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Dynamic response of pile foundations with flexible slabs

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Abstract. An elasto-dynamic model for pile-soil-pile interaction together with a simple plate model is used in this study to assess the effect of flexible foundation slabs on the dynamic response of pile groups. To this end, different pile configurations with various slab thicknessesare considered in two soil media with low and high elastic moduli. The analyses include dynamic impedances and seismic responses of pile-group foundations.The presented results indicate that the stiffness and damping of pile foundations increase with thickness of the foundation slab; however, the results approach those for rigid slab as the slab thickness approaches twice the pile diameter for the cases considered in this study. The results also reveal that pile foundations with flexible slabs may amplify the earthquake motions by as much as 10 percent in the low to intermediate frequency ranges.

Keywords: pile group; foundation slab; pile-soil-pile; interaction; stiffness; damping; earthquake; dynamic; impedance

1. Introduction

Over the past four decades, a suite of models and solutions have been advanced for dynamic analysis of piles and pile groups. These solutions range from analytical and approximate solutions mostly based on the Winkler medium and the concept of dynamic subgrade modulus (e.g. Novak 1974, Dobry and Gazetas 1988, Makris and Gazetas 1992) to rigorous analytical and numerical solutions (e.g. Waas and Hartmann 1981, Kaynia 1982, Roesset 1982, Wolf *et al.* 1984). The Winkler-type solutions have been further extended to more complex conditions (e.g. Mylonakis *et al.* 1997) and to nonlinear problems including development of gap at soil-pile interface and liquefaction (e.g. Boulanger *et al.* 1999). For an overview of these solutions and models, see Pender (1993).

While the simplified solutions have been elaborated over the years to handle complicated conditions, the rigorous analytical solutions have been necessary to provide the means to tune the simplified solutions, especially with regards to damping.

The dynamic analysis of piles has traditionally been based on linear elastic response of the soil. Moreover, in the case of pile groups, it has often been assumed that the piles are connected by rigid caps (Fig. 1(a)). These simplifications and assumptions have partly been motivated by the traditional soil-structure interaction solutions such as the three-step method (Kausel *et al.* 1978). Almost all of the existing solutions reported for the impedances of pile groups have been derived for rigid pile cap conditions. In reality, however, piles are often connected by foundation slabs which are far from

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rigid (Fig. 1(b)) except for special cases such as foundations of bridge piers. Many of the conclusions drawn on the basis of rigid pile-cap solutions are often used for ordinary pile foundations although the behavior of pile foundations with flexible slabs is different than that of foundations with rigid pile caps. The mode of slab/foundation deformation sketched in Fig. 1(b) is central in this subject, as discussed and illustrated by the results presented in this paper.

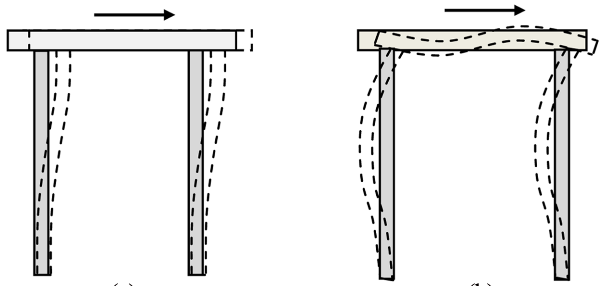
While, in principle, it is fairly straightforward to extend computational models of pile-soil-pile interaction to cases of flexible pile caps, no systematic set of results have been reported in the literature. This has motivated the present research which has the objective of computing new results in order to shed light on the role of flexible pile caps on the dynamic response of pile foundations.

2. Formulation

The computational model used in the present study is an extension of the model proposed by Kaynia (1982). The pile-soil tractions in this model are replaced by piecewise constant cylindrical loads on the pile shafts and circular loads at the pile tips. By using analytically-driven Green's functions for these loads in non-homogeneous, layered soil media, a soil stiffness matrix is established and is coupled to the dynamic stiffness of the piles. The responses of the piles are computed by imposition of the relevant boundary and traction conditions. The details can be found in Kaynia (1982) and in Kaynia and Kausel (1991).

The analyses are carried out under steady-state harmonic vibration in the frequency domain. Therefore, both the foundation impedances and the seismic response of the foundation are complex quantities. The seismic excitation is assumed to be due to vertically incident shear waves.

In the original solution by Kaynia (1982), the piles were assumed to be connected to a rigid pile cap (Fig. 1(a)). In the present study, it is assumed that the piles are connected by an elastic slab with uniform thickness h (Fig. 1(b)). The pile-soil stiffness matrix can simply be coupled to the stiffness matrix of the slab. While the ideal solution is to use advanced shell elements to model the foundation slab, a simpler engineering approach is adopted here. The slab foundation is modeled as equivalent beams spanning between the piles. For the in-plane (membrane) action, the slab is modeled as axial and shear-beam elements between the piles. This representation of the foundation slab is admittedly simplistic; however, it is believed that it will adequately capture the main mechanism of the interaction between the slab and the piles. The intension is not to provide design charts or curves for practical design, but rather cast light on the interaction between the piles and the slab.



(a) (b)

Fig. 1 Schematics of interaction of piles with stiff slab (left) and flexible slab (right)

There are a number of complex features in the dynamics of pile-soil-pile interaction which have been attributed to the assumption of rigid pile caps. One of these is the appearance of large peaks in the stiffness and damping of pile groups that stem from the out-of-phase soil particle motions between the neighboring piles. The present investigation will answer to this and other questions and will contribute to a better understanding of the dynamic behavior of pile foundations for practical conditions.

3. Parameters and analysis cases

A number of results are presented in the following to highlight the key features of the results. The objective of this study is twofold: (a) understand the mechanism by which the flexibility of the foundation slab in fluences the foundation response under external loads and earthquake excitation, and (b) give an approximate quantitative measure of this influence and the conditions under which the foundation slabs can be considered stiff.

The piles are considered to have a diameter d and an elastic modulus, E_p. The pile length, l, is kept constant equal to $20d$. It should be noticed that in the horizontal direction, the pile length, beyond typically 5 to 8 times the pile diameter, does not have any practical influence on the pile response. Therefore, this choice of pile length has little practical effect on the results and conclusions in this paper. Two homogeneous soil profiles with elastic moduli representing stiff and soft soil with respectively $E_p/E_s = 100$ and $E_p/E_s = 1000$ are considered. In all the models, the Poisson's ratio of the soil is taken equal to 0.4, and the ratio of the soil's mass density, ρ_s, to that of the pile, ρ_p, is taken equal to 0.75. It should be noticed that the elastic moduli of realistic soil profiles vary typically linearly with depth in normally-consolidated clays and parabolically with depth in sands. The uniform soil profiles selected here are thus not representative of realistic soil conditions. However, most results reported in the literature are for uniform soil media; therefore, only uniform soil profiles are considered here so that the presented results would be complimentary and comparable to the previous results, and could provide an understanding of the role of slab stiffness on pile-soil interaction.

Two pile configurations are considered: 4×4 and 8×8 square pile groups with pile spacing, s, equal to $5d$ and $10d$. Finally, four values of slab thickness, h, are considered as follows: $h/d = 0.25$, 0.5, 1.0 and 2.0 covering a range of thin (flexible) to thick (stiff) slabs.

The analyses are carried out by assuming concrete piles with diameter $d = 0.5$ m and $E_p = 30$ GPa. All the other parameters are selected on the basis of the normalized relations stated above, and the final results are also presented in normalized form so that they can be applied to other cases with similar non-dimensional soil/pile/slab parameters.

The presented results are the horizontal and vertical impedances of the foundation slabs together with the earthquake response of the slabs. In dealing with non-rigid pile caps, the notion of foundation response needs to be established. In contrast to pile groups with rigid pile caps where the piles move and rotate identically, the piles have different responses in a pile group with flexible pile cap. The degree by which this affects the total foundation response depends on the stiffness of the slab relative to that of the piles. In this study an *apparent average stiffness* is defined as follows. Unit loads are applied on the foundation at all the pile-heads and the average displacement of the pile heads is computed. The apparent stiffness is then computed as the inverse of the average pile responses. This is the same as the ratio between sum of the forces and sum of the pile-head displacements.

The impedances computed in this manner are presented as $K = k + ia_0c$ where $a_0 = \omega d/V_s$ is the normalized frequency, ω is the frequency of harmonic excitation and V_s is the shear wave velocity of the soil. Both the stiffness, k, and damping, c, are normalized by $N{\cdot}K^s$ where N is the number of piles and K^s is the static stiffness of a single pile in the group.

In the case of earthquake response, the pile-soil system is assumed to be excited by vertically-propagating shear waves. The response quantity of interest is the absolute value of the slab's transfer function; that is, the ratio of the average response of the pile heads divided by the motion of the soil surface in the free field. The computations can be extended to other types of seismic excitations such as surface waves and inclined shear waves (e.g. Kaynia and Novak 1992).

4. Dynamic response of foundations

Fig. 2 displays the average horizontal stiffness and damping of 4×4 pile groups with $s/d = 5$ (plots *a* and *b*) and $s/d = 10$ (plots *c* and *d*) in the stiff soil with $E_p/E_s = 100$. The figures show that the

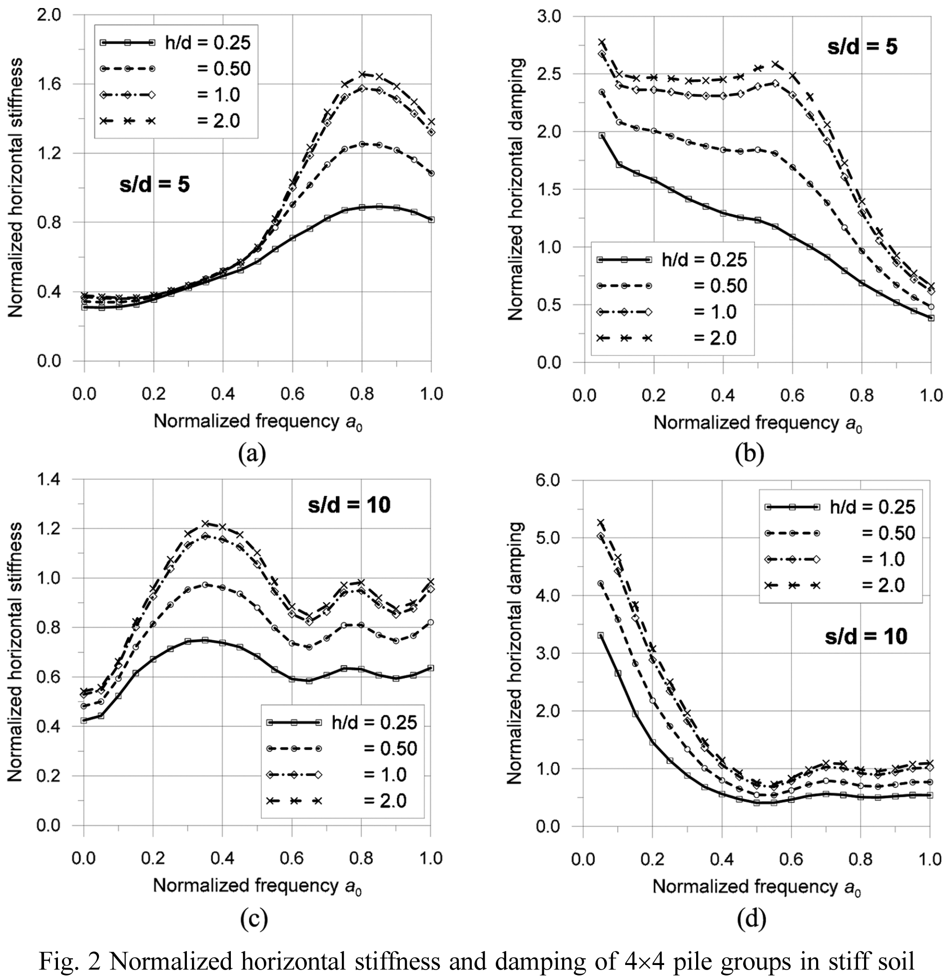


Fig. 2 Normalized horizontal stiffness and damping of 4×4 pile groups in stiff soil

impedances approach those for the rigid pile cap as h/d approaches 2. As expected, the horizontal stiffness increases with the slab thickness, especially around the frequency corresponding to the out-of-phase particle motions of the soil between adjacent piles (for example, around $a_0 = 0.8$ in Fig. 2(a) and around $a_0 = 0.4$ and 0.8 in Fig. 2(c)). What is less obvious is the fairly large effect of slab flexibility on the total foundation damping (plots *b* and *d*). The trends of the results for larger pile spacing are generally the same except that the damping in these cases drops more quickly with frequency.

The corresponding results for the vertical stiffness and damping are displayed in Fig. 3. More specifically, the figures present the average vertical stiffness and damping of 4×4 pile groups with $s/d = 5$ (plots *a* and *b*) and $s/d = 10$ (plots *c* and *d*) in the stiff soil with $E_p/E_s = 100$. As opposed to horizontal direction, the average stiffness and damping are practically independent of the slab flexibility. This is an interesting observation from a practical point of view. It should, however, be noted that this observation does not indicate that a flexible slab behaves like a rigid pile cap in the vertical direction. Indeed, the individual piles in foundations with flexible slabs have different responses as discussed later in this section.

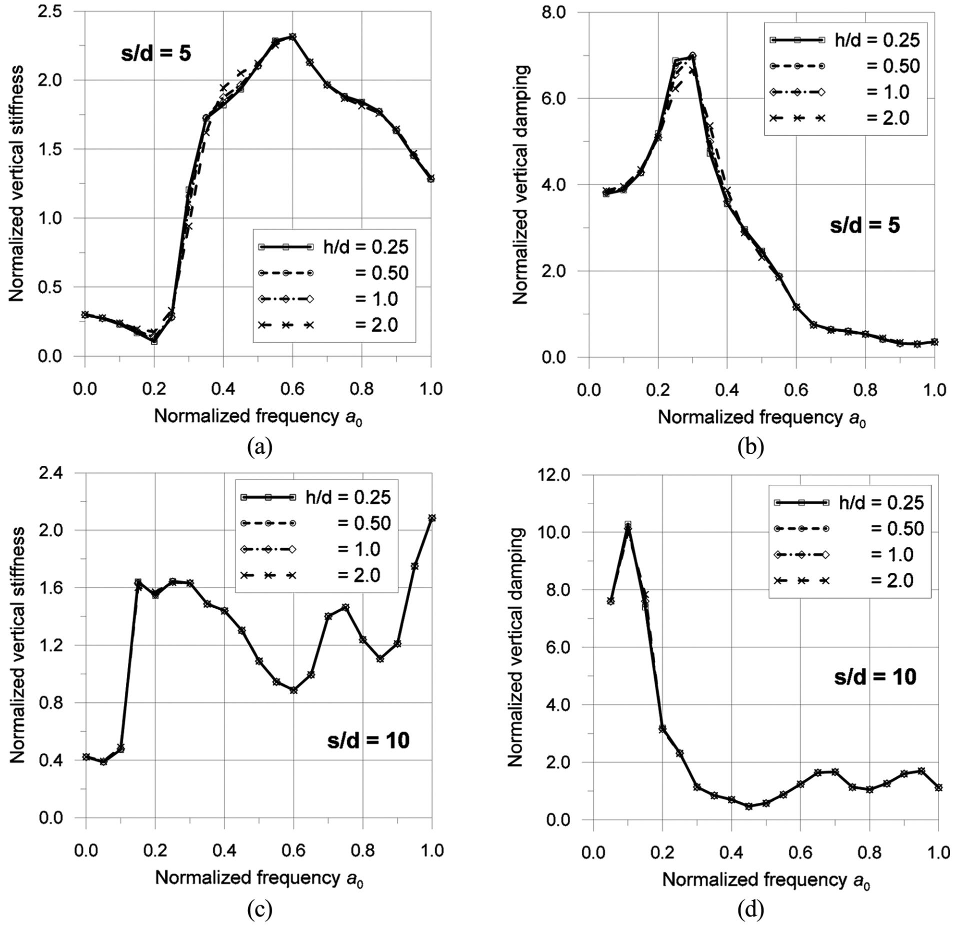


Fig. 3 Normalized vertical stiffness and damping of 4×4 pile groups in stiff soil

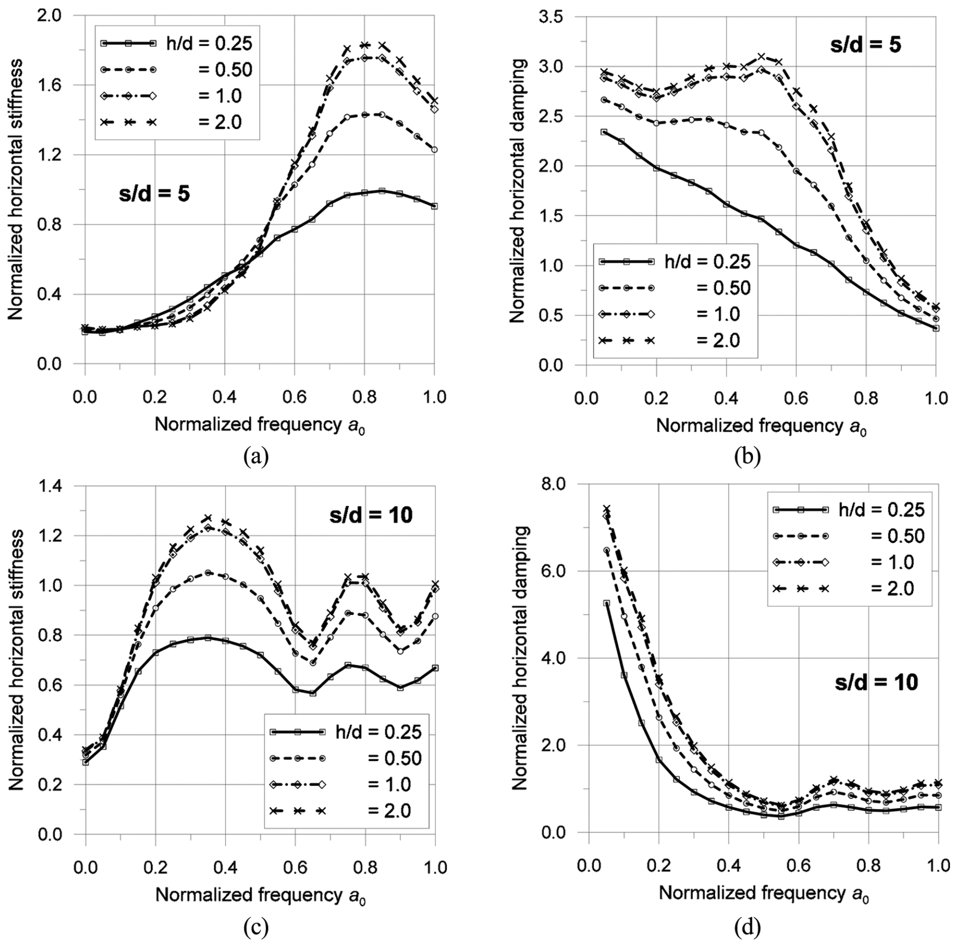


Fig. 4 Normalized horizontal stiffness and damping of 8×8 pile groups in stiff soil

The same set of results as in Figs. 2 and 3 are illustrated in Figs. 4 and 5 for 8×8 pile groups in the stiff soil medium. While the same trends observed previously in the smaller groups, including the role of flexible slab on the overall response, are also displayed by these results, it is worth noticing that the normalized damping generally increases in larger groups.

As stated above, the average vertical stiffness is rather independent of the slab flexibility although the individual piles have different responses. To verify this statement, the absolute vertical motions of the four piles closest to the middle of the 8×8 group with $s/d = 10$ are plotted as functions of frequency in Fig. 6. These piles are identified by their numbers (25 to 28) in a group of 64 piles. It should be noted that the insensitivity of the vertical response to the slab flexibility stems from the assumption that the dynamic loads are uniformly distributed on the foundation slab. In reality, this assumption is often not valid due to irregular structural loading and presence of stiffening systems such as walls and braces in the structure.

Fig. 7 displays the average horizontal stiffness and damping of 4×4 pile groups with $s/d = 5$ (plots *a* and *b*) and $s/d = 10$ (plots *c* and *d*) in the soft soil with $E_p/E_s = 1000$. The figures show that the impedances approach those of rigid pile caps as h/d approaches 2. Most of the general trends in

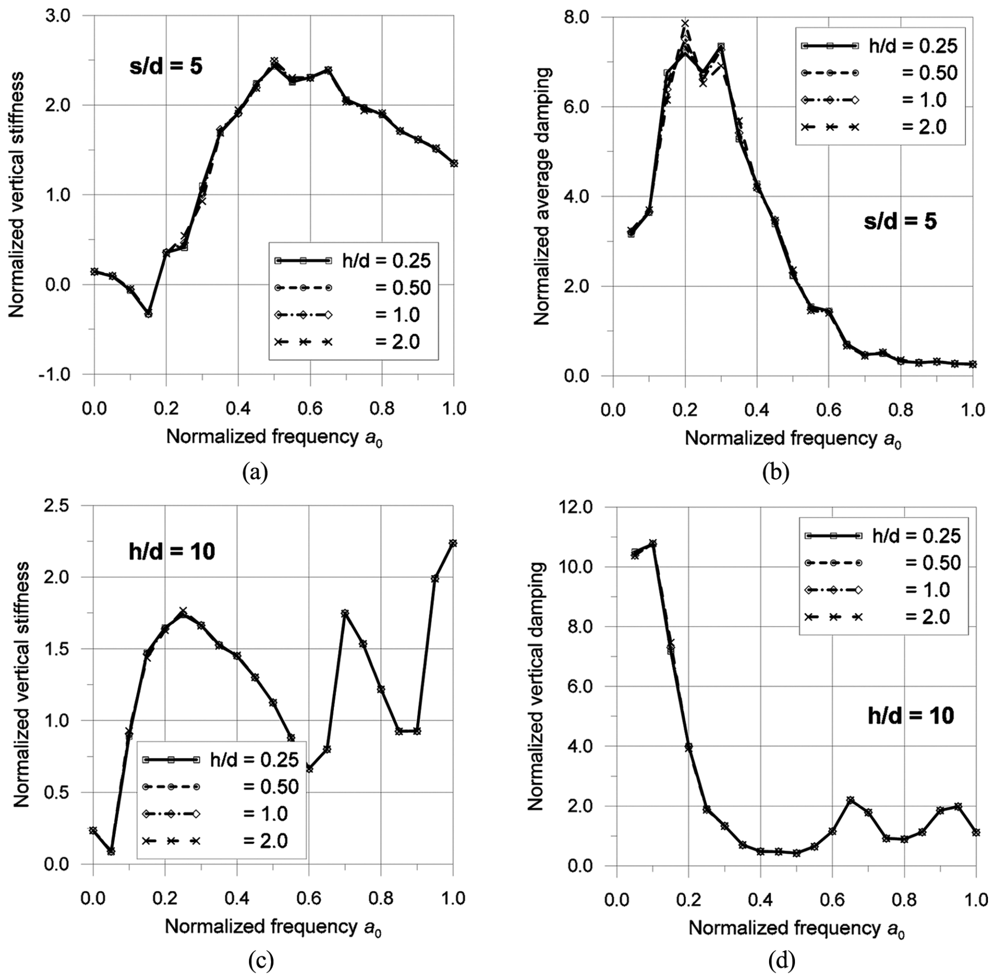


Fig. 5 Normalized vertlcal stiffness and damping of 8×8 pile groups in stiff soil

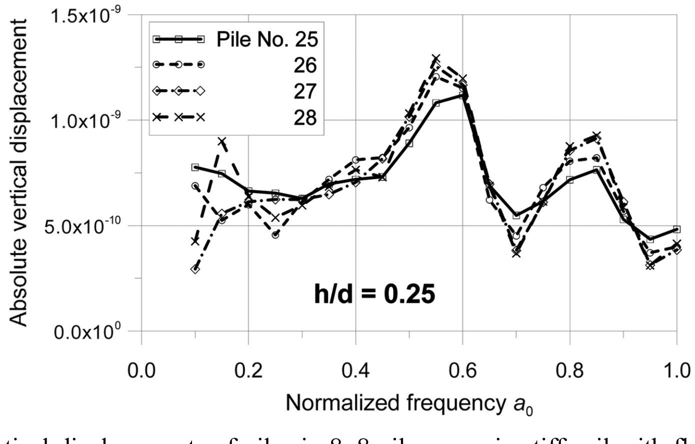


Fig. 6 Vertical displacements of piles in 8×8 pile group in stiff soil with flexible slab

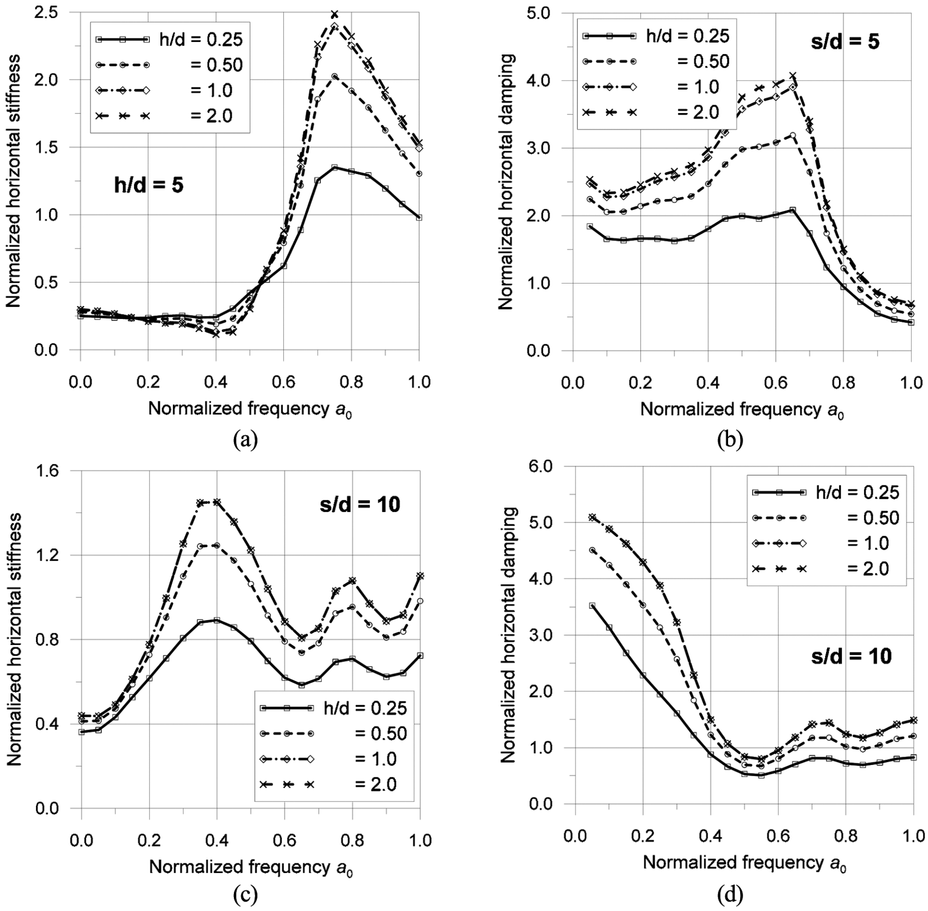


Fig. 7 Normalized horizontal stiffness and damping of 4×4 pile groups in soft soil

these results are similar to those for the pile groups in the stiff soil. As expected, and noted by earlier research (e.g. Kaynia 1982), the effect of pile-soil-pile interaction on the stiffness and damping is stronger in softer soil. What is new in these results is that in both soil types, the strong pile-soil-pile interaction effects observed at the out-of-phase response frequencies (for example, around $a_0 = 0.4$ and 0.8 in Figs. 2(c) and 4(c)) are moderated when using more flexible slabs.

The variations of the vertical stiffness and damping with frequency for pile groups in the soft soil ($E_p/E_s = 1000$) are plotted in Fig. 8. The conclusions and observations are generally similar to those for the stiff soil. The exception is the more noticeable effect of the slab flexibility on the vertical stiffness and damping for closely-spaced piles (see plots is Figs. 8(a) and 8(b)). The explanation can be found in the mechanism of flexible slab interaction with the piles. For close piles in soft soil and in the case of flexible slabs, the interaction between the piles is sufficiently strong to induce lateral and rotational displacements on the pile heads even under vertical loads (see Fig. 1(b)). As the pile spacing increases, the interaction diminishes and the piles tend to behave more independently. As explained for the case of stiff soil, the apparent insensitivity of the vertical response to the slab flexibility is a consequence of defining the response in terms of the average response of the piles in the group.

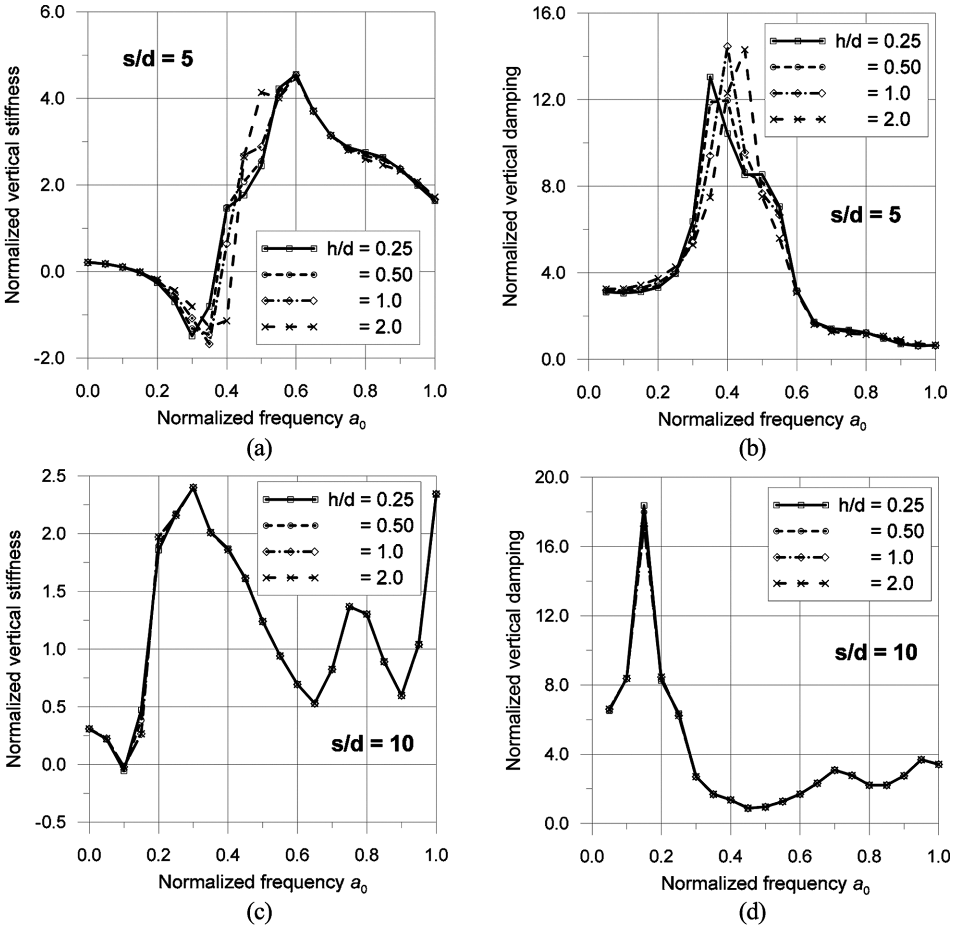
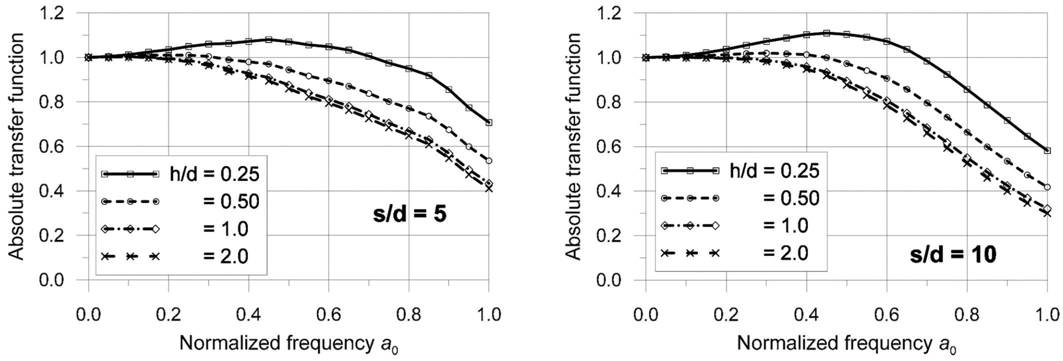


Fig. 8 Normalized vertical stiffness and damping of 4×4 pile groups in soft soil

5. Earthquake response of foundations

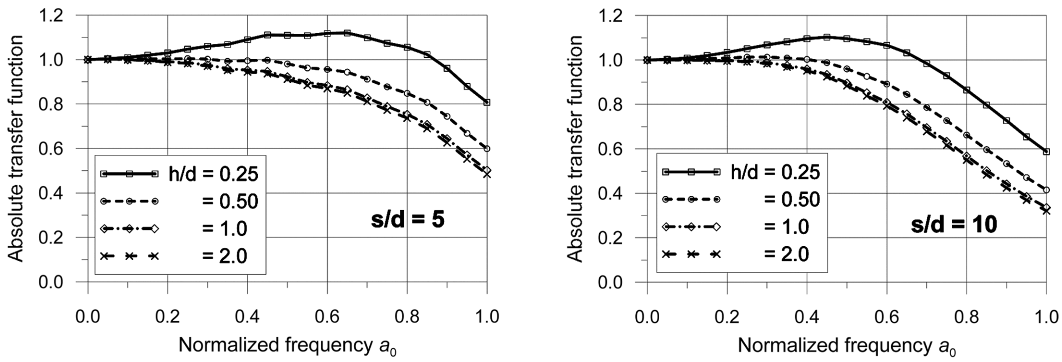
It has long been known that typical pile-group foundations tend to follow the earthquake motions in the free field although they tend to filter out the higher frequency components of the motions. This aspect of the response is often studied through the seismic transfer function of the foundation that is defined as the ratio between the absolute value of the horizontal motion of the foundation and that on the ground surface in the free field. For simplicity, this term is referred to as *foundation transfer function* in the following. The extension of the computational model to the earthquake response analysis is as outlined in Kaynia (1982).

Fig. 9 displays the foundation transfer functions for 4×4 pile groups with $s/d = 5$ and $s/d = 10$ in the stiff soil ($E_p/E_s = 100$) for different values of the slab thickness. As in previous analyses, the results tend to those for stiff pile cap as h/d approaches 2. An interesting feature is the amplification of the seismic response (transfer function exceeding unity) for flexible slabs. This can readily be explained with the help of the mechanism in Fig. 1. As the slab becomes more flexible, its



(a) (b)

Fig. 9 Absolute seismic transfer functions of 4×4 pile groups in stiff soil



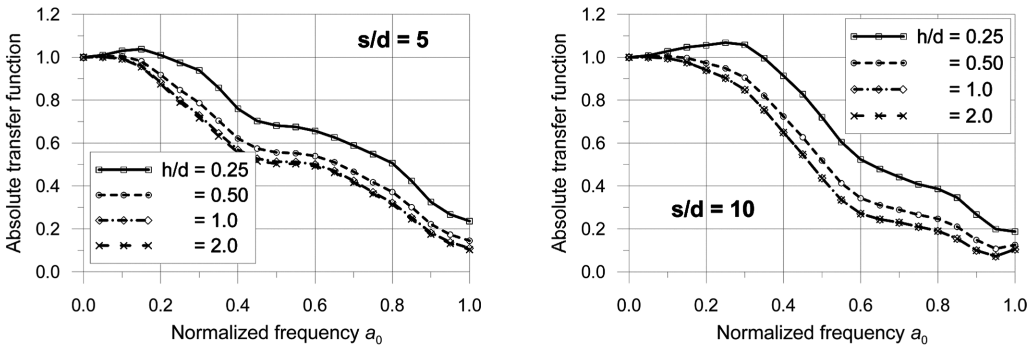
(a) (b)

Fig. 10 Absolute seismic transfer functions of 8×8 pile groups in stiff soil

restraining capability of the piles diminishes and the piles tend to behave more like free-head piles. The transfer functions for $h/d = 0.25$ in Fig. 9 are reminiscent of the transfer functions of single piles (e.g. Kaynia 1982).

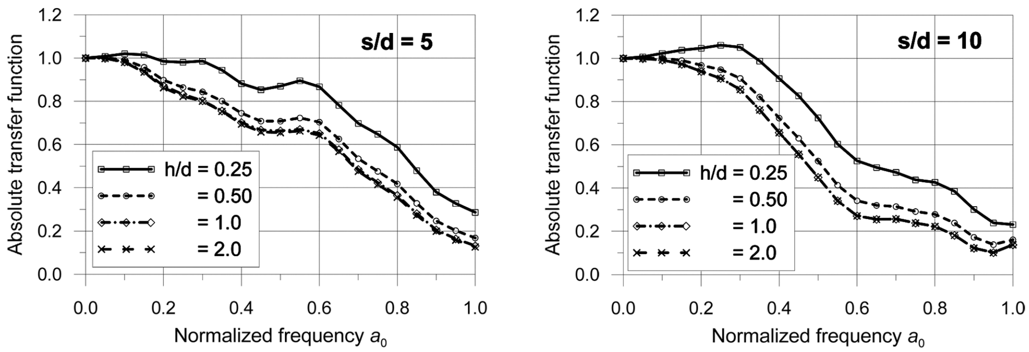
Fig. 10 displays the foundation transfer functions for 8×8 pile groups with $s/d = 5$ and $s/d = 10$ in stiff soil ($E_p/E_s = 100$). The trends of the results and the observations are consistent with those discussed above.

Finally, Figs. 11 and 12 display the seismic transfer functions for 8×8 pile groups with $s/d = 5$ and $s/d = 10$ in the soft soil ($E_p/E_s = 1000$). These results reveal the same general characteristics of transfer functions; however, their flat plateaus extend over shorter frequency ranges. This indicates that pile groups in soft soil tend to filter out the intermediate and large frequency content of an earthquake to a larger extent than pile groups in stiff soil. As in stiff soils, pile foundations with flexible slabs in soft soils tend to amplify the low to intermediate frequency ranges. An interesting conclusion drawn from Figs. 11 and 12 is that the transfer functions for largely-spaced piles are fairly independent of the number of piles. This can be discerned by comparing the plots in Figs. 11(b) and 12(b).



(a) (b)

Fig. 11 Absolute seismic transfer functions of 4×4 pile groups in soft soil



(a) (b)

Fig. 12 Absolute seismic transfer functions of 8×8 pile groups in soft soil

6. Conclusions

A Green's-function-based model for pile-soil-pile interaction together with a simple plate model was used in this study to evaluate the effect of flexible slabs on the dynamic and seismic responses of pile groups. To this end, different pile configurations with various slab flexibilities were considered in two soil media with low and high shear moduli. The analyses included computation of the dynamic impedances and the seismic response of pile foundations.

The presented results indicated that the stiffness and damping of pile foundations increase with the thickness of the foundation slab, and the results approach those for rigid slab as the slab thickness approaches $h/d = 2.0$ for the practical cases considered in this study. Moreover, the results revealed that, although the individual piles in foundations with flexible slab have different responses, the average stiffness and damping in the vertical direction, defined on the basis of the average load and average response of the slab, are fairly similar to those of stiff slabs. Finally, it was observed that flexible slabs tend to reduce the large peaks in the horizontal pile group stiffness observed at the out-of-phase pile response frequencies such as those appearing at around $a_0 = 0.4$ and 0.8 in Figs. 2(c) and 4(c).

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