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Dynamic analysis of concrete gravity dam-reservoir systems by wavenumber approach in the frequency domain

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Abstract. Dynamic analysis of concrete gravity dam-reservoir systems is an important topic in the study of fluid-structure interaction problems. It is well-known that the rigorous approach for solving this problem relies heavily on employing a two-dimensional semi-infinite fluid element. The hyper-element is formulated in frequency domain and its application in this field has led to many especial purpose programs which were demanding from programming point of view. In this study, a technique is proposed for dynamic analysis of dam-reservoir systems in the context of pure finite element programming which is referred to as the wavenumber approach. In this technique, the wavenumber condition is imposed on the truncation boundary or the upstream face of the near-field water domain. The method is initially described. Subsequently, the response of an idealized triangular dam-reservoir system is obtained by this approach, and the results are compared against the exact response. Based on this investigation, it is concluded that this approach can be envisaged as a great substitute for the rigorous type of analysis.

Keywords: concrete gravity dams; wavenumber; absorbing boundary conditions; truncation boundary.

1. Introduction

Dynamic analysis of concrete gravity dam-reservoir systems can be carried out rigorously by FE-(FE-HE) method in the frequency domain. This means that the dam is discretized by plane solid finite elements, while, the reservoir is divided into two parts, a near-field region (usually an irregular shape) in the vicinity of the dam and a far-field part (assuming uniform depth) which extends to infinity in the upstream direction. The former region is discretized by plane fluid finite elements and the latter part is modeled by a two-dimensional fluid hyper-element (Hall and Chopra 1982, Waas 1972). It is well-known that employing fluid hyper-elements would lead to the exact solution of the problem. However, it is formulated in the frequency domain and its application in this field has led to many especial purpose programs which were demanding from programming point of view.

On the other hand, engineers have often tried to solve this problem in the context of pure finite element programming (FE-FE method of analysis). In this approach, an often simplified condition is imposed on the truncation boundary or the upstream face of the near-field water domain. Thus, the fluid hyper-element is actually excluded from the model. Some of these widely used simplified conditions (Sommerfeld 1949, Sharan 1987), may result in significant errors if the reservoir length

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is small, and it might lead to high computational cost if the truncation boundary is located at far distances. The main advantage of these conditions is that it can be readily used for time domain analysis. Thus, they are vastly employed in nonlinear seismic analysis of concrete dams.

Of course, there have also been many researches in the last three decades to develop more accurate absorbing boundary conditions to be applied for similar fluid-structure or soil-structure interaction problems. Perfectly matched layer (Berenger 1994, Chew and Weedon 1994, Basu and Chopra 2003) and, high-order non-reflecting boundary condition (Higdon 1986, Givoli and Neta 2003, Hagstrom and Warbuton 2004, Givoli *et al.* 2006) are among the two main popular groups of methods which researchers have applied in their attempts. It is emphasized that these techniques have become very popular in recent years due to the fact that they could be applied in time domain as well as the frequency domain. However, it should be realized that they are not that attractive in the frequency domain. This is due to the fact that they are not very simple to be used and more importantly, they are compared with the hyper-element alternative which produces exact results no matter how small the reservoir near-field length is considered.

In the present study, the FE-FE analysis technique is chosen as the basis of a proposed method for dynamic analysis of concrete dam-reservoir system in the frequency domain, which is referred to as the wavenumber approach. The method is simply applying an absorbing boundary condition on the truncation boundary which is referred to as the wavenumber condition. It is as simple as employing Sommerfeld or Sharan condition on the truncation boundary. The method of analysis is initially explained. Subsequently, the response of an idealized triangular dam is studied due to horizontal ground motion for several alternatives employed as absorbing boundary condition. In each case, as well as the proposed option, the results are compared against the exact solution. The main parameter varied, is the length of reservoir near-field region to discuss the sensitivity of the results to this length for all those alternatives. Finally, it is mainly concluded that the wavenumber approach is ideal from programming point of view due to the local nature of wavenumber condition imposed on truncation boundary. Moreover, it can also be envisaged as a great substitute for the rigorous FE-(FE-HE) type of analysis in the frequency domain which is heavily relying on a fluid hyper-element as its main core.

2. Method of analysis

As mentioned, the analysis technique utilized in this study is based on the FE-FE method, which is applicable for a general concrete gravity dam-reservoir system. The coupled equations can be obtained by considering each region separately and then combine the resulting equations.

2.1 Dam body

Concentrating on the structural part, the dynamic behavior of the dam is described by the well-known equation of structural dynamics (Zienkiewicz and Taylor 2000)

$$\mathbf{M}\ddot{\mathbf{r}}+\mathbf{C}\dot{\mathbf{r}}+\mathbf{K}\mathbf{r} = -\mathbf{M}\mathbf{J}\mathbf{a}_g+\mathbf{B}^T\mathbf{P} \tag{1}$$

where **M, C** and **K** in this relation represent the mass, damping and stiffness matrices of the dam body. Moreover, **r** is the vector of nodal relative displacements, **J** is a matrix with each two rows equal to a 2×2 identity matrix (its columns correspond to a unit horizontal and vertical rigid body motion), and $\mathbf{a}_g$ denotes the vector of ground accelerations. Furthermore, **B** is a matrix which relates vectors of hydrodynamic pressures (i.e., **P**), and its equivalent nodal forces.

Let us now consider harmonic excitation with frequency ω, and limit the present study to the horizontal ground motion only. It is well known that the response will also behave harmonic (i.e., $\mathbf{r}(t)=\mathbf{r}(\omega)e^{i\omega t}$). Thus, Eq. (1) can be expressed as

$$(-\omega^2\mathbf{M}+(1+2\beta i)\mathbf{K})\mathbf{r} = -\mathbf{MJa}_g^h+\mathbf{B}^T\mathbf{P} \tag{2}$$

In this relation, it is assumed that the damping matrix of the dam is of hysteretic type. This means

$$\mathbf{C} = (2\beta/\omega)\mathbf{K} \tag{3}$$

Moreover, it should be emphasized that the superscript h on the acceleration vector refers to the horizontal type of excitation. That is

$$\mathbf{a}_g^h = \begin{pmatrix} a_g^x \\ 0 \end{pmatrix} \tag{4}$$

2.2 Water domain

Assuming water to be linearly compressible and neglecting its viscosity, its small irrotational motion (Fig. 1) is governed by the wave equation (Chopra 1967, Chopra *et al.* 1980)

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}-\frac{1}{c^2}\ddot{p} = 0 \ \text{ in } \Omega \ \text{ and } D \tag{5}$$

where p is the hydrodynamic pressure and c is the pressure wave velocity in water. The boundary conditions for reservoir surface and bottom are as follows

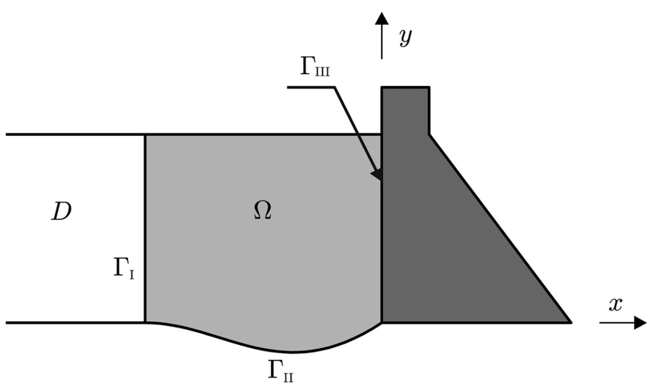

Fig. 1 Schematic view of a typical dam-reservoir system. The near-field reservoir domain Ω, the truncation boundary $\Gamma_{\rm I}$ and the far-field region D (excluded in the FE-FE type of analysis)

$$p = 0, \text{ on the water surface} \tag{6a}$$

$$\frac{\partial p}{\partial n} = -\rho a_g^n - q\dot{p}, \text{ at the reservoir's bottom} \tag{6b}$$

Herein, ρ is the water density and n denotes the outward (with respect to fluid region) perpendicular direction at the reservoir bottom. Moreover, the admittance or damping coefficient q utilized in the above equation, may be related to a more meaningful wave reflection coefficient α (Fenves and Chopra 1985)

$$\alpha = \frac{1-qc}{1+qc} \tag{7}$$

which is defined as the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a vertically propagating pressure wave incident on the reservoir bottom. For a fully reflective reservoir bottom condition, α is equal to 1 which leads to $q = 0$.

One can apply the weighted residual approach to obtain the finite element equation of the fluid domain, which may be written as

$$\mathbf{G}^e\ddot{\mathbf{P}}^e + \mathbf{H}^e\mathbf{P}^e = \mathbf{R}^e \tag{8}$$

With the following definitions

$$\mathbf{G}^e = \frac{1}{\rho c^2}\int_{\Omega_e}\mathbf{N}\mathbf{N}^T d\Omega \tag{9a}$$

$$\mathbf{H}^e = \frac{1}{\rho}\int_{\Omega_e}(\mathbf{N}_x\mathbf{N}_x^T + \mathbf{N}_y\mathbf{N}_y^T)d\Omega \tag{9b}$$

$$\mathbf{R}^e = \frac{1}{\rho}\int_{\Gamma^e}\mathbf{N}(\partial_n p)d\Gamma^e \tag{9c}$$

With $\mathbf{N}$ being the vector of element's shape functions, and $\mathbf{N}_x$, $\mathbf{N}_y$ denote its partial derivatives with respect to x, y, respectively. It is also worthwhile to emphasize that the superscript (^{e}) states that these matrices are related to the element level. The directional derivative $\partial_n p$ in Eq. (9c), can take three forms on different boundaries of the reservoir (Fig. 1)

- On the upstream boundary of the reservoir (Γ_{I}): One can apply different absorbing boundary conditions which will be discussed in next section.
- At the bottom of the reservoir (Γ_{II}), one can utilize Eq. (6b) as mentioned previously

$$\partial_n p = -\rho a_g^n - q\dot{p} \tag{10a}$$

- On the dam-reservoir interface (Γ_{III})

$$\partial_n p = -\rho\ddot{u}_n \tag{10b}$$

where $\ddot{u}_n$ is the total acceleration of fluid particles normal to the dam-reservoir interface. It is also noted that there must be compatibility of acceleration between the fluid and solid particles in that direction.

In general, an element may have all three above-mentioned boundary condition types. Thus, one can write $\mathbf{R}^e$ vector as follows

$$\mathbf{R}^e = \mathbf{R}_{\mathrm{I}}^e + \mathbf{R}_{\mathrm{II}}^e + \mathbf{R}_{\mathrm{III}}^e \tag{11}$$

Of course, it is possible that some of these boundary condition types are not applied for a certain element, which that part should be eliminated for that specific element. It is easily shown that one would obtain the following relations by utilizing Eqs. (10a) and (10b) in Eq. (9c), respectively

$$\mathbf{R}_{\mathrm{II}}^e = -\mathbf{B}_{\mathrm{II}}^e \mathbf{J}^e \mathbf{a}_g^h - q\mathbf{L}_{\mathrm{II}}^e \dot{\mathbf{P}}^e \tag{12a}$$

$$\mathbf{R}_{\mathrm{III}}^e = -\mathbf{B}_{\mathrm{III}}^e (\ddot{\mathbf{r}}^e + \mathbf{J}^e \mathbf{a}_g^h) \tag{12b}$$

With the following definitions

$$\mathbf{B}_i^e = \int_{\Gamma_i^e} \mathbf{N}\mathbf{n}^T \mathbf{N}_s^T d\Gamma^e \ ; \ i \in \{\mathrm{II}, \mathrm{III}\} \tag{13a}$$

$$\mathbf{L}_{\mathrm{II}}^e = \frac{1}{\rho} \int_{\Gamma_{\mathrm{II}}^e} \mathbf{N}\mathbf{N}^T d\Gamma^e \tag{13b}$$

Herein, $\mathbf{n}$ represents a unit outward normal vector. Moreover, $\mathbf{N}_s$ is the matrix of adjacent solid element shape functions utilized to interpolate accelerations in horizontal and vertical directions. It is worthwhile to mention that from practical point of view, the value of non-zero solid and fluid shape functions are essentially equal on the common fluid-solid interface.

Substituting Eqs. (12a) and (12b) into Eq. (11) will result in

$$\mathbf{R}^e = \mathbf{R}_{\mathrm{I}}^e - q\mathbf{L}_{\mathrm{II}}^e \dot{\mathbf{P}}^e - \mathbf{B}^e \ddot{\mathbf{r}}^e - \mathbf{B}^e \mathbf{J}^e \mathbf{a}_g^h \tag{14}$$

With the following definition

$$\mathbf{B}^e = \mathbf{B}_{\mathrm{II}}^e + \mathbf{B}_{\mathrm{III}}^e \tag{15}$$

It should also be noted that the relative acceleration at boundary Γ_{II} is identically equal to zero. Subsequently, Eq. (14) can be substituted in Eq. (8) which yields

$$\mathbf{G}^e \ddot{\mathbf{P}}^e + q\mathbf{L}_{\mathrm{II}}^e \dot{\mathbf{P}}^e + \mathbf{H}^e \mathbf{P}^e = \mathbf{R}_{\mathrm{I}}^e(t) - \mathbf{B}^e \ddot{\mathbf{r}}^e - \mathbf{B}^e \mathbf{J}^e \mathbf{a}_g^h \tag{16}$$

The equivalent form of this equation in the frequency domain would be

$$-\omega^2 \mathbf{G}^e \mathbf{P}^e + i\omega q \mathbf{L}_{\mathrm{II}}^e \mathbf{P}^e + \mathbf{H}^e \mathbf{P}^e = \mathbf{R}_{\mathrm{I}}^e(\omega) + \omega^2 \mathbf{B}^e \mathbf{r}^e - \mathbf{B}^e \mathbf{J}^e \mathbf{a}_g^h \tag{17}$$

Here, $\mathbf{L}_{\mathrm{II}}^e$ is a matrix, which corresponds to the absorption of energy at reservoir's bed. By assembling the element equations and imposing the free surface condition Eq. (6a), one would obtain the overall FE equation of the fluid domain

$$-\omega^2 \mathbf{G}\mathbf{P} + i\omega q \mathbf{L}_{\mathrm{II}}\mathbf{P} + \mathbf{H}\mathbf{P} = \mathbf{R}_{\mathrm{I}} + \omega^2 \mathbf{B}\mathbf{r} - \mathbf{B}\mathbf{J}\mathbf{a}_g^h \tag{18}$$

In this equation, $\mathbf{R}_{\mathrm{I}}$ is obtained by assembling the boundary integrals of Eq. (9c) on Γ_{I}.

2.3 Dam-reservoir system

The necessary equations for both dam and reservoir domains were developed in the previous sections. Thus, combining the main relations Eqs. (18) and (2) would result in the FE equations of the coupled dam-reservoir system in its initial form for the frequency domain

$$\begin{bmatrix} -\omega^2\mathbf{M} + (1+2\beta i)\mathbf{K} & -\mathbf{B}^T \\ -\omega^2\mathbf{B} & (-\omega^2\mathbf{G} + i\omega q\mathbf{L}_{\mathrm{II}} + \mathbf{H}) \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{M}\mathbf{J}\mathbf{a}_g^h \\ (-\mathbf{B}\mathbf{J}\mathbf{a}_g^h + \mathbf{R}_{\mathrm{I}}) \end{Bmatrix} \tag{19}$$

It is noted from the above equation that the vector $\mathbf{R}_{\mathrm{I}}$ still needs to be defined by some appropriate condition. This is related to the truncated boundary Γ_{I} which will be discussed below.

2.4 Modification due to truncation boundary contribution

The effect of truncation boundary will be treated in this section. For this purpose, let us now assume that this boundary (i.e., Γ_{I}) is vertical (i.e., along y-direction) and consider a harmonic plane wave with unit amplitude and frequency ω propagating along a direction which makes an angle θ with negative x-direction. This may be written in many different forms such as

$$p = e^{i(k'x + \lambda y + \omega t)} \tag{20a}$$

$$p = e^{(i\omega/c)[(\cos\theta)x + (\sin\theta)y + ct]} \tag{20b}$$

With the following relations being valid

$$k' = \frac{\omega}{c}\cos\theta \tag{21a}$$

$$\lambda = \frac{\omega}{c}\sin\theta \tag{21b}$$

$$k'^2 + \lambda^2 = \frac{\omega^2}{c^2} \tag{21c}$$

It is easily verified that the following condition is appropriate for the truncated boundary based on the assumed traveling wave (i.e., Eq. (20a))

$$\frac{\partial p}{\partial x} - ik'p = 0 \tag{22}$$

Employing Eq. (22) in Eq. (9c), it yields

$$\mathbf{R}_{\mathrm{I}}^e = -(ik')\mathbf{L}_{\mathrm{I}}^e\mathbf{P}^e \tag{23}$$

with the following definition

$$\mathbf{L}_{\mathrm{I}}^{e} = \frac{1}{\rho}\int_{\Gamma_{\mathrm{I}}^{e}} \mathbf{N}\mathbf{N}^{T} d\Gamma^{e} \tag{24}$$

Assembling $\mathbf{R}_{\mathrm{I}}^{e}$ for all fluid elements adjacent to truncation boundary leads to

$$\mathbf{R}_{\mathrm{I}} = -(ik')\mathbf{L}_{\mathrm{I}}\mathbf{P} \tag{25}$$

This can now be substituted in Eq. (19) to obtain the FE equations of the coupled dam-reservoir system in its final form for the frequency domain

$$\begin{bmatrix} -\omega^{2}\mathbf{M} + (1+2\beta i)\mathbf{K} & -\mathbf{B}^{T} \\ -\mathbf{B} & \omega^{-2}(-\omega^{2}\mathbf{G} + ik'\mathbf{L}_{\mathrm{I}} + i\omega q\mathbf{L}_{\mathrm{II}} + \mathbf{H}) \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} -\mathbf{M}\mathbf{J}\mathbf{a}_{g}^{h} \\ \omega^{-2}(-\mathbf{B}\mathbf{J}\mathbf{a}_{g}^{h}) \end{Bmatrix} \tag{26}$$

It is also noticed that the lower matrix equation of Eq. (26) is multiplied by ω^{-2} in this process to obtain a symmetric dynamic stiffness matrix for the dam-reservoir system.

2.5 Different options for defining parameter k'

The major remaining concept is the determination of parameter k' which will be discussed in this section. There are different available options. However, before a discussion on that, it is worthwhile to review some salient aspects on the exact analytical solution available for the domain D (Fig. 1). It is reminded that this domain is actually eliminated from our problem.

This is a regular semi-infinite region with constant depth H extending to infinity in the upstream direction. It is also assumed presently that the base of this region is completely reflective (i.e., $\alpha = 1$ or $q = 0$) and we are considering merely horizontal ground excitation. Under these circumstances, the exact solution for this region may be written as follows (Chopra *et al*. 1980)

$$p(x,y,t) = \sum_{j=1}^{\infty} B_{j} cos(\lambda_{j} y) e^{i(k'_{j}x + \omega t)} \tag{27}$$

It is noted that the solution is composed of different modes, and amplitude B_j depends on the existing conditions on the downstream face of that region. Moreover, parameters λ_j and k'_j are defined as

$$\lambda_{j} = \frac{(2j-1)\pi}{2H} \tag{28a}$$

$$k_{j}'^{2} + \lambda_{j}^{2} = \frac{\omega^{2}}{c^{2}} \tag{28b}$$

Herein, H represents the water depth. Based on Eq. (28b), one can actually determine k'_j (referred to as the j-th wavenumber) as follows

$$k'_{j} = \pm i\sqrt{\lambda_{j}^{2} - \frac{\omega^{2}}{c^{2}}} \tag{29}$$

Although, there are two options in this definition, the negative sign is merely admissible for a semi-infinite region extending to infinity in the negative x-direction as in our present case. This is due to the fact that we are only interested with evanescent modes (imaginary wavenumber) or traveling modes (real wavenumber) which are decaying or propagating towards the upstream direction. It is also noted that the j-th wavenumber (k'_j) becomes zero at a cut-off frequency referred to as the j-th natural frequency of the reservoir (i.e., ω_j^r). This is obtained by substituting Eq. (28a) in Eq. (29) under that condition which results in

$$\omega_j^r = \frac{(2j-1)\pi c}{2H} \tag{30}$$

Eq. (29) with the admissible negative sign, may also be written as

$$k_j' = \frac{\omega}{c}\left(\frac{-i(2j-1)}{\Omega}\sqrt{1-\frac{\Omega^2}{(2j-1)^2}}\right) \tag{31}$$

with the help of dimensionless frequency Ω

$$\Omega = \frac{\omega}{\omega_1^r} \tag{32}$$

Let us now describe some of the available options for selecting parameter k' in Eq. (26)

2.5.1 Alternative 1

The first option could be that one presumes the assumed planar wave is impinging the truncation boundary perpendicularly. Thus, the angle θ is zero, and k' is readily found from Eq. (21a)

$$k' = \frac{\omega}{c} \tag{33}$$

It is worthwhile to mention that, this may also be envisaged as the limiting case for each k_j' (Eq. 31) as Ω goes to infinity. Substituting Eq. (33) into Eq. (22) leads to what is known as Sommerfeld boundary condition for the frequency and time domains, respectively (Sommerfeld 1949)

$$\frac{\partial p}{\partial x} = \frac{i\omega}{c}p \tag{34a}$$

$$\frac{\partial p}{\partial x} = \frac{1}{c}\frac{\partial p}{\partial t} \tag{34b}$$

2.5.2 Alternative 2

The second option is to define k' based on an approximation of the first wavenumber. Thus, let us consider the first wavenumber (i.e., replace $j = 1$ in Eq. (31))

$$k_1' = \frac{\omega}{c}\left(\frac{-i}{\Omega}\sqrt{1-\Omega^2}\right) \tag{35}$$

By employing the estimate $\sqrt{1-\Omega^2}$ □ $(1 + i\Omega)$on Eq. (35) and utilizing Eq. (30) and Eq. (32), it yields

$$k' = \frac{\omega}{c} - i\frac{\pi}{2H} \tag{36}$$

Substituting Eq. (36) into Eq. (22) leads to what is known as Sharan boundary condition for the frequency and time domains, respectively (Sharan 1987)

$$\frac{\partial p}{\partial x} = \frac{i\omega}{c}p + \frac{\pi}{2H}p \tag{37a}$$

$$\frac{\partial p}{\partial x} = \frac{1}{c}\frac{\partial p}{\partial t} + \frac{\pi}{2H}p \tag{37b}$$

2.5.3 Alternative 3

The third option is what is proposed in this study. That is to define k' based on different wavenumbers for various frequency ranges. In particular, use the following strategy

$$k' = k_1' \;;\; for\ (0 \le \Omega \le 3)\ or\ (0 \le \omega \le \omega_2^r) \tag{38a}$$

$$k' = k_j' \;;\; for\ [(2j-1) \le \Omega \le (2j+1)\ or\ (\omega_j^r < \omega \le \omega_{j+1}^r)]\ and\ j \ge 2 \tag{38b}$$

2.5.4 Alternative 4

Finally, one might employ the j-th wavenumber for defining k' in all frequency ranges. This could be based on the first, second or any desired wavenumber. This might be written as

$$k' = k_j';\ \text{for}\ [0 \le \Omega \le \infty] \tag{39}$$

Of course, one would not expect to produce good results based on this option for all frequency ranges. However, this option could reveal the wavenumber which is most appropriate for each specific frequency range.

3. Modeling and basic parameters

The introduced methodology is employed to analyze an idealized dam-reservoir system. The details about modeling aspects such as discretization, basic parameters and the assumptions adopted are summarized in this section.

3.1 Models

An idealized triangular dam with vertical upstream face and a downstream slope of 1:0.8 is considered on a rigid base. The dam is discretized by 20 isoparametric 8-node plane-solid finite elements.

As for the water domain, two strategies are adopted (Fig. 2). For the FE-FE method of analysis

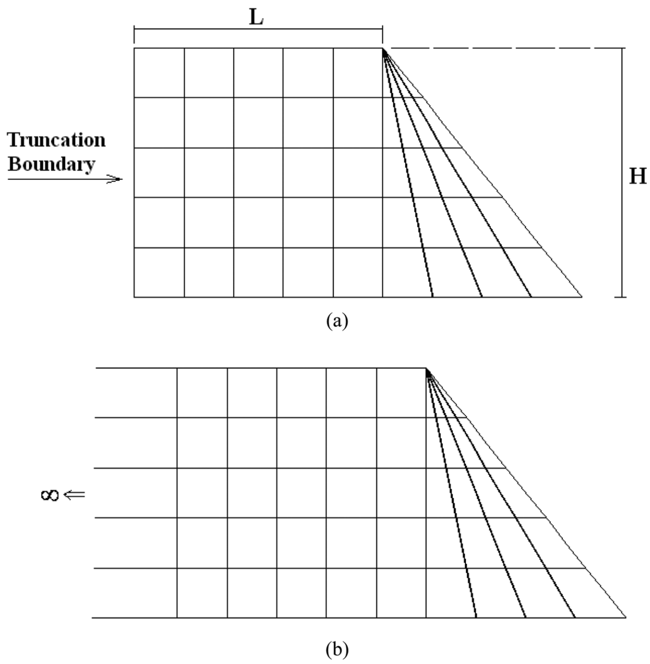

Fig. 2 The dam-reservoir discretization for $L/H = 1$; (a) FE-FE model and (b) FE-(FE-HE) model

which is our main procedure, only the reservoir near-field is discretized and the absorbing boundary condition is employed on the upstream truncation boundary according to different alternatives discussed. The length of this near-field region is denoted by L and water depth is referred to as H. Three cases are considered. These are in particular the L/H values of 0.2, 1 and 3 which represent low, moderate and high reservoir lengths. This region is discretized by 5, 25 and 75 isoparametric 8-node plane-fluid finite elements for three above-mentioned L/H values, respectively.

For the FE-(FE-HE) method of analysis, the reservoir domain is divided into two regions. The near-field region is discretized by fluid finite elements, and the far-field is treated by a fluid hyper-element. Of course, it should be emphasized that this option is merely utilized to obtain the exact solution (Lotfi 2001). Moreover, it is well-known that the results are not sensitive in this case to the length of the reservoir near-field region or L/H value.

3.2 Basic parameters

The dam body is assumed to be homogeneous and isotropic with linearly viscoelastic properties for mass concrete

Elastic modulus (E_d)	= 27.5 GPa
Poison's ratio	= 0.2
Unit weight	= 24.8 kN/m^3
Hysteretic damping factor (β_d)	= 0.05

The impounded water is taken as inviscid and compressible fluid with unit weight equal to 9.81 kN/m^3, and pressure wave velocity c = 1440 m/sec.

4. Results

It should be emphasized that all results presented herein, are obtained by the FE-FE method discussed, under different absorbing conditions applied on the truncation boundary. The only exception is for what is referred to as the exact response. That especial case is carried out by the FE-(FE-HE) analysis technique.

The first part of the study relates to a dam-reservoir system with a moderate near-field reservoir length (i.e., $L/H = 1$ (Fig. 2a)). Three cases are considered. The only difference between these cases, are the type of absorbing boundary condition imposed on the truncation boundary. These are in particular based on alternatives I, II and III (i.e., Sommerfeld, Sharan and wavenumber condition, respectively). The transfer function for the horizontal acceleration at dam crest with respect to horizontal ground acceleration, are presented in Fig. 3 for these three cases. It is noted that response in each case is plotted versus the dimensionless frequency. The normalization of excitation

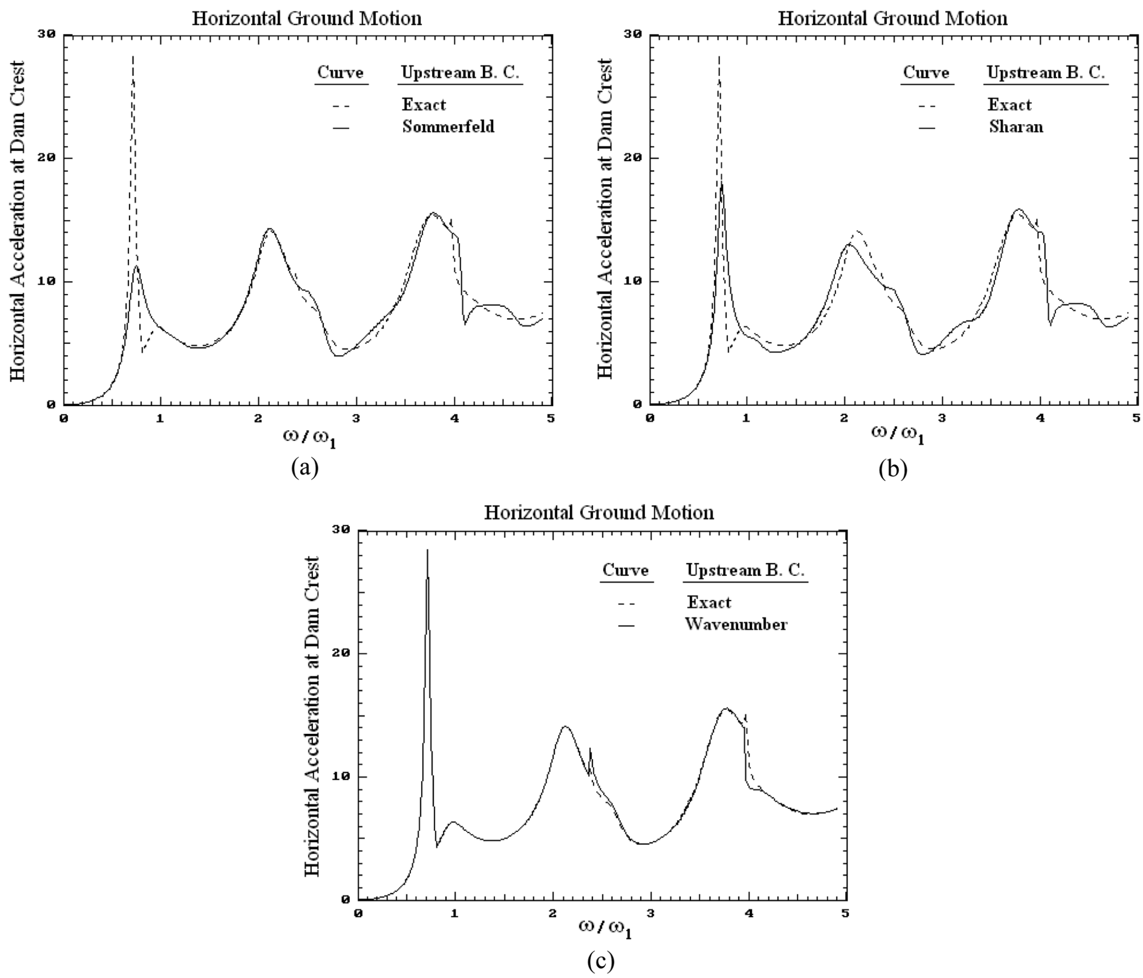

Fig. 3 Horizontal acceleration at dam crest due to horizontal ground motion for different absorbing boundary condition alternatives ($L/H = 1$); (a) Sommefeld, (b) Sharan and (c) Wavenumber

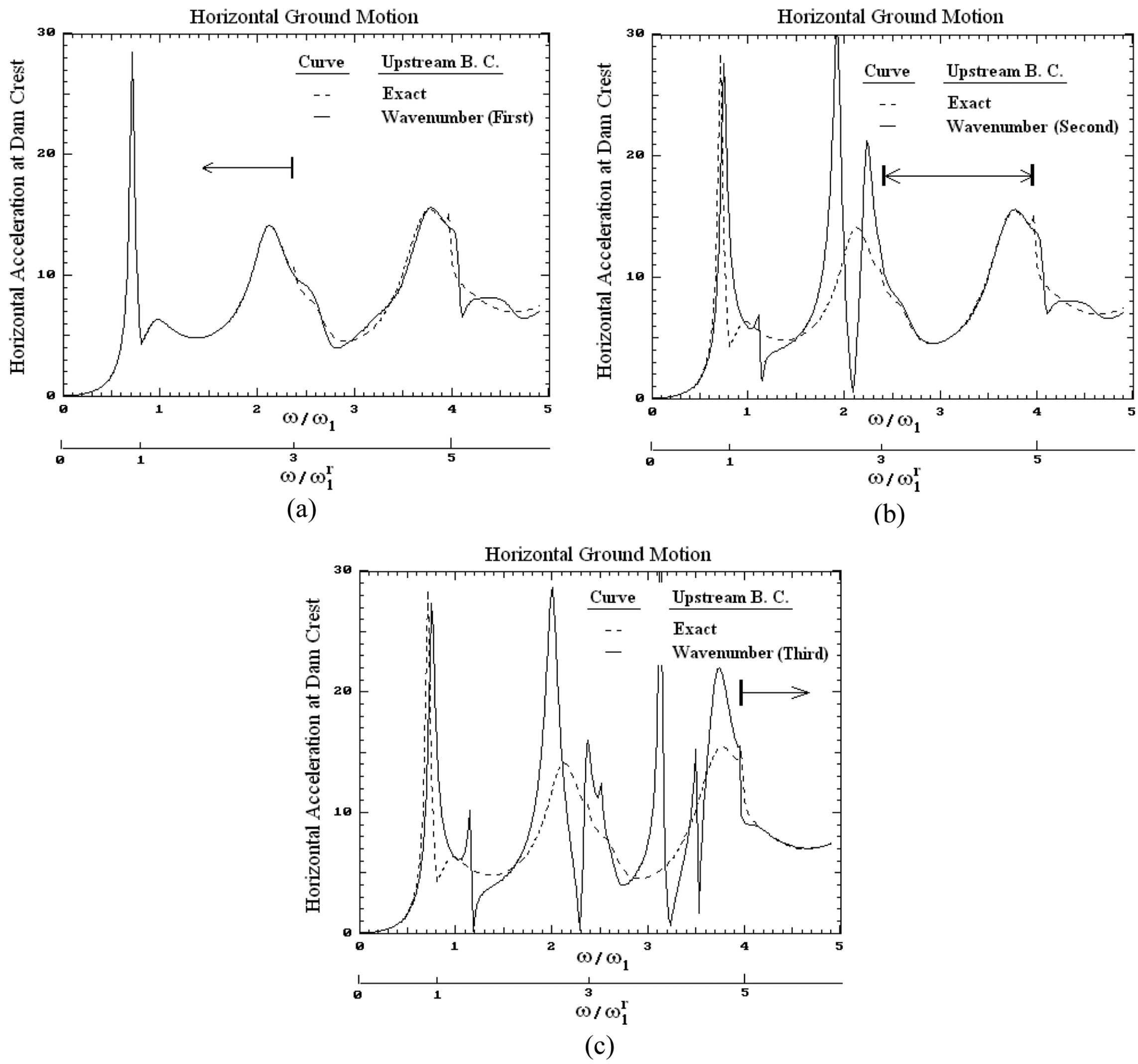

Fig. 4 Horizontal acceleration at dam crest due to horizontal ground motion for absorbing boundary condition alternative 4 ($L/H = 1$) in (a) First, (b) Second and (c) Third wavenumber selection

frequency is carried out with respect to ω_1, which is defined as the natural frequency of the dam with an empty reservoir on rigid foundation. Moreover, it is noticed that all cases are compared with the exact response.

It is observed that the response for Sommerfeld condition case has significant error near the fundamental frequency of the system where the first major peak occurs. For the second case (Sharan B.C.), the error reduces at the first major peak (compared to the first case); however, it is observed that the response maintains a similar pattern. Moreover, error increases at the second major peak for this case, which shows the deficiency of Sharan condition with respect to Sommerfeld B.C. for that frequency range.

The third case is related to the wavenumber approach. It is observed that the response agrees very well with the exact response and this initial result for this technique reveals the promising behavior

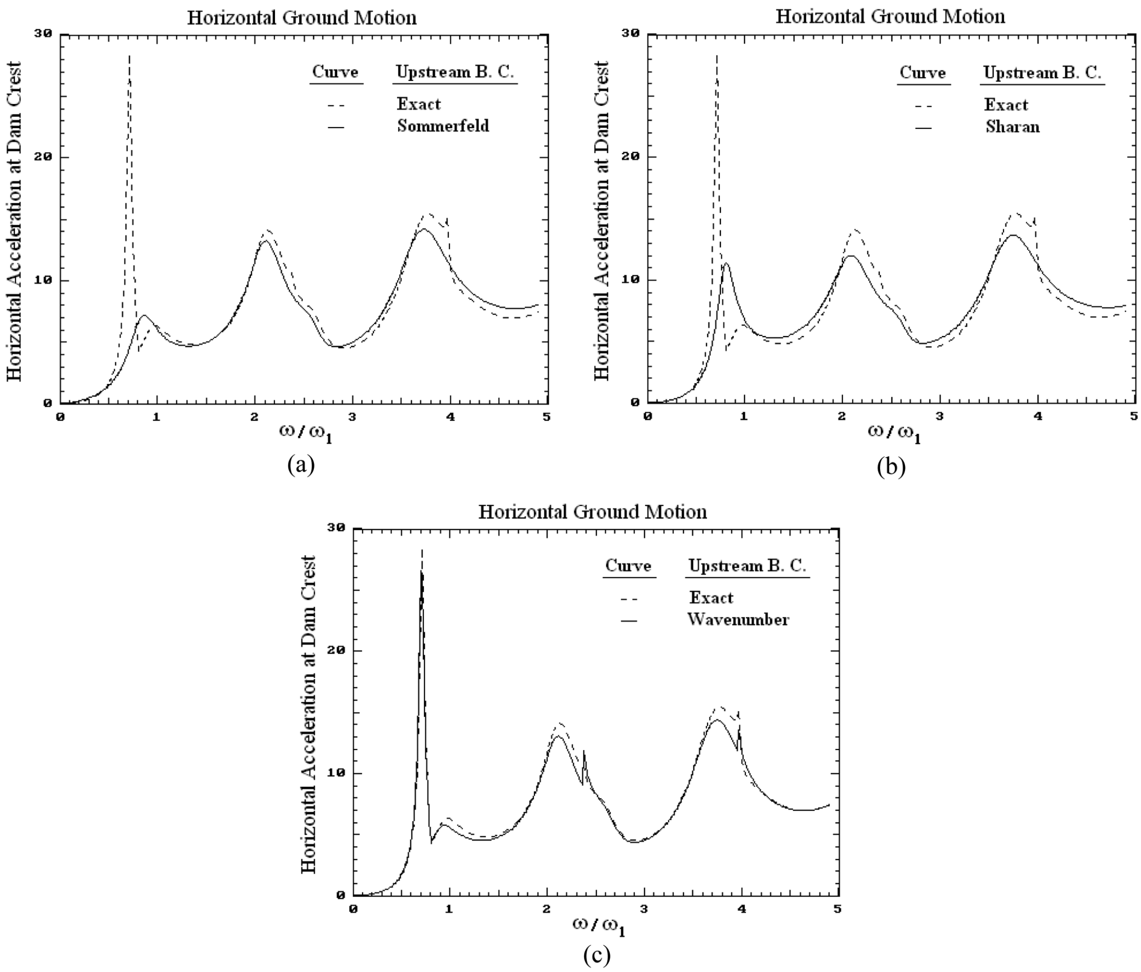

Fig. 5 Horizontal acceleration at dam crest due to horizontal ground motion for different absorbing boundary condition alternatives ($L/H = 0.2$); (a) Sommefeld, (b) Sharan and (c) Wavenumber

of this alternative.

For further investigation, it was decided to carry out three cases based on alternative 4. Each of these cases is utilizing a certain selective wavenumber for the whole frequency range. These were chosen based on the first, second and third wavenumbers. The responses for these cases are presented in Fig. 4. It is worthwhile to mention that another dimensionless frequency $\Omega\,(\Omega = \omega/\omega_1^r)$ is also shown for these plots to better visualize frequency ranges. It is reminded that first three natural frequencies of the reservoir occur at $\Omega = 1$, 3 and 5, respectively.

In respect to the three cases of alternative 4, it is observed that response is very accurate up to the second natural frequency of the reservoir (i.e., $0 \le \Omega \le 3$) when the first wavenumber is chosen. For the second case, the response is accurate between the second and third natural frequencies of the reservoir (i.e., $3 \le \Omega \le 5$). Similarly, the response is accurate between the third and fourth natural frequencies of the reservoir (i.e., $5 \le \Omega \le 7$) for the third wavenumber case. Of course, for this latter case, part of that frequency range is shown in the plot. The accurate part of the response is also

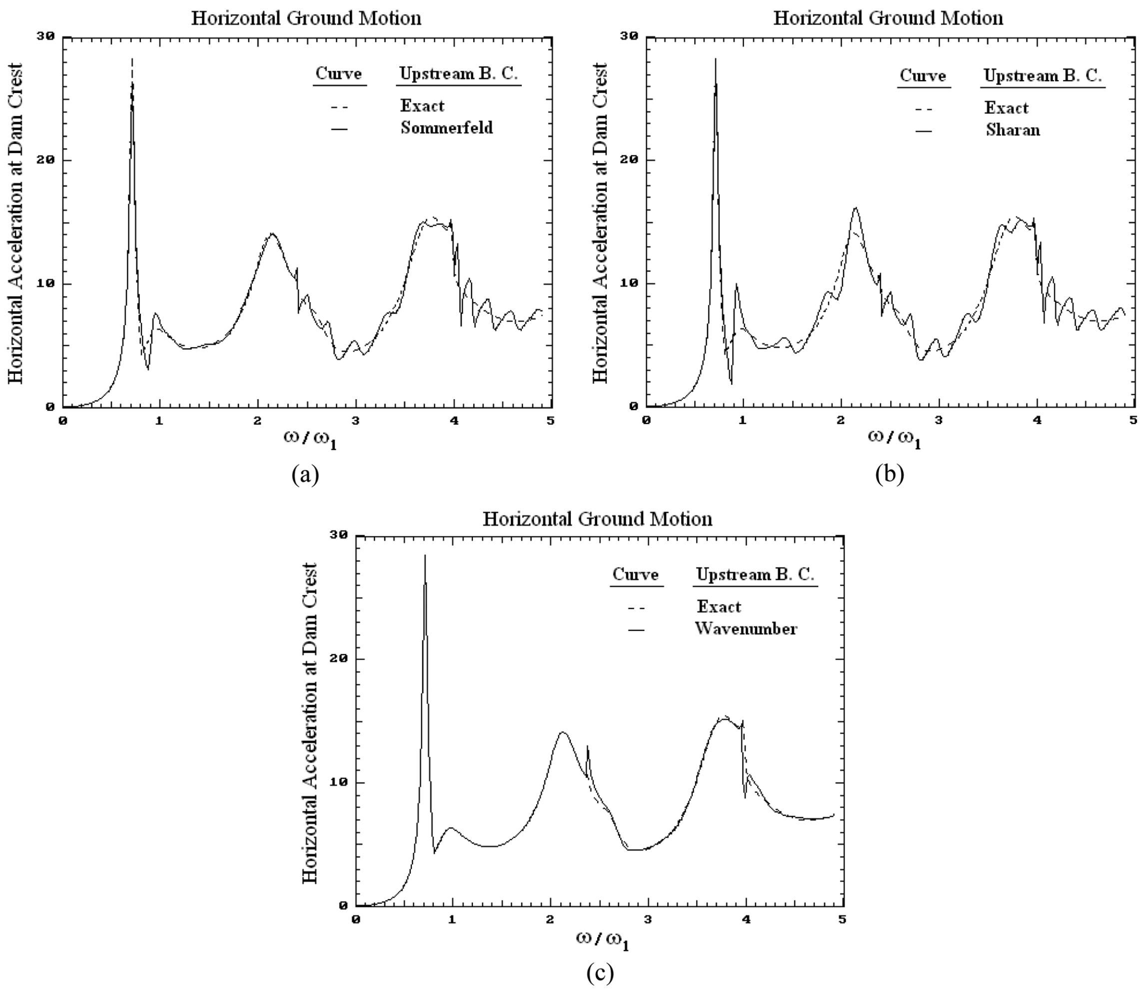

Fig. 6 Horizontal acceleration at dam crest due to horizontal ground motion for different absorbing boundary condition alternatives ($L/H = 3$); (a) Sommefeld, (b) Sharan and (c) Wavenumber

emphasized by the help of arrows in each plot.

The behavior discussed above for the alternative 4, help us to understand the justification for the strategy adopted for the alternative 3 (The wavenumber approach) and its effectiveness for all frequency ranges (Fig. 3c).

In the last part of this study, it was decided to investigate the behavior of three alternatives I, II and III (i.e., Sommerfeld, Sharan and wavenumber approaches) for other near-field reservoir lengths. At first a very low reservoir length (i.e., $L/H = 0.2$) is considered which is a challenging test for examining any type of absorbing boundary condition. Subsequently, a relatively high reservoir length (i.e., $L/H = 3$) was also selected.

The responses for the low length cases are presented in Fig. 5. It is observed that there are significant errors for the Sommerfeld and Sharan condition cases. Moreover, the errors have increased in comparison with the moderate reservoir length results (Fig. 5 versus Fig. 3). For the wavenumber approach, the response is still close to the exact response in most frequency ranges.

However, there exist errors in the range of 5% at the major peaks of the response. This is still believed to be remarkable result for such a challenging test.

Finally, let us examine the results for the high reservoir length (i.e., $L/H = 3$) which is illustrated in Fig. 6. It is noticed that responses for Sommerfeld and Sharan condition cases improves greatly in comparison with the low or even moderate length results discussed previously (Fig. 6 versus Fig. 5 or 3). However, some kinds of noise or distortion are noticed in the response of both these cases especially for higher frequencies. As for the wavenumber approach, it is noticed that the response agrees very well with the exact response, similar to the behavior noticed for the moderate reservoir length. Moreover, there are no signs of distortions in the response for the high reservoir length, contrary to what was noticed for the other two well-known alternatives I and II (Sommerfeld and Sharan condition cases).

5. Conclusions

The formulation based on FE-FE procedure for dynamic analysis of concrete dam-reservoir systems, was reviewed. Moreover, several options were discussed for imposing a local type of absorbing condition on the truncation boundary of the water domain. A special purpose finite element program was enhanced for this investigation. Thereafter, the response of an idealized triangular dam was studied due to horizontal ground motion for different alternatives employed as absorbing boundary condition. The main approach which was emphasized and proposed in this study is referred to as the wavenumber approach.

Overall, the main conclusions obtained by the present study can be listed as follows

In regard to Sommerfeld and Sharan absorbing condition

- There are significant errors occurring at the fundamental frequency of the system for low or even moderate reservoir lengths.
- The response improves greatly for the high reservoir lengths. However, some kinds of noise or distortion are noticed in the response of both these cases especially for higher frequencies.
- Obviously, the main advantage of these two conditions is that both of them can be readily utilized in time domain, as well as frequency domain.

In regard to wavenumber absorbing condition

- It is noticed that the response agrees very well in comparison with the exact response for the moderate as well as high reservoir lengths. However, errors in the range of 5% are noticed at the major peaks of the response for the low reservoir length ($L/H = 0.2$), which is still believed to be remarkable result for such a challenging test.
- There are no signs of distortions in the response under any circumstances.
- Obviously, the main disadvantage of this condition is that it cannot be utilized in time domain, and it is only suitable for frequency domain.
- The wavenumber approach is ideal from programming point of view due to the local nature of wavenumber condition imposed on truncation boundary. It can also be envisaged as a great substitute for the rigorous FE-(FE-HE) type of analysis which is heavily relying on a hyper-element as its main core. It is undeniable that the rigorous approach is significantly more complicated from programming point of view and also much more computationally expensive.

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