

## Probabilistic analysis of spectral displacement by NSA and NDA

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**Abstract.** Main objective of the present study is to determine the statistical properties and suitable probability distribution functions of spectral displacements from nonlinear static and nonlinear dynamic analysis within the frame work of Monte Carlo simulation for typical low rise and high rise RC framed buildings located in zone III and zone V and designed as per Indian seismic codes. Probabilistic analysis of spectral displacement is useful for strength assessment and loss estimation. To the author's knowledge, no study is reported in literature on comparison of spectral displacement including the uncertainties in capacity and demand in Indian context. In the present study, uncertainties in capacity of the building is modeled by choosing cross sectional dimensions of beams and columns, density and compressive strength of concrete, yield strength and elastic modulus of steel and, live load as random variables. Uncertainty in demand is modeled by choosing peak ground acceleration (PGA) as a random variable. Nonlinear static analysis (NSA) and nonlinear dynamic analysis (NDA) are carried out for typical low rise and high rise reinforced concrete framed buildings using IDARC 2D computer program with the random sample input parameters. Statistical properties are obtained for spectral displacements corresponding to performance point from NSA and maximum absolute roof displacement from NDA and suitable probability distribution functions viz., normal, Weibull, lognormal are examined for goodness-of-fit. From the hypothesis test for goodness-of-fit, lognormal function is found to be suitable to represent the statistical variation of spectral displacement obtained from NSA and NDA.

**Keywords:** nonlinear static analysis; nonlinear dynamic analysis; uncertainty modeling; statistical probability distribution functions; RC framed buildings

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### 1. Introduction

Over the past two decades structural engineering community is in the process of developing a new generation of design and rehabilitation procedures that will incorporate performance based engineering (PBE) concepts (ATC40 1996, FEMA273 1997). Combining the performance evaluation procedure with uncertainties in demand and capacity, Hazard United States Loss estimation program designated as HAZUS (Kircher *et al.* 1997, FEMA 2003) has been developed. Capacity spectrum method is adopted in HAZUS for the estimation of peak building response which is obtained as an intersection of median capacity curve and median demand curve. HAZUS (FEMA 2003) program has defined thirty six model building types and their capacity parameters for the development of capacity curves and for the development of fragility curves using capacity spectrum method. Depending on the seismic design level and approximate age of construction,

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four seismic design levels are provided to estimate the capacity of building viz., High-code (high seismic zone), Moderate code (moderate seismic zone), Low code (low seismic zone) and Pre code (buildings built before 1940 in California/before seismic codes are used for building design). No study has been reported in literature on variations in capacity parameters and spectral displacement for chosen building type and hazard level for Indian context. Any effort towards characterisation of capacity parameters in terms of yield and ultimate base shear and spectral displacement at yield and ultimate point for specific class of building is useful for vulnerability assessment and loss estimation of a city (Kamatchi *et al.* 2011). In the present study, an effort has been made to carry out nonlinear static and nonlinear dynamic analyses and determine the statistical properties in terms of mean and coefficient of variation (COV) of spectral displacements at the top of building for typical low rise and high rise buildings designed for zone III and zone V.

Nonlinear static analysis (NSA) procedures play a major role and capacity spectrum method is widely adopted for determination of inelastic demand in terms of spectral displacement and spectral acceleration in PBE. NSA procedures are preferred as an alternative to detailed nonlinear dynamic analysis (NDA) procedure in performance based engineering due to the complexities in nonlinear dynamic material modelling and unavailability of suitable acceleration time histories.

Studies on probabilistic models and analyses of bridges are reported in literature (Gardoni and Terjo 2013, Han and Bang 2012). Number of studies wherein the target spectral displacements are obtained by nonlinear static analysis and comparison has been made with the ones obtained through nonlinear dynamic analysis are reported in literature (Magliulo *et al.* 2007, Peter and Badoux 2000, Causevic and Mitrovic 2010, Mahdi and Gharaieb 2011, Moshref and Tehranizadeh 2011). Causevic and Mitrovic (2010) have discussed the recent developments in seismic analysis methods for the determination of target spectral displacements viz., nonlinear time history analysis procedure, nonlinear static procedure (FEMA 356, 2000), N2 nonlinear static method (Eurocode 8, 2004) and improved capacity spectrum method (FEMA 440, 2005). Causevic and Mitrovic (2010) have observed that maximum top displacement of the structure obtained by dynamic analysis with real time-history records (envelope) corresponds to 145% of the target displacement obtained using the non-linear static N2 method. Peter and Badoux (2000) have used capacity spectrum method for the determination of spectral target displacement for reinforced concrete (RC) framed building and obtained comparable results with nonlinear dynamic analysis for acceleration time histories compatible with Eurocode 8 (2004). Mahdi and Gharaieb (2011) have studied seismic behaviour of different classes of intermediate moment-resisting concrete space framed irregular buildings designed as per Iranian seismic code and stated that pushover analyses procedure needs further refinements and nonlinear time history analyses with suitable acceleration time histories may have to be carried out for torsionally stiff buildings. Moshref and Tehranizadeh (2011) have stated that force controlled procedure rather than displacement based procedure suggested by New Zealand code gives comparable results with nonlinear dynamic analysis.

Capacity and demand are the two important quantities in PBE and no study is reported in literature on the comparison of spectral displacement including the uncertainties in capacity and demand in Indian context. In the present study, uncertainties in capacity of the building is modeled by choosing cross sectional dimensions of beams and columns, density and compressive strength of concrete, yield strength and elastic modulus of steel, and live load as random variables. Uncertainty in demand is modeled by choosing peak ground acceleration as a random variable. In NSA, an ensemble of demand curves are generated using PGA as a random variable. Spectrum compatible time histories with PGA as random variable are used for NDA.

Typical three storey and ten storey reinforced concrete framed buildings designed as per Indian seismic code of practice IS 1893-2002 (Part 1) located in seismic zone V with maximum considered earthquake PGA of 0.36g and design basis earthquake PGA of 0.18g (representing severe earthquake) and seismic zone III with maximum considered earthquake PGA of 0.16g and design basis earthquake PGA of 0.08g (representing moderate earthquake) are chosen for the present study. NSA and NDA are carried out using IDARC 2D computer program (Valles *et al.* 1996) with the random sample input parameters. In NSA, capacity spectrum method is used to determine the spectral displacement with the demand spectra corresponding to design basis earthquake of Indian seismic code. Statistical properties are obtained for spectral displacements from NSA and suitability of probability distribution functions viz., normal, Weibull, lognormal are examined for goodness-of-fit. Similarly, the statistical properties are obtained for the spectral displacement corresponding to maximum roof displacement from nonlinear dynamic analyses with spectrum compatible time histories, and suitable probability distribution function is identified. The methodology adopted in this paper for arriving at the suitable probability distribution function for  $S_d$  using NSA and NDA is illustrated in Figs. 1 and 2 respectively.

## 2. Random variables for probabilistic seismic analysis

In the present study, uncertainties involved in various identified input parameters are modeled for nonlinear static and dynamic analysis. Cross sectional dimensions of beams and columns, density and compressive strength of concrete, yield strength and elastic modulus of steel, live load and PGA are chosen as random variables. Based on the recommendations given in literature on probabilistic models for random variables (Ranganathan 1990, Thomos and Trezos 2006, Der Kirureghian and Ang 1977) parameters are chosen for random variables as detailed below.

### 2.1 Cross sectional dimensions of beams and columns

As a result of variations in size, shape and the quality of formwork, concreting and vibrating operations during construction deviations in RCC member dimensions can be observed. The difference between the nominal and actual dimensions are best characterized by the mean deviations and standard deviation of deviations for different size ranges. In the present study mean deviations and standard deviation of deviations proposed by Ranganathan (1990) has been adopted as given in Table 1.

### 2.2 Density and Compressive strength of concrete

Variations in strength and density of concrete is observed in field due to variations in material quality, placing, supervision, weighing, mixing, curing and difference in actual strength of concrete in a structure compared to control specimens viz., cube or cylinder. Mean, COV and the type of probability distribution function proposed by Ranganathan, (1990) for Density and Compressive strength of concrete has been adopted in the present study as given in Table 1.

### 2.3 Yield strength and elastic modulus of steel

Since the steel bars are factory produced, the sizes of bars cannot vary significantly. Hence the

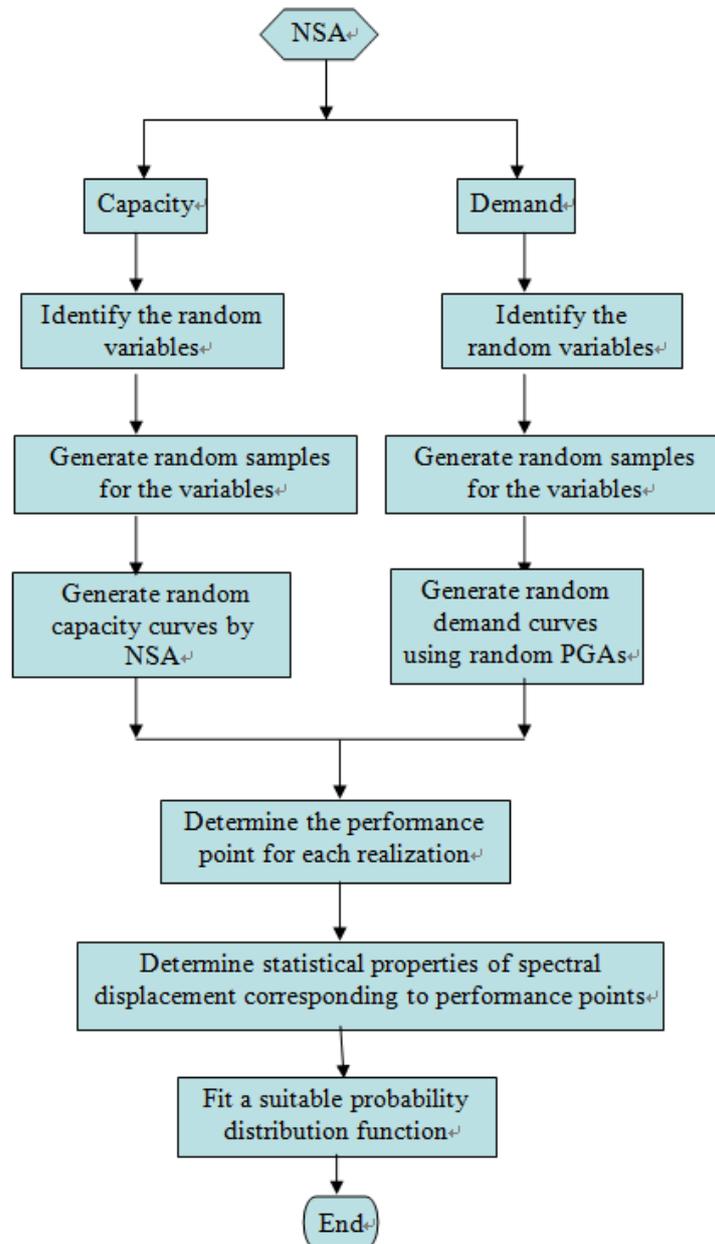


Fig. 1 Procedure adopted for getting the suitable probability distribution function for  $S_d$  through NSA

size of steel bar is not considered as random variable in the present investigation. The yield strength and the modulus of elasticity are the two main physical properties of steel that have been considered as random variables in the present study. Mean, COV and the type of probability distribution function for yield strength and modulus of elasticity of steel proposed by Ranganathan (1990) obtained based on the statistical analyses, has been adopted in the present study (see Table 1.).

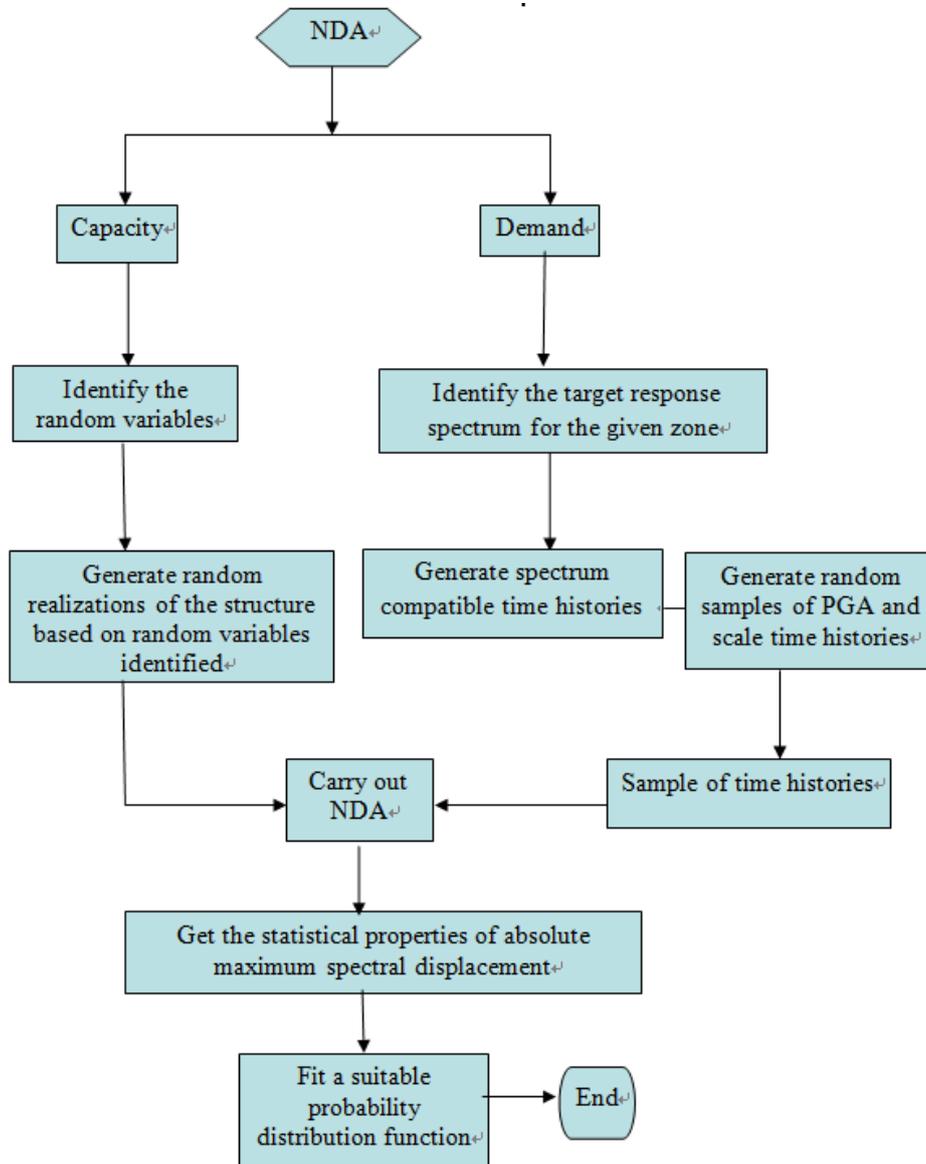


Fig. 2 Procedure adopted for getting the suitable probability distribution function for  $S_d$  through NDA

### 2.4 Live load

Variations in dead load is accounted by variations in density and dimensions of beams and columns. Weights of furniture and other heavy equipment constitute sustained part of live load and short duration loads viz., weight of people gathered in a place and all other short duration loads constitute intermittent part of live load. Gamma distribution for sustained load and exponential distribution for intermittent load proposed by Thomos and Trezos (2006) and Kirupakara *et al.* (2010) with mean and COV as given in Table 1 are adopted in the present study.

### 2.5 Peak Ground Acceleration (PGA)

PGA has been chosen as random variable for the generation of demand curve in NSA and in time history simulation for NDA, however, the design of the building has not been carried out for each simulation of PGA. The PGA corresponding to the design spectra as per IS 1893(Part 1)-2002 for Type I soil, for seismic zone V (0.18g) and seismic zone III (0.08g) are assumed as mean values for the generation of random samples. The COV for PGA has been chosen as 0.6 as specified in literature (Der Kirureghian and Ang 1977).

Random samples are generated with the respective mean, COV and the type of probability distribution function as given in Table 1 using Monte Carlo simulation. Using the random samples generated, one hundred NSA and NDA analyses are carried out for RC framed buildings using IDARC2D (Valles *et al.* 1996) computer program.

### 3. Description of RC framed buildings

Three storey building designed using IS 456:2000 and IS 1893(Part 1):2002 for seismic zone V is designated with caption B3Z5 and three storey building designed for seismic zone III is designated with caption B3Z3. Similarly, ten storey building designed for seismic zone V is designated with caption B10Z5 and ten storey building designed for seismic zone III is designated with caption B10Z3. Plan and elevation of the three storey RC framed buildings adopted in the present study is shown in Fig. 3. Table 2 gives the cross section details (Devandiran 2011) of B3Z5 and B3Z3. Plan and elevation and the cross section details of the ten storey building B10Z5 and B10Z3 are shown in Fig. 4 and Table 3 respectively.

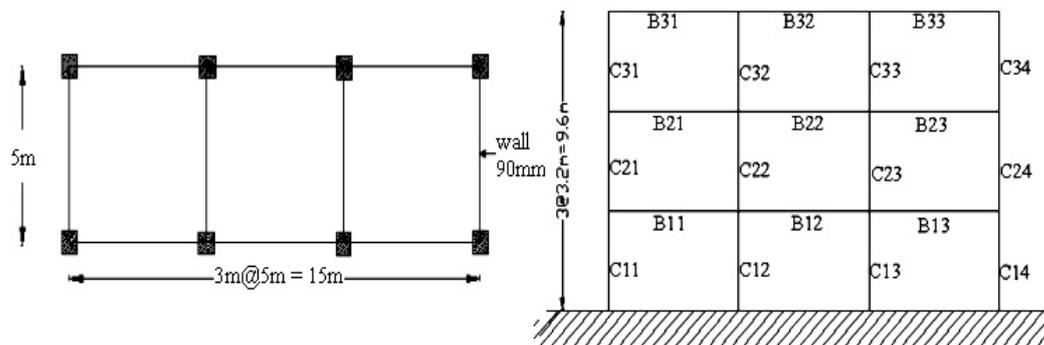


Fig. 3 Plan and elevation of B3Z5 and B3Z3

### 4. Nonlinear static analysis

#### 4.1 Capacity curves and capacity spectra

Since the deformation profile of the building under earthquake excitation which lasts only for a short duration is not known initially, force controlled analyses has been carried out. The nonlinear

Table 1 Parameters and distributions used for generation of random variables

Random variable		Probability distribution function	Mean	COV
Density of concrete		Lognormal	25 kN/m <sup>3</sup>	0.028
Compressive strength of concrete		Normal	20.29N/mm <sup>2</sup>	0.15
Yield strength of steel		Normal	468.9 N/mm <sup>2</sup>	0.073
Modulus of Elasticity of steel		Lognormal	2.041e5 N/mm <sup>2</sup>	0.076
Peak Ground Accelerations		Lognormal	0.08/0.16g	0.6
			Mean	Standard deviation
Live load		Gamma	0.3 kN/m <sup>2</sup>	0.34 kN/m <sup>2</sup>
		Exponential	0.3 kN/m <sup>2</sup>	
			Mean deviation	Standard deviation of deviation
Beam	Width	Normal	10.29(mm)	9.47(mm)
	Depth	Normal	14.37(mm)	9.38(mm)
Column	Width	Normal	-0.25(mm)	5.69(mm)
	Depth	Normal	0.11(mm)	7.89(mm)

Table 2 Design details B3Z5 and B3Z3

Details	B3Z5	B3Z3
Beam	0.3mx0.4m	0.25mx0.30m
Column	0.3mx0.5m	0.25mx0.35m
Floor slab thickness		0.13 m
Wall thickness		0.20 m
Storey height		3.20 m
Live load		2.3kN/m <sup>2</sup>
Earthquake load	As per IS-1893(Part 1)-2002	

Table 3 Design details B10Z5 and B10Z3

Details	B10Z5	B10Z3
Beam	All beams 1-6 stories	0.3mx0.7m
	7-10 stories	0.3mx0.5m
Column	1-3 stories inner columns	0.35mx0.6m
	Outer columns 1-3 stories, all columns 4-10 stories	0.35mx0.5m
		1-6 stories Inner columns 0.3mx0.6m
		Outer columns 1-6 stories, all columns 7-10 stories 0.3mx0.45m
Slab thickness	0.13m	0.13m
Wall thickness	0.20m	0.20m
Storey height	3.00m	3.00m
Live load	2.3 kN/m <sup>2</sup>	2.3 kN/m <sup>2</sup>

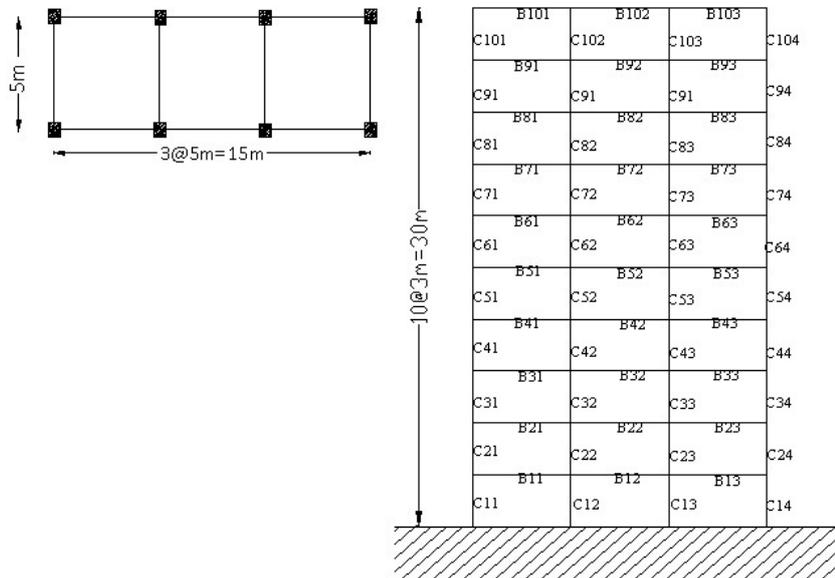


Fig. 4 Plan and elevation of B10Z5 and B10Z3

Table 4 Typical target base shear coefficients and top displacement percentages specified as input and actual output at the end of pushover analysis immediately before formation of collapse mechanism for one simulation\*

Building type	Input		Output at the end of pushover analysis	
	Target base shear coefficient	Target top displacement as a percentage of height	Maximum base shear coefficient	Maximum top displacement as a percentage of height
B10Z5	0.6	12	0.1583	4.669
B3Z5	0.6	12	0.4438	10.875
B10Z3	0.3	12	0.0974	9.889
B3Z3	0.4	17.5	0.2969	18.189

\*The values specified are only indicative. The program IDARC 2D terminates once mechanism forms or any of the target point is reached. Higher values are normally chosen so that structure is pushed till the mechanism forms.

static pushover analysis is carried out using the random samples generated and the ensemble of capacity curves are obtained using IDARC 2D computer program. In IDARC, by default possible plastic hinge locations are assumed to be at the ends of beams and columns of the frame. Different states of damage (collapse points) viz., cracking, yielding and failure of beams and columns are tracked during pushover analysis. Ultimate deformation capacity at the ends of beam and column is computed based on the criteria of reaching either one of the conditions viz., ultimate compressive strain in concrete or ultimate strength of one of the rebar. Pushover analysis will stop after reaching collapse mechanism as per damage states. This can be controlled by specifying target ultimate base shear coefficient (ratio of base shear to seismic weight of the building) or by

target top displacement percentage of total height. Typical target base shear coefficients and top displacement percentages specified as input and the actual output at the end of pushover analysis immediately before the formation of collapse mechanisms for one simulation each are listed in Table 4.

The capacity curve is a plot between base shears and roof displacements and the capacity curves for B3Z5, B3Z5, B10Z5 and B10Z3 are shown in Fig. 5. One hundred analyses have been carried out and from the ensemble of capacity curves, the mean capacity curve and mean ( $\pm$ ) sigma capacity curve for the all the four cases are obtained as shown in Fig. 6.

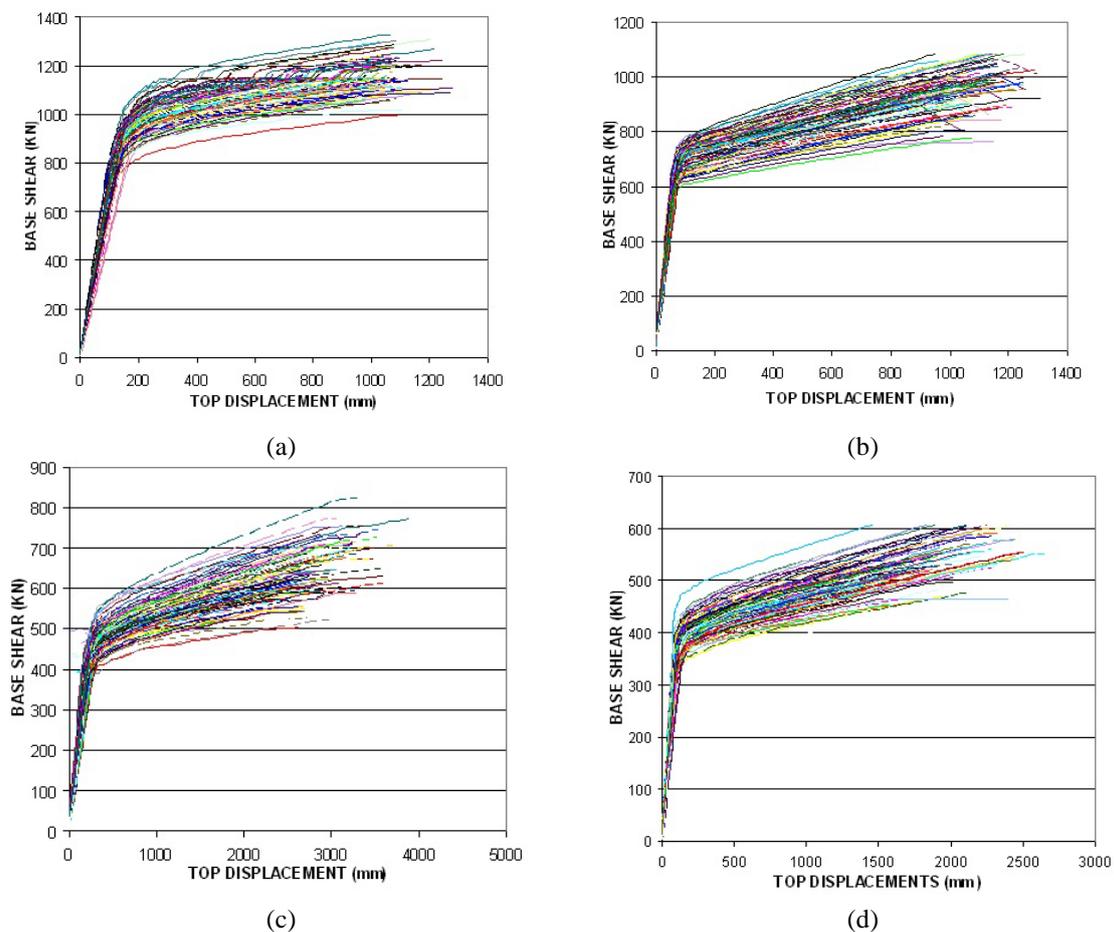


Fig. 5 Capacity curves from NSA

In capacity spectrum method, the capacity curve, which is in terms of base shear and roof displacement format, is converted to capacity spectrum, in Acceleration Displacement Response Spectra (ADRS) format in order to bring capacity and demand in the same platform. Capacity curves are transformed to capacity spectra using the spectral coordinates (ATC 40 1996, Kamatchi *et al.* 2010) corresponding to the first natural mode of the building using Eqs. (1) and (2).

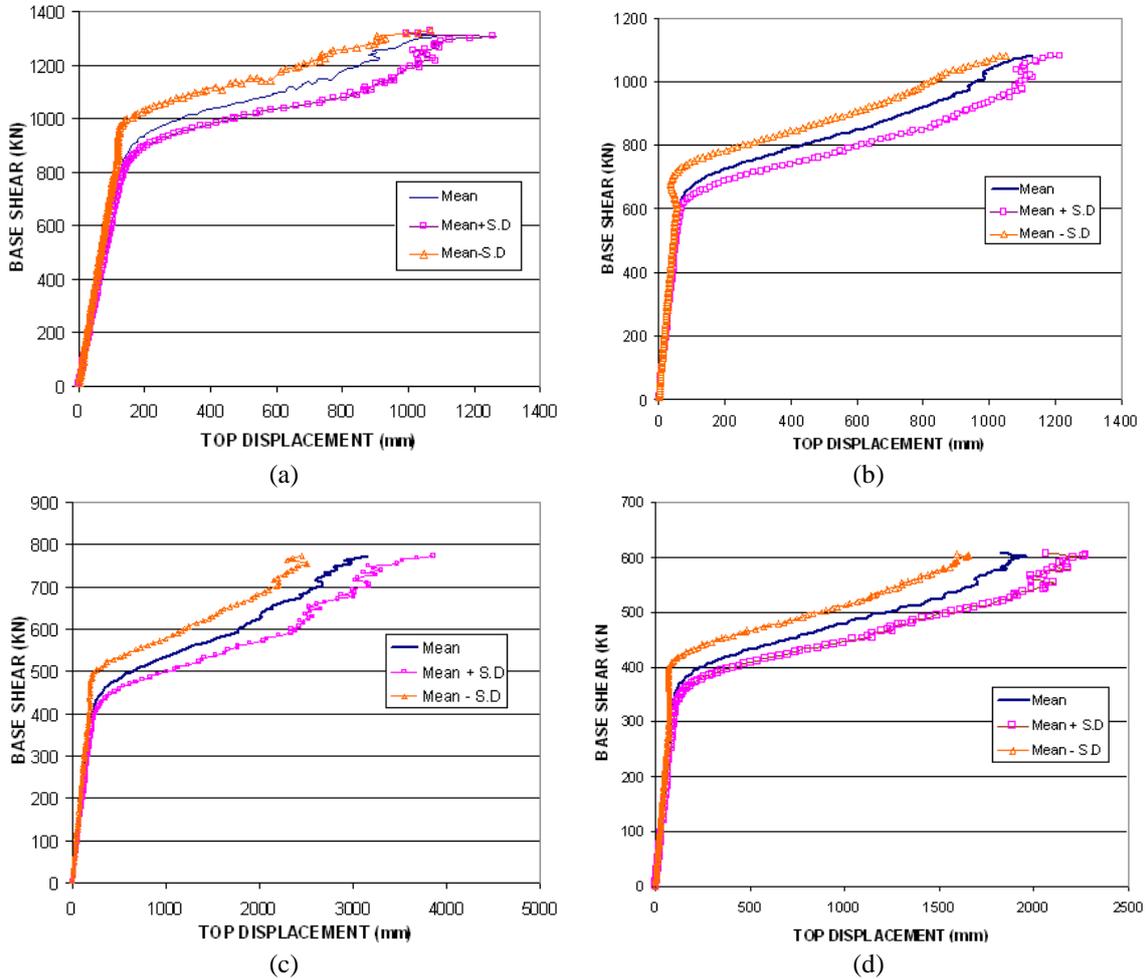


Fig. 6 Mean and standard deviation of the capacity curves

$$S_{aj} = \frac{V_j / W}{\alpha_1} \quad (1)$$

$$S_{dj} = \frac{\Delta_{roof}}{PF_1 \cdot \phi_{1,roof}} \quad (2)$$

where:

$V_j$  = base shear at the  $j^{\text{th}}$  point of the capacity curve

$W$  = weight of the building as sum of dead load and live load

$\alpha_1$  = modal mass coefficient for the first natural mode

$\Delta_{roof}$  = roof displacement

$PF_1$  = modal participation factor for the first natural mode as defined in Eq. (3) (ATC 40, 1996)

$\phi_{1,roof}$  = amplitude of roof in first natural mode as defined in Eq. (4) (ATC 40, 1996)

$S_{dj}$  =spectral displacement for each point on the curve  
 $S_{ai}$  =spectral acceleration for any time period of the building

$$PF_1 = \frac{\left[ \sum_{i=1}^N (w_i \phi_{i1}) / g \right]}{\left[ \sum_{i=1}^N (w_i \phi_{i1}^2) / g \right]} \quad (3)$$

$$\alpha_1 = \frac{\left[ \sum_{i=1}^N (w_i \phi_{i1}) / g \right]^2}{\left[ \sum_{i=1}^N w_i / g \right] \left[ \sum_{i=1}^N (w_i \phi_{i1}^2) / g \right]} \quad (4)$$

where,

$w_i/g$  - mass assigned to level i

$\phi_{i1}$  - amplitude of mode 1 at level i

$N$  -the level which is the uppermost in the main portion of the structure

#### 4.2 Demand curves and demand spectra

Peak ground acceleration (PGA) has been chosen as the random variable for the generation of random values for demand curve as explained in section 2.5. The response spectra generated using the random PGA values that form the samples of demand spectra are converted to ADRS format using Eq. (5).

$$S_{di} = \frac{T_i^2}{4\pi^2} S_{ai} g \quad (5)$$

Where:

$T_i$  = time period of the building in secs

$g$  = acceleration due to gravity

$i = i^{\text{th}}$  point of the spectra

#### 4.3 Capacity spectrum method

In capacity spectrum method the intersection of demand and capacity spectra is defined as the performance point of the building which represents the peak response of the building for the excitation corresponding to the demand curve. Schematic representation of performance points obtained from mean demand curve and mean capacity curve is shown in Fig. 7. The values of  $S_a$  and  $S_d$  corresponding to the performance point from the intersection of values obtained from the mean demand curve and mean capacity curve for all the four cases considered are given in Table 5.

Beams and columns of the chosen buildings are designed for demand spectra corresponds to the PGA of design basis earthquake and the envelope of all load combinations with load partial

safety factors of 1.5 and 1.2 including dead load, live load and earthquake load. Since the performance points lie in the elastic branch of capacity curve, process of reduction of demand spectra as per effective damping (Kamatchi *et al.* 2010) is not found to be required in the present study. Three storey building is rigid and ten storey building is flexible. Since the time period of three storey building is less the design spectral acceleration is more for three storey building compared to ten storey building. Though the absolute value of base capacity is more for ten storey building as shown in Fig. 5, base shear coefficient (base shear normalized by weight of the building) is more for three storey building compared to ten storey building. Demand in terms of spectral acceleration ( $S_a/g$ ) is more and demand in terms of spectral displacement ( $S_d$ ) is less at performance point for three storey building. On the other hand, demand in terms of spectral acceleration ( $S_a/g$ ) is less and demand in terms of spectral displacement is more at performance point for ten storey building.

**4.4 Statistical properties of spectral displacement corresponding to performance point by NSA**

The spectral displacement ( $S_d$ ) is a useful and important response parameter of building which is used for defining damage states in vulnerability and hazard analysis. Earlier in Table 5,  $S_a$  and  $S_d$  corresponding to performance points obtained from the intersection of mean demand and capacity curves for all the four cases are given. However from the intersection of each capacity

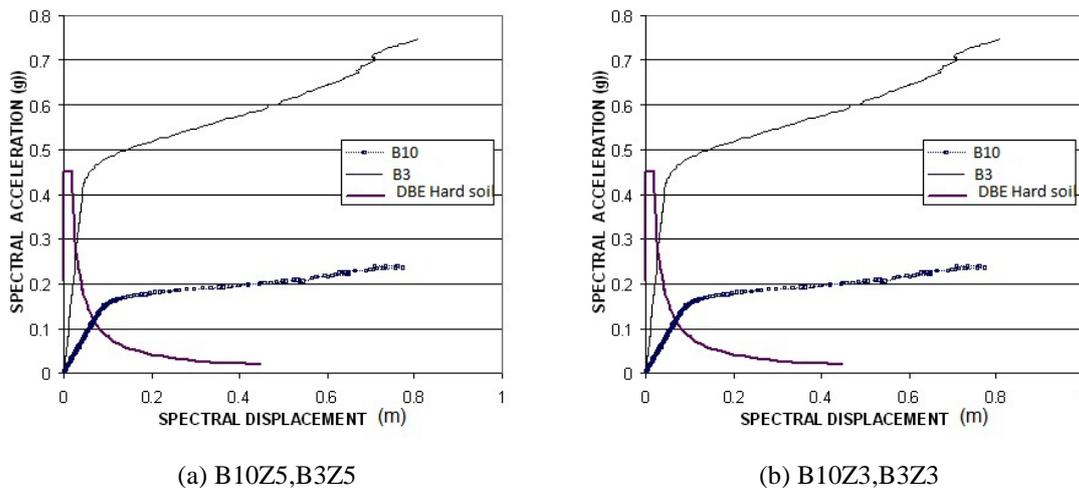


Fig. 7 Performance points using mean demand spectra and mean capacity spectra

Table 5  $S_a$  and  $S_d$  corresponding to performance points

Case	Type	Performance Points	
		$S_a(g)$	$S_d (m)$
1	B10Z5	0.109	0.074
2	B3Z5	0.272	0.029
3	B10Z3	0.026	0.061
4	B3Z3	0.076	0.021

Table 6 Statistical Properties of  $S_d$  from NSA

Case	Type	$S_d$ (m)			
		Minimum	Maximum	Mean	COV
1	B10Z5	0.018	0.404	0.068	0.721
2	B3Z5	0.007	0.154	0.028	0.679
3	B10Z3	0.010	0.245	0.052	0.673
4	B3Z3	0.004	0.092	0.019	0.684

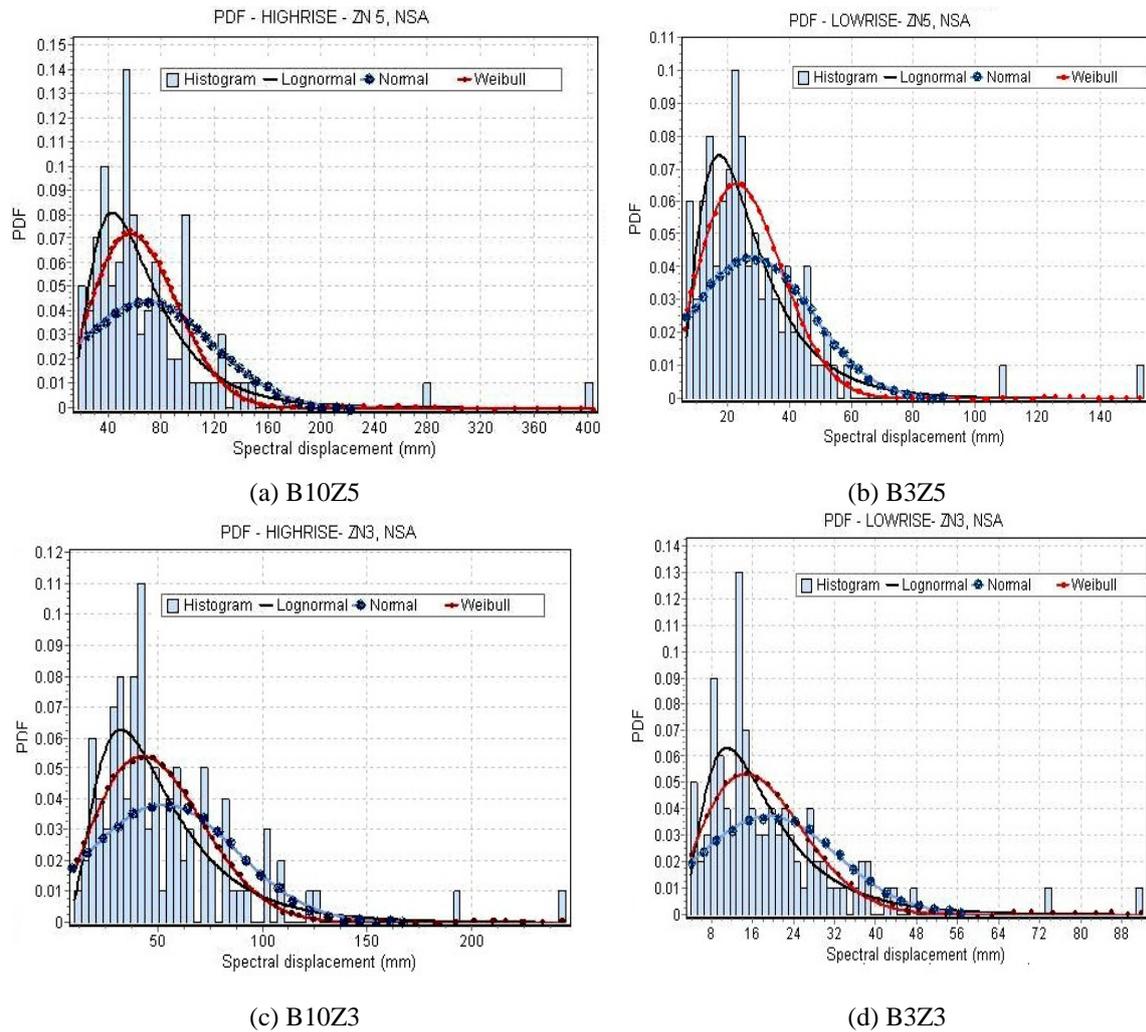


Fig. 8 Probability Density Functions for  $S_d$  by NSA for the four cases considered

curve and demand curve from the sample, an ensemble of one hundred performance points are obtained. The statistical properties viz., minimum, maximum, mean and COV of  $S_d$  from the ensemble of performance points are given in Table 6.

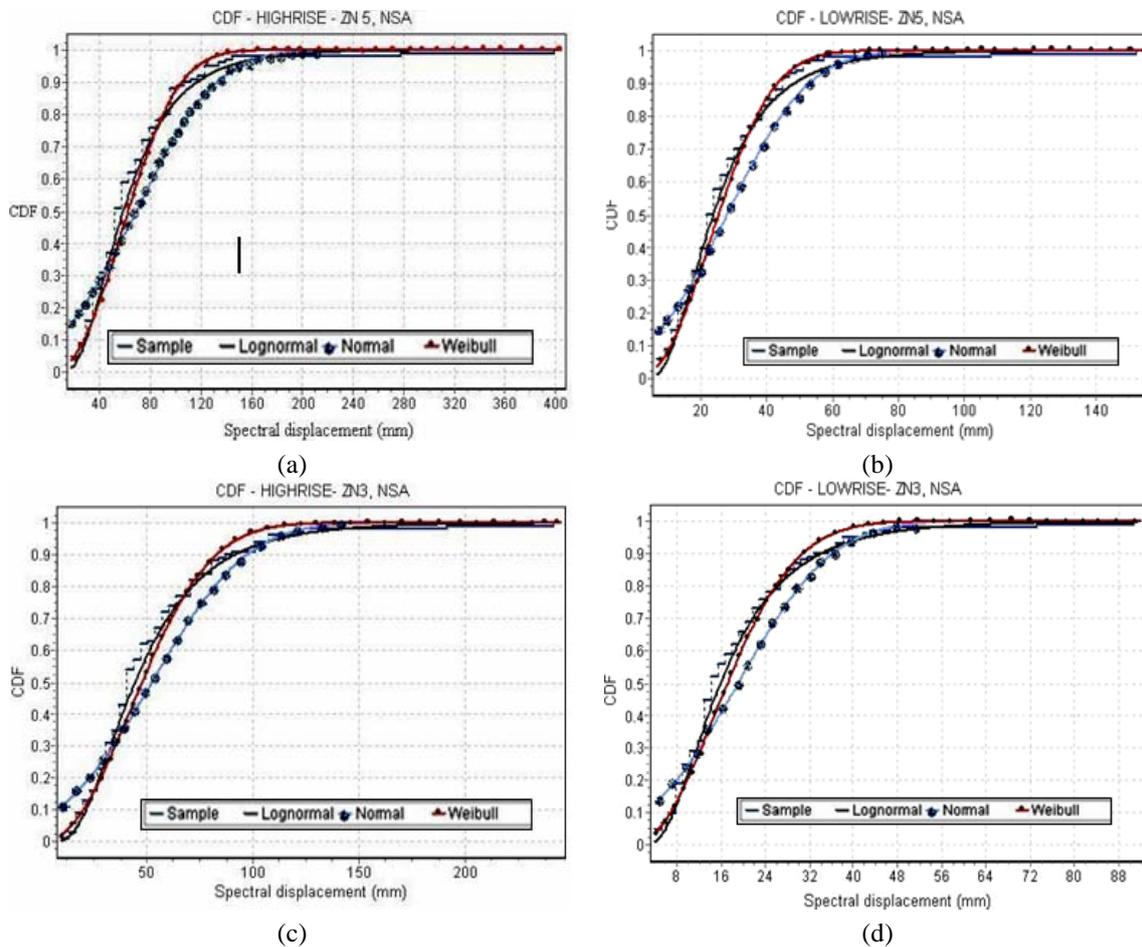
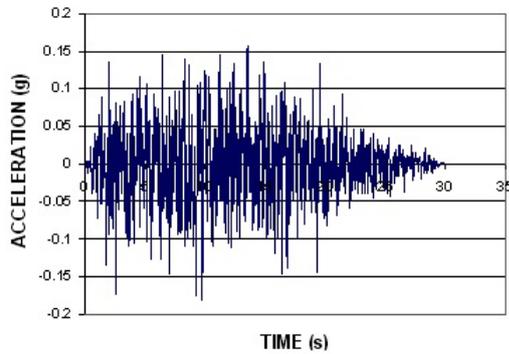


Fig. 9 Comparison of observed CDF of  $S_d$  and CDF obtained by fitting normal, lognormal and Weibull probability distribution functions

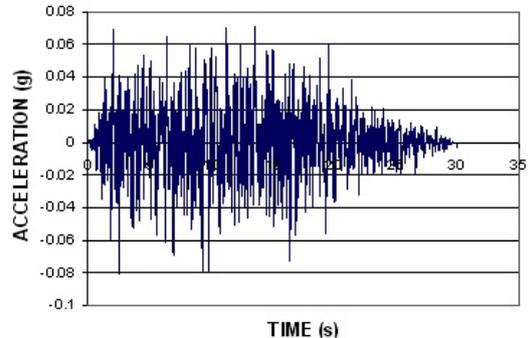
#### 4.5 Testing of hypothesis of distribution for spectral displacements obtained by NSA

From the sample of spectral displacement results, the histogram is plotted for the four cases considered as shown in Fig. 8. The solid lines in Fig. 8 indicates the probability density functions obtained using normal, lognormal and Weibull probability distribution functions for the sample data of  $S_d$ . The comparison of observed cumulative distribution function (CDF) from the sample and CDF by fitting normal, lognormal and Weibull distribution functions for the sample data of  $S_d$  is shown in Fig. 9. In order to check the given hypothesis, the difference to the observed data is checked with allowable error for 95% confidence with Kolmogorov-Smirnov (K-S) test (Papoulis and Pillai, 2002). The maximum error (difference between observed CDF and CDF obtained by applying chosen distribution) and the maximum allowable error for 95% confidence for normal, lognormal and Weibull probability distribution functions are compared.

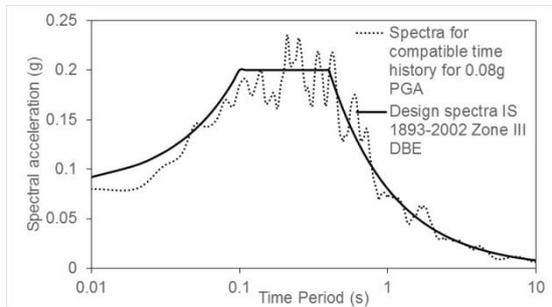
From the K-S test results, lognormal and Weibull probability distribution functions are found to pass the hypothesis test for 95 % confidence level. Out of the three probability distribution



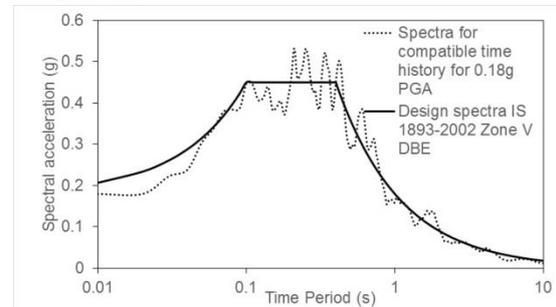
(a) Time history compatible with design spectra for Zone V



(b) Time history compatible with design spectra for Zone III



(c) Comparison of target design spectra of IS 1893-2002 for Zone III and spectra of simulated spectrum compatible time history



(d) Comparison of target design spectra of IS 1893-2002 for Zone V and spectra of simulated spectrum compatible time history

Fig. 10 Spectrum compatible time history and comparison of spectra

Table 7 Statistical properties of  $S_d$  from NDA

Case	Building	$S_d$ (m)			
		Minimum	Maximum	Mean	COV
1	B10Z5	0.015	0.230	0.065	0.600
2	B3Z5	0.007	0.103	0.030	0.567
3	B10Z3	0.008	0.191	0.045	0.689
4	B3Z3	0.004	0.080	0.017	0.706

functions chosen, lognormal probability distribution function has been found to be the best fit representing the statistical distribution of spectral displacement.

### 5. Nonlinear dynamic analysis

For the purpose of carrying out NDA, artificial time history of accelerations compatible with design spectra (Type I (Rock), damping 5%) of Indian seismic code with mean PGA of 0.18 g (zone V) and 0.08 g (zone III) are generated as shown in Fig. 10(a), 10(b) using SIMQKE (Gasparini and Vanmarcke 1976, Gelfi 2007) computer program. The comparison of target spectra

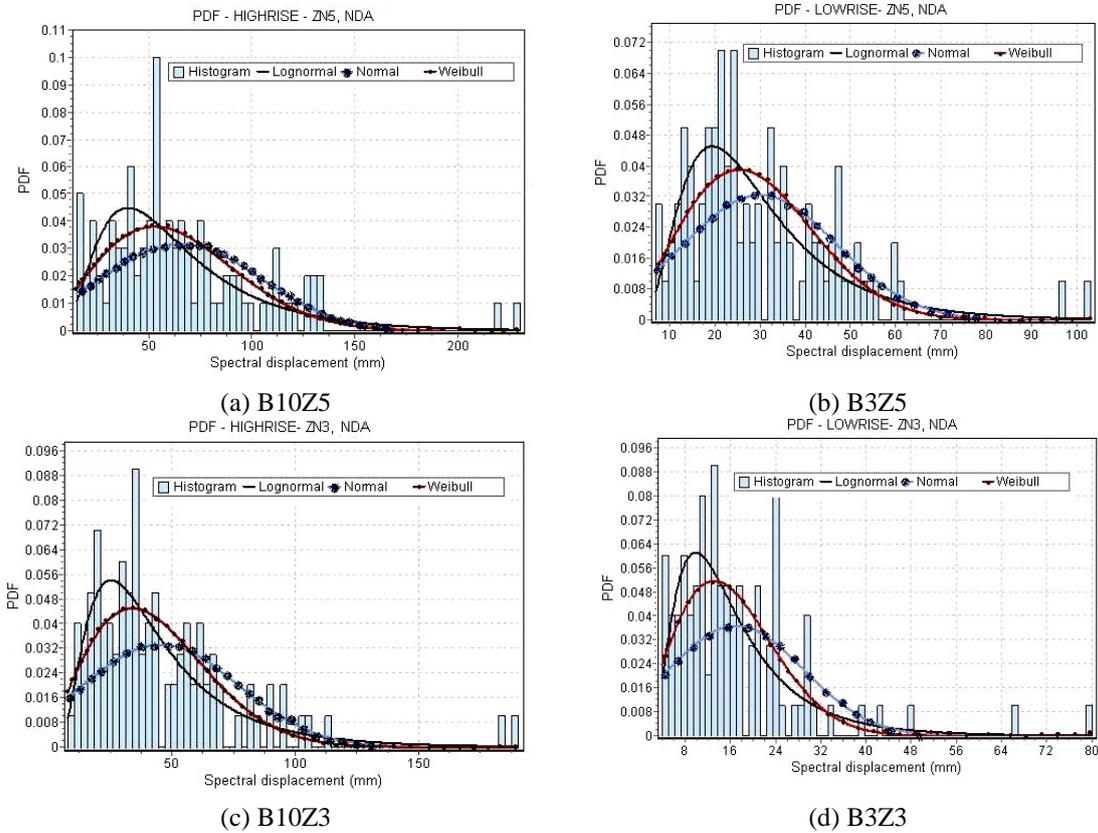


Fig. 11 Probability density functions for  $S_d$  by NDA for the four cases considered

and the spectra from simulated ground motion is also given in Fig. 10(c), 10(d) which shows that the simulation results are in satisfactory agreement with target response spectrum. Nonlinear Dynamic Analysis is carried out using IDARC 2D computer program wherein by default, Newmark beta algorithm with unconditionally stable constant average acceleration with parameters ( $\alpha = 1/4$  and  $\beta=1/2$ ) are adopted for numerical integration. Three parameter Park Model has been adopted for modelling hysteretic behavior of reinforced concrete beams and columns (Valles *et al.* 1996).

From the nonlinear time history analyses results carried out using the spectrum compatible time histories, and random sample of input parameters the absolute maximum roof displacement are obtained. Maximum roof displacements obtained for multi-degree freedom models are converted as spectral displacement for equivalent single degree freedom systems using Eq. (2) and the statistical properties viz., minimum, maximum, mean and COV are obtained.

### 5.1 Statistical distribution of spectral displacement corresponding to maximum absolute roof displacement obtained by NDA

From the ensemble of spectral displacements obtained from one hundred nonlinear time history analyses results, the statistical properties viz., minimum, maximum, mean and COV of  $S_d$

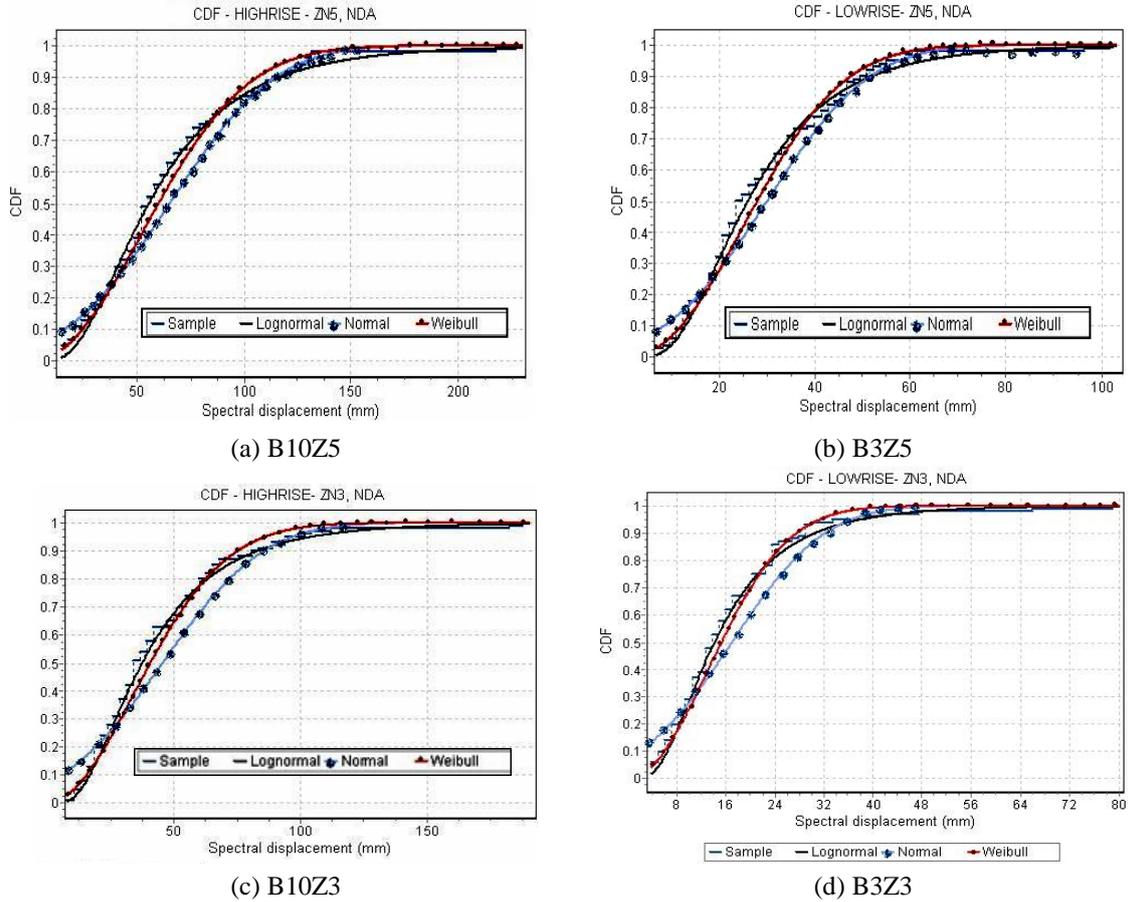


Fig. 12 Probability density functions for  $S_d$  by NDA for the four cases considered

Table 8 Examining the hypothesis of different probability distribution functions for  $S_d$

Building		Maximum absolute error between observed CDF from samples and probability distribution function predicted CDF			Best fit suggested based on K-S test results	
		Normal	Lognormal	Weibull		
B10Z5	NSA	0.154	0.056	0.106	Maximum error permitted for accepting the hypothesis for 95% confidence for the sample size chosen =0.136	
	NDA	0.123	0.065	0.066		
B10Z3	NSA	0.150	0.064	0.116		
	NDA	0.149	0.048	0.091		
B3Z5	NSA	0.142	0.058	0.095		Maximum error for lognormal distribution is small compared to other models hence lognormal function can be used for probabilistic modeling of $S_d$
	NDA	0.140	0.061	0.113		
B3Z3	NSA	0.152	0.057	0.109		
	NDA	0.149	0.050	0.096		

are obtained as shown in Table 7.

### 5.2 Testing of hypothesis of distribution for spectral displacements obtained by NDA

For the spectral displacement obtained through NDA, histograms in the form of probability density functions using normal, lognormal and Weibull probability distribution functions are shown in Fig. 11. The comparison of observed CDF from the sample and CDF obtained by fitting normal, lognormal and Weibull probability distribution functions for the sample data of  $S_d$  is shown in Fig. 12.

## 6. K-S test results

From the K-S test results, lognormal and Weibull probability distribution functions are found to pass the hypothesis test for 95 % confidence level. Out of the three probability distribution functions chosen lognormal probability distribution function has been found to be the best fit representing the statistical distribution of spectral displacement (Table 8).

## 7. Discussions

From the ensemble of capacity curves generated from NSA, it is seen that uncertainties in different input parameters have resulted in less variation in the linear part and more variations in the nonlinear part as given in Fig. 3. From the mean capacity curves, it is observed that, the ratio of ultimate to yield base shear is approximately 1.6, 1.95 and the ratio of ultimate to yield displacement is approximately 10.5, 15 in zone V and zone III respectively for B10. Similarly the ratio of ultimate to yield base shear and the ratio of ultimate to yield displacement of B3 is approximately 1.83, 1.88 and 22, 19 in zone V and zone III respectively. The variations are more in B10 compared to B3. The ratios of ultimate to yield base shears and ultimate to yield displacements for zone V and zone III will be useful in generation of capacity curve to this class of buildings for vulnerability assessment and loss estimation.

From the performance points obtained as an intersection of mean capacity and mean demand curve it is seen that, the ratio of  $S_a$  at performance point for zone V and zone III is 4.19 for B10 and 3.18 for B3, this may be viewed in light of the ratio of PGA for DBE for Zone V and Zone III is 2.25. The ratio of  $S_d$  at performance point for zone V and zone III is 1.21 for B10 and 1.38 for B3. The variation of mean  $S_d$  from intersection of mean capacity and mean demand curve (Table 5) and the mean from ensemble of  $S_d$  (Table 6) is 8.8% and 17.3% with respect to ensemble mean for zone V and zone III respectively for B10 and 3.6% and 10.5% with respect to ensemble mean for zone V and zone III respectively for B3. Often, mean or median capacity and demand curves are used in literature for loss estimation and vulnerability assessment. In the present study, the difference in performance points obtained from ensemble has been compared with the performance points obtained by the intersection of mean capacity and mean demand curve. The results suggests that first order approximation can be used for determination of  $S_d$ .

The variation of mean of  $S_d$  from NSA and NDA for B10 is 7.14% and 13.4% with respect to NDA for Zone V and Zone III respectively. Similarly, the variation of mean of  $S_d$  from NSA and NDA for B3 is 4.4% and 10.5% with respect to NDA for Zone V and Zone III respectively. The

difference in mean between NSA and NDA results are more for zone III compared to zone V for both B10 and B3.

The variation of COV of  $S_d$  from NSA and NDA for B10 is 20.1% and 2.3% with respect to NDA for Zone V and Zone III respectively. Similarly, the variation of COV of  $S_d$  from NSA and NDA for B3 is 19.7% and 3.1% with respect to NDA for Zone V and Zone III respectively. From the results it is seen that, the difference in COV between NSA and NDA results are more for zone V compared to zone III for both B10 and B3.

The CDF obtained by fitting lognormal distribution is found to be resulting in minimum error with observed CDF when compared to other distribution models for both NSA and NDA and passes the goodness-of-fit test at 95% confidence level as given in Table 8. The observation in the present study is in line with HAZUS, wherein capacity curve and spectral displacement are assumed to follow lognormal distribution.

## 8. Conclusions

In the present study an effort has been made to determine the statistical properties and suitable probability distribution functions of spectral displacements from nonlinear static and nonlinear dynamic analysis within the frame work of Monte Carlo simulation for typical low rise and high rise RC framed buildings located in zone III and zone V and designed as per Indian seismic codes.

Uncertainties in capacity of the building is modeled by choosing cross sectional dimensions of beams and columns, density and compressive strength of concrete, yield strength and elastic modulus of steel and live load as random variables. Uncertainty in demand is modeled by choosing peak ground acceleration as a random variable.

The percentage differences in mean and COVs of spectral displacement obtained from NSA and NDA are compared. It is seen that the percentage variations for mean are more for zone III with a maximum of 10.5% and percentage variations for COV is more for zone V with a maximum of 20.1%. The differences for mean as well as COV are more for ten storey building compared to three storey building. However, it can be concluded that NSA based statistical properties can be used in the absence of more accurate NDA based statistical properties.

Suitability of different probability distribution functions viz., normal, Weibull, lognormal are examined for goodness-of-fit. The mean and COV of spectral displacement obtained from NSA and NDA are compared and the results of NSA are found to be conservative as it is observed in literature. From the hypothesis test for goodness-of-fit lognormal function is found to be suitable to represent the statistical variation of spectral displacement obtained from NSA and NDA. This observation is in line with HAZUS, wherein the capacity curves and spectral displacements are found to follow lognormal distribution.

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