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Seismic induced damageability evaluation of steel buildings: a Fuzzy-TOPSIS method

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Abstract. Seismic resiliency of new buildings has improved over the years due to better seismic codes and design practices. However, there is still large number of vulnerable and seismically deficient buildings. It is not economically feasible to retrofit and upgrade all vulnerable buildings, thus there is a need for rapid screening tool. Many factors contribute to the damageability of buildings; this makes seismic evaluation a complex multi-criteria decision making problem. Many of these factors are non-commensurable and involve subjectivity in evaluation that highlights the use of fuzzy-based method. In this paper, a risk-based framework earlier proposed by Tesfamariam and Saatcioglu (2008a) is extended using Fuzzy-TOPSIS method and applied to develop an evaluation and ranking scheme for steel buildings. The ranking is based on damageability that can help decision makers interpret the results and take appropriate decision actions. Finally, the application of conceptual model is demonstrated through a case study of 1994 Northridge earthquake data on seismic damage of steel buildings.

Keywords: seismic evaluation; Fuzzy-TOPSIS; hierarchical structure; linguistic variables; Fuzzy sets; damageability; multi-criteria decision-making (MCDM)

1. Introduction

Recent earthquakes, such as 1994 Northridge earthquake and 1995 Kobe earthquake, highlights sombre reality that a large number of existing steel buildings are vulnerable to seismic loads and jeopardize human lives. The seismically vulnerable steel buildings are designed and built on the basis of older seismic codes (non-ductile buildings) that have to be screened and retrofitted to minimize damage and improve life safety. However, Mahin (1998) reported that from the 1994 Northridge earthquake, modern code-conforming steel moment resisting frames had brittle fractures in welded steel beam-column connections. Due to a large numbers of vulnerable buildings and availability of limited financial resources, a reliable and rapid screening tool is needed for retrofit prioritization.

Various vulnerability assessment techniques have been proposed for rapid screening of steel buildings. Seismic loss modelling requires quantification of occurrence and magnitude of hazard and corresponding building damage. Several studies dealt with different aspect related to seismic reliability and risk assessment for steel buildings (Chang *et al.* 2009, Reyes-Salazar *et al.* 2012). Various techniques are proposed to assess building vulnerability and loss estimation, which entails

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empirical method (Tesfamariam and Liu 2010), heuristic method (Tesfamariam and Saatcioglu 2008a, 2010) and analytical method (Cornell *et al*. 2002, Fragiacomo *et al*. 2004). A point scoring method was first proposed in California in the mid seventies (Boissonnade and Shah 1985), subsequently, in the mid eighties, it is expanded into expert derived damage probabilities (ATC 1985). A rapid visual screening (RVS) is developed by FEMA 154 (ATC 2002). A three-tier process is developed by FEMA 310 (ASCE 1998). Other reported regional damage estimations are Canada (NRC 1992, 1993), New Zealand (NZSEE 2006).

For initial screening of steel buildings, many factors contribute to the damageability of a building; this makes seismic evaluation a multi-criteria decision making (MCDM) problem. The availability of hard data for these contributory factors is difficult and many of these factors are generally non-commensurate and also involve subjectivity in their evaluation. Since the data obtained from experts' judgment are mainly qualitative, therefore the evaluation process requires handling uncertainties related to vagueness or imprecision. For example, in a visual evaluation, an inspector assesses the quality of construction of a building and provides subjective/qualitative judgment such as good, average, or poor (Hadipriono and Ross 1991). Thus, the aggregation of non-commensurate factors and qualitative data induced uncertainty can be best handled using fuzzy-based methods (Zadeh 1965). A good introduction to the fuzzy based MCDM methods can be found in Carlsson and Fuller (1996). Ribeiro (1996) discussed about the processes for identifying, measuring and combining criteria and alternatives to create a conceptual model for decision and evaluation in fuzzy environments.

Tesfamariam and Saatcioglu (2008a) have introduced a heuristic-based risk assessment tool for RC buildings. This evaluation framework requires less data which can be obtained through a walk down survey. The present study uses Tesfamariam and Saatcioglu's hierarchical structure (Fig. 1) for damageability assessment of steel buildings which has two modules: (1) structural deficiency

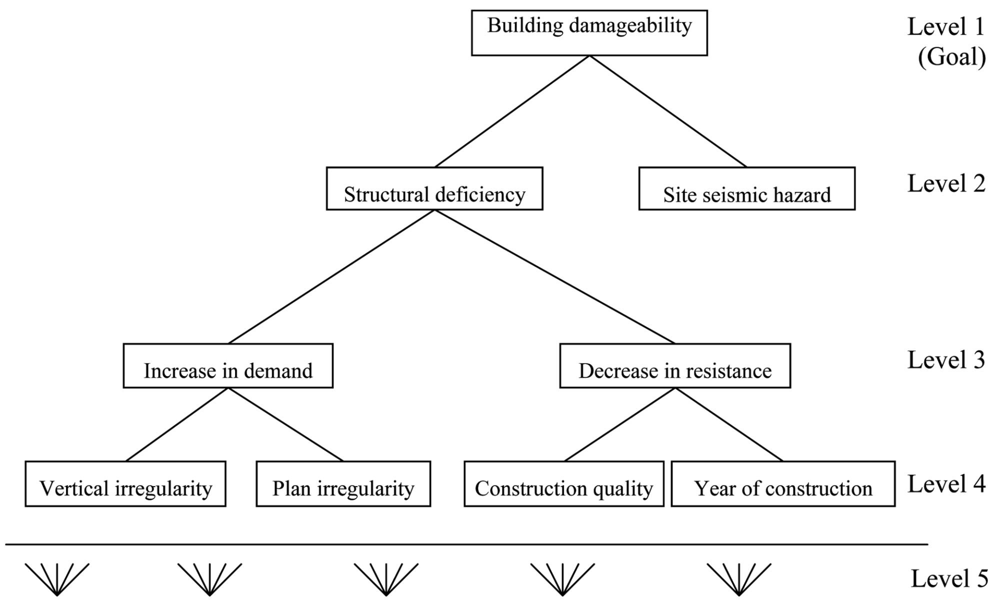


Fig. 1 Hierarchy structure for seismic damageability assessment of steel buildings (modified after Tesfamariam and Saatcioglu 2008a)

and (2) site seismic hazard. The structural deficiency are quantified by considering (1) vertical irregularity (*VI*), (2) plan irregularity (*PI*), (3) construction quality (*CQ*) and (4) year of construction (*YC*). Fig. 1 provides a five-level hierarchical structure. *Level 1* of the hierarchy represents the overall goal of the analysis, i.e., to identify the most vulnerable building which can be computed by integrating the parameters at *Level 2*, e.g. site seismic hazard and structural deficiency. The structural deficiency can be computed by integrating the parameters at *Level 3* that relates to the increase in the demand and decrease in the resistance (capacity). Parameters that contribute towards the increase in seismic demand and the decrease in structural resistance are presented in *Level 4*. *Level 5* entails the buildings to be screened and ranked for detailed evaluation and retrofitting.

Each parameter throughout the hierarchical structure is integrated using Fuzzy-TOPSIS methods. The vulnerability assessment consists of three phases. Phase 1 entails quantification and

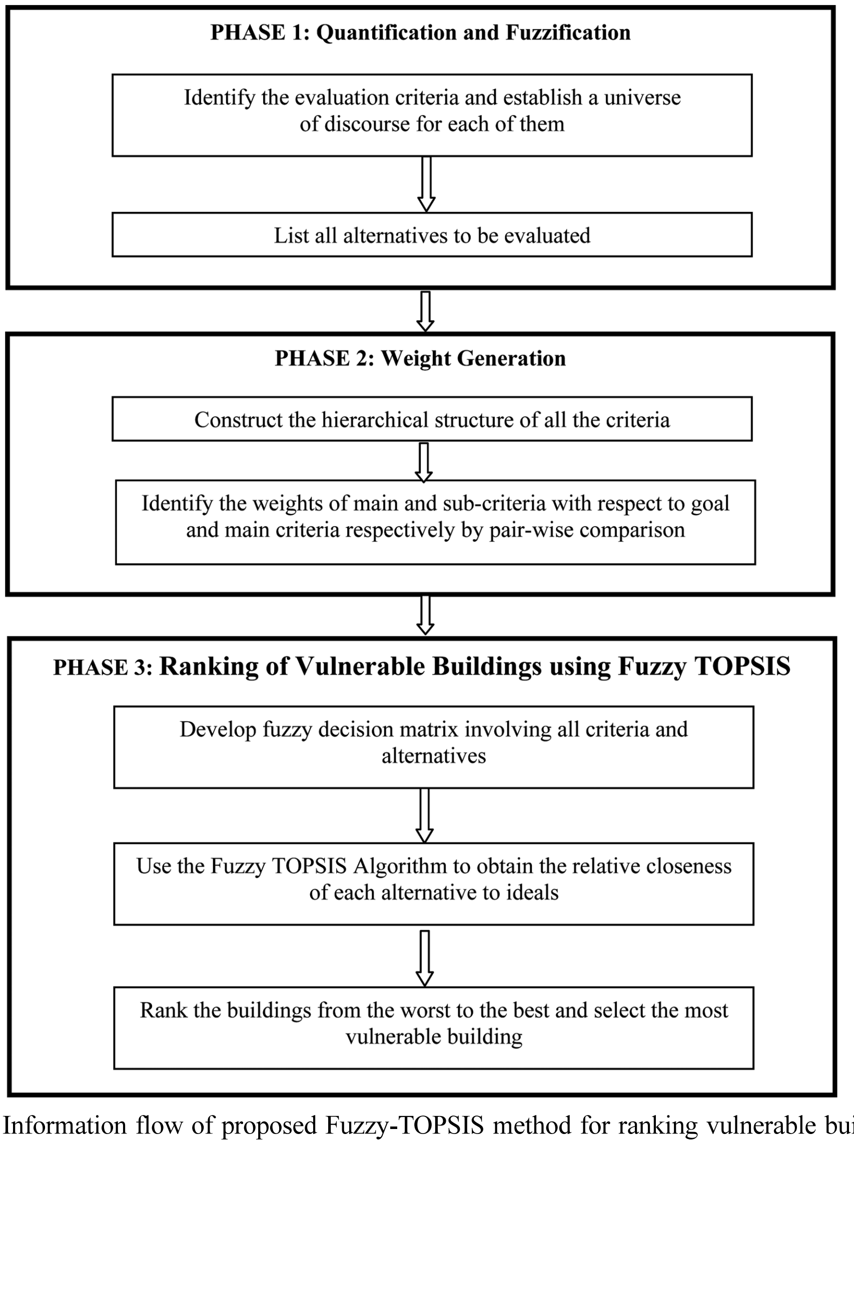


Fig. 2 Information flow of proposed Fuzzy-TOPSIS method for ranking vulnerable building

fuzzification of the basic input parameters obtained from a walk down survey. In Phase 2, the weight of each criteria and sub-criteria are determined (assigned) using AHP. Phase 3 involves an identification of the most vulnerable buildings through ranking based on evaluation using Fuzzy-TOPSIS method. Fig. 2 shows the sequence of information flow for Fuzzy-TOPSIS based seismic vulnerability assessment model.

The remainder of the paper is organized as follows: Section 2 provides background information on fuzzy sets. Section 3 presents a step-by-step procedure of Fuzzy-TOPSIS method. The efficacy and utility of the proposed model is discussed in section 4 through an illustrative case study of the 1994 Northridge earthquake.

2. Fuzzy sets

A fuzzy set is a collection of ordered pair $A = \{x, \mu_x\}$ that describes the relationship between an uncertain quantity x and a membership function μ_x which ranges between 0 and 1. The fuzzy set theory is an extension of the traditional set theory. In the traditional set theory, x is either a member of the set A or not. But in the fuzzy set theory x can be a member of set A with a certain degree of membership μ_x. Fuzzy sets are qualified as fuzzy numbers, if they are normal, convex and bounded (Klir and Yuan 1995). Fuzzy numbers can be bell, triangular, trapezoidal, Gaussian in shape. However, the selected shape should be justified by available information. Generally, triangular or trapezoidal fuzzy numbers (TFN or ZFN) are used for representing linguistic variables (Kenarangui 1991, Rivera and Barón 1999). In the proposed approach, the subjective judgment of various criteria involved in building vulnerability is assumed linguistic and described using a TFN for the sake of simplicity. Other type of fuzzy number may increase the computational complexity without substantially affecting the significance of the results (Wang and Elhag 2006, Yang and Hung 2007, Malekly *et al*. 2010).

A ZFN $\tilde{A}$ is represented by four points (a, b, c, d) on the universe of discourse, representing the "minimum", "most likely interval" and "maximum value", respectively. The *TFN* is a special type of *ZFN*, where $b = c$. The membership function $\mu_{\tilde{A}}(x)$ for *TFN* and *ZFN* are defined as

$$TFN{:}\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-c}{b-c}, & b \le x \le c \\ 0, & x > c \end{cases}$$

$$ZFN{:}\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{x-d}{c-d}, & c \le x \le d \end{cases} \tag{1}$$

Common fuzzy arithmetic operations for two *TFN*, $\tilde{A}_1$ and $\tilde{A}_2$ are given as following

$$\tilde{A}_1(+)\tilde{A}_2 = (a_1, b_1, c_1)(+)(a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_3) \quad (2)$$

$$\tilde{A}_1(-)\tilde{A}_2 = (a_1, b_1, c_1)(-)(a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_3) \quad (3)$$

$$k.\tilde{A}_1 = (ka_1, kb_1, kc_1) \quad (4)$$

3. Fuzzy-TOPSIS

TOPSIS (Technique for order preference by similarity to ideal solution) is a powerful tool for handling ranking multi-attribute/criteria decision making (MCDM) problems. Hwang and Yoon (1981) described the TOPSIS concept, with the reference to the positive and negative ideal solutions, as the ideal and anti-ideal solutions, respectively. The TOPSIS method defines an index called similarity (or relative closeness) to rank the alternatives based on the distance (or similarity)

Table 1 Computational differences in the extension of Fuzzy TOPSIS method

References	Criteria weights	Type of fuzzy numbers	Normalization methods	Ranking methods
Chen and Hwang (1992)	Fuzzy	Trapezoidal	Linear normalization	Lee and Li's (1988) generalized mean method
Liang (1999)	Fuzzy	Trapezoidal	Manhattan distance	Chen's (1985) ranking with maximizing set and minimizing set
Chen (2000)	Fuzzy	Triangular	Linear normalization	Chen (2000) assumes the fuzzy positive and negative ideal solutions as (1, 1, 1) and (0, 0, 0), respectively
Chu (2002)	Fuzzy	Triangular	Modified Manhattan distance	Liou and Wang's (1992) ranking method of total integral value with α = 1/2
Tsaur *et al.* (2002)	Crisp	Triangular	Vector normalization	Zhao and Govind's (1991) center of area method
Chu and Lin (2003)	Fuzzy	Triangular	Linear normalization	Kaufmann and Gupta's (1988) mean of the removals method
Zhang and Lu (2003)	Crisp	Triangular	Manhattan distance	Chen's (2000) fuzzy positive and negative ideal solutions: as (1, 1, 1) and (0, 0, 0), respectively
Ertuğrul and Karakaşoğlu (2008)	Fuzzy	Triangular	Vertex distance	Chen's (2000) fuzzy positive and negative ideal solutions: as (1, 1, 1) and (0, 0, 0), respectively
Chen and Lee (2010)	Interval type 2	Interval type 2	-	The positive ideal solution and the negative ideal solution based on Lee and Chen's (2008) concept of ranking values of trapezoidal interval type-2 fuzzy sets.

of their evaluated score from the ideal solution in a MCDM problem. Though TOPSIS is designed to capture expert knowledge/opinions, the conventional TOPSIS (crisp) does not reflect human thinking style or judgement and fails to incorporate the uncertainties associated with decision making.

In order to deal with fuzzy MCDM problems, classic TOPSIS method proposed by Hwang and Yoon (1981) has been extensively extended by many researchers. Chen (2000) extended this method to solve group decision-making problems under fuzzy environment, where the fuzzy positive and negative ideal solutions were defined in order to calculate the closeness coefficient for each alternative. Fuzzy-TOPSIS method furnishes decision makers with a ranking tool in a case where the data are not expressed in crisp numerical values and are qualitative or linguistic in nature. The simplicity of the Fuzzy-TOPSIS method has made it a popular tool for ranking the alternatives in real applications in different areas of expertise, such as weapons selection (Dağdeviren *et al.* 2009), project selection (Salehi and Tavakkoli-Moghaddam 2008), supplier selection (Wang *et al.* 2009), robot selection (Chu and Lin 2003) and bridge risk assessment and management (Wang and Elhag 2006).

Fuzzy-TOPSIS method has been extensively used and modified by many researchers to deal with fuzzy MCDM problems. Table 1 provides a summary of computational differences in the extensions of TOPSIS method under fuzzy environment. A six steps process of Fuzzy-TOPSIS method is discussed further below following Triantaphyllou and Lin (1996).

3.1 Step 1: Construct the normalized decision matrix

In the first step, a normalized decision (or evaluation) matrices is constructed. Suppose a decision committee are asked to analyze a problem involving i alternatives ($A_1, A_2,...,A_i$) and j criteria ($C_1, C_2,...,C_j$), which results in the following evaluation matrices

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ \vdots \\ A_i \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & \cdots & \cdots & C_j \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & - & - & \tilde{x}_{1j} \\ \tilde{x}_{21} & \tilde{x}_{22} & - & - & \tilde{x}_{2j} \\ - & - & - & - & - \\ - & - & - & - & - \\ \tilde{x}_{i1} & \tilde{x}_{i2} & - & - & \tilde{x}_{ij} \end{bmatrix} \end{array} \tag{5}$$

where $\tilde{x}_{ij}$ is the fuzzy rating of alternative A_i with respect to criterion C_j. In our case, the inspectors provide the data for decision matrices in the form of linguistic variables. These variables entering the decision matrix can be either triangular fuzzy numbers or can be membership values showing the degree of relationship between the factors and the evaluations. These values can take any value between 0 and 1 (Ross 2005).

If all the criteria, $C_{k\ (k=1,\ 2,...,\ j)}$ are assessed by the same set of fuzzy linguistic variables, then the fuzzy decision matrix $\tilde{D}$ is of the same dimension and normalization is not required. Otherwise $\tilde{D}$ has to be normalized to transform the various criteria scales into a comparable scale. The linear scale transformation is used to obtain the following normalized fuzzy decision matrix $\tilde{R}$

$$\tilde{R} = [\tilde{r}_{ij}]_{i \times j} \tag{6}$$

where, $\tilde{r}_{ij}$ is the normalized fuzzy evaluation value of alternative A_i with respect to criterion C_j which can be obtained by Eqs. (7) or (8) depending on benefit criteria (B) or cost criteria (C), respectively.

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}\right), d_j^* = \max_i c_{ij}, j \in B \tag{7}$$

$$\tilde{r}_{ij} = \left(\frac{d_j^-}{c_{ij}}, \frac{d_j^-}{b_{ij}}, \frac{d_j^-}{a_{ij}}\right), d_j^- = \min_i a_{ij}, j \in C \tag{8}$$

3.2 Step 2: Construct the normalized decision matrix

A set of weights $W = (w_1, w_2, ..., w_j)$ such that $\sum_1^j w_j = 1$ derived using analytic hierarchical process (AHP) is used (see Appendix A) in conjunction with the above mentioned normalized decision matrix to determine the weighted normalized decision matrix $\tilde{V}$

$$\tilde{V} = [\tilde{v}_{ij}]_{i \times j} \tag{9}$$

The elements of the weighted normalized decision matrix, $\tilde{v}_{ij}$ is obtained as: $\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j$, where $\tilde{w}_j$ is the fuzzy weight of criterion.

3.3 Step 3: Determine the positive-ideal and the negative-ideal solutions

The fuzzy positive ideal solution (FPIS, A^*) and the fuzzy negative ideal solution (FNIS, A^-) are determined as follows

$$A^* = \{v_1^*, v_2^*, v_3^*, ..., v_j^*\} = \{(\max_j c_{ij} \mid j \in B), (\min_j a_{ij} \mid j \in C)\} \tag{10}$$

$$A^- = \{v_1^-, v_2^-, v_3^-, ..., v_j^-\} = \{(\min_j a_{ij} \mid j \in B), (\max_j c_{ij} \mid j \in C)\} \tag{11}$$

In this study, A^* indicates the most preferable alternative or ideal solution which refers to the lowest possible vulnerability membership function {1,0,0,0,0} that corresponds to five-tuple membership values (μ_{VL}, μ_L, μ_M, μ_H, μ_{VH}), where, *VL, L, M, H* and *VH* denote, respectively, very low, low, medium, high and very high. Similarly, A^- indicates the least preferable alternative or negative-ideal solution which is the highest possible vulnerability membership function {0,0,0,0,1}.

3.4 Step 4: Calculate distances from the ideal solutions

The distance of each alternative from FPIS and FNIS are calculated as follows

$$D_i^* = \sum_j d(\tilde{v}_{ij}, \tilde{v}_j^*) \tag{12}$$

$$D_i^- = \sum_j d(\tilde{v}_{ij}, \tilde{v}_j^-) \tag{13}$$

where $d(.)$ is the distance measurement between two fuzzy numbers or fuzzy sets.

Different methods used for measuring the distance between two fuzzy numbers, such as geometric distance, Hausdorff metric, dissemblance index method and Bhattacharyya distance have been summarized elsewhere (Zwick *et al.* 1987). In this study, because of its simplicity, the vertex method is used to measure the separation distances of each alternative to the ideal solution and negative-ideal solution (Eqs. (14) and (15)).

$$D_i^* = \sum_j \sqrt{\frac{1}{5}[(\mu_{VL}-\mu_{VL}^*)^2+(\mu_L-\mu_L^*)^2+(\mu_M-\mu_M^*)^2+(\mu_H-\mu_H^*)^2+(\mu_{VH}-\mu_{VH}^*)^2]} \tag{14}$$

$$D_i^- = \sum_j \sqrt{\frac{1}{5}[(\mu_{VL}-\mu_{VL}^-)^2+(\mu_L-\mu_L^-)^2+(\mu_M-\mu_M^-)^2+(\mu_H-\mu_H^-)^2+(\mu_{VH}-\mu_{VH}^-)^2]} \tag{15}$$

3.5 Step 5: Calculate the relative closeness to the ideal solution

A closeness coefficient (CC_i) is used to rank all possible alternatives. The relative closeness coefficient of each alternative with respect to the FPIS (A^*) and FNIS (A^-) is calculated as follows

$$CC_i = \frac{D_i^-}{D_i^* + D_i^-} \tag{16}$$

3.6 Step 6: Rank the preference order

The best satisfied alternative can now be decided according to preference rank order of CC_i. It is the one which has the shortest distance from the ideal solution. The way the alternatives are processed in the previous steps reveals that if an alternative has the shortest distance from the ideal solution, then this alternative is guaranteed to have the longest distance to the negative-ideal solution.

4. Seismic vulnerability assessment of buildings: Fuzzy-TOPSIS method

The Northridge earthquake with a moment magnitude M_w6.7 struck the San Fernando Valley on January 17, 1994. The ATC-38 (ATC 2001) building performance and strong motion data is used to demonstrate the application of proposed Fuzzy-TOPSIS method. The buildings are divided into four distinct groups based on their structural systems (Table 2). The steel moment resisting frame systems (SMRF) are classified as group 1. The steel frames with concrete shear walls (SFSW) are grouped together as group 2. Since FEMA 273 (1997) explicitly distinguishes between the behaviour of a steel frame with concrete shear wall and a frame with masonry infill shear wall and their design, group 3 includes steel frames with infill shear walls (SFISW). Finally, the light weight steel frame buildings (LWF) are classified under group 4. Fig. 3 represents the histogram extracted from the damage database of Northridge earthquakes for each of the basic risk items provided in Fig. 1.

Table 2 Structural system groups

Group	Building types (FEMA 237 1997)	Group name
Group 1	*S*1*, *S*1*A*	SMRF
Group 2	*S*2, *S*2*A*, *S*4	SFSW
Group 3	*S*5	SFISW
Group 4	*S*3	LWF

**S*1= Steel moment frame with stiff diaphragms
*S*1*A*: Steel moment frame with flexible diaphragms
*S*2: Steel braced frame with stiff diaphragms
*S*2*A*: Steel braced frame with flexible diaphragms
*S*3: Steel light frame
*S*4: Steel frame with concrete shear walls
*S*5: Steel frame with infill masonry shear walls

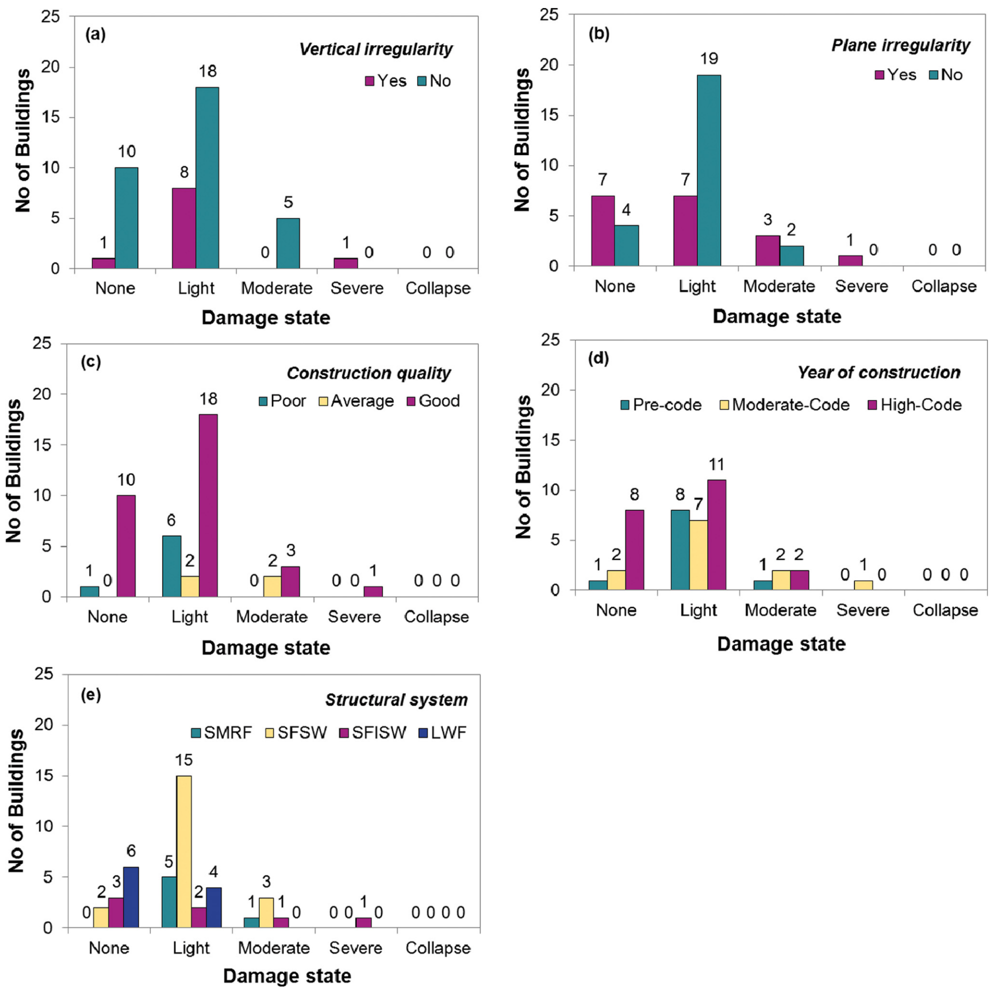
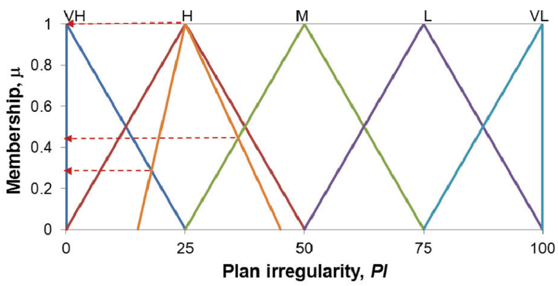


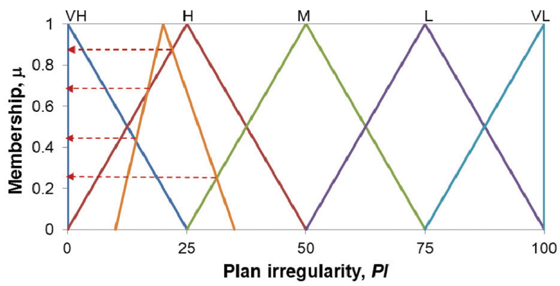
Fig. 3 Basic risk items and mapping over different damages states for steel buildings: (a) vertical irregularity, (b) plan irregularity, (c) construction quality, (d) year of construction and (e) structural system (type of building)

4.1 Phase 1: Quantification and fuzzification

In the proposed approach, various criteria involved in building vulnerability is described using TFN and the granularity is associated with the level of damage states; and the input value of each attribute/criterion is expressed by five-tuple fuzzy set (μ_{VL}, μ_L, μ_M, μ_H, μ_{VH}) (Fig. 4), where μ_i refers to the membership to each fuzzy subsets and the subscript describes the corresponding risk level. The membership values are calculated using Eq. (1). The coordinates of TFN (a, b, c) or ZFN (a, b, c, d) for different influencing factors in different granularity levels have been presented in Table 3. For brevity the rationale behind choosing the corresponding TFN for each parameter is not discussed here, however the interested readers are referred to Tesfamariam and Saatcioglu (2008b). For each influencing input, the spread or range of fuzziness (support), specified by the difference between the maximum and minimum [c-a] is assigned subjectively. For example, the values for the construction quality are selected to vary from 0-10, where 0 and 10 corresponds to very high and very low risk, respectively. The aggregation is performed on a commensurable interval [0, 1] of the membership values μ_i. For example, the TFN coordinates for a medium construction quality can be represented as TFN (2.5, 5, 7.5). This TFN is selected arbitrarily, since the most likely value is 5, which is the midpoint between very high (0) and very low (10) risk. Similarly, the values for plan irregularity and vertical irregularity are selected such that it varies from 0-100, in accordance with the general description of the soft story, and extreme soft story specified in NEHRP design



(a)



(b)

Fig. 4 Granulation of plan irregularity: (a) for SMRF and (b) for SFSW

Table 3 The Coordinates of fuzzy numbers for different influencing factors in different granularity levels (after Tesfamariam and Saatcioglu 2008b)

Input	Range	*VH*	*H*	*M*	*L*	*VL*
Vertical irregularity (*VI*)	0-100	(0, 0, 25)	(0, 25, 50)	(25, 50, 75)	(50, 75, 100)	(75, 100, 100)
Plan irregularity (*VI*)	0-100	0, 0, 25	0, 25, 50	25, 50, 75	(50, 75, 100)	(75, 100, 100)
Construction quality (*CQ*)	0-10	0, 0, 2.5	0, 2.5, 5.0	2.5, 5.0, 7.5	(5.0, 7.5, 10)	(7.5, 10, 10)
Year of construction (*YC*)	1910-2010	(0, 1960, 1970)	(1960, 1970, 1980)	(1970, 1980, 1990)	(1980, 1990, 2000)	(1990, 2000, 2010)
Spectral acceleration, g (S_a)	0-5	(0, 0.05, 0.1)	(0.05, 0.1, 0.4)	(0.1, 0.4, 0.55)	(0.4, 0.55, 2)	(0.55, 2, 2.5)

guideline (FEMA 454-1 2004).

The *YC* has been divided into three distinct group based on NIBS (1999) as pre-code ($YC \leq 1941$), moderate-code ($1941 < YC < 1975$) and high-code ($YC \geq 1975$) (Tesfamariam and Saatcioglu 2008a). The presence or absence of *VI* and *PI* are linguistically evaluated as Yes or No, respectively. The *CQ* is evaluated linguistically as: Poor, Average and Good. However, in the fuzzification of the *VI*, *PI* and *CQ*, the system structural system groups (Table 2) is taken into consideration. It is assumed that, a SMRF building with *VI* = {Yes} for example, tend to have higher damage than a braced frame building with *VI* = {Yes}. Thus, different TFNs are subjectively selected for *PI*, *VI* and *CQ* corresponding to each group of structural system (Table 2). For example, presence of *PI* is assigned TFNs (15, 25, 45) and (10, 20, 35), respectively, for SMRF and SFSW. These TFNs are traced over the granules provided in Fig. 4 and corresponding fuzzification is summarized in Table 4. It should be noted that, Fig. 4 is drawn by using the coordinates provided in Table 3 for granulation of *PI*. Thus, the fuzzification of *PI* for SMRF is obtained from Fig. 3(a)

$$(\mu_{VL}, \mu_L, \mu_M, \mu_H, \mu_{VH}) = (0, 0, 0.44, 1.0, 0.29)$$

Based on Fig. 4(b), it can be seen that *PI* for SFSW intersects the high granule at two locations. In this case, the maximum value is selected. Thus the fuzzification becomes

$$(\mu_{VL}, \mu_L, \mu_M, \mu_H, \mu_{VH}) = (0, 0, 0.25, \max(0.88, 0.65), 0.43) = (0, 0, 0.25, 0.88, 0.43)$$

The process of quantification of the input parameter and corresponding fuzzification of the 1994

Table 4 Five-tuple fuzzy sets for plan irregularity, vertical irregularity and construction quality for evaluation of (a) group 1 & 2 and (b) group 3 & 4 of structural systems

	(a)	Group 1 (MRF systems)					Group 2 (frames braced or with concrete shear walls)				
		VL	*L*	*M*	*H*	*VH*	*VL*	*L*	*M*	*H*	*VH*
PI	yes	0	0	0.44	1.00	0.29	0	0	0.25	0.88	0.43
	no	0.33	0.89	0.56	0	0	0.50	0.91	0.45	0	0
VI	yes	0	0	0.44	1.00	0.29	0	0	0.25	0.88	0.43
	no	0.33	0.89	0.56	0	0	0.50	0.91	0.45	0	0
CQ	poor	0	0	0.14	0.86	0.43	0	0	0	0.71	0.57
	average	0	0.38	1.00	0.38	0	0	0.44	1.00	0.44	0
	good	0.43	0.86	0.14	0	0	0.57	0.71	0	0	0

	(b)	Group 3 (frames with infill shear walls)					Group 4 (light weight steel frames)				
		VL	*L*	*M*	*H*	*VH*	*VL*	*L*	*M*	*H*	*VH*
PI	yes	0	0	0.4	0.9	0.43	0	0	0.13	0.75	0.57
	no	0.38	1	0.5	0	0	0.57	0.75	0.13	0	0
VI	yes	0	0	0.4	0.9	0.43	0	0	0.13	0.75	0.57
	no	0.38	1	0.5	0	0	0.57	0.75	0.13	0	0
CQ	poor	0	0	0	0.71	0.57	0	0	0	0.57	0.71
	average	0	0.44	1	0.44	0	0	0.29	1	0.29	0
	good	0.57	0.71	0	0	0	0.71	0.57	0	0	0

Table 5 Summary of 1994 Northridge earthquake database for steel structures and their fuzzification

Building ID	*YC*	*VI*	*PI*	*CQ*	*Sa*	*D*	*CQ*	*VI*	*PI*	*Sa*	*YC*
CDMG087-CG-06	1980	N	N	N	0.305	2	(0.72,0.58,0,0,0)	(0.58,0.76,0.13,0,0)	(0.58,0.76,0.13,0,0)	(0,0.32,0.69,0,0)	(0,0,1,0,0)
CDMG231-ER-05	1976	N	N	U	0.207	3	(0,0.38,1,0.38,0)	(0.34,0.89,0.56, 0.01,0)	(0.34,0.89,0.56, 0.01,0)	(0,0.65,0.36,0,0)	(0,0,0.6,0.4,0)
CDMG231-GZ-01	1970	N	N	N	0.305	2	(0.43,0.86,0.15, 0,0)	(0.34,0.89,0.56, 0.01,0)	(0.34,0.89,0.56, 0.01,0)	(0,0.32,0.69, 0,0)	(0,0,0,1,0)
CDMG231-GZ-07	1990	Y	N	N	0.111	2	(0.58,0.72,0.01, 0,0)	(0,0,0.26,0.88, 0.43)	(0.51,0.91,0.46, 0.01,0)	(0,0.97,0.04, 0,0)	(0,1,0,0,0)
CDMG231-GZ-22	1990	N	Y	N	0.722	1	(0.58,0.72,0.01, 0,0)	(0.51,0.91,0.46, 0.01,0)	(0,0,0.26, 0.88,0.43)	(0,0,0,0.89, 0.12)	(0,1,0,0,0)
CDMG279-ER-15	1980	N	Y	N	0.722	1	(0.72,0.58,0,0,0)	(0.58,0.76,0.13,0,0)	(0,0,0.13,0.76,0.58)	(0,0,0,0.89,0.12)	(0,0,1,0,0)
CDMG303-JH-04	1960	Y	Y	U	0.893	2	(0,0.38,1,0.38,0)	(0,0,0.45,1,0.29)	(0,0,0.45,1,0.29)	(0,0,0,0.77,0.24)	(0,0,0,0,1)
...	...	...									
...											
USGS081-GZ-07	1940	N	N	N	0.675	2	(0.72,0.58,0,0,0)	(0.58,0.76,0.13,0,0)	(0.58,0.76,0.13,0,0)	(0,0,0,0.92,0.09)	(0,0,0,0,1)
USGS082-ER-11	1974	Y	Y	N	0.506	4	(0.58,0.72,0.01, 0,0)	(0,0,0.26,0.88, 0.43)	(0,0,0.26,0.88,0.43)	(0,0,0.3,0.71,0)	(0,0,0.4,0.6,0)
USGS233-GZ-01	1980	Y	Y	N	0.113	2	(0.43,0.86,0.15, 0,0)	(0,0,0.45,1,0.29)	(0,0,0.45,1,0.29)	(0,0.96,0.05,0,0)	(0,0,1,0,0)
USGS233-GZ-18	1980	Y	N	U	0.123	2	(0,0.38,1,0.38,0)	(0,0,0.45,1,0.29)	(0.34,0.89,0.56, 0.01,0)	(0,0.93,0.08,0,0)	(0,0,1,0,0)
USGS284-HY-01	1990	N	Y	N	0.202	1	(0.72,0.58,0,0,0)	(0.57,0.8,0.3,0,0)	(0,0,0.13,0.76,0.58)	(0,0.66,0.34,0,0)	(0,1,0,0,0)

YC = Year of construction; *VI* = Vertical irregularity; *PI* = Plan irregularity; *CQ* = Construction quality; *D* = Damage state (1 = None,..., 5 = Collapse)

Table 6 Basic risk items and fuzzification for the illustrative example

Basic risk item	Field observation	Fuzzification
Structural system (*SS*)	Steel light frame (LWF)	
Vertical irregularity (*VI*)	No	(0.57, 0.75, 0.13, 0, 0)
Plan irregularity (*PI*)	No	(0.57, 0.75, 0.13, 0, 0)
Construction quality (*CQ*)	Good	(0.71, 0.57, 0, 0, 0)
Year of construction (*YC*)	1980	(0, 0, 1.0, 0, 0)
Spectral acceleration (S_a)	0.305	(0, 0.32, 0.68, 0, 0)

Northridge earthquake walk down survey is illustrated in Table 5. The illustration is carried out for a steel Building ID = CDMG087-CG-06 (Table 5), where basic risk items obtained from the walk down survey along with their corresponding fuzzifications are summarized in Table 6. For *PI*, *VI* and *CQ*, the fuzzification is obtained from Table 4 corresponding to the appropriate structural system. In case of spectral acceleration (S_a) and *YC*, the values are directly traced over the granules obtained from Table 3. Thus, the basic risk items for other buildings have been fuzzified and snapshots of the results are summarized in Table 5.

4.2 Phase 2: Weight generation

Tesfamariam and Saatcioglu (2008b) have proposed relative weights generated using AHP (see Appendix A) for seismic vulnerability assessment of reinforced concrete buildings, and the weights are adopted in this article. The relative importance of each criterion at each level of the hierarchical structure is provided in Table 7. To highlight sensitivity of weights on the CC_i values, in Table 7, three scenarios, 1, 2 and 3, are considered. Finally, the weights for each of the criteria at Level 4 (Fig. 3) are obtained by multiplying the weights of this level by the criteria above them to the highest level in the hierarchical structure. For instance, the weight for construction quality will be 0.69×0.75×0.75 = 0.388. Level 5 of the hierarchy represents the buildings to be ranked.

Table 7 Weights for building damageability assessment

Basic parameter	Scenario 1 AHP weights*	Scenario 2	Scenario 3
Vertical irregularity	0.69	0.5	0.31
Plan irregularity	0.31	0.5	0.69
Construction quality	0.69	0.5	0.31
Year of construction	0.31	0.5	0.69
Increase in demand	0.25	0.5	0.75
Decrease in resistance	0.75	0.5	0.25
Structural deficiency	0.75	0.5	0.75
Site seismic hazard	0.25	0.5	0.25

*Adopted from Tesfamariam and Saatcioglu (2008b)

4.3 Phase 3: Ranking of vulnerable buildings

The ranking of vulnerable buildings were performed using Fuzzy-TOPSIS method outlined in Section 3. The ranking of various buildings options is the last step in deciding which building is the most vulnerable among available. The closeness coefficient (CC_i) was calculated to determine the ranking order of the vulnerable buildings. Since the goal is to identify the most vulnerable building, the ideal solution is the one which is closest to the negative ideal solution. Therefore, the buildings are ranked in ascending order of CC_i and the building with minimum CC_i will be the most vulnerable. For the most vulnerable building, the calculated CC_i should be close to zero and for the least vulnerable building the calculated CC_i should be close to the unit value. Therefore, the calculated CC_i value has been normalized between 0 and 1 by using

$$CC_N = \frac{CC_i - CC_{i_{\min}}}{CC_{i_{\max}} - CC_{i_{\min}}} \tag{17}$$

where CC_N is the normalized CC_i for any condition; $CC_{i_{\min}}$ is the minimum CC_i for the most vulnerable building under extreme unfavourable condition; and $CC_{i_{\max}}$ is the maximum CC_i for least vulnerable building under extreme favourable condition calculated by Fuzzy-TOPSIS methodology outlined in Section 3. Basic risk items used in calculating $CC_{i_{\min}}$ and $CC_{i_{\max}}$ are outlined in Table 8.

5. Results and discussions

The damageability ranking of the buildings based on damage state are presented in Table 9. The damage was classified into five discrete states: none (*N*), light (*L*), moderate (*M*), severe (*S*) and collapse (*C*) (Tesfamariam and Saatcioglu 2008b). Fig. 5 presents the CC_N vs. damage stage of the buildings. In general, the CC_N is found to decrease with an increase of damage state. This implies that the CC_N values correlate with observed damage and is an indication that the proposed methodology describes the vulnerability of buildings effectively. It should be noted that lower CC_N value refers to higher vulnerability. The scattered CC_N value at each damage state reflects the model uncertainty and highlights the need to gather more information on the potential causes of building vulnerability and site seismic hazard. Fig. 5 also shows results of the three weight scenarios, scenario 2 (compromising situation), to variation of importance on each input parameter (scenarios

Table 8 Basic risk items used in calculating $CC_{i_{\min}}$ and $CC_{i_{\max}}$

Basic risk item	$CC_{i_{\min}}$	$CC_{i_{\max}}$
Structural system (*SS*)	LWF	SMRF
Vertical irregularity (*VI*)	Yes	No
Plan irregularity (*PI*)	Yes	No
Construction quality (*CQ*)	Poor	Good
Year of construction (*YC*)	1940	2010
Spectral acceleration (S_a)	0.90	0.05

Table 9 Damageability rank of the buildings

Ranking order	General damage	Building ID	CC_N
1	1	USC021-GTZ-15	0.153114
2	2	CDMG567-GZ-08	0.16859
3	2	CDMG567-GZ-06	0.214276
4	2	CDMG567-GZ-13	0.214363
5	2	CDMG567-GZ-03	0.251119
6	2	CDMG303-JH-04	0.296048
7	2	CDMG386-SH-14	0.375117
8	2	CDMG567-GZ-01	0.400756
9	3	CDMG567-GZ-04	0.400947
10	2	CDMG538-SH-01	0.471808
11	2	USGS233-GZ-18	0.512443
12	3	CDMG370-MF-15	0.530558
...	...	...	…
40	1	CDMG463-AC-14	0.813278
41	1	CDMG538-SH-06	0.845601
42	2	CDMG087-CG-06	0.845704
43	1	CDMG463-AC-02	1

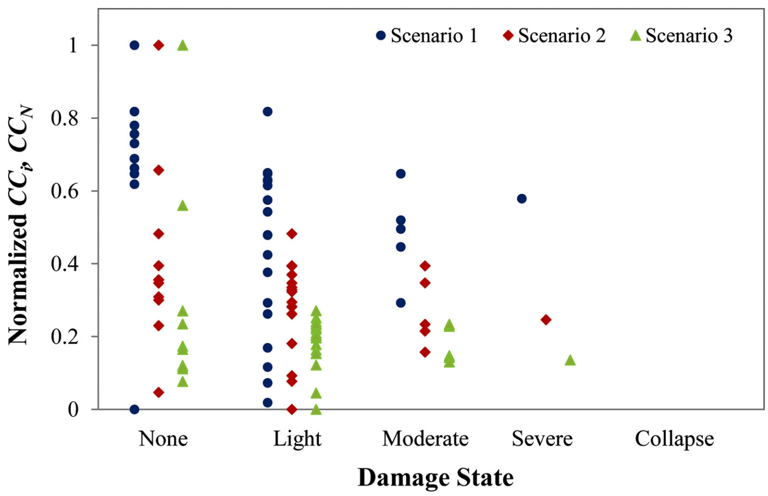


Fig. 5 Closeness coefficient versus damage state

1 and 3). Overall, for all scenario wrights, there is a similar trend in CC_N value, decrease with increasing damage levels. However, for lower damage levels, scenario1 furnishes a higher CC_N values and under estimates higher damage levels. Whereas, scenarios 2 and 3, overestimate lower damage levels and shows a better prediction at the higher damage levels.

The effect of spectral acceleration (S_a) on CC_N value for different type of structural system is plotted in Fig. 6. It can be observed that irrespective of the structural system, the higher the S_a, the

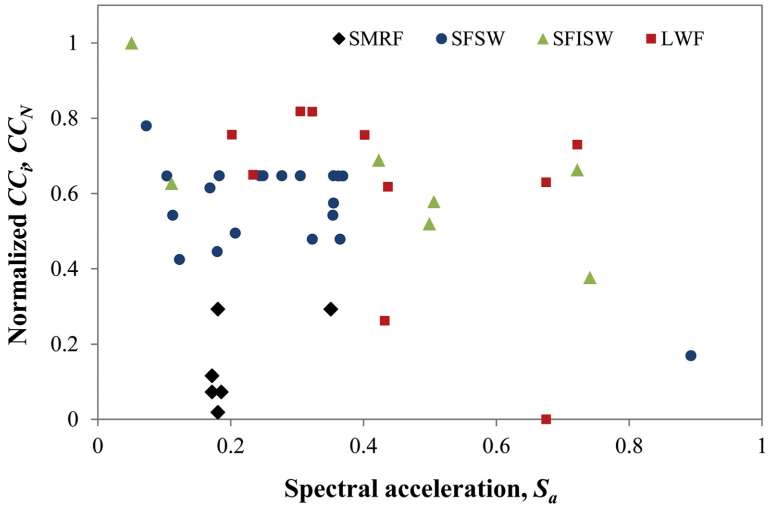


Fig. 6 Closeness coefficient versus spectral acceleration for different structural system

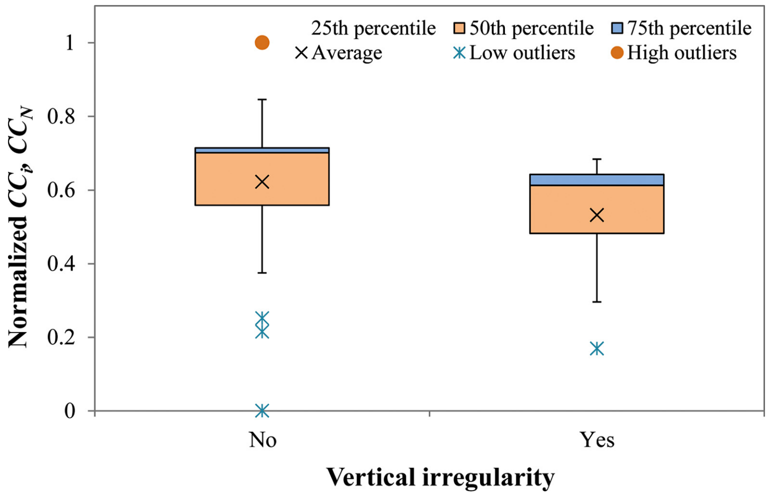


Fig. 7 Closeness coefficient versus vertical irregularity

smaller is the CC_N value, as expected implies greater damage with increasing S_a. Scattered CC_N value, however, implies that degree of damage or vulnerability depends not only on the S_a values but also on inherent system deficiency, structural system, e.g. shear wall or moment resisting frame buildings and structural deficiency, e.g. *VI*.

The effects of structural system, *VI, PI, YC* and *CQ* on CC_N values are presented, respectively, in Figs. 7-10 using Box-and-whisker diagram. Box-and-whisker diagram represents a set of data using median, lower quartile (25th percentile), upper quartile (75th quartile), maximum and minimum value (Appendix B). Presence of *VI* and/or *PI* contributes to an increase in seismic demand which is reflected, respectively, in Figs. 7 and 8 by lower CC_N value. Fig. 9 shows the effect of *YC* on the vulnerability of the buildings. As illustrated in this figure, older buildings are more vulnerable as

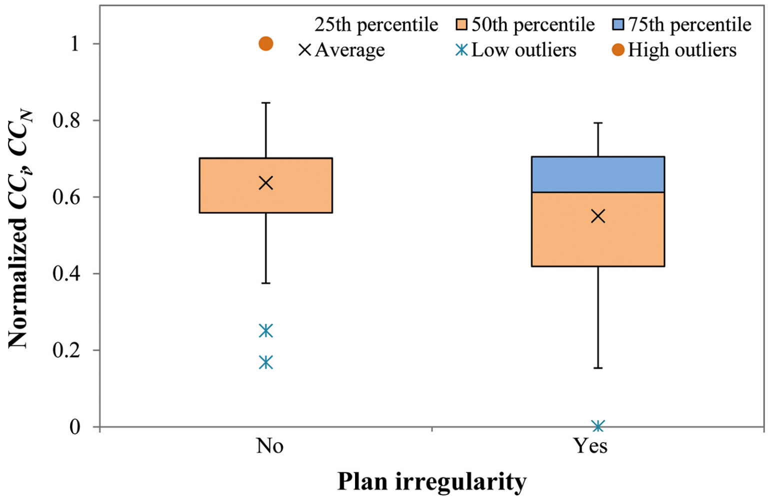


Fig. 8 Closeness coefficient versus plan irregularity

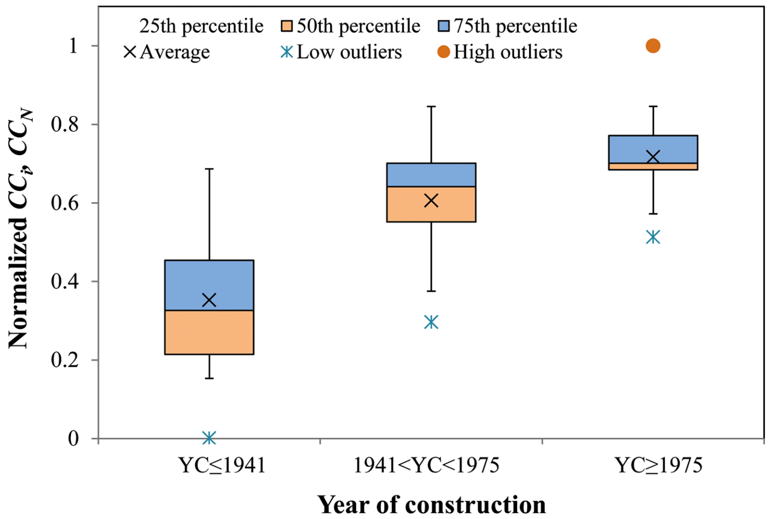


Fig. 9 Closeness coefficient versus year of construction

indicated by lower CC_N values. This is probably because the resistance of new buildings has improved with the implementation of more recent design codes. Moreover, the quality of construction has improved to a large extent in the recent decades due to achievements in construction materials and also improvements in construction techniques. The effect of *CQ* on building vulnerability is presented in Fig. 10 which conforms that good quality construction improves the resistance of a building and in turn building becomes less susceptible to damage.

In order to evaluate the effectiveness of the structural system on the final rank of the buildings, a plot of the structural system versus final rank has been provided in Fig. 11 that clearly shows that relatively, the SMRF is the most damageable. The second damageable types of structures are moment resisting frames. Interestingly, it can be seen that light weight steel frames showed good

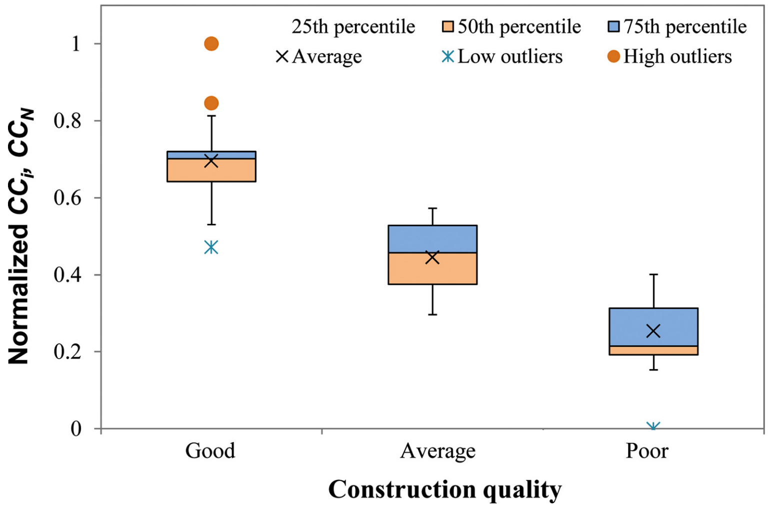


Fig. 10 Closeness coefficient versus construction quality

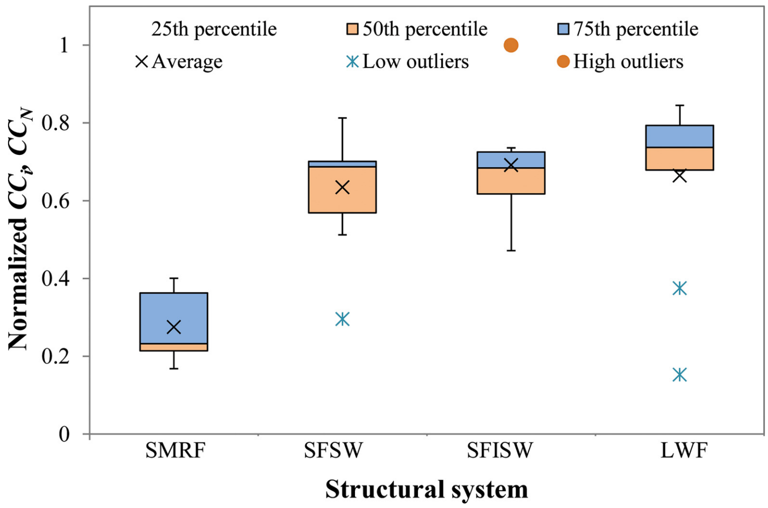


Fig. 11 Effect of type of structural system on closeness coefficient CC_N

performance. This suggests the importance of reducing the weight of the structures for reducing its potential damage.

6. Conclusions

The building seismic vulnerability assessment and their subsequent repair and rehabilitation are a challenging task. Different building vulnerability assessment techniques have been reported, ranging from a simple scoring method to more complex methods of nonlinear structural analyses. Data for a large number of contributory factors are required for the assessment which generally bear uncertainties and warrant fuzzy-based MCDM methods.

In this study, a hierarchical structure of various levels of contributory factors is presented which is evaluated using Fuzzy-TOPSIS method. The output of the proposed model is a ranking order based in vulnerability of the buildings which is used as a surrogate for damageability of buildings. Finally, the model is illustrated through the 1994 Northridge earthquake.

Some highlights of this study are as follows:

- New method of Fuzzy-TOPSIS is a very helpful tool to deal with qualitative judgments and uncertain evaluations expressed in linguistic variables.
- Subjectivity is inherent in any kind of MCDM problem. One of the strengths of the proposed approach for evaluation of the buildings is that it is capable of considering and reflecting the views of any number of decision makers in the process of decision making. To get the best result, it is suggested that experts from different areas related to seismic evaluation of steel structures be involved in the process to avoid biased evaluations and comparisons as much as possible.
- As the closeness coefficients (CC_N) for alternatives are calculated, special attention must be paid to the final ranking of the alternatives. The highest CC_N closest to the fuzzy positive ideal solution (FPIS) is not always corresponding to the first rank and the best solution. In this study we were interested in the vulnerable buildings; therefore the desirable alternatives were those which were close to the fuzzy negative ideal solution (FINS) (smallest CC_N).
- The final rank confirmed the damage states assigned to the buildings by the inspectors. It showed that buildings with higher damage states have smaller $CC_{N.}$ However the damageability of the building depends on a number of other factors such a structural system, structural deficiency etc.
- The ranking of the buildings survived from the Northridge earthquake showed that the older buildings were more damageable than the relatively newer buildings and this is due to the improvements in seismic design codes and also in construction quality of the buildings.
- The most damageable buildings in the region were: (1) steel moment resisting frames, (2) steel braced frames, (3) light weight steel frames and (4) steel frame with infill masonry shear walls.
- The proposed method for the prioritization of buildings for repair or rehabilitation integrates the site seismic hazard and structural deficiency. This method is flexible enough to incorporate other contributing parameters. With proper modifications, this model can be extended to risk assessment of other civil infrastructure systems.

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Appendix A

The Analytic hierarchical process (AHP) method introduced by Saaty (1980) is useful in determining the relative importance of each criterion in a multi-level and hierarchical set of criteria. AHP estimates the relative importance of each criteria in a group using pairwise comparisons based on a scale of 1 to 9 (Saaty 1980), where "1" represents two criteria are equally important, while "9" represents that one criteria is absolutely more important than the other (Table A1). The pairwise judgment matrix thus developed, indicates dominance or relative importance of one element over another (Saaty 1980). The result of the pairwise comparison on n criteria is summarized in an $n \times n$ matrix as follows

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, a_{ii} = 1, a_{ji} = 1/a_{ij}, a_{ij} \neq 0 \tag{A1}$$

The final weights of the criteria in each level of the hierarchy are determined by taking the geometric mean of each column of the final judgement matrix and then normalizing the derived matrix. Finally, the weights at the lowest level will be obtained by multiplying the weights of the corresponding criteria in higher levels from the highest level to that level. In a case of n criteria, a set of weights in each level of hierarchy could be written as

$$W = (w_1, w_2, \dots, w_n) \text{ where } \sum_1^n w_n = 1 \tag{A2}$$

Table A1 AHP importance scale (Saaty 1980)

Comparative importance	Definition	Explanation
1	Equally important	Two decision elements (e.g. indicators) equally influence the parent decision element.
3	Moderately more important	One decision element is moderately more influential than the other.
5	Strongly more important	One decision element has stronger influence than the other.
7	Very strongly more important	One decision element has significantly more influence over the other.
9	Extremely more important	The difference between influences of the two decision elements is extremely significant.
2, 4, 6, 8	Intermediate judgment values	Judgment values between equally, moderately, strongly, very strongly and extremely.
Reciprocals		If v is the judgment value when i is compared to j, then $1/v$ is the judgment value when j is compared to i.

Appendix B

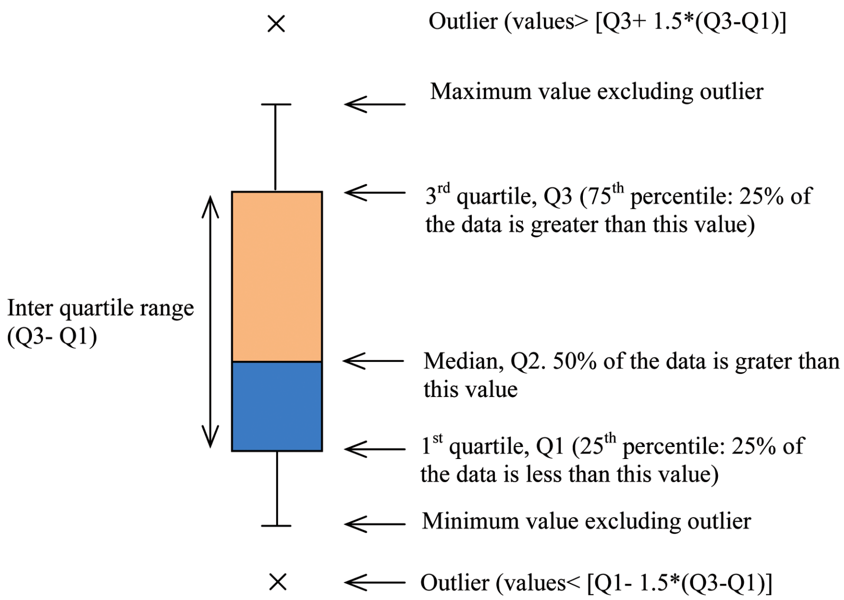

Fig. B1 Illustration of box-and-whisker diagram