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Vertical response spectra for an impact on ground surface

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Abstract. An impact on the ground surface may represent several phenomena, such as a crash of an airplane or an explosion or the passage of a train. In order to analyze and design structures and equipment to resist such a type of shocks, the response spectra for an impact on the ground must be given. We investigated the half-space motions due to impact using the finite element method. We performed extensive parametric analyses to define a suitable finite element model and arrive at displacement time histories and response spectra at varying distances from the impact point. The principal scope of our study has been to derive response spectra which: (a) provide insight and illustrate in detail the half-space response to an impact load, (b) can be readily used for the analysis of structures resting on a ground subjected to an impact and (c) are a new family of results for the impact problem and can serve as reference for future research.

Keywords: impact; shock; spectra; soil; aircraft impact; explosion; heavy tamping; urban vibration

1. Introduction

The problem of impact on the ground consists of computing the motions induced to it by a suddenly applied force on the surface. In general, the ground is modeled in the literature as a uniform elastic isotropic half-space. An impact on the ground surface may represent several phenomena, such as a crash of an airplane, or an explosion, or the passage of a train. In order to analyze and design structures and equipment to resist such types of shocks, we need the acceleration, and displacement response spectra of the ground motions caused by the impact. It is obvious that the spectra must be computed for the site of each structure. The problem arises frequently for several types of structures such as general purpose structures and nuclear power plant facilities (Class I buildings, tunnels and conduits) for the cases of explosion and aircraft impact.

This problem is called Lamb's problem because it was described for the first time by Lamb in his pioneering work in 1904, and it has been studied by many researchers ever since. Pekeris (1955)

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was the first who computed the vertical and horizontal components of displacement in closed form. His computation was for Poisson's ratio ν=0.25 and, subsequently, Mooney (1974) extended that work for any value of Poisson's ratio. The work of Gakenheimer and Miklowitz (1969) and Gakenheimer (1970) involves, for the first time, the solution of the transient problem in the space-time domain for both the ground surface and the interior but still the results are not readily useable in engineering. Johnson (1974) regarded the problem as a Green's function and presented both a collection of solutions and numerical implementations. Georgiadis *et al*. (1999) describe a procedure for numerical computation of the integral-transform solution of Lamb's problem and present graphs for the particular case of Poisson's ratio ν=0.25.

The impact problem is a very difficult subject of elastodynamics and almost all researchers proceed analytically with ingenious transformations but at some point they arrive at integrals that must be evaluated numerically.

In all publications the results are reported in terms of integral functions, dimensionless displacement time histories, or Green's functions, all of which constitute a solution to the problem but they are not readily useable for actual applications. Finally, in almost all cases the writers have used material properties that are not close to reality, such as a Poisson's ratio of 0.25.

In summary, a designer can find little in the results that provide directly a measure of the vibrations induced in the soil by an impact although many papers address and solve successfully the mathematics of Lamb's problem. The goal of this investigation is to provide this type of missing information.

2. Scope and methodology

The preceding brief literature review shows that the ground motions due to an impact have to be computed with numerical integration, irrespective of how elegant, or innovative, was the particular mathematical approach that the writers proposed. Thus, eventual shortcomings of the numerical methods, such as the requirement to select a finite time step, are present in all solutions. In addition, the researchers make simplifying assumptions for proceeding with the exact (analytical) solution, such as that the material is undamped, and the effects of those assumptions are present in the results.

With due consideration to the above, we opted to investigate the problem using exclusively numerical methods. In this context we used the finite element method and performed extensive parametric analyses to arrive at displacement time histories and shock spectra at various distances from the impact point. Our approach allowed for the consideration of impact functions closer to reality than Lamb's Heaviside loading, while we used realistic soil properties, namely Poisson's ratio representative of actual soils and inclusion of material damping in the analyses.

It is well known that the response spectra are a powerful tool for the dynamic analysis and design of soil and structure systems. Furthermore, they are familiar to the engineering community and provide not only the required design quantities but also a valuable insight on the structural behavior. For this reason we used the computed displacement time histories to obtain response spectra suitable for design and included them in this paper.

The principal scope of our study has been to derive response spectra which:

· Provide insight and illustrate in detail the half-space response to an impact load,

· Can be readily used for the analysis of structures resting on a ground subjected to an impact,

· Are a new family of results for the impact problem and can serve as reference for future research.

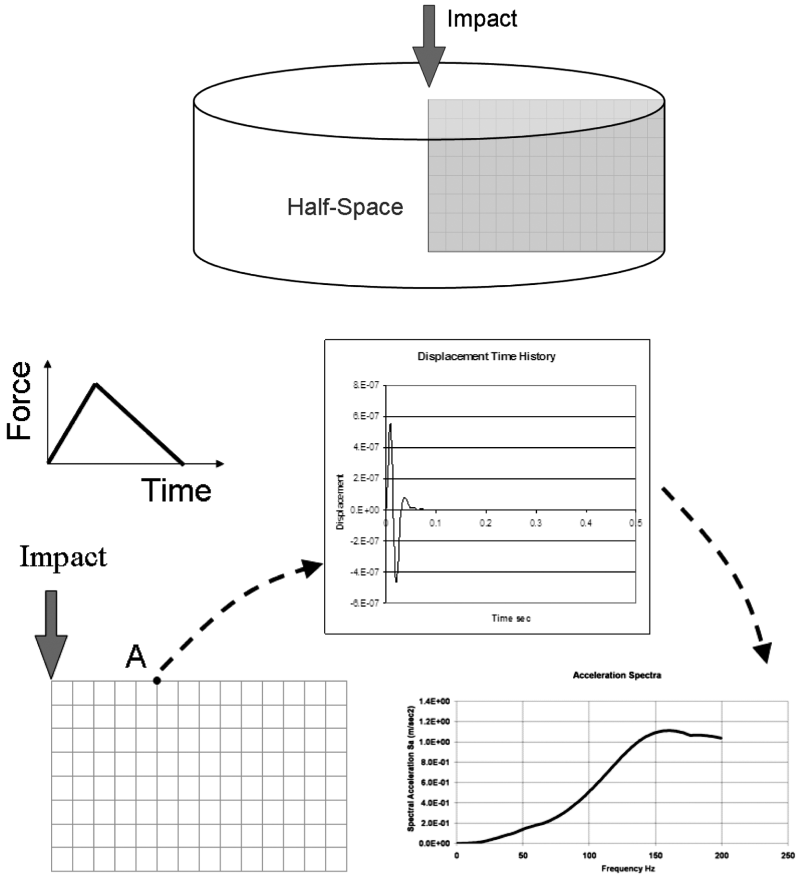


Fig. 1 Description of the problem and methodology of the study

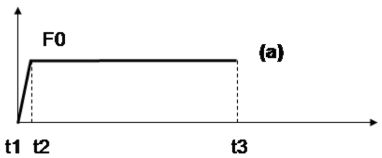
For this purpose a comprehensive study (VanWessem 2008) provided an insight into the half-space high frequency response (to about 100 Hz) and subsequent research summarized in this paper extended the range to 200 Hz.

The model used for these analyses is three-dimensional (axisymmetric) with the impact along the axis as it is shown schematically in Fig. 1 below. A meridional plane is discretized with finite elements and the definition of the analysis as axisymmetric accounts for the overall cylindrical model (Fig. 1). The dynamic analyses of the model produced displacement time histories at selected points on the ground surface spaced at distances of 10, 20, 30, 40 and 60 m from the impact point. Subsequently, with those time histories as input to an ANSYS module the response spectra were computed as schematically shown in Fig. 1.

3. Impact loading

Lamb's problem is defined for a Heaviside loading, namely an applied force at full magnitude within zero rise time and that remains constant thereafter. Available publications assume this type of loading which, though, is not representative of actual problems because, in reality, the force has

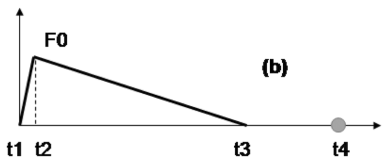
Table 1 Parameters considered for suddenly applied constant loading



Suddenly applied loading (quasi Heaviside)			
Loading	*t*1(sec)	*t*2(sec)	*t*3(sec)
LCs	10-7	0.001	0.10
LC	10-7	0.01	0.10
LAs	10-7	0.001	0.20
LA	10-7	0.01	0.20
L2s	10-7	0.001	0.40
L2	10-7	0.01	0.40

Note: Time *t*3 is *Ttot* that we discuss in Section 4.2 below.

Table 2 Parameters considered for non-symmetric triangular loading



Non-symmetric triangular loading				
Loading	*t*1(sec)	*t*2(sec)	*t*3(sec)	*t*4(sec)
L4s	10-7	0.001	0.02	0.40
L4f	10-7	0.01	0.02	0.40
L5s	10-7	0.001	0.04	0.40
L5f	10-7	0.01	0.04	0.40
L6s	10-7	0.001	0.08	0.40
L6f	10-7	0.01	0.08	0.40

Note: Time *t*4 is *Ttot* that we discuss in Section 4.2 below.

finite rise time, and the force is reduced to almost zero after a short time, regardless if the shock is due to an impact *per se* of an object or an explosion.

Thus, two of Lamb's assumptions are not satisfied in real life. Furthermore, industry standard software require that a certain, even very small, rise-time exist for the description of the load. In our study of the impact problem we considered loading functions with a finite rise time which are closer to reality and are shown in Tables 1 and 2.

A numerical implementation of a quasi Heaviside loading was also employed for verification of our models and methodology against literature results. In a finite element implementation of a suddenly applied force the rise time cannot be, in general, null and we therefore used a sufficiently short time which resulted in satisfactory agreement against published results (see also *Model verification* section below).

Another issue concerning the impact load is the area of its application because a point load is simply a mathematical device. A load distributed over a circular area is appealing but we explicitly decided to use a point load for the following reasons:

- A typical impact area is not circular and therefore assuming it would be just another approximation.
- Proper modeling of the half-space with finite elements for a distributed load (circular for axisymmetric conditions) would result in a very large number of elements while the gain in accuracy is questionable since the St Venant principle applies for the points where we are interested to obtain the spectra. The St Venant effect is clearly observed on the response spectra curves that evolve smoothly from the distance of 10 m to 60 m, in addition to the fact that

distances of 10 m, or longer, from the impact, are large with respect to the loading.

· There is a considerable uncertainty as to the magnitude of the impact loading and therefore the potential enhancement of accuracy by considering distributed loads does not seem to be very promising.

4. Development of analytical model

Numerical simulation of a continuum such as a soil profile and the pursuit of a cost-effective solution pose several serious and conflicting considerations with regard to modeling and solution parameters as briefly described in the following paragraphs.

4.1 Frequencies to be included in the analyses and element size

The finite element model representing the soil in the sense of a half-space must be capable of generating high frequency response in order to simulate sufficiently the soil behavior under an impact load which is typically rich in high frequencies. While in a typical seismic analysis frequencies beyond about 25 Hz have little influence on the overall structural response, it has been shown that due to the very short rise time and duration of an impact, high frequency modes are excited and contribute to the local response. In a comprehensive study Wolf *et al.* (1978) concluded that structural response for impact on a concrete wall converge to peak acceleration at about 150~200 Hz and that modes up to 100 or even 200 Hz have to be included. It was further shown by Constantopoulos *et al.* (1981) that these hold true for underground structures as well whereby response spectra frequencies in the range of 50 to 70 or even 150 Hz are present and have to be included for the case of an impact on the ground surface. Response spectra to 100 Hz were presented by (VanWessem 2008) and we extend the range to 200 Hz in this study.

As a consequence the element size of the model must allow for representation of corresponding high frequencies while the elements should not be needlessly very small because the problem size would increase beyond available numerical capabilities. We considered 1 m and 2 m square

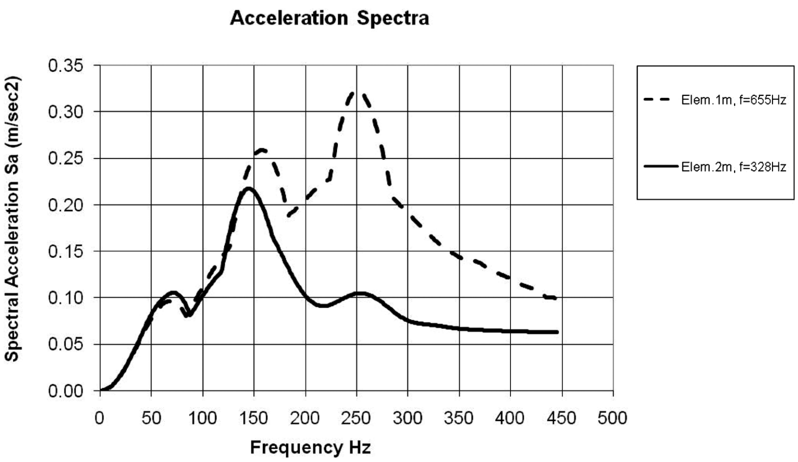


Fig. 2 Influence of element size on frequency content of ground respone

elements which according to the usual rule of 3 to 4 elements per wave length would transmit 150~200 Hz and 75~100 Hz, respectively, for shear waves. In order to further check the frequencies transmitted by the model we computed its response at a certain point with a mesh with 1 m square elements and another with 2 m elements. The first model had highest mode at 655 Hz and the other at 328 Hz.

The results of these analyses for a quasi Heaviside impact are shown in Fig. 2 from which we can deduce:

· The response of both models is practically identical to about 140~150 Hz.
· The spectra converge to a peak ground acceleration at frequencies well above 250~300 Hz
· Both models generate response at frequencies well over 150 Hz
· Very high frequencies are present which, though, may not be representative of reality but the result of the modeling of the load as a quasi Heaviside.

Based on the above we conclude that a finite element model with 2 m square elements is suitable for computing response spectra up to 200 Hz. Furthermore, it is well known that when solving for the eigenvalues the highest modes of a model are in appreciable error. We therefore limited the extracted modes to a maximum frequency of about 225 Hz while the model had modes up to 328 Hz.

4.2 Integration time step and loading function duration

Dynamic analyses, even linear elastic as performed for this study, are affected substantially by the time step *dt* used for the modal integration and superposition as well as the total duration *Ttot* of the loading. Total duration, marked as *t3* in Table 1 and *t4* in Table 2, control the inclusion in the analysis of the residual vibrations.

In order to establish the appropriate values for *dt* and *Ttot* we varied them and computed the response of our model at a certain point. The solution of these cases included very high frequency modes, namely 328 Hz, so that eventual effects of different values of *dt* and *Ttot* would become evident.

The response spectra for a surface point at a distance of 12 m from the point of impact are shown in the figures above for a quasi Heaviside loading and the following conditions:

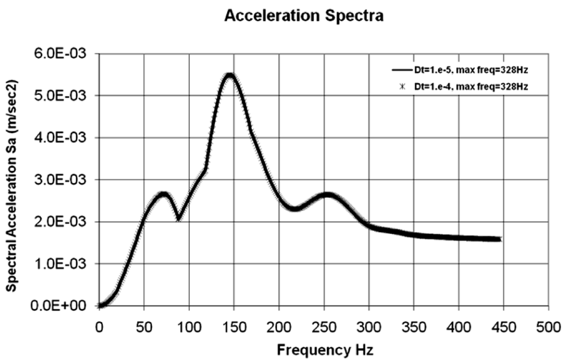


Fig. 3 Spectra comparison for time steps *dt* 0.1 and 0.01 msec

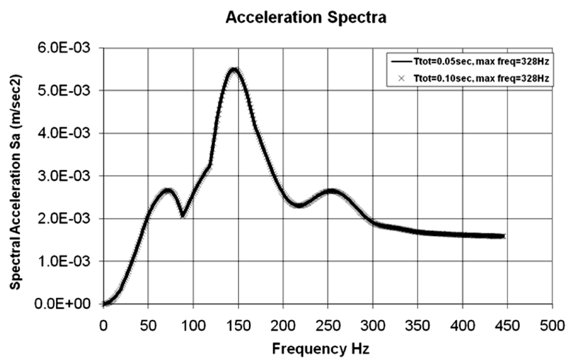


Fig. 4 Spectra comparison for load duration *Ttol* 50 and 100 msec

· In Fig. 3 the spectra were computed from analyses with *dt*=0.1 msec and 0.01 msec and it is obvious that they are indistinguishable from each other.
· The same observation is true for analyses with a total duration of 50 and 100 msec (Fig. 4).

In conclusion, a time step of 0.1 msec and a minimum total loading duration of 50 msec are appropriate.

4.3 Boundary conditions

A finite element model is bounded and this poses a fundamental limitation when modeling a continuum because the boundaries affect the solution obtained. This is particularly true for a dynamic analysis because the waves propagating from the source of excitation are reflected at the boundaries and upon return they interact with outgoing waves causing an erroneous response. In this work the lateral and bottom boundaries of the model were placed at such a distance so that reflected waves did not affect the area of interest within the time span the response was computed.

The adequacy of the distances of the boundaries from the points of interest can be assessed by inspection of the displacement time histories at those points.

An example of such an assessment is shown in Fig. 5. The displacement time histories for two points at 10 and 20 m from the impact are shown for a quasi Heaviside load and *Ttot*=0.30 sec. The displacement should be essentially constant because the load remains constant with time. However, we observe that towards the end (circled by the ellipses) the displacement increases and this is due to the interaction with waves that returned after reflection at the boundaries. Thus, in this case, the combination of the location of the boundaries and the time span *Ttot*=0.30 sec for which the response was computed is not satisfactory. On the other hand, for time span *Ttot*=0.20 sec (thick line) the boundaries are at sufficient distance because reflected waves do not affect the points of interest.

In summary, displacement time history plots provided valuable information to define safe distances for the boundaries of the finite element mesh. In addition, comparison with literature added a further check on the model (see also *model verification* section below).

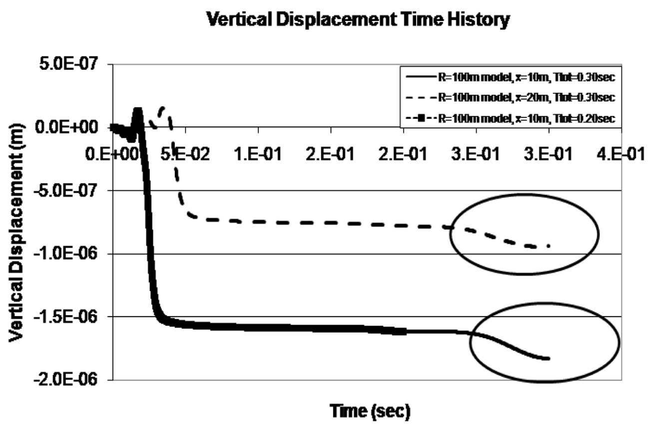


Fig. 5 Boundaries at inadequate distance for 0.3 sec, interaction of reflected waves and adequate distance for 0.2 sec

It is well known that finite element models are stiffer than the continuum they simulate. In order to compensate for this effect to some degree the boundary conditions at the bottom of the model were modeled with horizontal rollers while at the outer (periphery) surface all degrees of freedom were left free.

4.4 Linear elastic analysis and nonlinear effects

At the point of impact the soil certainly fails, undergoes large irreversible displacements, and, typically, a crater is formed. However, these nonlinearities are limited to the impact area and the propagating waves do not cause nonlinear behavior of the soil as the data in Fig. 6 below suggests.

A large site consisting of a layer of dredged coral and silty sand overlying the original coral bed was improved by the dynamic compaction method. The method consisted of raising a 16 ton weight to a height of 24 m and letting it fall freely on the ground where it produced, typically, craters 1.5 m deep. The vibrations caused in the ground were recorded at many points and the of Fig. 6 are reproduced from Constantopoulos and Sotiropoulos (1988) who presented a brief summary of the recordings.

Both figures present the ground particle velocities at points A and B, 5 and 50 m, respectively, from the point of impact. Left Fig. 1(a) corresponds to the first impact while right Fig. 1(b) corresponds to the 15 th impact.

It can be seen that the recordings in Figs. 1(a) and 1(b) are practically identical which suggests that the local failure and compaction of the soil did not influence appreciably the waves and soil movements.

In addition, we consider that the uncertainties involved in determining the impact force are greater than eventual effects of nonlinearities. Finally, we believe that a reliable three dimensional nonlinear model for soil does not really exist and therefore potential nonlinear analyses would be at an unknown margin of error. Analyses with parametric variation of soil properties would be more representative of real soil than any nonlinear analysis.

Based on the above, we decided to perform linear elastic analyses

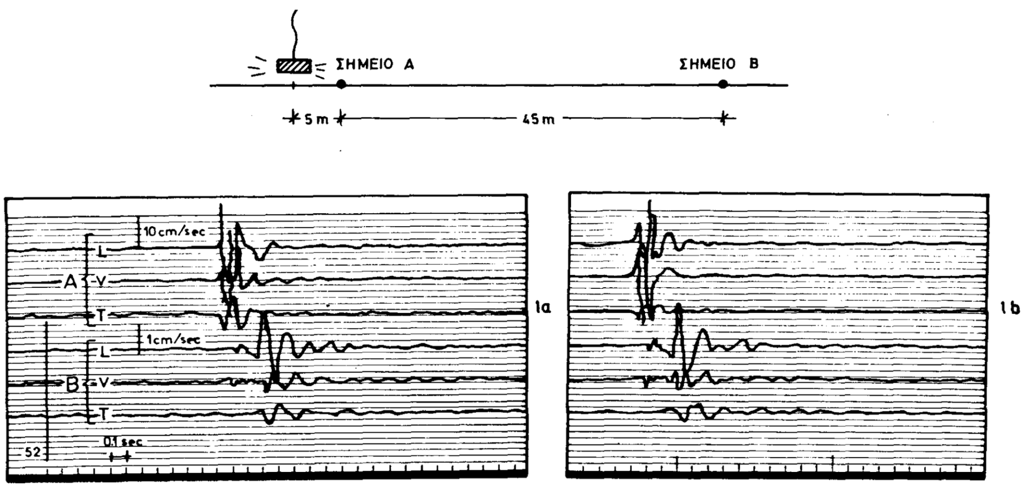


Fig. 6 Vibration recordings for dynamic compaction of loose ground

4.5 Software and hardware

All analyses, including response spectra computations, were performed with industry standard software ANSYS, Academic versions 11 and 12, in two phases. First phase analyses were performed on Intel Pentium 4 computers under Windows XP while second phase required much stronger computation capabilities, impossible for a PC, and were performed on an IBM 4-Way 2.1 GHz Server running under AIX 5L V5.3. The project was organized in such a way as to have ANSYS, either on a PC or the IBM Server, output the results on files which were subsequently input to Excel for multiple comparisons and graphs.

4.6 Response spectra

The response spectra at selected points on the ground surface were computed for 5% damping.

4.7 Finite element models

Two groups of finite element models were used in this study (Fig. 7).

Group A is a 200 by 200 m domain discretized with 2 m square elements. This group was employed for the parametric solutions and computation of the soil response at various selected points which were distributed at distances ranging from 10 to 60 m from the point of impact.

Group B simulates a soil profile 200 m high as well, but its lateral extent varies in order to allow very fine discretization with finite elements. Consistent with the considerations described in boundary conditions section above the elements for Group B were always square and this group was used for verification purposes.

The study presented in this paper deals with the dynamic analysis of an axisymmetric finite element model subjected to the loading defined earlier. The dynamic analysis of the model consisted of mode frequency analysis and synthesis of the response in each mode to obtain the overall response at selected points of the model. The eigenvalue or mode-frequency problem was solved by ANSYS using the Lanczos method. Several thousand modes of vibration were computed and considered for the dynamic analysis of the system and it was assured that modes extended to 225 Hz. This, of course, meant that the number of modes computed for the model for Soil 2 was

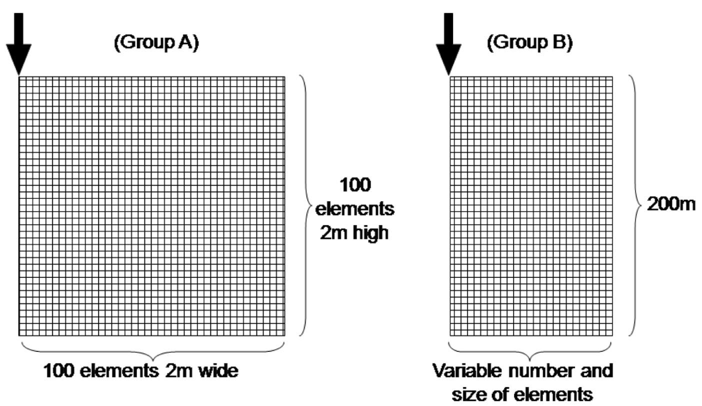


Fig. 7 Finite element models

considerably higher than that for Soil 1 because it is softer and therefore there are more modes below 225 Hz.

Subsequent to the extraction of normal modes and mode shapes, each mode is treated independently to obtain its response to the loading. In computing the response in a single mode, ANSYS incorporates the appropriate modal damping which we defined with a quite small value, 4% for all modes. The response of a single mode is computed in ANSYS by integrating the corresponding second order differential equation. For effective inclusion of higher modes in the solution, an appropriately small time step had to be selected, otherwise the integration errors effectively "filter" the high mode response out of the solution. For the present study the integration time step was selected to be very small, namely 0.1 msec which we established to be appropriate as discussed above.

The response at any point of the system is then obtained by combining the in accordance with the participation of each mode. The response of several points is thus obtained in terms of displacement time histories. Finally, the response spectra for the points in question are obtained for the desired structural damping value, which we selected to be 5%.

4.8 Model verification

Our work introduces for the first time a new family of results, the acceleration response spectra, to the literature related to the impact problem and therefore there is not any similar data to compare with. Verification of our results was also difficult because the great majority of available papers present their solutions in forms which are not readily useable with our methodology. The best source of data for comparison with the results from our methodology was Gakenheimer (1970), which was also used for verification by Georgiadis *et al.* (1999). In short, the verification of our results was performed as follows:

a. Gakenheimer (1970) presents normalized displacement time histories for various rays in the half-space starting from the point of impact and for certain material properties, that is Poisson's ratio ν=0.25 which results that Lamé constants are equal.

b. Thus, the only quantity that we can compare for verification are time histories. We computed

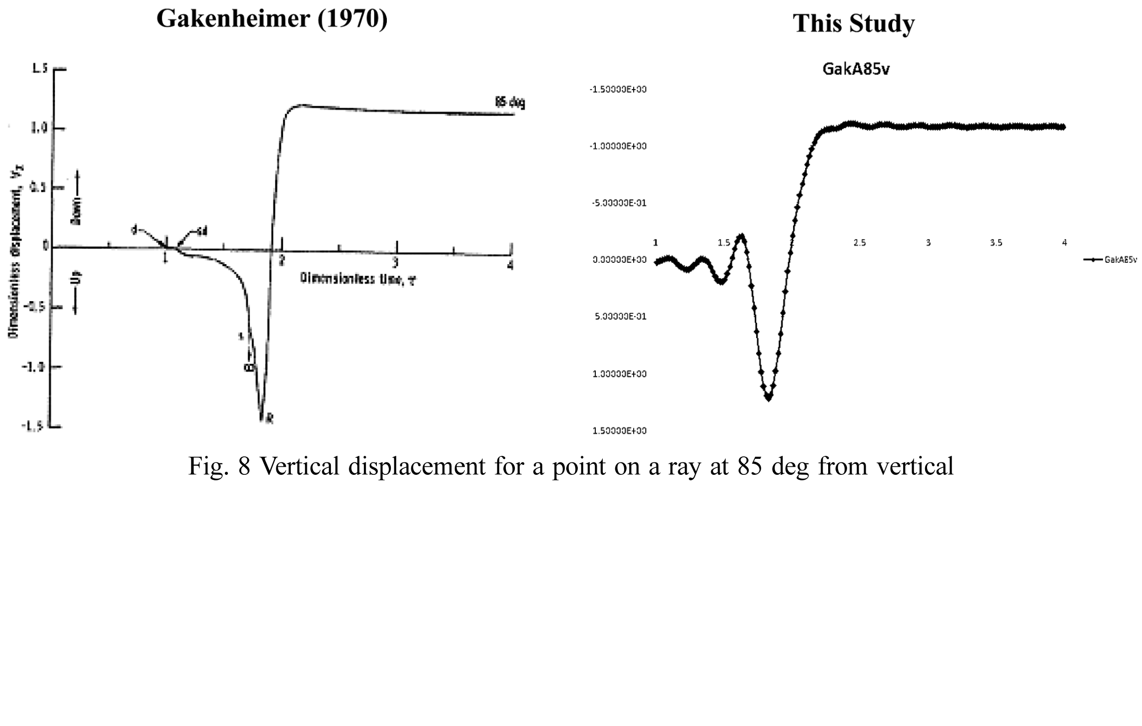


Fig. 8 Vertical displacement for a point on a ray at 85 deg from vertical

displacement time histories at various points with a finite element model and material properties the same as used by Gakenheimer.

c. Subsequently we normalized the displacement time histories the same way Gakenheimer did and compared our curves with his publication.

d. It can be seen in Fig. 8 below reproduced from (VanWessem 2008) that the curves have essentially the same shape and values and we therefore conclude that the displacement time histories we computed are verified against the Gakenheimer paper.

e. Furthermore, the verification implies that the boundary conditions, details of the finite element model, and methodology of our research are satisfactory.

Once the displacement time histories were verified we proceeded with well established analytical methods to derive the response spectra which we consequently consider verified.

5. Material

We consider a homogeneous, linearly elastic and isotropic half-space subjected to an axisymmetric impact loading at time t=10-7. Two sets of material properties were used in the analyses, one corresponding to a medium stiff and the other to a stiff soil.

Table 3 Soil properties used in the study

Property	Soil 1 (stiff)	Soil 2 (medium stiff)
Shear wave velocity Vs (m/sec)	600	450
Poisson's ratio	0.45	0.45
Dilatational wave velocity Vp (m/sec)	1989.97	1492.48
Unit weight (N/m^3)	20000.00	19500.00
Density (kg/m^3)	2038.74	1987.77
Lamé μ - G mod (N/m^2)	7.339E+08	4.025E+08
Young's modulus E mod (N/m^2)	2.128E+09	1.048E+09

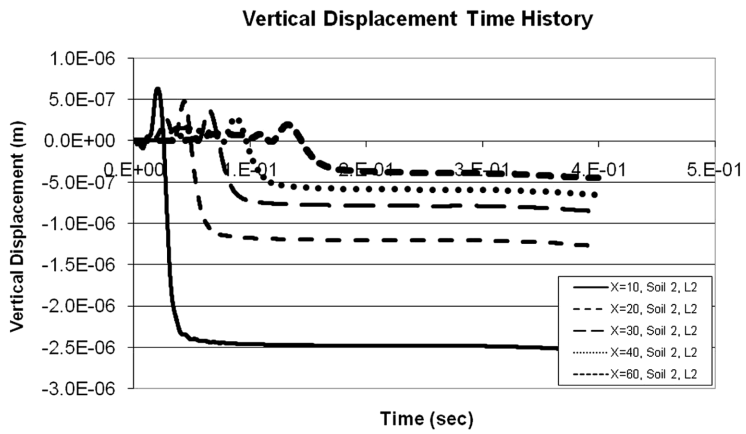


Fig. 9 Typical plot of displacement time histories of all points of interest on the ground surface

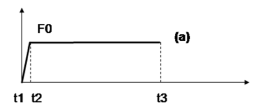
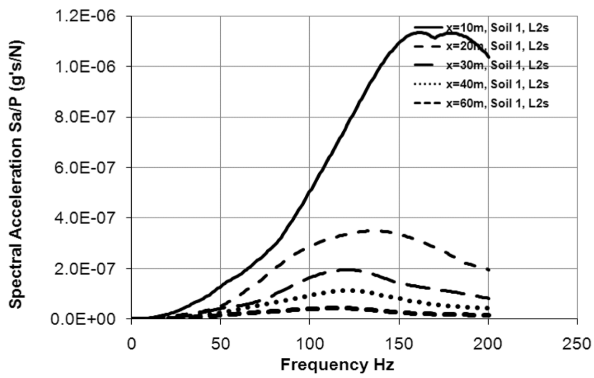
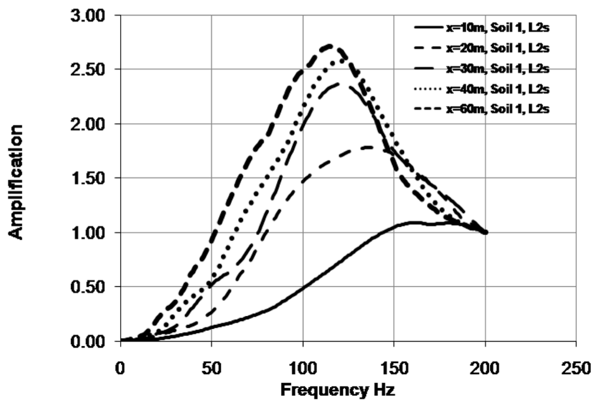
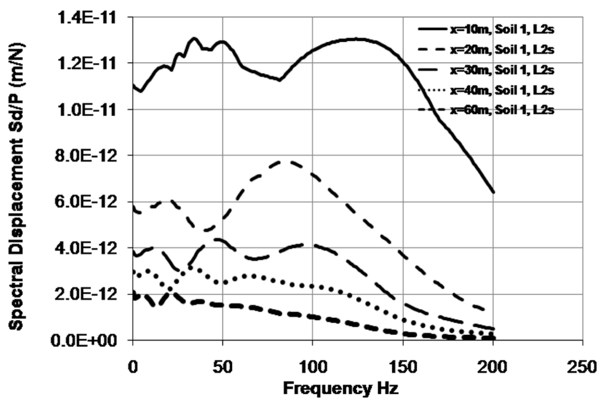
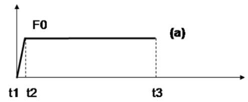
Rise time 0.001 sec ***Ttot***=0.40 sec	
Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.	
Amplification factors for the Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. PGA is considered the spectral value at 200 Hz.	
Vertical Displacement Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.	

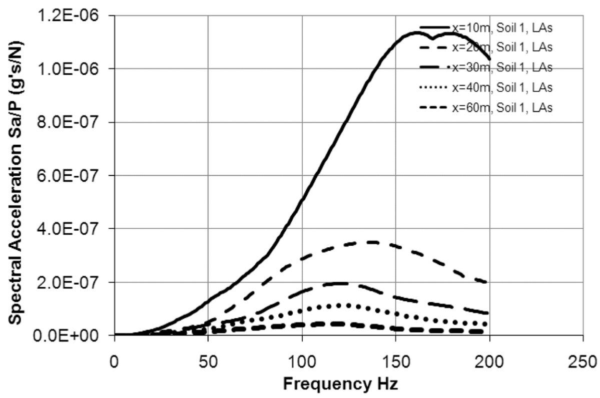
Fig. 10 Spectra for loading L2s. Quasi-Heaviside, Rise time 0.001sec, *Ttot*=0.40 sec, Soil 1

Rise time 0.001 sec

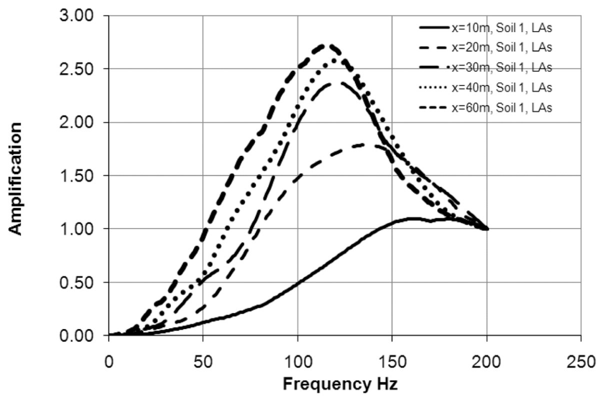
Ttot=0.20 sec



Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.



Amplification factors for the Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. PGA is considered the spectral value at 200 Hz.



Vertical Displacement Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.

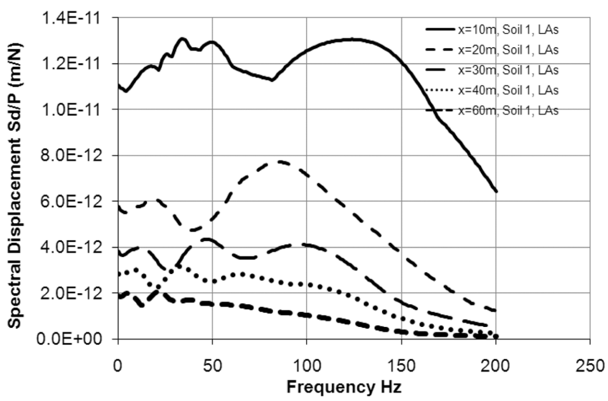


Fig. 11 Spectra for loading LAs. Quasi-Heaviside, Rise time 0.001 sec, *Ttot*=0.20 sec, Soil 1

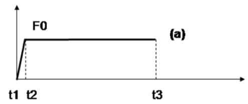
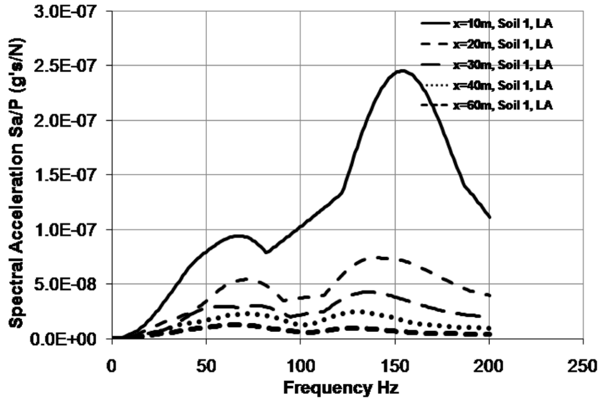
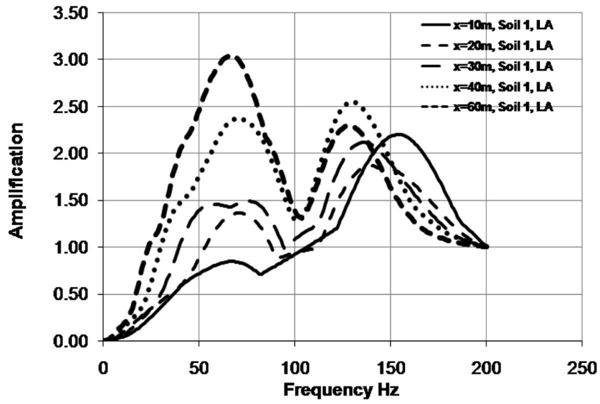
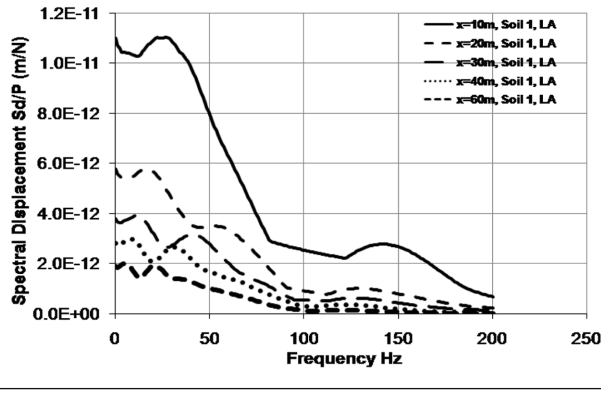
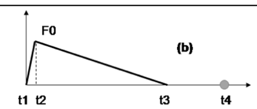
Rise time 0.01 sec ***Ttot***=0.20 sec	
Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.	
Amplification factors for the Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. PGA is considered the spectral value at 200 Hz.	
Vertical Displacement Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.	

Fig. 12 Spectra for loading LA. Quasi-Heaviside, Rise time 0.01sec, *Ttot*=0.20 sec, Soil 1

<table>
<tr>
<td>Rise time 0.001 sec

T3=0.02 sec, Ttot=0.4 sec</td>
<td>
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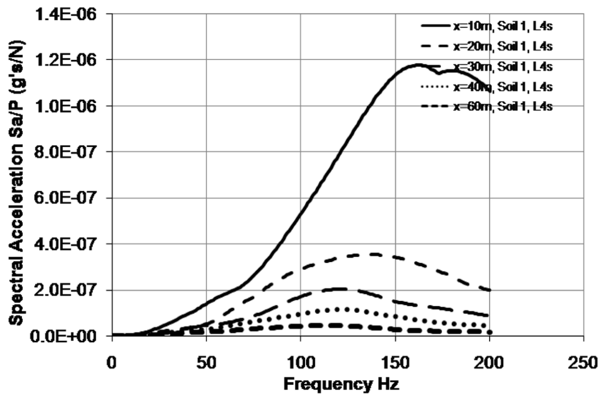
<td>Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.</td>
<td>
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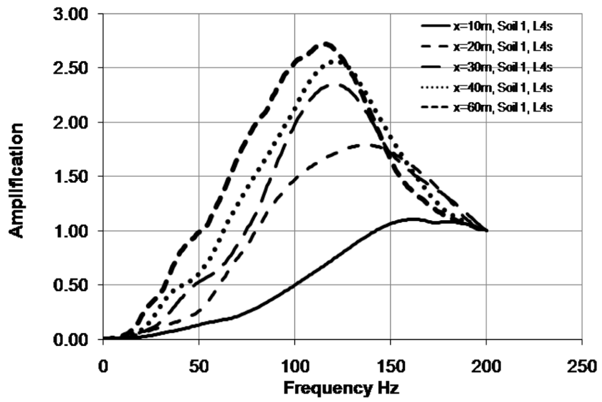
</tr>
<tr>
<td>Amplification factors for the Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. PGA is considered the spectral value at 200 Hz.</td>
<td>
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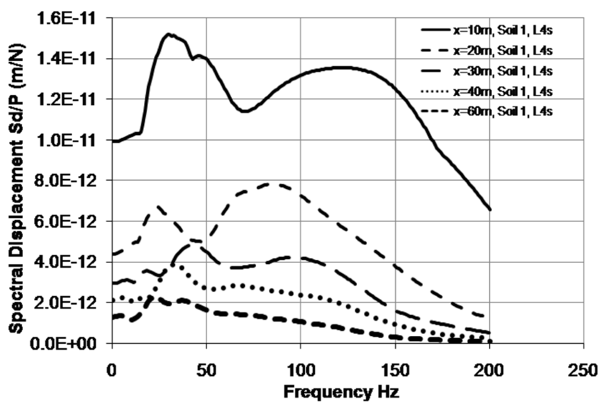
<tr>
<td>Vertical Displacement Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load.</td>
<td>
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</table>

Fig. 13 Spectra for loading L4s. Asymmetric triangle, Rise time 0.001 sec, *Ttot*=0.4 sec, Soil 1

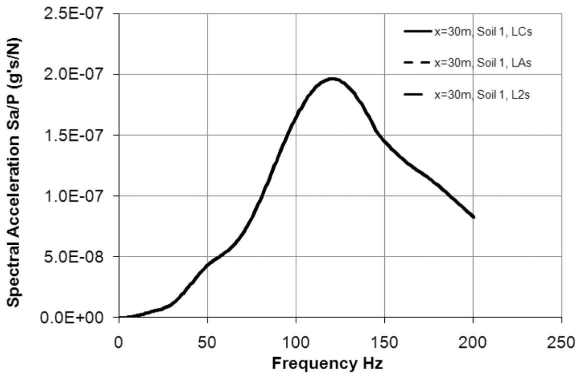
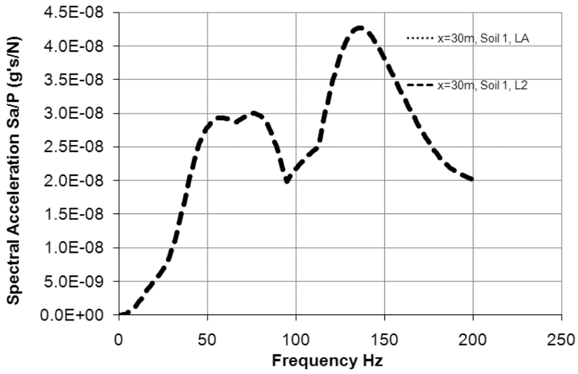
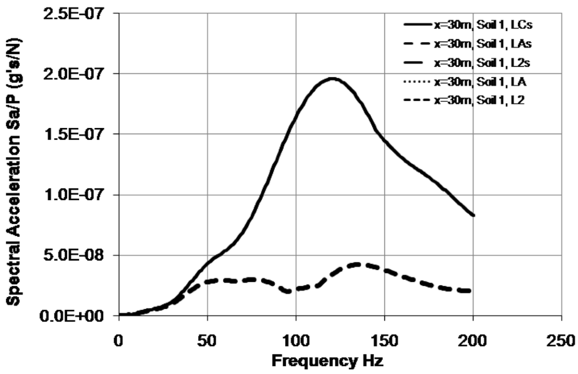
Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load Loadings LCs, LAs, L2s, rise time 0.001 sec.	
Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load. Loadings LA, L2, rise time 0.01 sec	
Vertical Pseudo-Acceleration Response Spectra for points on the ground surface and 5% damping. Values are normalized to the peak value of applied load. All loadings	

Fig. 14 Spectra for loading LAs. Quasi-Heaviside, Rise time 0.001 sec and 0.01 sec, *Ttot*=0.10 to 0.40 sec

6. Results and discussion

Publications that provide practical results for an impact on the ground are very scarce while several are available for structural pounding/impact (e.g. Mahmoud and Jankowski 2010, Polycarpou and Komodromos 2011). The goal of this research has been to provide this information and the results were obtained in terms of displacement time histories and response spectra at five selected points on the surface of the model.

The displacement time histories at all five points of interest on the ground surface are plotted together and are presented as a typical case in Fig. 9. It is obvious from that figure that:

· The traveling wave nature of the problem and the time lag of the wave arrivals at different distances from the exciting force are correctly simulated

· The displacements remain constant after the transient vibration which correctly simulates the phenomenon and proves that wave reflections at boundaries do not affect the solution.

6.1 Influence of distance from impact point

Figs. 10-13 present the response spectra computed from the displacement time histories. Each of those figures corresponds to a single particular loading case and comprises three plots:

Acceleration response spectra: The first is the vertical pseudo-acceleration response spectra for the points on the ground surface and 5% damping. The values are reported in fraction of the acceleration of gravity (g's) and correspond to a force of 1*N* and the respective soil properties and loading function used for each plot. There is a clear, and anticipated, trend that the spectral values decrease with increasing distance from the point of impact.

Amplification factors: The second is the variation of the amplification factors with frequency for the above pseudo-acceleration response spectra, namely the ratio at each frequency of the spectral (pseudo) acceleration to the Peak Ground Acceleration (PGA) at the respective point on the ground surface. We adopted for simplicity, but no compromise in accuracy, as PGA the spectral acceleration at the maximum frequency of the spectrum (i.e., 200 Hz). It is interesting to note that within the range of properties and loadings we used:

· The amplification factors almost invariably increase with distance

· Their maximum value is between 2.5 and 3.5

An interpretation of the above behavior is that near the point of impact there is an abundance of high frequencies in the dynamic motions and there is not any frequency lower that those carrying the PGA that is particularly amplified. In fact careful observation of the amplification factors shows that for frequencies quite close to the maximum frequency there is an amplification but it is quite small.

On the other hand, at larger distances several high frequency components have been filtered out and therefore some of the lower frequencies which remain become predominant and amplified. This is also seen in the “hump” observed in lower frequencies.

Displacement response spectra: The third is the vertical displacement response spectra for the points on the ground surface and 5% damping. The values are reported in meters and correspond to a force of 1*N* and the respective soil properties and loading function used for each plot. There is a clear, and anticipated, trend that the spectral values decrease with increasing distance from the point of impact.

6.2 Dimensionless response spectra

It would be interesting if the response spectra were expressed in terms of dimensionless accelerations. Given that our problem is defined by nine independent variables i.e., Vs, v, β, ρ, $t1$, $t2$, P, r, t and the dimensional matrix is of rank three (i.e., mass, time and length), Buckingham’s

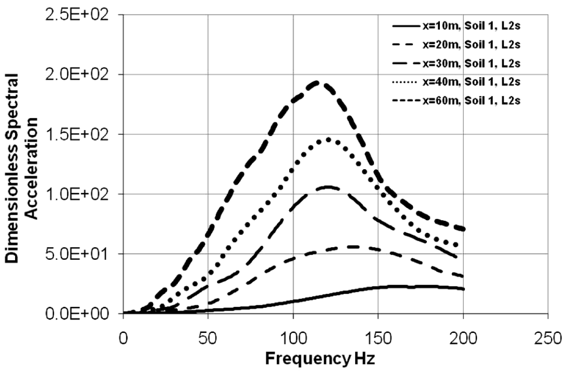


Fig. 15 Dimensionless acceleration response spectra, Soil 1, Loading L2s

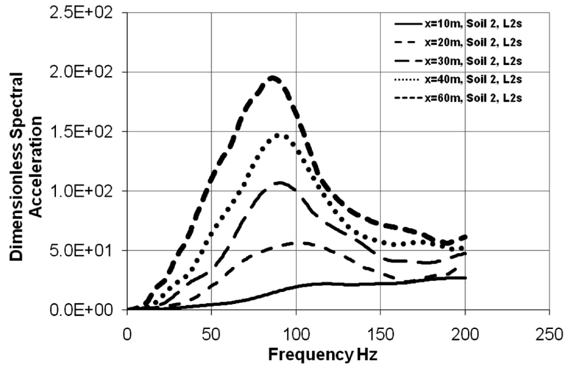


Fig. 16 Dimensionless acceleration response spectra, Soil 2, Loading L2s

(1914) theorem suggests that the solution to the problem can be fully described by six independent dimensionless parameters.

We tried for dimensionless acceleration the quantity $(a\ Vs^{2}\ \rho\ r\ t1^{2})/P$, where $t1 = r/Vs$

We believe negative results are as significant as positive and, we therefore, present our effort although our success has been limited so far as it can be seen in the figures above. The figures pertain to the response of five points on the ground surface for loading L2s and soils 1 and 2. The ordinates are the dimensionless acceleration quantity presented above.

It is clear that the two sets of curves do not coincide and therefore we cannot deduce the response for, say, soil 2 from the response of soil 1. Furthermore, the figures give the reader the first impression that acceleration increases with distance, which is not the case, of course. Thus, it is necessary to consider two soils in order to bracket the results for a stiff and a medium stiff soil encountered in practice.

6.3 Influence of rise time and total duration of loading function

Fig. 14 involves a very interesting comparison of pseudo-acceleration response spectra computed for the response at a certain point but for different loading cases.

Acceleration response spectra at a certain point for three different loadings with same rise-time: The first plot is the vertical pseudo-acceleration response spectra for an arbitrary point on the ground surface at a distance of 30 m from the point of impact. All spectra are for 5% damping. The values are reported in fraction of the acceleration of gravity and correspond to a force of 1*N*. In all cases the soil properties are the same and the loadings are described as quasi-Heaviside functions with a rise-time of 0.001 sec and total duration 0.10, 0.20 and 0.40 sec. The three spectra are indistinguishable.

Acceleration response spectra at a certain point for two different loadings with different than above rise-time: The second plot is the response spectra for the same point as above but for two loading functions which have the same rise-time of 0.01 sec which is 10 times longer than above. Again, the two spectra are indistinguishable.

Acceleration response spectra at a certain point for all five two different loadings: The third plot is the compilation plot of the above response spectra for the same point but for five different loading functions. The spectra in this plot correspond to:

· Five quasi-Heaviside loadings
· Three of them, LCs, LAs and L2s have the same rise-time of 0.001sec but different total duration, 0.10, 0.20 and 0.40 sec, respectively. The spectra are indistinguishable among the three.
· Two of them, LA and L2 have the same rise-time of 0.01sec but different total duration, 0.20 and 0.40 sec, respectively. The spectra are indistinguishable between them.
· The first group (LCs, LAs and L2s) have the same rise-time of 0.001sec that is $1/10^{th}$ the rise time of the second group (LA and L2). The spectra of these two groups are very different between them.

The above imply that within the range of properties and quasi-Heaviside loadings we used, the shape and magnitude of the vertical pseudo-acceleration spectra are uniquely defined by the rise-time while the total duration does not seem to have any considerable influence, as long as the boundaries are far enough to avoid the effects of wave reflection. Mathematically, both rise-time and total duration affect the frequency content of the loading but it certainly appears the rise-time fully prevails in the half-space dynamic response.

An important implication of this finding is that solutions based on Lamb's problem, namely zero rise time, probably result into misleadingly high response values. The value of 0.01 sec for rise-time that we used corresponds to an impact of fast-flying military aircraft upon a rigid wall (Drittler and Gruner 1976) and we suggest this be considered as a minimum in impact analyses.

7. Conclusions

The motions induced to a half-space by a suddenly applied force, i.e., an impact or an explosion or a passing train, has been studied in this paper using the finite element method. Many researchers have dealt with the problem and their work was primarily based on integral transformations or Green functions and arrived to solutions which require further numerical implementations. However, there is little which can serve a designer for readily obtaining a measure of the vibrations induced in the soil by an impact.

On this basis, the authors of this paper opted to investigate the problem using exclusively numerical methods. In this context we used the finite element method and performed extensive parametric analyses to define a suitable finite element model and compute the response at varying distances from the impact point. Our approach allowed for the consideration of impact functions closer to reality than Lamb's Heaviside loading, while we used realistic soil properties, namely Poisson's ratio representative of actual soils and inclusion of material damping in the analyses.

It is well known that the response spectra are a powerful tool for the dynamic analysis and design of soil and structure systems. Furthermore, they are familiar to the engineering community and provide not only the required design quantities but also a valuable insight on the structural behavior. For this reason we used the computed displacement time histories to obtain response spectra suitable for design.

The principal scope of our study has been to derive response spectra which:
· Provide insight and illustrate in detail the half-space response to an impact load,
· Can be readily used for the analysis of structures resting on a ground subjected to an impact,
· Are a new family of results for the impact problem and can serve as reference for future research.

Thus, we arrived at the following data and conclusions:

· Vertical response spectra in terms of pseudo-accelerations and displacements for several points at the ground surface.

· These acceleration response spectra show a clear, and anticipated, trend that the spectral values decrease rapidly with increasing distance from the point of impact.

· Amplification factors of the spectra exhibit the following interesting behavior, at least for the range of properties and loadings we used:

-They almost invariably increase with distance

-Their maximum value is between 2.5 and 3.5

An interpretation of the above behavior is that near the point of impact there is an abundance of high frequencies in the dynamic motions and there is not any frequency lower that those carrying the PGA that is particularly amplified. In fact careful observation of the amplification factors shows that for frequencies quite close to the maximum there is a slight amplification.

On the other hand, at larger distances several high frequency components have been filtered out and therefore some of the lower frequencies which remain in the motion become predominant and amplified. This is also seen in the "hump" that we observe in lower frequencies as the distance increases.

· Displacement response spectra which show a similar trend with the accelerations.

· Within the range of properties and quasi-Heaviside loadings we used, the shape and magnitude of the vertical pseudo-acceleration spectra are uniquely defined by the rise-time while the total duration does not seem to have any considerable influence. This behavior is quite similar to the response of a linear SDOF system to an impact force.

· An important implication of the above is that solutions based on Lamb's problem probably result into misleadingly high response values.

· We suggest that the value of 0.01 sec for rise-time be considered as a minimum in impact analyses, unless it is otherwise documented. This value that we used in our analyses corresponds to an impact of fast-flying military aircraft upon a rigid wall.

· It is more important for regulators and code authors who specify impact loading time histories to specify realistic rise-time values rather than total duration.

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