Earthquakes and Structures, Vol. 4, No. 1 (2013) 1-10
DOI: http://dx.doi.org/10.12989/eas.2013.4.1.001

Analysis of wave motion in an anisotropic initially stressed fiber-reinforced thermoelastic medium

Raj Rani Gupta[*1] and Rajani Rani Gupta[2]

[1]*Department of Mathematics, MMU, Mullana, Ambala, Haryana, India*
[2]*Department of Mathematics and Applied Sciences, MEC, Muscat, Oman*

(Received August 22, 2010, Revised December 23, 2011, Accepted January 6, 2012)

Abstract. The present investigation deals with the analysis of wave motion in the layer of an anisotropic, initially stressed, fiber reinforced thermoelastic medium. Secular equations for symmetric and skew-symmetric modes of wave propagation in completely separate terms are derived. The amplitudes of displacements and temperature distribution were also obtained. Finally, the numerical solution was carried out for Cobalt and the dispersion curves, amplitudes of displacements and temperature distribution for symmetric and skew-symmetric wave modes are presented to evince the effect of anisotropy. Some particular cases are also deduced.

Keywords: wave propagation; initially stressed; fiber-reinforced; transversely isotropic; amplitudes.

1. Introduction

The influence of pre-existing stress on elasticity of solids referred as initial stress and strain or external stress is quite important subject and has been investigated by number of researchers. The propagation of elastic waves of a fiber-reinforced medium plays a great role in the practical problems of civil engineering and geophysics. Effect of earthquake on artificial structures near the surface of the earth is also of prime importance. A structure is excited during an earthquake and similar disturbances, which may cause more or less violent vibrations. These vibrations depend on the ground vibration as well as on the physical properties of the structures (Richter 1958). Most concrete construction on or near the surface of the earth includes steel reinforcing. The characteristic property of reinforced concrete member is that its components, namely concrete and steel act together as a single anisotropic unit as long as they remain in the elastic condition, i.e., the components are bound together without relative displacement. However, due to the mismatch of material properties, there exists a residual stress during the manufacture process of fiber-reinforced material. On the contrary, to prevent the fiber-reinforced material from brittel fracture, the layered structure is usually pre-stressed during the manufacture process. During the last five decades considerable attention has been directed towards this phenomenon. Biot (1965) in his work depicted the difference between the acoustic propagation under initial stress and in stress free state.

[*]Corresponding author, Assistant Professor, E-mail: raji.mmec@gmail.com

Kumar and Gupta (2010) discussed the wave motion in micropolar transversely isotropic thermoelastic half space without energy dissipation. To the author's knowledge, no work has been carried out so far to discuss the effect of initial stress on the propagation behavior of Lamb waves in the layered fiber-reinforced structure.

In this article, analytic investigation of the propagation of Lamb waves in the layer of fiber-reinforced transversely isotopic thermoelastic initially stressed medium is considered. The effect of the anisotropy on the phase velocity, attenuation coefficient and specific loss is presented and illustrated graphically, for symmetric and skew-symmetric modes. The amplitude ratios of displacements and temperature distribution are also obtained, to evince the effect of anisotropy. This study has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy.

2. Basic equations

The linear equations governing thermoelastic interactions in homogeneous transversely isotropic initially stressed fibre-reinforced thermoelastic solid are

Constitutive relations

$$t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta(a_k a_m e_{km} a_i a_j) - \beta_{ij}T, \tag{1}$$

The deformation tensor is defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i, j = 1, 2, 3. \tag{2}$$

Balance law: The balance laws for initially stressed fiber-reinforced linearly elastic medium whose preferred direction is that of **a** are (Dhaliwal and Sherief 1980, Kumar and Gupta 2010, Kolsky 1963)

$$t_{ij,j} - P\omega_{ij,j} = \rho\ddot{u}_i, \tag{3}$$

Equation of heat conduction: Following, Lord and Shulman (1967)

$$K_{ij}T_{,ij} = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(T_0\beta_{ij}\dot{u}_{i,j} + \rho C_e\dot{T}), \quad i, j = 1,2,3. \tag{4}$$

where ρ is the mass density, t_{ij} are components of stress, u_i the mechanical displacement, e_{ij} are components of infinitesimal strain, P is the normal initial stress, $\omega_{ij} = (u_{j,i} - u_{i,j})/2$, T the temperature change of a material particle, T_0 the reference uniform temperature of the body, K_{ij} the heat conduction tensor, β_{ij} the thermal elastic coupling tensor, c^* the specific heat at constant strain, a_j are components of **a**, all referred to cartesian coordinates. The vector **a** may be a function of position. The coefficients $\lambda, \mu_L, \mu_T, \alpha$ and β are elastic constants with the dimension of stress. We choose **a** (Lord and Shulman 1967) so that its components are (1, 0, 0). The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

3. Problem formulation

Following Slaughter (2002), appropriate transformations have been used on the set of Eq. (1), for deriving the equations for transversely isotopic medium and restricted our analysis to the two dimensional problem.

In the present paper, an infinite layer with traction free surfaces at $x_2 = \pm H$ (layer of thickness $2H$), which consists of homogeneous, initially stressed, fiber reinforced transversely isotropic thermoelastic material is considered. The origin of the coordinate system (x_1, x_2, x_3) is taken on the middle surface of the layer. The $x_1 - x_3$ plane is chosen to coincide with the middle surface and x_2 -axis normal to it along the thickness. For the two-dimensional problem, we assume the components of the displacement vector of the form

$$\vec{u} = (u_1, u_2, 0) \tag{5}$$

and assume that the solutions are explicitly independent of x_3, i.e., $\partial / \partial x_3 \equiv 0$. Thus the field equations and constitutive relations for such a medium reduces to

$$c_{11}\frac{\partial^2 u_1}{\partial x_1{}^2} + c_{55}\frac{\partial^2 u_1}{\partial x_2{}^2} + (c_{12} + c_{55})\frac{\partial^2 u_3}{\partial x_1 \partial x_2} - \frac{P}{2}(\frac{\partial^2 u_2}{\partial x_1 \partial x_2} - \frac{\partial^2 u_1}{\partial x_2{}^2}) - \beta_1 \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{6}$$

$$c_{55}\frac{\partial^2 u_2}{\partial x_1{}^2} + c_{22}\frac{\partial^2 u_2}{\partial x_2{}^2} + (c_{12} + c_{55})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} - \frac{P}{2}(\frac{\partial^2 u_1}{\partial x_1 \partial x_2} - \frac{\partial^2 u_2}{\partial x_2{}^2}) - \beta_2 \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{7}$$

$$K_1 \frac{\partial^2 T}{\partial x_1{}^2} + K_2 \frac{\partial^2 T}{\partial x_2{}^2} - \rho c^* (\frac{\partial T}{\partial t} + \tau_o \frac{\partial^2 T}{\partial t^2}) = T_o (\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2})(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_2 \frac{\partial u_2}{\partial x_2}), \tag{8}$$

$$t_{22} = c_{12}\frac{\partial u_1}{\partial x_1} + c_{22}\frac{\partial u_2}{\partial x_2} - \beta_2 T,\ t_{21} = c_o (\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}), \tag{9}$$

where $\beta_1 = (c_{11} + c_{13})\alpha_1 + c_{13}\alpha_2$, $\beta_2 = (c_{33} + c_{13} - c_{55})\alpha_1 + c_{33}\alpha_2$, $c_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta$, $c_{13} = \lambda + \alpha$, $c_{22} = c_{33} = \lambda + 2\mu_T$, $c_o = c_{44}/2$, $c_{44} = c_{66} = 2\mu_L$, $c_{55} = 2\mu_T$, $c_{23} = c_{33} - c_{55}$, and λ, α, β, μ_L, μ_T are material constants, K_1, K_2 are coefficients of thermal conductivity, τ_o is thermal relaxation time, u_1, u_2 are the components of displacement vector.

For further considerations, it is convenient to introduce the non-dimensional quantities defined by

$$x_i^{'} = \frac{\omega_1^* x_i}{v_1}, u_i^{'} = \frac{\rho v_1 \omega_1^* u_i}{\beta_1 T_o},\ t_{ij}^{'} = \frac{t_{ij}}{\beta_1 T_o}, t' = \omega_1^* t, T' = \frac{T}{T_o}, \tag{10}$$

where $\omega_1^* = \frac{c^* c_{11}}{k_1}, v_1^2 = \frac{c_{11}}{\rho}.$

4. Boundary conditions

The boundary conditions for the thermally insulated, initially stressed fiber-reinforced transversely isotropic layer are the vanishing of normal stress, tangential stress and temperature distribution. Therefore, we consider the following non-dimensional boundary conditions at $x_2 = \pm H$ are given by

$$t_{22} = 0,\ t_{21} = 0, \frac{\partial T}{\partial x_2} = 0. \tag{11}$$

5. Normal mode analysis and solution of the problem

The solution for (u_1, u_2, T) representing propagating waves in the $x_1 - x_2$ plane is assumed to be of the form

$$(u_1, u_3, T) = (\bar{u}_1, \bar{u}_3, \bar{T}) e^{i\xi(x_1 + m x_3 - ct)} \tag{12}$$

where ξ is the wave number , $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave, *m* is the unknown parameter which signifies the penetration depth of the wave.

With the help of Eqs. (10) and (12), Eqs. (6)-(8) reduced to (after suppressing primes)

$$[(a_2 m^2 + a_1)\bar{u}_1 + m a_3 \bar{u}_2 + a_4 \bar{T}] e^{i\xi(x_1 + m x_2 - ct)} = 0,$$

$$[m a_5 \bar{u}_1 + (a_6 m^2 + a_7)\bar{u}_2 + a_8 m \bar{T}] e^{i\xi(x_1 + m x_2 - ct)} = 0,$$

$$[a_9 \bar{u}_1 + m a_{10} \bar{u}_2 + (a_{11} + a_{12} m^2)\bar{T}] e^{i\xi(x_1 + m x_2 - ct)} = 0, \tag{13}$$

where

$$a_1 = \frac{(\rho c^2 - c_{11})\xi^2}{c_{55}}, a_2 = (\frac{P}{2c_{55}} - 1)\xi^2, a_3 = -(\frac{c_{12} + c_{55}}{c_{55}} + \frac{P}{2c_{55}})\xi^2, a_4 = -\frac{i\beta_1 \xi}{c_{55}},$$

$$a_5 = (-\frac{c_{12} + c_{55}}{c_{55}} + \frac{P}{2c_{55}})\xi^2, a_6 = -(1 + \frac{c_{22}}{c_{55}})\xi^2, a_7 = (\frac{c_{11}}{c_{55}} - \frac{P}{2c_{55}})\xi^2, a_8 = -\frac{i\beta_2 \xi}{c_{55}},$$

$$a_9 = -\frac{T_o \beta_1}{c_{55}}(\omega - i\xi\tau_o\omega^2)\xi,\ a_{10} = a_8(\xi\tau_o\omega^2 - i\omega), a_{11} = -\frac{k_1\xi^2}{c_{55}} - \frac{\rho c^*(i\omega + \tau_o\omega^2)}{c_{55}}, a_{12} = -\frac{k_2\xi^2}{c_{55}}$$

The condition for the non-trivial solution of system of Eq. (13), yields a cubic equation in m^2 as

$$Am^6 + Bm^4 + Cm^2 + D = 0, \tag{14}$$

where

$$A = a_2 a_6 a_{12}, B = a_1 a_6 a_{12} + a_2 a_6 a_{11} + a_2 a_7 a_{12} - a_2 a_8 a_{10} - a_3 a_5 a_{12}, D = a_7 (a_1 a_{11} - a_4 a_6),$$
$$C = a_1 (a_6 a_{11} + a_7 a_{12}) + (a_2 a_7 - a_3 a_5) a_{11} + (a_3 a_9 - a_1 a_{10}) a_8 + a_4 (a_5 a_{10} - a_6 a_9).$$

The roots of this equation give three values of c^2. Three positive values of c will be the velocities of propagation of three possible waves. The waves with velocities c_1, c_2, c_3 correspond to three types of quasi waves propagating into the medium. Let us name these waves as, quasi-longitudinal displacement (qLD) wave, quasi transverse displacement (qTD) and quasi thermal wave (qT) wave.

So Eq. (14) leads to the following solution for displacements and temperature distribution

$$(u_1, u_2, T) = \sum_{j=1}^{3} [A_j \cos(\xi m_j x_2) + B_j \sin(\xi m_j x_2)](1, r_j, t_j) e^{i\xi(x_1 - ct)}, \tag{15}$$

where

$$r_j = \frac{-[(a_8 a_9 - a_5 a_{11}) m_j - a_5 a_{12} m_j^3]}{a_6 a_{12} m_j^4 + (a_6 a_{11} + a_7 a_{12} - a_8 a_{10}) m_j^2}, \; t_j = \frac{(a_5 a_{10} - a_6 a_9) m_j^2 - a_6 a_7}{a_6 a_{12} m_j^4 + (a_6 a_{11} + a_7 a_{12} - a_8 a_{10}) m_j^2}.$$

6. Derivation of secular equation

Substituting the values of u_1, u_2 and T in the boundary conditions Eq. (11) at the surfaces $\pm H$ of the layer

$$\begin{gathered}
\sum_{j=1}^{3} \{A_j [(g_1 + g_{2j}) c_j + g_{3j} s_j] + B_j [g_{3j} c_j - (g_1 + g_{2j}) s_j]\} = 0, \; \sum_{j=1}^{3} \{-A_j g_{6j} s_j + B_j g_{6j} c_j\} = 0 \\
\sum_{j=1}^{3} \{A_j [(g_1 + g_{2j}) c_j - g_{3j} s_j] + B_j [g_{3j} c_j + (g_1 + g_{2j}) s_j]\} = 0, \; \sum_{j=1}^{3} \{A_j g_{6j} s_j + B_j g_{6j} c_j\} = 0 \\
\sum_{j=1}^{3} \{A_j [g_{4j} s_j + g_{5j} c_j] + B_j [g_{4j} c_j - g_{5j} s_j]\} = 0, \sum_{j=1}^{3} \{A_j [-g_{4j} s_j + g_{5j} c_j] + B_j [g_{4j} c_j + g_{5j} s_j]\} = 0,
\end{gathered} \tag{16}$$

where

$$s_j = \sin(m_j x_2 \xi), c_j = \cos(m_j x_2 \xi), g_1 = c_{13} i\xi, g_{2j} = -\beta_2 t_j,$$

$$g_{3j} = r_j m_j c_{33} \xi, g_{4j} = c_o m_j \xi, g_{5j} = r_j i\xi, g_{6j} = m_j t_j \xi, \; j = 1,2,3.$$

In order that the six boundary conditions given by Eq. (11) be satisfied simultaneously, the determinant of the coefficients of A_j and B_j (j=1, 2, 3) in Eq. (16) vanishes. This gives an equation for the frequency of the layer oscillations. The frequency equation for the waves in the present case, after applying lengthy algebraic reductions and manipulations of the determinant leads to the following secular equations

$$[T_1]^{\pm}[g_{61}g_{42}(g_1+g_{23})-g_{61}g_{43}(g_1+g_{22})]+[T_2]^{\pm}[g_{62}g_{43}(g_1+g_{21})-g_{62}g_{42}(g_1+g_{23})]$$
$$+[T_3]^{\pm}[g_{63}g_{41}(g_1+g_{22})-g_{63}g_{42}(g_1+g_{21})]=0. \quad (17)$$

These are the frequency equations which correspond to the symmetric and skew symmetric mode with respect to the medial plane $x_3 = 0$. Here, the superscript '+' corresponds to skew symmetric and '-' refers to symmetric modes and $T_j = \tan(m_j x_2 \xi),\ j = 1,2,3.$

6.1 Amplitudes of displacements and temperature distribution

In this section the amplitudes of displacement components and temperature distribution for symmetric and skew symmetric modes of plane waves can be obtained as

$$[(u_1)_{sym},(u_1)_{asym}]=\sum_{j=1}^{3}[A_j\cos(m_jx_2\xi),B_j\sin(m_jx_2\xi)]e^{i\xi(x_1-ct)},$$

$$[(u_2)_{sym},(u_2)_{asym}]=\sum_{j=1}^{3}r_j[A_j\sin(m_jx_2\xi),B_j\cos(m_jx_2\xi)]e^{i\xi(x_1-ct)},$$

$$[(T)_{sym},(T)_{asym}]=\sum_{j=1}^{3}t_j[A_j\sin(m_jx_2\xi),B_j\cos(m_jx_2\xi)]e^{i\xi(x_1-ct)}. \quad (18)$$

6.2 Specific loss

The specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum. Kolsky (1963), shows that specific loss $(\Delta W / W)$ is, c times the absolute value of the ratio of the imaginary part of wave number to the real part of wave number i.e.,

$$\frac{\Delta W}{W}=4\pi\left|\frac{\mathrm{Im}(k)}{\mathrm{Re}(k)}\right| \quad (19)$$

He noted that specific loss is the most direct method of defining internal friction for a material.

6.3 Particular case

(1) Isotropic Elastic Case**:** Taking $\mu_L = \mu_T = \mu$ and $\alpha = \beta = 0$ the Eq. (17), the corresponding expression for initially stressed isotropic fiber-reinforced elastic solid are obtained.

(2) In the limiting case if on neglecting the effect of initial stress, the results for transversely isotropic fiber-reinforced thermoelastic solid are recovered.

7. Numerical results and discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. For the purpose of numerical computations, we have used Matlab's Programming. The following relevant physical constants are taken for a fiber-reinforced transversely isotopic material

$$\rho = 2.66\times 10^{3}\, Kg/m^{3},\;\; \lambda = 5.65\times 10^{10}\, N/m^{2},\;\; \mu_T = 2.46\times 10^{10}\, N/m^{2},\;\; \mu_L = 5.66\times 10^{10}\, N/m^{2}$$

$$\alpha = -1.28\times 10^{10}\, N/m^{2},\quad \beta = 220.90\times 10^{10}\, N/m^{2},\quad K_1 = .0921\times 10^{3}\, Jm^{-1}\deg^{-1} s^{-1},$$

$$K_2 = .0963\times 10^{3}\, Jm^{-1}\deg^{-1} s^{-1},\;\; \alpha_1 = .017\times 10^{4}\deg^{-1},\;\; \alpha_2 = .015\times 10^{4}\deg^{-1},$$

$$c^{*} = 0.787\times 10^{3}\, J\, Kg^{-1}\deg^{-1},\;\; T_0 = 293K,\; P = 100,\; \tau_0 = .05s,\; \omega = 2s^{-1}.$$

The plots of non-dimensional phase velocity, attenuation coefficient and specific loss with non-dimensional wave number restricted to thickness H=0.1 for symmetric and skew symmetric modes are shown in Figs. 1-6. Here, solid line with and without center symbol represent the variations corresponding to initially stressed fiber-reinforced thermoelastic transversely isotropic (ISFRTIT) and, for comparison, broken lines with and without center symbol represent the variations corresponding to initially stressed fiber-reinforced thermoelastic isotropic (ISFRIT). The lines shown in the figures without center symbol represent the variations corresponding to initial mode (n=1) of wave propagation, lines with center symbol (-*o*-) represent the variations corresponding to second mode (n=2) and lines with center symbol (-*x*-) represent the variations corresponding to final mode (n=3) of wave propagation.

Figs. 1 and 4 show the variations of phase velocity with respect to wave number for symmetric and skew symmetric modes, respectively. It is depicted from these figures that for higher modes of wave propagation (n=2, n=3) there is a sharp increase in the phase velocity over the interval (0, 0.5), but later on their values get decreases slowly and ultimately become constant with further increase in wave number. Whereas for initial mode of wave propagation, its value start with slow initial increase and become constant with further increase in wave number. The variations are almost similar with slight difference in the amplitudes for the cases of ISFRTIT and ISFRIT.

The variation of attenuation coefficient with respect to wave number for symmetric and skew symmetric modes can be depicted from Figs. 2 and 5, respectively. It is seen from Fig. 2 that for initial mode (n=1), the value of attenuation coefficient initially increase with small oscillation in

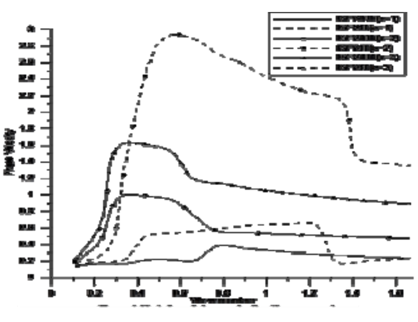

Fig. 1 Variation of phase velocity with wave number for symmetric mode

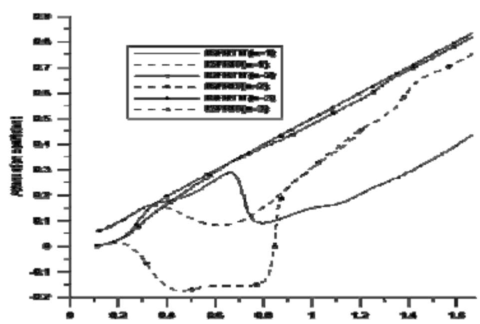

Fig. 2 Variation of attenation coefficient with wave number for symmetric mode

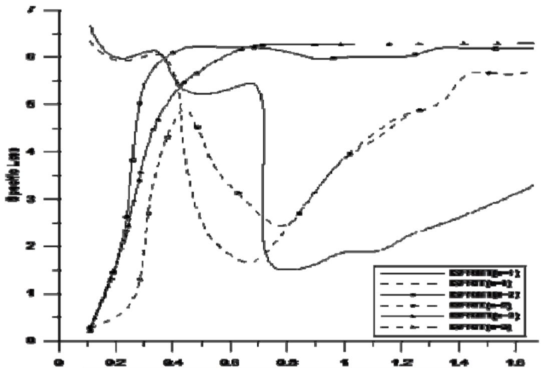

Fig. 3 Variation of specific loss with wave number for symmetric mode

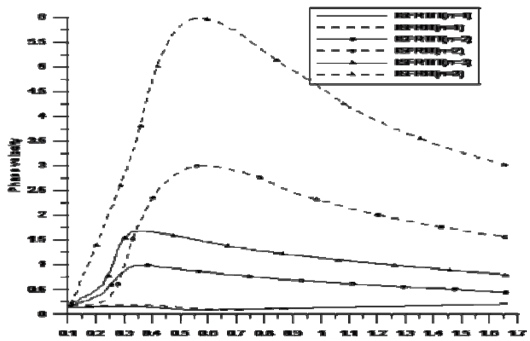

Fig. 4 Variation of phase velocity with wave number for skew symmetric mode

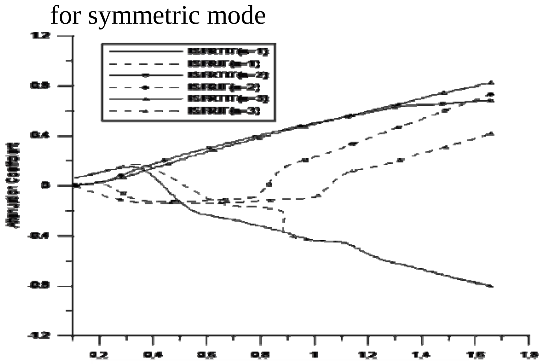

Fig. 5 Variation of attenuation coefficient with wave number for skew symmetric mode

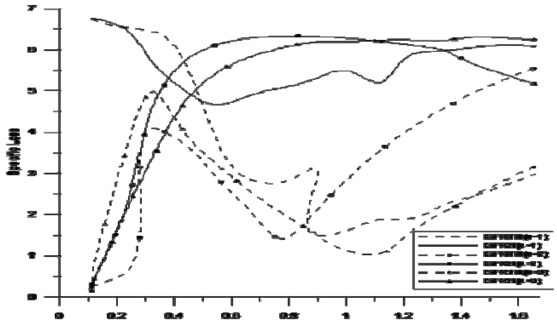

Fig. 6 Variation of specific loss with wave number for skew symmetric mode

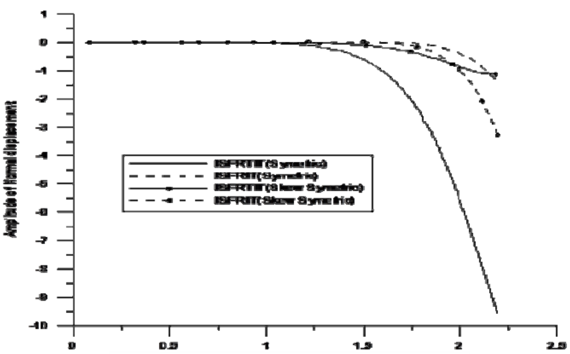

Fig. 7 Variations of amplitude of normal displacement with thickness of the layer

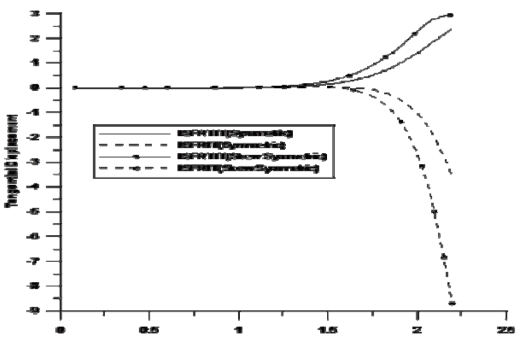

Fig. 8 Variations of amplitude of tangential displacement with thickness of the layer

the interval (0, 0.6), decreases sharply over the interval (0.6, 0.8) and then increases sharply with increase in wave number. For the next mode (n=2) and in the case of ISFRIT, its value decreases sharply in the interval (0, 0.4) and within the interval (0.4, 0.8) it nearly become constant and then

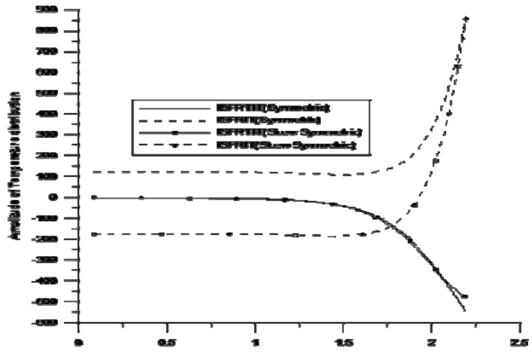

Fig. 9 Variations of amplitude of temperature distribution with thickness of the layer

increases sharply with increase in wave number. However for ISFRTIT and in both the case of higher mode (n=3), its value goes increases uniformly with wave number.

Figs. 3 and 6 illustrate the variations of specific loss with wave number for symmetric and skew symmetric modes. It is depicted from these figures that for the initial mode its value start with sharp initial decrease and then oscillate to attain a constant value, while for the higher modes (n=2, 3), its value sharply increases with increase in wave number and attain a constant value at the end.

Figs. 7-9 indicate the trend of variations of amplitudes of normal displacement, tangential displacement and temperature distribution with respect to thickness H of the layer. It is depicted from Fig. 7 that the amplitude of normal displacement remains constant initially up to 1.5 and then decreases sharply with increase in thickness of the layer. The variation pattern for both ISFRTIT and ISFRIT remain same. Figs. 8 and 9 depict the variation of tangential displacement and temperature distribution with thickness of the layer. It can be seen from Fig. 8 that for the case of ISFRTIT and for both symmetric and skew symmetric mode, its value initially remains constant and then goes on increasing with increase in depth. While for the case of ISFRIT reverse behavior is depicted. Also, the variations of temperature distribution are similar to those of tangential displacement, but with opposite behavior for ISFRTIT and ISFRIT after reaching the value 1.6.

8. Conclusions

The expression for the propagation of waves in an infinite layer of initially stressed fiber reinforced thermoelastic transversely isotropic medium after deriving the secular equation is derived. The phase velocity of higher modes of wave propagation for symmetric and skew-symmetric modes attain quite large values at vanishing wave number, which sharply flattens out to become steady with increasing wave number. The value of attenuation coefficient initially increases and then tends to zero at higher values of wave number. An appreciable of anisotropy is evinced from all the curves. The values of phase velocity get decreased with increase in anisotropy, while that of attenuation coefficient and specific loss oscillates arbitrarily.

References

Belfield, A.J., Rogers, T.G. and Spencer, A.J.M. (1983), "Stress in elastic plates reinforced by fibres lying in concentric circles", *J. Mech. Phys. Solids*, **31**(1), 25-54.
Biot, M.A. (1965), *Mechanics of incremental deformation*, John Wiley and Sons, Inc., New York.
Dhaliwal, R.S. and Sherief, H.H. (1980), "Generalized thermoelasticity for anisotropic media", *Q Appl. Math.*, **38**, 1-8.
Kumar, R. and Gupta, R.R. (2010), "Analysis of wave motion in micropolar trasnversely isotropic thermoelastic half space without energy dissipation", *Interact. Multiscale Mech.*, **3**(2), 145-156.
Kolsky, H. (1963), *Stress waves in solids*, Clarendon Press, Oxford; Dover Press, New York.
Liu, X. and Hu, G. (2004), "Inclusion problem of microstretch continuum", *Int. J. Eng. Sci.*, **42**(8-9), 849-860.
Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids*, **15**(5), 299-306.
Montanaro, A. (1999), "On singular surface in isotropic linear thermoelasticity with initial stress", *J. Acoust. Soc. Am.*, **106**(3), 1586-1588.
Richter, C.F. (1958), *Elementary seismology*, San Francisco, London: W.H. Freeman.
Singh, B. (2006), "Wave propagation in thermally conducting linear fibre-reinforced composite materials", *Arch. Appl. Mech.*, **75**(8-9), 513-520.
Slaughter, W.S. (2002), *The linearized theory of elasticity*, Birkhauser.
Spencer, A.J.M. (1972), *Deformation of fibre-reinforced materials*, London: Oxford University Press.

CC