Ductility demand of partially self-centering structures under seismic loading: SDOF systems

Xiaobin Hu¹ and Yunfeng Zhang^{*2}

¹School of Civil Engineering and Architecture, Wuhan University, Wuhan City, Hubei Province 430072, China ²Department of Civil and Environmental Engineering, University of Maryland, College Park, MD 20742, USA

(Received March 23, 2011, Revised November 25, 2011, Accepted July 30, 2012)

Abstract. In this paper, a numerical simulation study was conducted on the seismic behavior and ductility demand of single-degree-of-freedom (SDOF) systems with partially self-centering hysteresis. Unlike fully self-centering systems, partially self-centering systems display noticeable residual displacement after unloading is completed. Such partially self-centering behavior has been observed in a number of recently researched self-centering structural systems with energy dissipation devices. It is thus of interest to examine the seismic performance such as ductility demand of partially self-centering systems. In this study, a modified flag-shaped hysteresis model with residual displacement is proposed to represent the hysteretic behavior of partially self-centering structural systems. A parametric study considering the effect of variations in post-yield stiffness ratio, energy dissipation coefficient, and residual displacement ratio on the displacement ductility demand of partially self-centering systems was conducted using a suite of 192 scaled ground motions. The results of this parametric study reveal that increasing the post-yield stiffness, energy dissipation coefficient or residual displacement ratio of the partially self-centering systems generally leads to reduced ductility demand, especially for systems with lower yield strength.

Keywords: ductility; earthquake response; hysteresis; residual displacement; self-centering system

1. Introduction

Large residual deformation associated with conventional ductile structural design can often make the structure appear unsafe to occupants, impair its ability to resist subsequent aftershock earthquakes and significantly increase the cost of post-earthquake retrofit (Ruiz-Garcia and Miranda 2006a, 2006b). Residual structural deformation thus starts to be recognized as a complementary parameter in the evaluation of structural (and non-structural) damage in performance-based earthquake engineering (Pampanin *et al.* 2003, Christopoulos and Pampanin 2004). Recognizing the importance of controlling the residual deformation, self-centering seismic resisting system has recently been attracting considerable attention from the community (e.g. Kurama *et al.* 1999, Ricles *et al.* 2001, Christopoulos *et al.* 2002b, Mahin *et al.* 2006). Such self-centering system is characterized by a flag-shaped hysteresis loop with certain energy dissipation capability and small or zero residual structural deformation after strong earthquakes.

^{*}Corresponding author, Professor, E-mail: zyf@umd.edu



Fig. 1 Hysteretic model of (a) self-centering and (b) partially self-centering systems

Special metals like shape memory alloys (SMAs) also exhibit a flag-shaped stress-strain curve (Desroches and Smith 2004, Song *et al.* 2006). Due to its unique energy dissipation behavior and high fatigue life, SMA has been studied for use as a self-centering damping device in seismic resistant structures (Dolce *et al.* 2005, Zhu and Zhang 2008, Miller *et al.* 2011).

Though the self-centering system has the appealing behavior of reducing or eliminating the residual structural deformation, it may exhibit certain shortcoming if not properly designed. Particularly, the self-centering system has a lower energy dissipation capacity compared to the bilinear elasto-plastic hysteretic system, which is evidenced by the smaller enclosed area of its hysteresis loop. To address this issue, supplemental damping mechanism such as friction damping (Dolce *et al.* 2000, Zhu and Zhang 2008), hysteretic damping and viscous damper (Kam *et al.* 2010) is often implemented to augment the energy dissipation capacity of self-centering structural systems. However, increased energy dissipation capacity with the afore-mentioned measures such as friction damper is often accompanied with residual displacement in the system, after the system is completely unloaded.

Research works on self-centering systems with increased energy dissipation (Kurama 2001, Rodgers *et al.* 2007, Cardone *et al.* 2008, Wolski *et al.* 2009, Tremblay *et al.* 2010, Kam *et al.* 2010) have shown partially self-centering behavior, as schematically illustrated in Fig. 1(b). Residual displacement became more pronounced with increasing energy dissipation in self-centering systems. However, although partially self-centering system exhibits residual displacement after unloading (which is a serviceability issue in seismic design), this has a beneficial effect of increased hysteretic damping capacity and may make such systems superior to fully self-centering systems in terms of energy dissipation capacity and ductility demand. No research has been reported on the interplay between residual displacement and ductility demand of partially self-centering structural systems. But understanding how such structural systems behave under seismic loading is important to develop cost-effective seismic designs.

In this paper, a modified flag-shaped hysteresis model is proposed for partially self-centering systems, with an additional parameter representing residual displacement in comparison with the flag-shaped hysteresis model. This modified flag-shaped model was then used for a comprehensive parametric study conducted to investigate the ductility demand of partially self-centering systems under seismic excitation through nonlinear time history analysis. For systems with given values of initial period and yield strength, a favorable combination of post-yield stiffness ratio, energy

dissipation coefficient and residual displacement ratio can be determined from this parametric study results, in order to achieve economical design with lower ductility demand. These three parameters are chosen here to study the interplay between post-yield stiffness ratio, energy dissipation coefficient, residual displacement ratio and the system's ductility demand.

2. Analysis procedure: Overview

2.1 General description

Performance-based seismic design often involves the use of an equivalent single-degree-of-freedom (SDOF) system that represents the dominant vibration mode of the concerned structure, often multistory (Fajfar and Krawinler 1997). Analysis of such an equivalent SDOF system under seismic excitation not only provides insight into its inelastic seismic response behavior but also forms the basis for design parameter selection. The governing equation of motion of a nonlinear SDOF system under earthquake-induced base excitation can be expressed as

$$m\ddot{x} + c\dot{x} + f(x) = -m\ddot{x}_{\sigma} \tag{1}$$

where *m* is the mass and *c* is the viscous damping coefficient of the SDOF system; f(x) is the nonlinear restoring force of the SDOF system. *x* and \dot{x} are the displacement and velocity of the SDOF system relative to the ground; \ddot{x}_g is the ground acceleration caused by earthquake. A five percent viscous damping ratio is assigned in the time-history analysis. The P-Delta effect is not considered to avoid the complication from P-Delta effect on post-yield stiffness reduction in this study. It is noted that P-delta effect could lead to reduced effective post-yield stiffness and cause structural instability especially for flexible structures (Adam *et al.* 2004). The results of this study can be used if the reduction in post-yield stiffness due to P-Delta effect is known from other analysis.

In this study, the yield strength f_{y} of the nonlinear SDOF system is defined as

$$f_y = \frac{F_e}{R} \tag{2}$$

where *R* is the strength reduction factor, which represents the actual strength level of the nonlinear SDOF system relative to the ground motion intensity, and F_e is the elastic design strength which can be obtained from the elastic response spectrum as follows

$$F_e = mS_a \tag{3}$$

where S_a is the elastic spectral acceleration. An *R* value equal to or less than 4.0 corresponds to a relatively high lateral strength, and an *R* value greater than 4.0 indicates relatively low lateral strength (Seo and Sause 2005).

In performance-based earthquake engineering, the inelastic displacement is one of the primary response indices to determine both the structural and non-structural damage to buildings under seismic loading. The ductility demand is defined here as the ratio of the maximum inelastic displacement x_m to the yield displacement x_y for a specified *R* value as follows

$$\mu_R = \frac{x_m}{x_v} \tag{4}$$

In this study, the ductility demand is used as the primary response index for evaluating the seismic performance of the partially self-centering SDOF system.

2.2 Hysteresis model for partially self-centering system

For a fully self-centering system, a flag-shaped hysteresis model has been widely used to represent its cyclic behavior for the sake of simplicity (e.g. Christopoulos *et al.* 2002, Seo and Sause 2005), as shown in Fig. 1(a), where k_e is the initial elastic stiffness. Key to defining this hysteresis model are two independent parameters: post-'yield' stiffness ratio α and energy-dissipation coefficient β . The coefficient β reflects the energy dissipation capacity of the self-centering system, as shown in Fig. 1. For example, a lower bound value of zero produces a piecewise nonlinear elastic system while an upper bound β value of 1.0 corresponds to a self-centering system with the greatest possible energy dissipation capacity. For a given self-centering system with known initial stiffness and 'yield' strength values, the flag-shaped hysteresis model can be fully defined with these two parameters: α and β .

An alternative way to enhance the energy dissipation capacity of the system, as shown in Fig. 1(b), is to allow the system to unload to a nonzero displacement. This hysteresis model is referred to as modified flag-shaped model here to represent the hysteretic behavior of partially self-centering systems, which could also arise due to relatively low initial stiffness. Most self-centering system consists of a restoring component (through post-tensioning tendons or shape memory alloy bar) and an energy dissipation component (metallic yielding or friction devices). Thus the flag-shaped hysteresis is the superposition of these two hysteresis loops. An example illustrating this concept can be found in the self-centering friction damping brace developed by Zhu and Zhang (2008). Therefore, if the initial stiffness corresponding to the restoring component is too low, superimposing the energy dissipation component hysteresis would cause a residual displacement in the overall hysteresis loop. Compared to the flag-shaped model, the modified flag-shaped model needs an additional parameter for definition, i.e., the residual displacement, x_r . Apparently, if the residual displacement is set to zero, the modified flag-shaped model can be considered as a general hysteresis model for describing self-centering systems.

2.3 Earthquake ground motions

A total of 192 historical earthquake ground motion records were selected from the PEER NGA database (http://peer.berkeley.edu/nga/) and scaled to be compatible with a target spectrum. The target spectrum adopted in this study is the uniform hazard response spectrum derived by Somerville (2002) for the design basis earthquake intensity level with a probability of exceedance of 10% in 50 years in Van Nuys, California. The selected earthquake records are free of any forward directivity effects (i.e., near-fault effects). All earthquake ground motion accelerograms in this ensemble were recorded on soil type D, and were generated by earthquakes of moment magnitude Mw ranging from 5.7 to 7.3. The hypocentral distance for these records ranges between 3.4 and 59.7 km.

368



Fig. 2 Elastic response spectrum (spectral acceleration) of the 192 scaled earthquake records

The scaling factor for each of the selected earthquake records was calculated by minimizing the sum of the squared error between its 5%-damped spectral accelerations and the target spectrum at the selected periods of 0.1, 0.5, 1.0, 1.5 and 2.0 seconds. The ensemble average and the average plus/minus one standard deviation spectral accelerations over the 192 scaled records along with the target spectrum are shown in Fig. 2. A good match can be seen between the ensemble average spectral values and the target spectrum in the range of periods greater than 0.4 seconds. In this study, the ensemble average spectral acceleration ordinates over the selected 192 scaled earthquake records was used to calculate the elastic spectral acceleration in Eq. (3).

3. Parametric study

3.1 Analysis parameters

In this study, the residual displacement, x_r , of the partially self-centering system is related to the maximum inelastic displacement by the following relation

$$x_r = \gamma x_m \tag{5}$$

where γ is the residual displacement ratio. However, the value of x_m is generally not available in advance before the simulation is done. According to the well-known equal displacement rule (Veletsos and Newmark 1960), it can be stated that the ductility demand is approximately equal to the strength reduction factor for structures with moderate or long periods (Seo and Sause 2005). Therefore, the residual displacement can be estimated from Eqs. (4) and (5) as,

$$x_r = \gamma R x_v \tag{6}$$

In this parametric study, the strength reduction factor *R* values considered are 2, 4 and 6, which are intended to be representative of strong systems, medium-strength systems and weak-strength systems relative to the ground motion intensity respectively. Four different values are considered for α , β and γ respectively as listed in Table 1. The initial system periods used in this study range between 0.1 to 1.9 sec. with the interval of 0.2 sec.

Table 1 The values of the parameters considered in the parametric study



Fig. 3 Constant-R ductility demand spectra

The values of α , β and γ produces a total of 64 different combinations for fully self-centering systems ($\gamma = 0$) or partially self-centering systems. For convenience, these combinations are denoted as SC α - β or PSC α - β - γ , where SC and PSC are the short names denoting self-centering and partially self-centering systems respectively; α , β and γ are the corresponding parameter values in percent. For example, SC5-25 represents a fully self-centering system with α equal to 0.05 and β equal to 0.25; PSC5-25-10 denotes a partially self-centering system with α equal to 0.05, β equal to 0.25 and γ equal to 0.10.

3.2 Ductility demand spectrum

The ensemble average ductility demand spectra over the suite of 192 scaled earthquake records for the SC 5-25 and PSC 5-25-10 systems with *R* equal to 2.0, 4.0 and 6.0 are shown in Fig. 3, where $\overline{\mu}_R$ denotes the ensemble average value of the ductility demand over the 192 records. It is seen that in general the $\overline{\mu}_R$ spectrum can be divided into two distinctive regions, one is the short-period region where $\overline{\mu}_R$ is strongly period-dependent and tends to increase as the period decreases, while the other region is in the long-period region where $\overline{\mu}_R$ tends towards a constant value as the period increases. It is observed that $\overline{\mu}_R$ is around 2, 5 and 7 for the strength reduction factor *R* values of 2.0, 4.0 and 6.0 in the moderate- and long-period region (e.g. T > 1.2 sec.) respectively, implying that the "equal displacement rule" well-established for elasto-plastic systems is also valid for the moderate- and long-period region of the partially self-centering systems, the equal displacement rule may not yield conservative estimate. More extensive research need to be done to quantify the *R*- μ relationship for partially self-centering system in the future.



(a) $\overline{\mu}_R$ spectra for PSC systems with different α (b) Ratios of $\overline{\mu}_R$ for PSC systems with different α values to that of PSC5-25-10 system

Fig. 4 Effect of α on $\overline{\mu}_R$ spectra (*R*=4.0)

Fig. 3 also shows that $\overline{\mu}_R$ increases with increasing *R* values. Also, the period separating the two regions on the spectrum curve increases with the increase of *R* values. It is also worth noting that the three ductility demand spectral curves of the partially self-centering system all fall below the corresponding curve of the fully self-centering system. For *R* values equal to 2.0, 4.0 and 6.0, the average reduction in $\overline{\mu}_R$ of PSC 5-25-10 compared to SC 5-25 is 0.242, 0.611 and 0.808 respectively. This reduction is greater for *R* value of 6.0, meaning that the partially self-centering system, especially for those cases with relatively lower strength values.

3.3 Effect of parameter variation on displacement ductility demand: single variable change

3.3.1 Effect of post-yield stiffness ratio α

To investigate the effect of post-yield stiffness α on $\overline{\mu}_R$ of the partially self-centering systems, the $\overline{\mu}_R$ spectra for the partially self-centering systems within the range of α values considered in this parametric study are computed and plotted in Fig. 4(a). In all cases, β , γ and R are set to be 0.25, 0.10 and 4 respectively. It can be seen that $\overline{\mu}_R$ generally decreases with the increase of α over the period range considered, although in practice increasing the post-yield stiffness of partially self-centering systems may be costly, for example, by using more high-strength pre-tensioning tendons, thus increasing cost due to additional steel tendon use and anchoring requirements for these pre-tension steel tendon.

Fig. 4(b) shows the ratios of $\overline{\mu}_R$ for the PSC10-25-10, PSC20-25-10 and PSC30-25-10 systems to that of the PSC5-25-10 system respectively. It can be seen that the effect of α on $\overline{\mu}_R$ depends on the system's initial period. For periods less than 0.5 s, increasing α values is more effective in reducing $\overline{\mu}_R$ and the reduction becomes smaller with the increase of the period.



(a) $\overline{\mu}_R$ spectra for PSC systems with different β (b) Ratios of $\overline{\mu}_R$ for PSC systems with different values β values to that of PSC5-25-10 system



However, for periods longer than 0.5 s, the effect of α is less effective and the reduction seems to be independent of the period value.

3.3.2 Effect of energy dissipation coefficient β

Fig. 5(a) shows the $\overline{\mu}_R$ spectra for the partially self-centering systems over the range of β values considered in this parametric study. For all cases, α , γ and R are kept to be constant equal to 0.05, 0.10 and 4 respectively. Similar to the above observations made for the variation of α value, $\overline{\mu}_R$ generally decreases with the increase of β over the period range. However, the change in $\overline{\mu}_R$ is relatively small (within 10 percent of each other) as seen in Fig. 5(a).

Fig. 5(b) shows the ratios of $\overline{\mu}_R$ for the PSC5-50-10, PSC5-75-10 and PSC5-100-10 systems to that of the PSC5-25-10 system. In comparison with the effect of α , the reduction of $\overline{\mu}_R$ with increasing β values remains largely unchanged over the concerned period range, implying that the effect of β on $\overline{\mu}_R$ is independent of the system's period. Additionally, the effect of energy dissipation coefficient β appears to be slightly larger than the effect of post-yield stiffness α on $\overline{\mu}_R$ of the partially self-centering systems, for the period range greater than 0.5 sec. Increasing the value of energy dissipation coefficient β requires the use of supplemental dampers and thus may not be a cheap option. Furthermore, increasing energy dissipation capacity of partially self-centering systems by certain measures such as friction dampers may also lead to increased residual displacement, as observed in previous research works described in the Introduction section.

3.3.3 Effect of residual displacement ratio γ

Fig. 6(a) shows the $\overline{\mu}_R$ spectra for the partially self-centering systems with the range of γ values considered in this parametric study. In all cases, α , β and R values are equal to 0.05, 0.25 and 4 respectively. It can be seen that $\overline{\mu}_R$ generally decreases with the increase of γ over the



(a) μ_R spectra for PSC systems with different γ values

ms with different γ (b) Ratios of $\overline{\mu}_R$ for PSC systems with different γ values to that of SC5-25 system Fig. 6 Effect of γ on $\overline{\mu}_R$ spectra (R=4.0)



Fig. 7 Effect of combined change of α and β on $\overline{\mu}_R$ ($\gamma=0.10$)

period range, and this change in $\overline{\mu}_R$ is more pronounced than the effects of α and β , for the period range greater than 0.5 sec.

Fig. 6(b) shows the ratios of $\overline{\mu}_R$ for the PSC5-25-10, PSC5-25-20 and PSC5-25-30 systems to that of the SC5-25 system. It can be seen that for a small value of γ (for example, $\gamma = 0.1$), its effect on $\overline{\mu}_R$ is approximately independent of the system's period; however, for a larger value of

 γ (e.g. $\gamma = 0.3$), the effect is most effective at the period value around 0.3 s. For system with a period of 0.3 s, the value of $\overline{\mu}_R$ for partially self-centering system with γ of 0.3 drops by 28 percent compared with the corresponding self-centering system. This can be partially explained by the fact that when the residual displacement ratio γ increases, more energy dissipation through hysteretic damping is available to the system, thus reducing the system's displacement ductility demand. Apparently, when γ increases, the merits of self-centering systems, that is, small residual displacement, gradually fade away.

3.4 Effect of parameter variation on ductility demand: multivariate change

From the observations made above, increasing the individual values of α , β or γ respectively can reduce the ductility demand of the partially self-centering systems. In this section, combined variation of these parameters is examined in order to provide guidance on selecting design parameter values to achieve a desired ductility for the partially self-centering systems.

3.4.1 Effect of post-yield stiffness ratio and energy dissipation coefficient combined

To investigate the effect of the combined change of α and β on $\overline{\mu}_R$ for the partially self-centering systems, the $\overline{\mu}_R$ spectra for the partially self-centering systems with the range of α and β values considered in this study are shown in Fig. 7. The initial periods of these systems are 0.5 s and 1.5 s for *R* equal to 2 and 6 respectively. In all cases, γ is set to be 0.10. It is seen that increasing α or β values generally deceases the $\overline{\mu}_R$ value, which is more significant for the systems with larger *R* values (systems with relatively low strength). For partially self-centering systems with smaller *R* values (systems with relatively high strength), increasing α or β is not very effective in reducing the $\overline{\mu}_R$ value.

Fig. 7 also shows that for the partially self-centering systems with lower strength, the effect of β on $\overline{\mu}_R$ depends on the particular value of α . The smaller value α is, the more rapidly $\overline{\mu}_R$ decreases with the increase of β . Similarly, the effect of α on $\overline{\mu}_R$ also depends on to the value of β . For a smaller value of β , a larger reduction of $\overline{\mu}_R$ can be achieved with the increase of α values.

3.4.2 Effect of post-yield stiffness ratio and residual displacement ratio combined

Fig. 8 shows the $\overline{\mu}_R$ values for the partially self-centering systems with the range of α and γ values considered in this parametric study. The initial periods of these systems are 0.5 and 1.5 s with *R* equal to 2 and 6 respectively. In all cases, β is set to be 0.25. It is seen that increasing α or γ generally leads to deceased $\overline{\mu}_R$ values, and it is more significant for systems with larger *R* values (systems with relatively lower strength). For the partially self-centering systems with smaller *R* values (systems with relatively higher strength), increasing α or γ values is not as effective.

Fig. 8 also shows that for the partially self-centering systems with lower strength, the effect of γ on $\overline{\mu}_R$ is largely independent of α . Similarly, increasing α values reduces $\overline{\mu}_R$ uniformly over



Fig. 8 Effect of combined change of α and γ on $\overline{\mu}_R$ (β =0.25)



Fig. 9 Effect of combined change of β and γ on $\overline{\mu}_R$ (α =0.05)



Fig. 10 Effect of combined changes of α , β and γ on $\overline{\mu}_R$ (*R*=4.0, *T*=1.1 s)

the range of γ considered in this parametric study, implying that the effect of α on $\overline{\mu}_R$ is also independent of γ .

3.4.3 Effect of energy dissipation parameter and residual displacement ratio combined

Fig. 9 shows the $\overline{\mu}_R$ values for the partially self-centering systems with the range of β and γ values considered in this parametric study. The initial periods of these systems are 0.5 and 1.5 s for R equal to 2 and 6 respectively. For all cases, α is equal to 0.05. It is seen that increasing β or γ generally results in deceased $\overline{\mu}_R$ values, and this is more significant for the systems with larger R values (systems with relatively lower strength). For the partially self-centering systems with smaller R values (systems with relatively higher strength), increasing β or γ values is not as effective.

Fig. 9 shows that for the partially self-centering systems, the effect of γ on $\overline{\mu}_R$ depends on the value of β . The smaller value β has, the more rapidly $\overline{\mu}_R$ decreases with the increase of γ . For example, for system with the initial period of 0.5 s and R equal to 6, the difference of $\overline{\mu}_R$ between partially self-centering system with γ of 0.3 and self-centering system is 2.194 for β equal to 0.25; however, for β equal to 1.00 the corresponding value is 0.882. Similarly, the effect of β on $\overline{\mu}_R$ is also related to the specific value of γ . For a smaller value of γ , a larger reduction in $\overline{\mu}_R$ can be achieved with the increase of β .

3.4.4 Effect of the three parameters combined

To investigate the combined effect of the three parameters, α , β and γ on $\overline{\mu}_R$ of the partially self-centering systems, variations of $\overline{\mu}_R$ with the range of α , β and γ values considered in this parametric study are shown in Fig. 10. The initial periods of these concerned systems are 1.1 s for *R* equal to 4. Similar observations as the above can be made.

For any partially self-centering system with a specified initial period and strength reduction factor, the relationship between $\overline{\mu}_R$ and the three parameters can be determined, which can be utilized to provide a basis to select a favorable combination of these parameters for partially self-centering systems. The values of these parameters can be determined with the objective to minimize the ductility demand of partially self-centering systems under seismic excitation while considering the following requirements: (1) these values should be achieved economically and (2) the residual deformation is below the accepted level. For example, suppose the initial period of a partially self-centering system is 1.1 s and the strength reduction ratio is equal to 4. Two possible combinations of the three parameters can be considered as candidates for design. For the first combination, α , β and γ are set to be 0.05, 1.0, 0 respectively; and the second candidate involves that α , β and γ values are equal to 0.20, 0.25, 0.1 respectively. From Fig. 10, it can be derived that the ductility demands for these two combinations are both around 3.9. Depending on the relative cost of realizing energy dissipation, post-yield stiffness, and technical challenges in eliminating residual displacements from the system, one of the two combinations can be selected for seismic design.

4. Comparative study of time history responses

For comparison purpose, two cases including one partially self-centering system and one self-centering system were analyzed subjected to a scaled version of the 1940 El Centro N-S earthquake record and the displacement responses are plotted in Fig. 11(a). A scaling factor of 1.73 is used. As shown in Fig. 11(b), the 5% damped elastic response spectrum of the scaled record is in good agreement with the target spectrum used in this study.

The two systems considered both have an initial period of 1.0 s and a mass of 4.0×10⁶ kg,



Fig. 11 Acceleration time history and elastic response spectrum for the scaled 1940 El Centro record



Fig. 12 Responses of the SC and PSC systems subjected to the scaled 1940 El Centro record (T=1.0 s,

which are intended to be representative of a 7-story steel moment resisting frame (MRF). For the two systems, the strength reduction factor *R* for both systems is set to be 6, and the post-yielding stiffness ratio α and energy-dissipation coefficient β are both set to 0.05 and 0.25 respectively. The residual displacement ratio is set to 0.10 for the partially self-centering system, while the self-centering system has no residual displacement by definition.

The displacement time history and hysteresis curves of the two systems subjected to the scaled 1940 El Centro record are shown in Fig. 12. It can be seen that though the partially self-centering system has a non-zero residual displacement (about 0.0115 m), its peak displacement is decreased by 12% compared to the fully self-centering system due to its increased energy dissipation capacity. This can be partially explained by the fact that more hysteretic energy dissipation occurs in the partially self-centering system as seen in the hysteresis loops shown in Fig. 12(b). Therefore, for certain cases, for example, those with small energy dissipation coefficient values, partially self-centering systems have the potential to achieve lower peak displacement values. The practical implication of this observation is: for certain self-centering systems, adding energy dissipation at a price of increased residual displacement may be desirable since peak displacement would also be reduced. Therefore, in practical application, a favorable combination of post-yield stiffness ratio, energy dissipation coefficient and residual displacement ratio for partially self-centering systems needs to be determined based on a prescribed criterion that considers ductility demand, relative cost of realizing energy dissipation, post-yield stiffness, and technical challenges in eliminating residual displacements from the system.

5. Conclusions

In this paper, a modified flag-shaped hysteresis model is presented for partially self-centering systems. Compared to fully flag-shaped hysteresis model, an additional parameter is included to account for the residual displacement of the system. With given initial period and yield strength, the hysteresis behavior of a partially self-centering system can be fully defined by post-yield stiffness ratio, energy dissipation coefficient and residual displacement ratio. In order to

 $R=6.0, \alpha=0.05, \beta=0.25$)

investigate the influence of these three parameters on the displacement ductility demand of partially self-centering systems, a parametric study has been conducted through nonlinear time history analysis based on an ensemble of 192 historical earthquake records scaled to the design basis earthquake (DBE) in southern California. The findings from this parametric study can be summarized as follows:

- (1) "Equal displacement rule" is verified for the moderate- and long-period region of strong partially self-centering systems. For medium or weak-strength partially self-centering systems, the equal displacement rule may not give conservative estimate. Definition for medium and weak-strength partial self-centering systems is based on the strength reduction factor R value, which is given after Eq. (6). Larger R value corresponds to weaker system since its strength is lower. More extensive research need to be done to quantify the R- μ relationship for partially self-centering system in the future.
- (2) In general, increasing post-yield stiffness ratio, energy dissipation coefficient or residual displacement ratio all lead to reduced displacement ductility demand for partially self-centering systems, especially for systems with lower yield strength. The effect of post-yield stiffness ratio on the ductility demand of partially self-centering systems depends on the initial period of the system. However, in general, increasing energy dissipation coefficient or residual displacement ratio reduces the ductility demand of the partially self-centering systems by an almost constant ratio over the period range considered in this parametric study.
- (3) The effect of energy dissipation coefficient on the ductility demand of partially self-centering systems is related to the post-yield stiffness ratio and residual displacement ratio. However, the latter two parameters are independent of each other in affecting the ductility demand of the partially self-centering systems considered in the parametric study.
- (4) With given initial period and strength level, a favorable combination of post-yield stiffness ratio, energy dissipation coefficient and residual displacement ratio can be determined for partially self-centering systems based on a prescribed criterion that considers ductility demand, relative cost of realizing energy dissipation, post-yield stiffness, and technical challenges in eliminating residual displacements from the system.

The P-Delta effect is not considered to avoid the complication from P-Delta effect on post-yield stiffness reduction in this study. The results of this study can be used if the reduction in post-yield stiffness due to P-Delta effect is known from other analysis.

Acknowledgments

The authors are grateful to the University of Maryland for providing partial financial support for this research project. However, the opinions and conclusions expressed in this paper are solely those of the writers and do not necessarily reflect the views of the sponsors.

References

Adam, C., Ibarra, L.F. and Krawinkler, H. (2004), "Evaluation of p-delta effects in non-deteriorating MDOF structures from equivalent SDOF systems", Proc. 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada.

Cardone, D., Di Cesare, A., Ponzo, F.C. and Blonna, B. (2008), "Evaluation of behavior factor for

flag-shaped hysteretic model", Proc. International Conference on Engineering Optimization, Rio de Janeiro, Brazil.

- Christopoulos, C., Filiatrault, A. and Folz, B. (2002a), "Seismic response of self-centering hysteretic SDOF systems", *Earthq. Eng. Struct. D.*, **31**, 1131-1150.
- Christopoulos, C., Filiatrault, A., Folz, B. and Uang, C.M. (2002b), "Post-tensioned energy dissipating connections for moment-resisting steel frames", *ASCE J. Struct. Eng.*, **128**(9), 1111-1120.
- Christopoulos, C. and Pampanin, S. (2004), "Towards performance-based seismic design of MDOF structures with explicit consideration of residual deformations", *ISET J. Earthq. Technol.*, **41**(1), 53-73.
- Desroches, R. and Smith, B. (2004), "Shape memory alloys in seismic resistant design and retrofit: a critical review of their potential and limitations", J. Earthq. Eng., 8(3), 415-429.
- Dolce, M., Cardone, D. and Marnetto, R. (2000), "Implementation and testing of passive control devices based on shape memory alloys", *Earthq. Eng. Struct. D.*, 29(7), 945-968.
- Dolce, M., Cardone, D., Ponzo, F.C. and Valente, C. (2005), "Shaking table tests on reinforced concrete frames without and with passive control systems", *Earthq. Eng. Struct. D.*, **34**(14), 1687-1717.
- Kurama, Y., Sause, R., Pessiki, S. and Lu, L.W. (1999), "Lateral load behavior and seismic design of unbonded post-tensioned precast concrete walls", ACI Struct. J., 96(4), 622-632.
- Kam, W.Y., Pampanin, S., Palermo, A. and Carr, A.J. (2010), "Self-centering structural systems with combination of hysteretic and viscous energy dissipations", *Earthq. Eng. Struct. D.*, **39**(10), 1083-1108.
- Kurama, Y.C. (2001), "Simplified seismic design approach for friction-damped unbonded post-tensioned precast concrete walls", ACI Struct. J., **98**(5), 705-716.
- Mahin, S., Sakai, J. and Jeong, H. (2006), "Use of partially prestressed reinforced concrete columns to reduce post-earthquake residual displacements of bridges", *Proc., 5th National Seismic Conf. on Bridges & Highways*, San Francisco.
- Miller, D.J., Fahnestock, L.A. and Eartherton, M.R. (2011), "Self-centering buckling-restrained braces for advanced seismic performance", *ASCE Struct. Congress*, Las Vegas, Nevada.
- Pampanin, S., Christopoulos, C. and Priestley, M.J.N. (2003), "Performance-based seismic response of frame structures including residual deformations. Part II: multi-degree of freedom systems", J. Earthq. Eng., 7(1), 119-147.
- Ricles, J.M., Sause, R., Garlock, M.M. and Zhao, C. (2001), "Posttensioned seismicresistant connections for steel frames", J. Struct. Eng., 127(2), 113-121.
- Rodgers, G.W., Chase, J.G., Mander, J.B., Dhakal, R.P. and Solberg, K.M. (2007), "DAD post-tensioned concrete connections with lead dampers: analytical models and experimental validation", *Proc. 8th Pacific Conference on Earthquake Engineering (8PCEE)*.
- Ruiz-Garcia, J. and Miranda, E. (2006a), "Evaluation of residual drift demands in regular multi-storey frames for performance-based seismic assessment", *Earthq. Eng. Struct. D.*, 35(13), 1609-1629.
- Ruiz-Garcia, J. and Miranda, E. (2006b), "Residual displacement ratios for assessment of existing structures", *Earthq. Eng. Struct. D.*, **35**(3), 315-336.
- Seo, C.Y. and Sause, R. (2005), "Ductility demands on self-centering systems under earthquake loading", ACI Struct. J., 102(2), 275-285.
- Somerville, P. and Collins, N. (2002), *Ground motion time histories for the Van Nuys building*, URS Corporation.

380

- Song, G., Ma, N. and Li, H.N. (2006), "Applications of shape memory alloys in civil structures", *Eng. Struct.*, **28**(9), 1266-1274.
- Tremblay, R., Christopoulos, C., Erochko, J. and Kim, H. (2010), "Experimental validations and design of self-centering energy dissipative (SCED) bracing systems", Proc. 9th U.S. National and 10th Canadian Conference on Earthquake Engineering, Toronto, Canada.
- Veletsos, A.S. and Newmark, N.M. (1960), "Effect of inelastic behavior on the response of simple systems to earthquake motions", *Proceedings of the Second World Conference on Earthquake Engineering*, 2, 895-912.
- Wolski, M., Ricles, J.M. and Sause, R. (2009), "Experimental study of a self-centering beam-column connection with bottom flange friction device", J. Struct. Eng., 135(5), 479-448.
- Zhu, S. and Zhang, Y. (2008), "Seismic analysis of concentrically braced frame systems with self-centering friction damping braces", *ASCE J. Struct. Eng.*, **134**(1), 121-131.

JL