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Seismic design of structures using a modified non-stationary critical excitation

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Abstract. In earthquake engineering area, the critical excitation method is an approach to find the most severe earthquake subjected to the structure. However, given some earthquake constraints, such as intensity and power, the critical excitations have spectral density functions that often resonate with the first modes of the structure. This paper presents a non-stationary critical excitation that is capable of exciting the main modes of the structure using a non-uniform power spectral density (PSD) that is similar to natural earthquakes. Thus, this paper proposes a new method to estimate the power and intensity of earthquakes. Finally, a new method for the linear seismic design of structures using a modified non-stationary critical excitation is proposed.

Keywords: random vibration; critical excitation; spectral density function; non-stationary input

1. Introduction

The seismic design of structures based only on previous earthquakes is not adequate because future earthquakes may be more destructive. However, in recent seismic design procedures, the static and modal seismic designs of structures are based on the design spectrum produced by previous earthquakes. Presently, critical excitation methods have been developed to generate worst-case critical excitations (Takewaki 2007, Takewaki *et al*. 2012). The consideration of important constraints of earthquakes such as power and intensity can achieve this purpose. The critical excitation method is an optimization problem that maximizes the structural response as an objective function that is subject to some constraints.

Critical excitation was initially proposed by Papoulis (1967). Then, Drenick (1970) used the method of critical excitation for structures in the time domain. For this method, an input excitation was obtained that produced the maximum response. In addition, Shinozuka (1970) applied the same method in the frequency domain and presented a narrower upper bound of the maximum response.

Previously, many people have studied various constraints and objective functions. Iyengar (1972), Manohar and Sarkar (1995, 1998) and Takewaki (2001, 2002) developed a new optimization problem in the frequency domain to determine the input excitation by considering the

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response resonance of higher modes of the structure with input excitation. Some proposed constraints include the power and the intensity limit of earthquakes that were presented by Takewaki (2007). Also, Abbas and Manohar (2002) considered a set of constraints with respect to the stochastic analysis. Ben-Haim and Elishakoff (1990) and Pantelides and Tzan (1996) presented several interesting convex models. Ghodrati and Ashtari (2006) used the wavelet theory to generate artificial earthquake excitations and introduced more excitations by using a linear combination of resultant artificial earthquakes to obtain the critical excitation.

Takewaki (2001a, 2001b, 2007) proposed the optimum line as a geometric technique for solving stationary problems. Following Takewaki (2001a, 2001b, 2007), Ashtari (2004, 2006) introduced the Simple Optimum Line method which applied some approximations to find a simpler formula in this case. Then, an exact method for SDOF systems and a simple numerical technique were proposed to determine the stationary critical excitation for MDOF systems. Finally, Ashtari and Ghasemi proposed a more realistic expression of critical excitation (2010a, 2010b).

Then, Moustafa *et al*. (2010) considered the modeling of critical excitation for an inelastic structure with regarding damage indices. Afterward, Moustafa and Takewaki (2009) with respect to the probability measurements and entropy concept introduced an approach to find resonance records. Nevertheless, it is worth mentioning that regarding to the Fujita *et al*. (2010) research, by finding optimal placement of viscoelastic dampers the objective drifts function of a linear structure can be more minimized. Finally, according to the drift and input energy demands, Takewaki (2011) scaled ground motions for designing tall buildings.

Some of the proposed critical excitation methods focused on the primary frequency range of the structure. Based on the maximization problem, the optimization issue seems to be evident, but the result of the power spectral density function has no similarity to the power spectral density of a natural earthquake. This research aims to determine a more realistic critical input excitation. Therefore, this research proposes a modified critical excitation that is similar to natural earthquakes and follows the previous PSD of earthquakes that can be used in the linear seismic design of structures.

2. Expression of earthquake ground motion in terms of the more general approach

Several PSD functions have been proposed in the past. The function presented by Kanai (1957) and Tajimi (1960), which is known as the Kanai-Tajimi PSD function, has been widely used in previous studies. The PSD of ground acceleration, $G_1(\omega)$ which depends on three parameters and represents filtered white noise, is given by

$$S_{g1}(\omega) = G_1(\omega)S_0 \tag{1}$$

$$G_1(\omega) = \frac{\omega_g^4 + 4F_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4F_g^2\omega_g^2\omega^2} \tag{2}$$

where $\omega(rad/s)$ is the frequency, S_0 is the intensity of the ideal white noise, $\omega_g(rad/s)$ is the filter frequency that determines the dominant range of the input frequencies and the damping coefficient ξ_g indicates the sharpness of the power spectral density shape. The intensity of the white noise S_0 tends to zero, which is not consistent with the PSD of actual earthquake records. Furthermore, singularities are present at ω equal to zero for PSD functions of the ground velocity and displacement (see Ashtari and Ghasemi (2010a)). To overcome this inconsistency, modifications to the Kanai-Tajimi PSD function were proposed by Lai (1982) and Clough (1975). The modification suggested by Lai (1982) was obtained by filtering $S_{g1}(\omega)$ through an SDOF system, whereas the modification suggested by Clough (1975) was obtained by filtering $S_{g1}(\omega)$ through an SDOF system without mass.

$$S_{g2}(\omega) = G_2(\omega)S_{g1}(\omega) = G_2(\omega)G_1(\omega)S_0 \tag{3}$$

where

$$G_2(\omega) = \frac{\omega^2}{\omega^2 + \omega_O^2} \tag{4}$$

where ω_O is a variable that determines the low-frequency content of the ground motion. The larger amount of ω_O is the least amount of the low-frequency content in the ground motion result. This paper aims to present a more realistic critical excitation by employing an earthquake PSD. For this purpose, by using the Kanai-Tajimi PSD, some frequencies can be excited more than other frequencies, such as the PSD of a natural earthquake, and the new method can easily estimate the PSD constraints of the earthquake. Eventually, a new method for the linear seismic design of structures is proposed that uses a modified critical excitation.

3. Calculating the power of an earthquake

The integral of the PSD function in the frequency range is equal to the power limit.

$$Power = \bar{S} = \int_0^{\omega} S_g(\omega)d\omega \tag{5}$$

In other words, during the time domain, the power of the earthquake is equal to the mean square of the acceleration of the earthquake.

$$Power = S = E\left(\ddot{u}_g^2(t)\right) \tag{6}$$

$K(\omega)$ can be defined as a modified filter; therefore, by the multiplication of $G_1(\omega)$ and $G_2(\omega)$

$$K(\omega) = G_1(\omega)G_2(\omega) \tag{7}$$

According to Eq. (7), the spectral density function of the ground motion can be expressed as

$$S_g(\omega) = K(\omega)S_0 \tag{8}$$

where S_0 is the PSD of the white noise. Thus, the power of the earthquake can be written as

$$Power = \bar{S} = K_2 S_0 \tag{9}$$

$$K_2 = \int_0^{\omega} K(\omega) d\omega \tag{10}$$

If the average values of ω_g, ω_c and ξ_g from previously recorded earthquakes (see the average value in Ahmadi (1979) and Hong and Wang (2002)) are substituted into the modified filter $K(\omega)$, then the value of K_2 is equal to 61.81. (K_2 = 61.81).

4. Maximum intensity of the power spectral density of an earthquake

The maximum power spectral density of the Modified Kanai-Tajimi PSD is equal to

$$\bar{s} = \mathrm{Max}_{\omega=0} S_g(\omega) \tag{11}$$

By utilizing the K_1 coefficient, $\bar{s}$ can be directly related to S_0 as follows

$$\bar{s} = K_1 S_0 \tag{12}$$

Using the average values of ω_g, ω_c, and ξ_g from previous earthquakes (Ahmadi (1979) and Hong and Wang (2002)), the value of K_1 is then equal to 3.638 (K_1 = 3.638).

Finally, based on Eq. (9) and Eq. (12), the relationship between $\bar{s}$ and $\bar{S}$ can be expressed as

$$\bar{s} = K_3 \bar{S} \tag{13}$$

where

$$K_3 = \frac{K_1}{K_2} \tag{14}$$

Additionally, by replacing the average values of ω_g, ω_c and ξ_g, which were obtained from previous earthquakes, K_3 will be equal to 0.0589 for the *Kanai – Tajimi* PSD modified by the Lai filter. (K_3 = 0.0589).

5. Modified non-stationary critical excitation

The optimization problem for the non-stationary critical excitation method can be defined by

$$Max: E\left[\sigma_{d_{k0}}(t_n)^2\right] = \int_{-\infty}^{+\infty} F_{m,n}(\omega) S_w(\omega) d\omega \tag{15}$$

which is subjected to

$$\int_0^{\infty} S_w(\omega) d\omega < \bar{S}_w \tag{16}$$

$$S_W(\omega) \leq \bar{s}_w \tag{17}$$

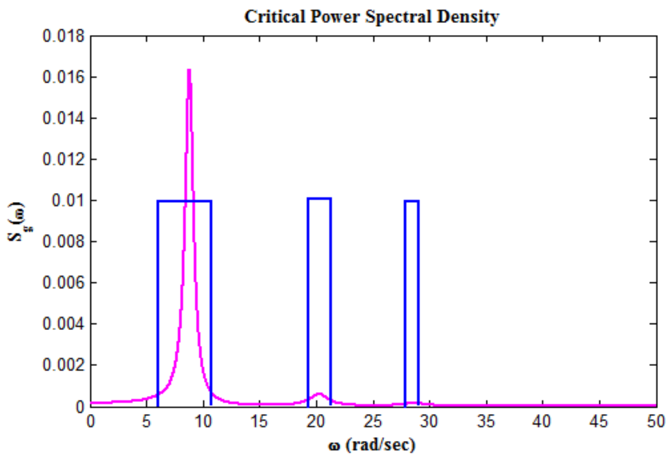


Fig. 1 Critical excitation PSD function obtained from the previous method

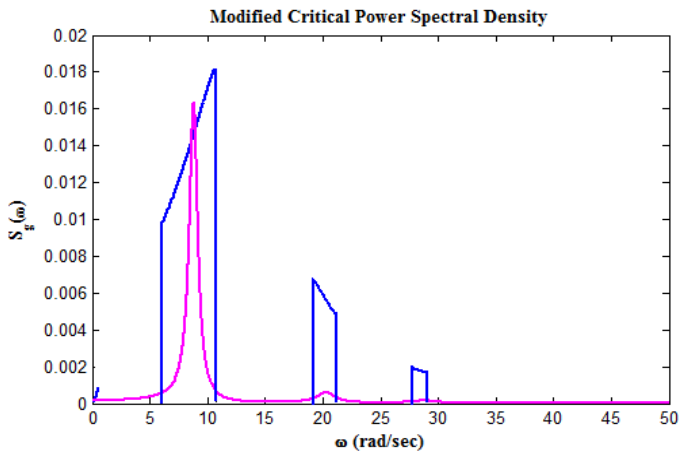


Fig. 2 Modified form of the critical excitation PSD function

Based on the Takewaki (2007) to maximize the objective function in the frequency domain, Eq. (16) results in a critical PSD function similar to a multi-rectangular shape (see Fig. 1).

However, the PSD functions of the recorded earthquakes have a variable intensity, i.e., the natural PSD functions of the earthquakes are not the same and exhibit a constant intensity along their entire frequency content. According to the random vibration theory, the following relationship can be obtained for a non-stationary critical excitation by assuming $\ddot{u}_g(t) = c(t).w(t)$.

$$S_g = c(t)^2.S_w(\omega) \tag{18}$$

If $c(t)$ is a constant, then $S_W(\omega)$ becomes similar to the power spectral density function of a natural earthquake. Accordingly, the intensity constraint (Eq. (16)) changes to a more realistic form. Here, $S_W(\omega)$ resembles the *Kanai – Tajimi* PSD.

$$S_w(\omega) \le G_1(\omega)G_2(\omega)S_{0w} \le K_s\bar{s}_w \tag{19}$$

By assuming K_S = 1, the input spectrum will be exactly the *Kanai – Tajimi* PSD. However, if K_S > 1, then the area of part of the modified PSD becomes equal to the power of $S_w(\omega)$. Therefore, the final form of the optimization problem of the non-stationary critical excitation can be modified as Eqs. (15) and (16) which is subjected to Eq. (19).

Eq. (19) indicates that if the mode of the frequency tends towards the predominant frequency of the *Kanai-Tajimi* PSD function (the peak of the *Kanai – Tajimi* PSD), then this mode has a stronger effect. Thus, the higher modes of the structure can be excluded from the critical excitation.

6. Seismic design using the modified critical excitation

This paper introduces a new method for the linear seismic design of structures by means of risk analysis and modified critical excitation. The algorithm of the proposed method is developed as follows

Step 1) Determination of the power constraint by considering probabilistic risk analysis ($\bar{S}$).

Step 2) Determination of the power constraint of the Gaussian stationary function (S_w) by utilizing $\bar{S}$.

Step 3) Determination of the $\bar{s}_w$ constraint with respect to the relationship between $\bar{S}_w$ and $\bar{s}_w$.

Step 4) Determination of drifts of each floor at the critical moment with respect to $\bar{S}_w$ and $\bar{s}_w$.

Step 5) Comparison of the obtained drifts with the allowable drifts and strengthening the structure if the former drifts exceed the allowable amount and are unacceptable.

6.1 Determination of the power constraint in a stationary state

The value of $\bar{S}$ is equal to the mean square value of the acceleration.

$$\bar{a} = \sqrt{E(a^2)} \tag{20}$$

For the probabilistic risk analysis, by creating elements for the linear and surface sources, the value of the PGA can be obtained for every definite probability on the basis of the damping relationship and the Richter-Gutenberg relationship. In a typical damping relationship, the value of the PGA is obtained for the interval moment of the earthquake. By simplifying, the damping relations can be calculated from $a = \sqrt{E(a^2)}$. For example, in Code 2800, the value of $\bar{a}$, which is the same as $\sqrt{\bar{S}}$, is obtained for a 10% probability of the occurrence of an earthquake during a 50-year period after a structure is built.

If the value of the PGA is obtained by using risk analysis, then there is a simple method to obtain the value of $\bar{S}$ by employing a simple proportion coefficient.

$$\bar{S} = \left(\frac{PGA}{n}\right)^2 \tag{21}$$

Table 1 Value of n for selected earthquakes

Earthquake	Year	Location	PGA (m/s^2)	$\bar{S}$ (m^2/s^4)	$\bar{s}$ (m^2/s^3)	n
El-Centro 270	1940	Imperial Valley, CA, USA	2.11	0.186	0.011	4.88
El-Centro 180	1940	Imperial Valley, CA, USA	3.07	0.266	0.0305	5.95
Kobe KJM 000	1995	Hyōgo, Japan	8.06	1.092	0.0528	7.71
Kobe KJM 090	1995	Hyōgo, Japan	5.87	0.707	0.0416	6.98
Taft021	1952	Kern County, CA, USA	1.53	0.0627	0.0056	5.69
Taft111	1952	Kern County, CA, USA	1.74	0.0722	0.0043	6.49
Bajestan-L1	1918	Quchan, Iran	0.92	0.0237	0.0014	5.97
Ferdos-L1	1968	Dasht bayaz, Iran	0.86	0.0296	0.0017	4.98
Tabas	1978	Tabas, Iran	9.15	2.861	0.2017	5.41
						6.006 = n

Table 2 Value of K_0 for selected large earthquakes

Earthquake	$\bar{S}$	$\bar{s}$	$\bar{\delta}_w$	δ_w	Points	K_0
El-Centro 270	0.266	0.305	0.544	0.3879	1024	20.5
El-Centro 180	2.861	0.2071	87.59	6.201	1024	30.6
Kobe KJM 000	1.0917	0.528	31.86	1.704	512	29.2
Naghan	3.698	0.0772	71.42	1.422	128	19.3

According to Table 1, the coefficient n is obtained for nine accelerograms in which the average value of n is 6.0.’

6.2 Determination of the power constraint in a non-stationary state

The value of $\bar{S}_w$ is equal to the mean square of the Gaussian stationary function $w(t)$, which can be expressed by

$$\bar{S}_w = E[w(t)] = E\left[\frac{a(t)^2}{o(t)^2}\right] \tag{22}$$

where $a(t)$ is the natural acceleration function of the earthquake. This equation can also be written as

$$\bar{S} = E\left[a(t)^2\right] \tag{23}$$

For the definite function $c(t)$, both of the parameters $\bar{S}_w$ and $\bar{S}$ are dependent of the $a(t)$ function. Hence, the mean coefficient can be determined statistically as follows

$$\bar{S}_w = K_0.\bar{S} \tag{24}$$

To determine K_0, the values of $\bar{S}_w$ and $\bar{S}$ should first be calculated for different earthquakes, and eventually, K_0 will be equal to $\bar{S}_w / \bar{S}$.

To calculate $\overline{S}_w$ and $\overline{s}_w$, weak accelerations were excluded from the accelerogram, and the remaining accelerations were subjected to the PSD function calculations. Additionally, the coefficients α and β in the Bolotin envelope function $c(t) = e^{-\alpha*t} - e_1 \cdot e^{-\beta*t}$ were determined under the condition that the peak of $c(t)$ occurs at the moment of the PGA, and the duration of the earthquake is equal to the limitation of the $c(t)$ function. Additionally, the total number of selected points is equal to

$$N_1 = 2^{[\log_2 N_0]} \tag{25}$$

where N_0 represents the number of points that remain after weak accelerations were excluded from the beginning and end of the accelerogram.

6.3 Determination of the amplitude constraints in a non-stationary state

According to Eq. (15), regarding several earthquakes, the shape of $S_w(\omega)$ is similar to $S_g(\omega)$. Therefore, the modified *Kanai-Tajimi* PSD function can also be used for $S_w(\omega)$. Thus,

$$\overline{s}_w = K_3 \overline{S}_w \tag{26}$$

In Table 2, the values of $\overline{s}_w$ were obtained from earthquake records. A comparison of the results of Table 2 with the results of Eq. (26) indicates that this assumption provides good results (with the exception of the Naghan earthquake). Because the Naghan earthquake contains only a few points of the acceleration of the earthquake (128 points), the possibility of errors in the calculation of $\overline{S}_w$ *and* $\overline{s}_w$ is high. If values of zero are added to the end of the accelerogram, and 256 points of acceleration are used to calculate $\overline{S}_w$ *and* $\overline{s}_w$ instead of 128 points, the values of $\overline{S}_w$ *and* $\overline{s}_w$ will be equal to 43.296 and 0.8583, respectively.

7. Design example

The linear seismic design of a two-story commercial building in Tehran on soil type II (Iranian code (2005)) is considered. The span and story height are shown in Fig. 3. The system of the building consists of a simple framework with steel concentric bracing. The damping ratio is 0.02. A bracing check is desired.

(The duration of the earthquake is assumed be 30 seconds, and its PGA is assumed to occur during the 5th second.)

Solution

Considering the mass and stiffness of the structure

The stiffness of the stories in the x and y directions for the first and second stories is equal to

$$K_{1x,y} = 2K_{BR_1} = 2 \times 37.98 = 75.96(KN/cm)$$
$$K_{2x,y} = 2K_{BR_2} = 2 \times 68.78 = 137.56(KN/cm)$$

Furthermore, the mass of each story is equal to

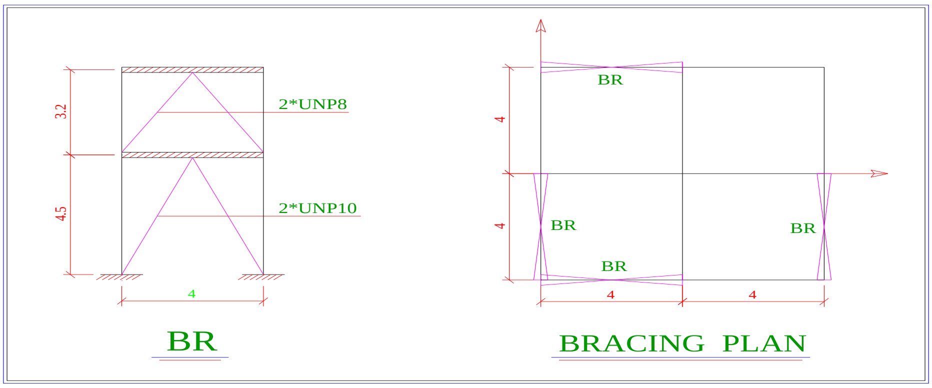


Fig. 3 Bracing plan and Bracing elevation of a commercial building, (UNP represents the cross-section of the braces' profile)

$$m_1 = 83.23\,tons$$
$$m_2 = 63.49\,tons$$

7.1 Design using static analysis

Using the static analysis method, the basic shear force (earthquake force) is equal to

$$V = \frac{ABI}{R}W = \frac{(0.35 * 2.5 * 1)}{6} * (83.23 + 63.49) = 21.4KN$$
$$F_2 = \frac{W_2 h_2}{\sum Wh}V = \frac{63.49 * (4.5 + 3.2)}{63.49 * 7.7 + 83.23 * 4.5} * 21.4 = 12.12KN$$

The story drift is calculated by

$$\Delta_j = V_j / K_j$$

$$\Delta_j = \frac{V_1}{K_1} = \frac{21.4}{75.96} = 0.28cm < \frac{0.03h_1}{R} = 0.03 * \frac{450}{6} = 2.25cm$$

$$\Delta_2 = \frac{V_2}{K_2} = \frac{12.12}{137.56} = 0.088cm < \frac{0.03h_2}{R} = 0.03 * \frac{320}{6} = 1.16cm$$

$$F_{BR1} = \frac{V_1}{2} * 1.5 = 16.18 < F_{a1} = 16.2KN\ O.K.$$

$$F_{BR2} = \frac{V_2}{2} * 1.5 = 9.09 < F_{a2} = 18KN\ O.K.$$

And, in chevron bracing, the coefficient K = 0.85 should be applied as

$$F_{a1} = \left(2B_1F_{\max_1}\right) = 16.8KN$$

$$F_{a2} = \left(2B_2F_{\max_2}\right) = 19.5KN$$

The coefficient K is assumed to be equal to 0.85 for the purpose of applying the joint area of the bracing to the related beam or column, which accounts for the existing partial local supports (such as walls). The 2UNP60 can also be used, but with the consideration that the necking effect (stipulated in Appendix 2 of the Iranian Code 2800) does not correspond to the standard value. Consequently, the 2UNP80 is used as the bracing profile for the second floor.

$KL/r \leq 6035/\sqrt{F_y} \cong 123$(Necking checks as stipulated in Appendix 2 of the Iranian Code 2800)

7.2 Design using the modified critical excitation

The modified critical excitation method results in the value of the maximum inter-story drift by considering the following assumption

$$\omega_{range} = 50(rad/\sec),\ \ \delta_w = 0.2(rad/\sec),\ \ t_{end} = 7(\sec),\ \text{ and }\ t_{start} = 3(\sec)$$

Considering that Tehran has a very high relative risk of an earthquake occurrence (PGA=0.35 g), and given the discussion in Section 5, we obtain

$$\omega_{range} = 50(rad/\sec),\ \ \delta_w = 0.2(rad/\sec),\ \ t_{end} = 7(\sec),\ \text{ and }\ t_{start} = 3(\sec)$$

Thus, the constraints of the Gaussian static variable ($a(t)/c(t)$) = $w(t)$ were obtained as follows

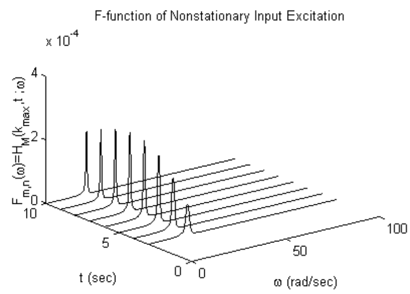
$$\bar{S}_w = 30\bar{S} \Rightarrow \bar{S}_w = 9.82(m^2/s^4),\ \text{ and }\ \bar{s}_w = 0.0589\bar{S}_w \Rightarrow 0.0589\bar{S}_w \Rightarrow \bar{s}_w = 0.578(m^2/s^3)$$

Finally, we obtain

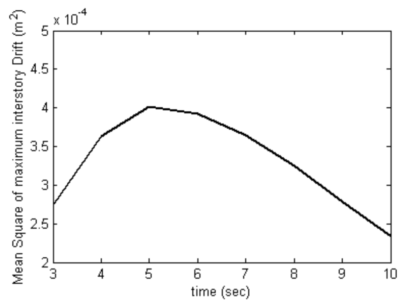
$$Max = \left\{\sigma_{d1} = \sqrt{E(d_1^2)}\right\} = 2.00(cm)\ \ at\ \ t = 5\sec$$

$$Max = \left\{\sigma_{d2} = \sqrt{E(d_2^2)}\right\} = 0.55(cm)\ \ at\ \ t = 5\sec$$

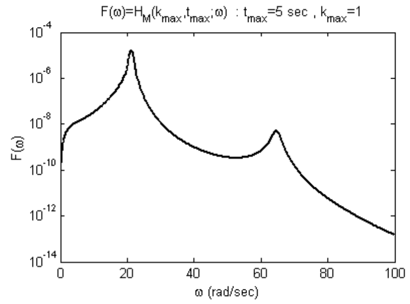
Fig. 4 illustrate the PSD function for the modified non-stationary critical excitation. As expected, according to Fig. 4, the drift of the first story is greater than that of the second story, and the maximum response occurs at the moment when the earthquake reaches its *PGA* moment. In addition, Fig. 4 includes the diagram for the $F(\omega)$ function, which has two peaks related to the first and second modes of the structure. Fig. 5 shows the frequency content of the modified critical excitation; the band frequency of the critical excitation approaches the peak of the *Kanai-Tajimi* PSD function.



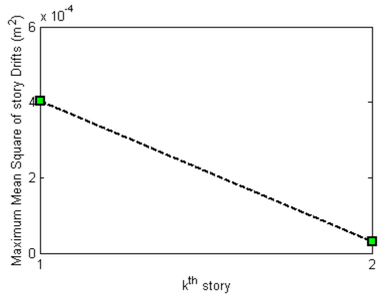
(a) Critical excitation in time-frequency domains



(b) Mean-square response of the structure



(c) Square frequency response function of the structure



(d) Maximum mean square of story drifts Square frequency response function of the structure

Fig. 4 Modified critical excitation and structural response

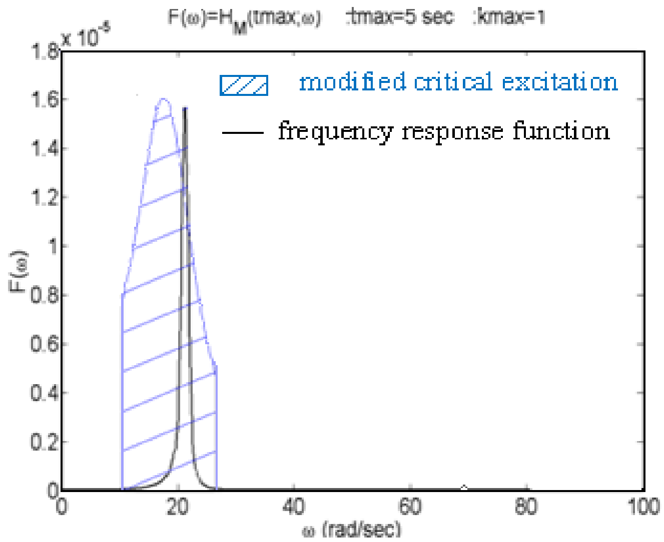


Fig. 5 Frequency content of the modified critical excitation

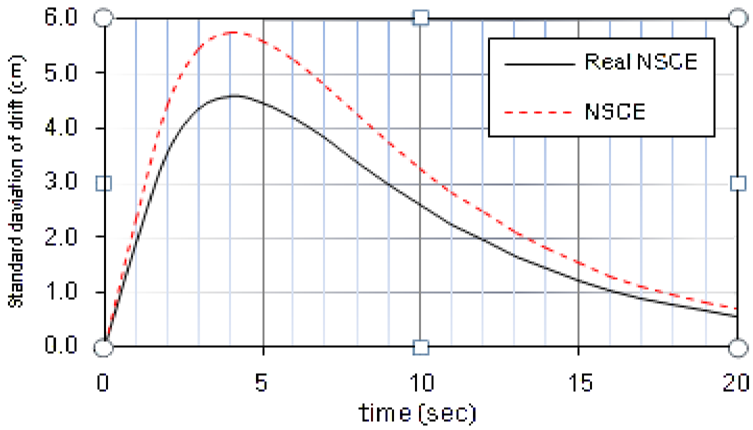


Fig. 6 Standard deviation of inter-story drift for the non-stationary critical excitation

8. Comparison of the responses obtained in the modified non-stationary critical excitation method with the conventional method

This paper proposes a modified critical excitation in which its intensity constraint is a function of the frequency (ω). Fig. 6 shows the standard deviation of the inter-story drift that was obtained from the conventional method compared to the modified non-stationary critical excitation methods. As shown in this diagram, the responses obtained from the modified method (Real NSCE) are less than the conventional method (NSCE) (about 80 percent).

The intensity and power constrains were obtained from the El-Centro Earthquake (1940)

$$\left(\bar{S}_w = 8.34\left({m^4}/{\sec^4}\right) and\ \bar{s}_w = 0.7363\left({m^2}/{\sec^3}\right)\right).$$

9. Conclusions

1) The modified critical excitation can be employed for the frequency content of the ground on which the structure was built, such as a natural earthquake excitation. Furthermore, the proposed method can generate a critical structural response.
2) In addition to providing the algorithm for obtaining the response using the modified non-stationary critical excitation method, this paper identified a method that has advantages such as the ease of calculation and more realistic responses compared to conventional methods.
3) In this paper, a new method was proposed to determine the power and amplitude constraints of earthquakes in non-stationary conditions that was based on the *PGA* of desirable earthquakes.
4) Finally, this paper provides a new seismic design of structures that utilizes the modified critical excitation method and can be used by considering only one earthquake parameter (PGA), which is in conformance with many other linear seismic design codes for structures.

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