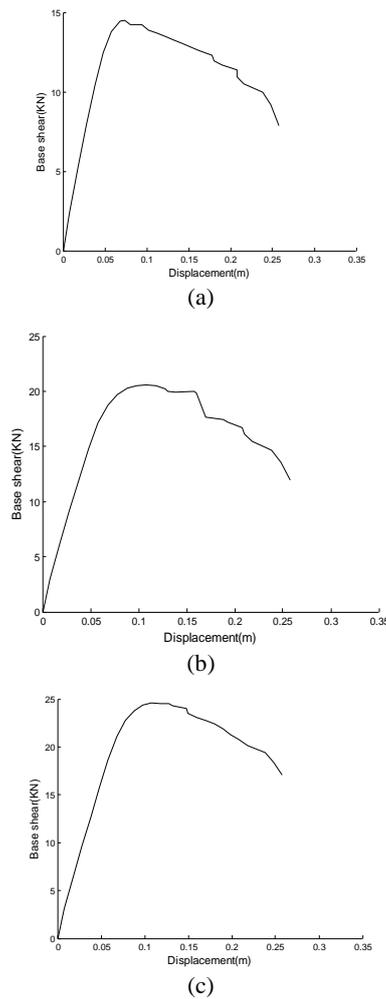
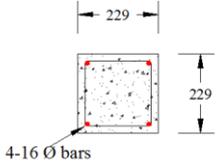
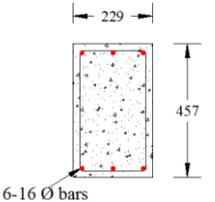
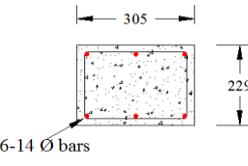
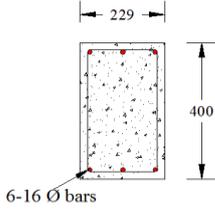
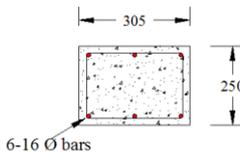
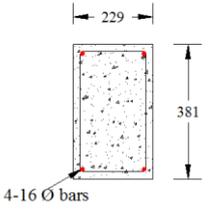
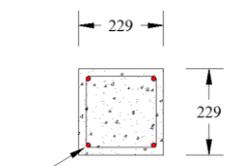
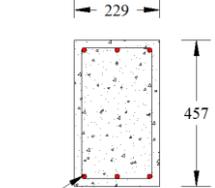
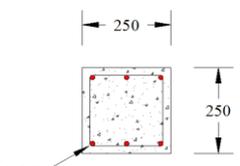
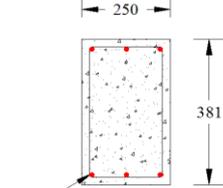
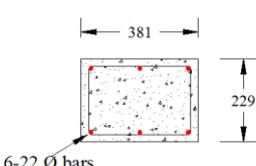
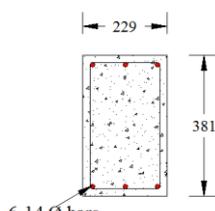


Fig. 10 Examples of pre-seismic analytical capacity curves of selected simulation frames: (a) sub-design 1, (b) sub-design 2, (c) sub-design 3

Table 4 Beam and column design for pre-seismic and 1st generation design period

Sr. No.		Pre-seismic	
		Column	Beam
1	Sub-design 1		
2	Sub-design 2		
3	Sub-design 3		
1 st generation			
4	Sub-design 1		
5	Sub-design 2		
6	Sub-design 3		

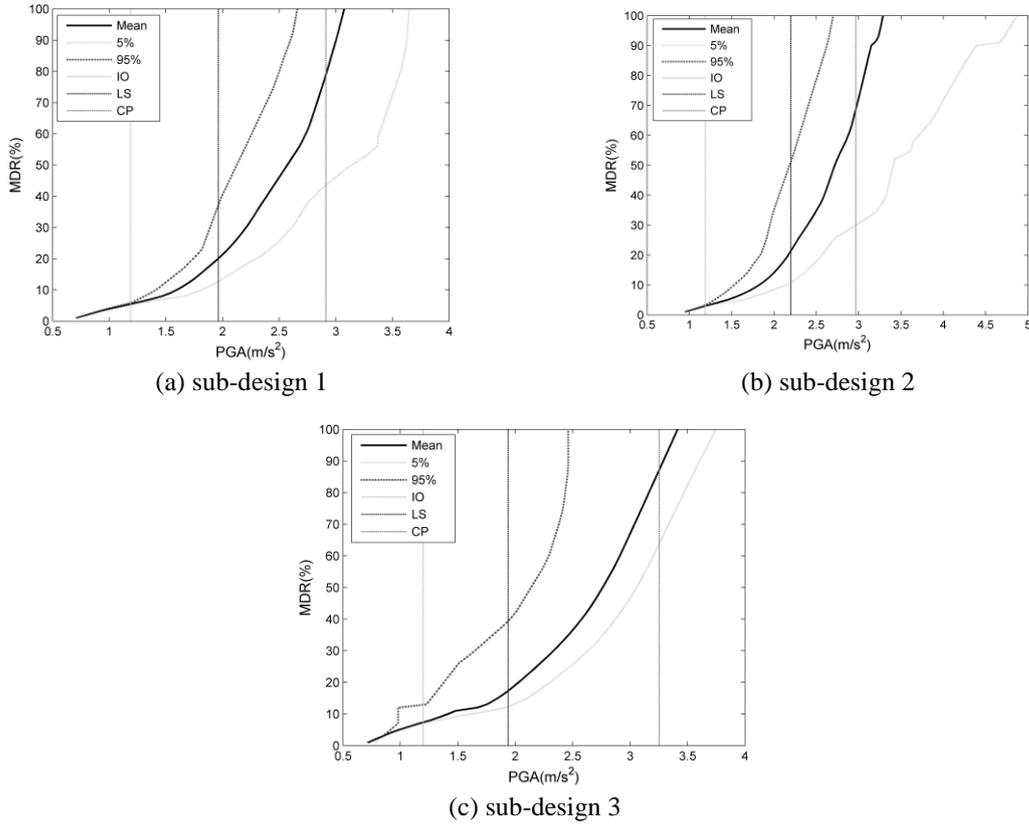


Fig. 11 1st generation LR RC structure vulnerability curves

Table 5 Probabilistic/statistical data for parameter Δ_{cp} in different design periods

Collapse drift, Δ_{cp} (%), normal density function				
	Mean (μ)	Standard deviation (σ)	Minimum value	Maximum value
All CDP (engineered, Bare)	4.28	1.8	$\mu - \sigma$	$\mu + \sigma$

4.3 Probabilistic analytical vulnerability curves

Vulnerability curves were derived as a function of PGA and include mean (50%), 95% and 5% confidence interval curves. For verification purposes, the derived curves are compared with the 3 acceptance criteria defined in FEMA356 (2000) for immediate occupancy (IO), life safety (LS) and collapse prevention (CP). The mean PGA values (corresponding to mean confidence interval curves) associated with each criterion were determined from the analysis results and were plotted for comparison purposes on the derived vulnerability curves in Figs. 9, 11, 12 and 13.

4.3.1 LR (Pre-seismic)

The vulnerability curves for pre-seismic LR sub-design 1, 2 and 3 are shown in Fig. 9a, 9b and

9c respectively. For these curves, corner periods in accordance with the EC8 type 1 spectrum for stiff site ($T_c=0.5$) were used for the PGA evaluation. The PDF of the key capacity parameters are the same for all the sub-designs of the pre-seismic curves. The difference in design causes slight variation in the curves (Fig. 9). The $T_{failure}$ point of each design category structure was determined based on a bi-criteria limit. The first criterion which was set as the ultimate limit was a drift value of $ISD_{max} = 4.00\%$, which is in agreement with the findings of Rossetto and Elnashai (2003) from their evaluation of empirical vulnerability curves for non-ductile structures, based on empirical data. This value was also treated in a probabilistic manner as shown in Table 5. The second criterion was the amount of degradation in the capacity curve due to loss of strength. A 20% loss of strength was assumed to correspond to the collapse of the structure. As expected, this was the dominant criterion for most of the simulation frames. The cyclic analysis was conducted up to the maximum displacement, as defined by the $ISD = 4.00\%$. If strength loss was observed in the capacity curve, the secant period corresponding to 20% loss was calculated and treated as the $T_{failure}$. Examples of the capacity curves for the 3 sub-designs are shown in Fig. 10a-c. It is clear from these graphs that strength loss was the dominant criterion for the collapse of the simulation frames as anticipated for sub-standard buildings.

The vulnerability curves for sub-design 1 showed early accumulation of damage at low PGA levels as compared to the two other designs. In the first two figs. (Fig. 9a-b) the 95% confidence interval curves reach collapse at low PGA in comparison to the PGA calculated corresponding to FEMA356 (2000) collapse prevention (CP) damage state, which highlights the very brittle nature of these structures with pre-seismic sub-design 1 and 2. Moreover, large variability can be seen in sub-design 1 and 2 curves as compared to sub-design 3, which utilizes larger sections and higher reinforcement ratios leading to structures with predominantly flexural behaviour and thus less variability. The probability of achieving the collapse limit state is also reduced for this design type.

4.3.2 LR (1st generation)

The vulnerability curves for 1st generation LR sub-design 1, 2 and 3 (outlined in Table 4) are shown in Fig. 11a-c, respectively. The curves for the buildings of this CDP were derived using the same spectral corner periods and $T_{failure}$ was defined as for pre-seismic structures. As compared to

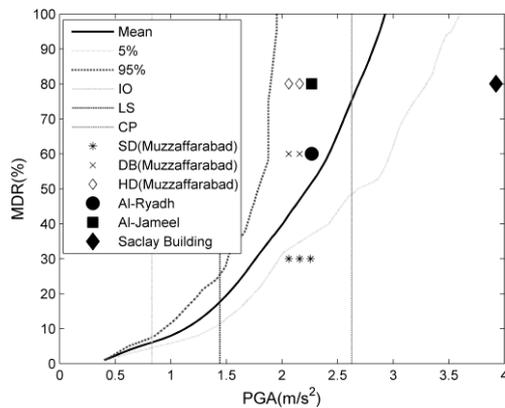


Fig. 12 General curve for pre-seismic LR RC buildings

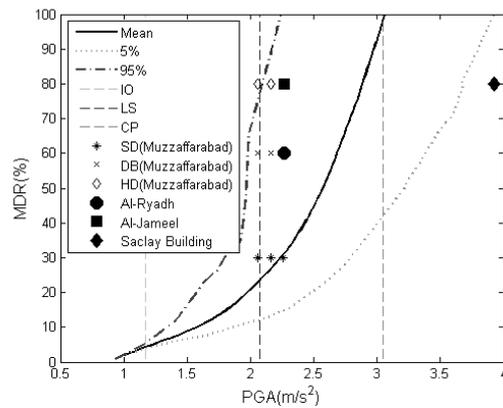


Fig. 13 General curve for 1st generation LR RC buildings

the pre-seismic curves, the 95% confidence interval 1st generation building curves in general show collapse at higher PGA for buildings with larger column sizes and higher reinforcement ratio (1st generation sub-design 3) and are closer to the PGA associated with FEMA356 (2000) collapse damage state. However, there is still high variability due to brittle behaviour in all three curves.

4.3.3 General curves (LR structures)

Finally, general curves are derived for pre-seismic and 1st generation buildings by integrating the outcomes of the three design sub-categories as shown in Fig. 12 and 13. Moreover, empirical damage data obtained from the Kashmir earthquake (UNDP-ERRP 2008, A.N.Associates 2006b, A.N.Associates 2006a) and the “Saclay building” tested on a shaking table (Chaudat *et al.* 2005) are compared with the derived curves. The comparison shows that the buildings from Pakistan lie in the region between mean and 95% confidence interval curves confirming the large damage at small PGA levels. As expected, the shaking table building with better design outperforms both the pre-seismic general curves and 1st generation structures.

5. Conclusions

A probabilistic analytical seismic vulnerability assessment framework has been proposed for the development of analytical vulnerability curves for substandard RC frame structures typical of the majority of the existing building stock in many South and Eastern European countries but also in developing countries like Pakistan and India. Complex degradation behaviour due to brittle failure modes is accounted for in the modelling stage and a new idealisation procedure is proposed for degrading behaviour.

A modified reverse capacity demand diagram method is used for the determination of the structural response. A period based damage index is proposed based on the principle of change in the frequency of the structure with increased damage.

To illustrate the proposed vulnerability framework, seismic vulnerability curves are generated for LR buildings with no (pre-seismic design codes) and moderate (1st generation design codes) seismic design consideration. The pre-seismic structures vulnerability curves predict early damage accumulation due to brittle failure modes for all the three sub-design categories. Moreover, high variability among 95% confidence interval, 5% confidence interval and mean curves was observed for both CDP buildings. LR buildings with improved seismic design (1st generation) showed more gradual damage accumulation and less variability, due to more flexural failure modes. The severe damage condition according to mean vulnerability curves for both building categories is in general agreement with the PGA associated with the collapse limit state by FEMA356.

To improve further the seismic vulnerability predictions, the limit states of brittle reinforced concrete structures need to be further studied experimentally, using typical materials and construction practices.

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