Earthquakes and Structures, *Vol. 19, No. 2 (2020) 119-128*
DOI: https://doi.org/10.12989/eas.2020.19.2.119

A simplified method for estimating fundamental periods of pylons in overhead electricity transmission systems

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(Received February 22, 2020, Revised June 23, 2020, Accepted August 6, 2020)

Abstract. In seismic design of a pylon supporting transmission lines in an overhead electricity transmission system, an estimation of the fundamental periods of the pylon in two orthogonal vertical planes is necessary to compute the seismic forces required for sizing pylon members and checking pylon deflections. In current practice, the fundamental periods of a pylon in two orthogonal vertical planes are typically obtained from eigenvalue analyses of a model consisting of the pylon of interest as well as some adjacent pylons and the transmission lines supported by these pylons. Such an approach is onerous and numerically inconvenient. This research focused on development of a simplified method to determine the fundamental periods of pylons. The simplified method is rooted in Rayleigh's quotient and is based on a single-pylon model. The force vectors that can be used to generate the shape vectors required in Rayleigh's quotient are presented in detail. Taking three pylons selected from representative overhead electricity transmission systems having different design parameters as examples, the fundamental periods of the chosen pylons predicted from the simplified method were compared with those from the rigorous eigenvalue analyses. Result comparisons show that the simplified method provides reasonable predictions and it can be used as a convenient surrogate for the tedious approach currently adopted.

Keywords: pylon; period; Rayleigh's quotient; vibration; seismic

1. Introduction

As an essential part of our modern life, electrical energy generated from a power plant is usually delivered by overhead electricity transmission systems to destination substations. An overhead electricity transmission system consists of transmission lines (i.e., conductors and ground wires) supported by a group of pylons (i.e., steel lattice towers). Due to the overwhelming dependencies of our modern society on electricity, overhead electricity transmission systems are required to cover the regions with seismicity. Although wind load will have a huge impact on the stable operation of the transmission tower-line system (Altalmas and El Damatty 2014, Li *et al*. 2013, Li and Yu 2019, Tian *et al*. 2020), inadequate seismic designs of the pylons among an overhead electricity transmission system can lead to post-earthquake interruptions of electrical power supply, causing socioeconomic impacts in the affected region. For example, estimated direct costs for fully restoring the damaged electricity transmission systems were said to be approximately $500 million and $4 billion for the 1994 Northridge and 1995 Kobe earthquakes, respectively (Shinozuka *et al*. 1999).

When performing the seismic design of a pylon, an estimation of its fundamental periods in two orthogonal vertical planes is necessary to determine the seismic forces required for sizing pylon members and checking pylon deflections. Unlike conventional building structures for which many design standards such as ASCE/SEI 7-16 (ASCE 2016) recommend empirical formulas to approximate their fundamental periods, existing knowledge to achieve adequate estimates of the fundamental periods for pylons is fairly limited. Note that many of the formulas to approximate the fundamental periods of ordinary building structures were developed from regression analyses of the fundamental periods identified based on the data recorded from instrumented buildings or simplified analysis models (Goel and Chopra 1997, Goel and Chopra 1998, Liu *et al*. 2013, to list a few). Although there are some formulas for the basic period of ordinary building structures (Asteris *et al*. 2017, De *et al*. 2018, Kim *et al*. 2007, Sangamnerkar and Dubey 2017, Zhao *et al*. 2017, Shatnawi *et al*. 2019). These equations are generally inapplicable to pylons due to the significant differences in stiffness and mass distributions in pylons in comparison with typical building structures.

At present, considerable seismic studies have been carried out on transmission tower-line system (Chen *et al*. 2018, Wei *et al*. 2019, Tian *et al*. 2020), and none of the analysis could work without the fundamental period of the structure. According to the current seismic design practice, to glean the fundamental periods of a pylon in an overhead

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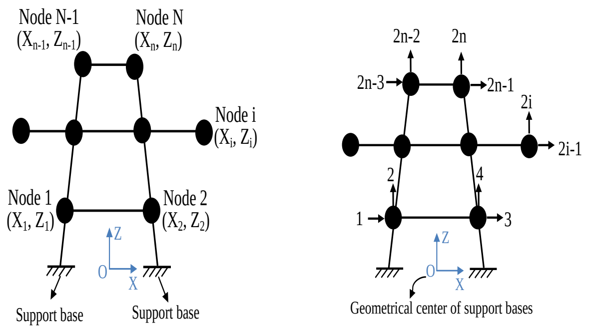
http://www.techno-press.com/journals/was&subpage=7
ISSN: 2092-7614 (Print), 2092-7622 (Online)



(a) Node ID and node coordinates

(b) DOF ID

Fig. 1 A simple system illustrating the nodes and DOFs

electricity transmission system, a sophisticated Finite Element (FE) model consisting of not only the pylon of interest but also a few adjacent pylons as well as the transmission lines supported by these pylons has to be developed for eigenvalue analyses. Such a model is referred to herein as the cluster model. Note in the cluster model that the transmission lines and the adjacent pylons are included to create the ideal boundary conditions for the pylon of interest. Unlike the FE models for conventional civil structures in which stiffness gradually varies in the system, the pylons and the transmission lines included in the cluster model have significantly different stiffness features. Consequently, eigenvalue analyses using the cluster model can be numerically challenging. Moreover, the low-order eigenvalues and eigenvectors obtained from the cluster model are usually associated with the vibration modes of the transmission lines rather than these of the pylons since the transmission lines are much more flexible compared with the pylons. To achieve the fundamental periods of the pylon of interest, extremely-high-order eigenvalues (e.g., higher than the 480th order in an example that will be discussed in detail in a following section) have to be extracted. Further, identifying the specific eigenvalues associated with the fundamental vibration modes the pylon of interest among a huge number of eigenvalues obtained is formidably tedious. Therefore, the approach adopted in the current seismic design practice for determination of the fundamental periods of pylons involves excessive modelling efforts and high computational costs. An urgent need exists to develop a simplified method which is convenient to implement and provides reasonable approximates of the fundamental periods of pylons.

The objective of this investigation was to fill the critical knowledge gap described above. Specifically, this technical note formulates a simplified method which is rooted in Rayleigh's quotient to estimate the fundamental periods of a pylon from an overhead electricity transmission system. Taking three pylons from representative overhead electricity transmission systems with different design parameters as examples, the authors further assess accuracy of the proposed simplified method against the onerous but more rigorous approach adopted in the current practice. Recommendations to implement the simplified method in practice are presented as well.

2. Development of a simplified method

This section derives a simplified method for estimating the fundamental periods of pylons in overhead electricity transmission systems. The theoretical basis for the simplified method, Rayleigh's quotient, is briefly revisited and is subsequently extended into the simplified method. Implementation procedures and strategies to properly select the parameters for the simplified method are then presented.

2.1 Revisit of Rayleigh's quotient

For a Multiple-Degree-of-Freedom (MDOF) system with lumped masses, Rayleigh's quotient is given as (Chopra 2011)

$$\omega_{\mathrm{n}}^2 = \frac{\phi^{\mathrm{T}}\mathbf{k}\phi}{\phi^{\mathrm{T}}\mathbf{m}\phi} \tag{1}$$

where ω_n represents the circular frequency of the system; ϕ represents the assumed shape vector that defines the deflected shape of the system; and $\mathbf{m}$ and $\mathbf{k}$ represent the mass and stiffness matrices of the system, respectively.

Note that Rayleigh's quotient can be derived by equating the maximum potential energy and the maximum kinetic energy of the system executing simple harmonic motion (Chopra 2011, Clough and Penzien 1993). Although Rayleigh's quotient is valid for any vibration frequency of the MDOF system, its common engineering application is to evaluate the lowest or fundamental frequency with a trial shape vector that is chosen based on physical insight (Chopra 2011).

2.2 Extension to the simplified method

To obtain the shape vector, ϕ, a vector of external forces, **f**, can be assumed on the system in practice. Note that the i_{th} element in **f** represents the force applied at degree of freedom (DOF) i in the system. Accordingly, ϕ can be computed as

$$\phi = \mathbf{k}^{-1}\mathbf{f} \tag{2}$$

Substituting Equation (2) into Equation (1) gives

$$\omega_{\mathrm{n}}^2 = \frac{\phi^{\mathrm{T}}\mathbf{f}}{\phi^{\mathrm{T}}\mathbf{m}\phi} \tag{3}$$

Obviously, accuracy of the fundamental frequency predicted by Rayleigh's quotient depends on the vector of external forces, **f**, considered in the calculation. Conceptually, **f** should be selected to generate a deflected shape of the system which best approximates its fundamental vibration mode shape. Here, the authors propose the following two candidates for **f** which are calculated based on the mass matrix, **m**, and two assumed acceleration influence vectors, $\mathbf{A_v}$ and $\mathbf{A_r}$, respectively

$$\mathbf{f} = \mathbf{m}\mathbf{A_v} \tag{4}$$

$$\mathbf{f} = \mathbf{m}\mathbf{A_r} \tag{5}$$

The acceleration influence vectors, $\mathbf{A_v}$ and $\mathbf{A_r}$ can be determined according to the following criteria: holding the entire system of interest as a rigid body, the i_{th} element in $\mathbf{A_v}$ can be determined as the acceleration along DOF i due to application of a unit acceleration at the support bases of the system along the vertical direction; while the i_{th} element in $\mathbf{A_r}$ repents the acceleration along DOF i due to application of a unit angular acceleration about the geometrical center of the support bases of the system in the vertical plane of interest.

To illustrate the above criteria, consider an example two-dimensional system shown in Fig. 1, which has N lumped masses and 2N DOFs in the XOZ plane. Assuming that the coordinates of the i_{th} lumped mass is (x_i, z_i), one can determine $\mathbf{A_v}$ and $\mathbf{A_r}$ as

$$\mathbf{A_v} = \left\{ 0,\quad 1 \ldots \quad \underbrace{0,\quad 1}_{\text{DOFs } 2i\text{-1 and } 2i} \quad \ldots\ 0,\quad 1 \right\}^T \cdot 1.0\, m/s^2 \tag{6}$$

$$\mathbf{A_r} = \left\{ -r_1 \sin\alpha_1, r_1\cos\alpha_1, \ldots, \underbrace{-r_i \sin\alpha_i}_{\text{DOF } 2i\text{-1}}, \underbrace{r_i\cos\alpha_i}_{\text{DOF } 2i}, \ldots, -r_n \sin\alpha_n, r_n \cos\alpha_n \right\}^T \cdot 1.0\, rad/s^2 \tag{7}$$

$$r_i = \sqrt{x_i^2 + z_i^2} \tag{8}$$

$$\sin(\alpha_i) = \frac{z_i}{r_i} \tag{9}$$

$$\cos(\alpha_i) = \frac{x_i}{r_i} \tag{10}$$

Mathematically, Eq. (7) can be further simplified as

$$\mathbf{A_r} = \left\{ -z_1,\quad x_1 \ldots \quad \underbrace{-z_i,\quad x_i}_{\text{DOFs } 2i\text{-1 and } 2i} \quad \ldots\ z_n,\quad x_n, \right\}^T \cdot 1.0\, rad/s^2 \tag{11}$$

Accordingly, the periods of the system adopting $\mathbf{A_v}$ and $\mathbf{A_r}$ which are denoted as $T_{n\mathbf{Av}}$ and $T_{n\mathbf{Ar}}$ respectively, can be determined as

$$T_{n.\mathbf{A_v}} = 2\pi\sqrt{\frac{\phi^T \mathbf{m}\phi}{\phi^T \mathbf{m}\mathbf{A_v}}} \tag{12}$$

$$T_{n.\mathbf{A_r}} = 2\pi\sqrt{\frac{\phi^T \mathbf{m}\phi}{\phi^T \mathbf{m}\mathbf{A_r}}} \tag{13}$$

The larger value from the above two predictions is taken as the fundamental period of the system, T_n, which is

$$T_n = \max(T_{n.\mathbf{A_v}},\quad T_{n.\mathbf{A_r}}) \tag{14}$$

Beyond $\mathbf{A_v}$ and $\mathbf{A_r}$ introduced above for the simplified method, another acceleration influence vector, denoted as $\mathbf{A_h}$, is also considered in this research for comparison purpose. Holding the entire system of interest as a rigid body, the ith element in $\mathbf{A_h}$ can be determined as the acceleration along DOF i due to application of a unit acceleration at the support bases of the system along the horizontal direction in a vertical plane of interest. As will be discussed in a following section, the force vector associated with $\mathbf{A_h}$ is sometimes used to determine the fundamental periods of regular building structures from Rayleigh's quotient. Inclusion of $\mathbf{A_h}$ in this research helps confirm whether a force vector suitable for regular building structures remains applicable for pylons.

Taking the example system shown in Fig. 1 for consideration, $\mathbf{A_h}$ can be given as

$$\mathbf{A_h} = \left\{ 1,\quad 0 \ldots \quad \underbrace{1,\quad 0}_{\text{DOFs } 2i\text{-1 and } 2i} \quad \ldots\ 1,\quad 0 \right\}^T \cdot 1.0\, m/s^2 \tag{15}$$

Similarly, the period of the system associated with $\mathbf{A_h}$ denoted as $T_{n\mathbf{Ah}}$, can be computed as

$$T_{n.\mathbf{A_h}} = 2\pi\sqrt{\frac{\phi^T \mathbf{m}\phi}{\phi^T \mathbf{m}\mathbf{A_h}}} \tag{16}$$

2.3 Implementation procedure

In practice, a pylon should be first isolated from an overhead electricity transmission system and discretized as a MDOF system to implement the simplified method. Fig. 2 shows such a pylon for developing the MDOF system. The MDOF system is referred to thereafter as the single-pylon model. As shown, the pylon has two major portions - the trunk and the crossarms. The members in the trunk can be differentiated into three categories: the primary post members, the diaphragm members, and the diagonal members; while these in the cross-arms can be separated into three categories: the upper chord members, the lower chord members, and the web members.

The nodes at the ends of each primary post member and these at the ends of each bottom chord member are selected to lump the masses tributary from the adjacent pylon members. In a vertical plane of interest (e.g., the XOZ plane as shown in Fig. 2), each of the nodes lumping the masses from pylon members has two DOFs – the x and z translation components.

Aside from the masses from the pylon members, the masses from the transmission lines supported by the pylon of interest should also be included into the single-pylon model. Fig. 3 shows how transmission lines are physically supported by a pylon. As shown, the transmission lines are attached to the bottom chord members through an insulator. A spherical bearing is used at each end of the insulator, making the insulator actually a two-force member. It is proposed to lump the tributary masses of the transmission lines to the ends of the bottom chord members (i.e., where the corresponding insulators are attached). Considering the fact that each insulator can rotate freely about its ends and the swaying vibrations of the insulator and the attached transmission lines relative to the pylon crossarm are not of interest, each of the lumped masses tributary from the

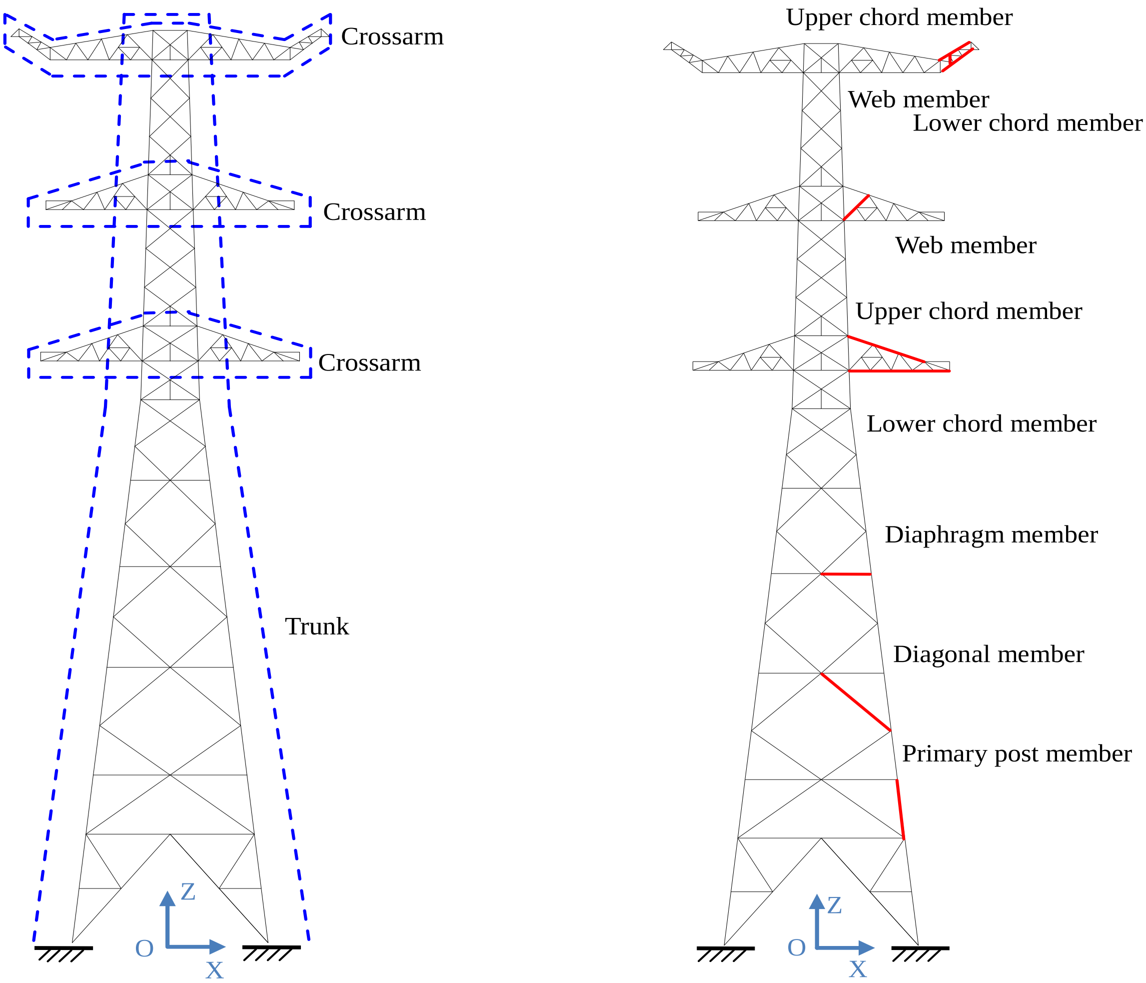


Fig. 2 Components in a pylon

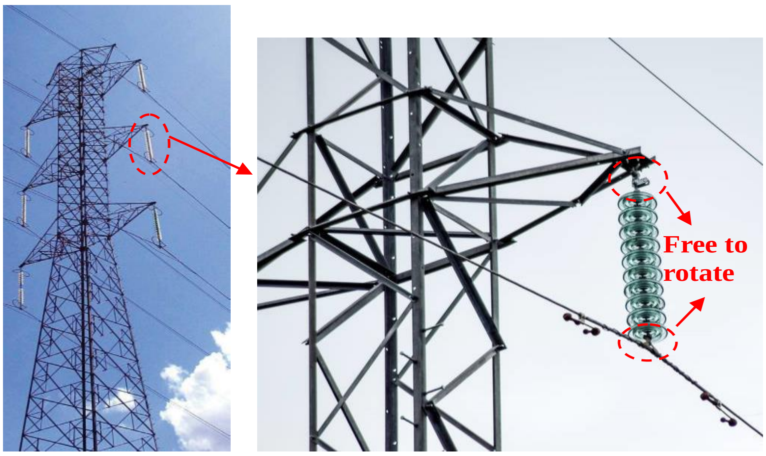
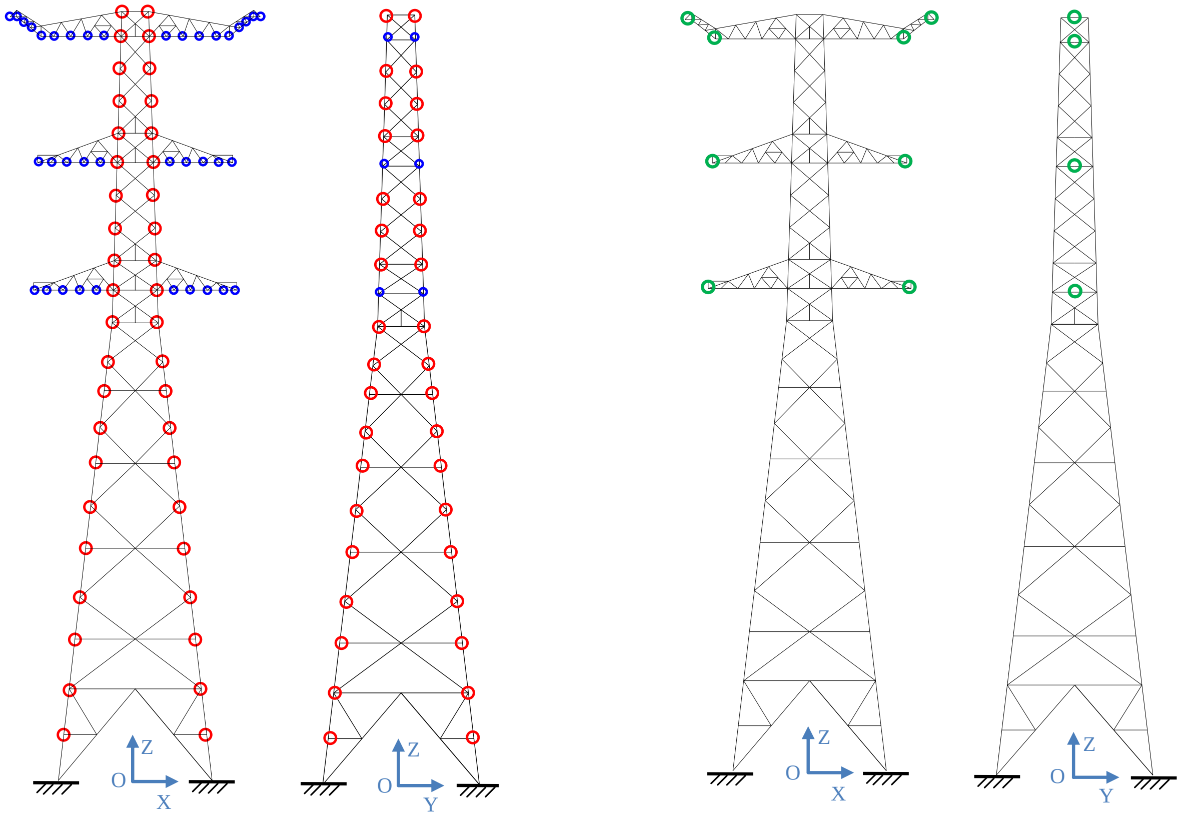


Fig. 3 Illustration of an insulator

transmission lines only has one DOF along the axial direction of the insulator (i.e., the vertical direction). Taking a typical pylon for example, Figs. 4 (a) and 4(b) show distributions of the lumped masses from the pylon members and these from the transmission lines, respectively. For the vibration mode of the pylon in the XOZ plane, each lumped mass in Fig. 4(a) has two DOFs (the x and z translations) and each lumped mass in Fig. 4(b) only has one DOF (the z translation). Accordingly, if the model has N_p lumped masses from pylon members and N_t lumped masses from transmission lines, $\mathbf{m}$ for determining the fundamental period of the system in the XOZ plane is a $(2\,N_p + N_t) \times (2\,N_p + N_t)$ matrix.



(a) Lumped masses from pylon members (b) Lumped masses from transmission lines

Fig. 4 Lumped masses in the single-pylon model

After establishing the mass matrix, the acceleration influence vectors, $\mathbf{A}_v$ and $\mathbf{A}_r$ should be determined based on Eqs. (6) and (11). Next, the external force vectors (i.e., $\mathbf{f}$) should be determined based on Eqs. (4) and (5) followed by application of the forces to the single-pylon model. The displacements gleaned at the identified DOFs from elastic static analyses of the single-pylon model are subsequently used to form ϕ. With $\mathbf{m}$, ϕ, and the corresponding acceleration influence vector ($\mathbf{A}_v$ or $\mathbf{A}_r$), the fundamental period of the pylon can determined from Eqs. (12) to (14). Notably, when $\mathbf{A}_h$ is considered, the above procedure can be similarly implemented. Moreover, the procedure can be similarly implemented in the orthogonal vertical plane (i.e., the YOZ plane for the example shown in Fig. 4) if vibration of the pylon in that plane is of interest.

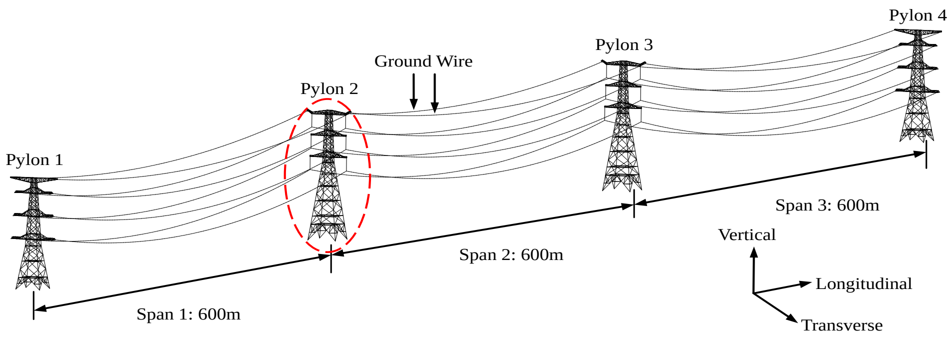
3. Validation of the simplified method

Three representative pylons from three different overhead electricity transmission systems (denoted as Prototypes A to C) were chosen as examples to validate the simplified method developed in the prior section. Specifically, the fundamental periods of the example pylons determined from the simplified method based on the single-pylon models were compared with these computed from the rigorous eigenvalue analyses using the cluster models. This section first describes the prototypes and example pylons and then reports comparisons of the analysis results.

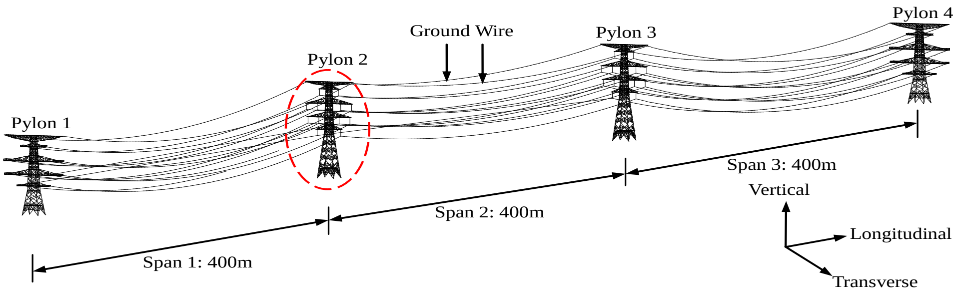
Fig. 5 shows the cluster models for the example pylons. Note that Pylon 2 from each cluster model is of interest. Fig. 6 illustrates elevation of each example pylon. The member configurations of the example pylons selected from Prototypes A to C are categorized as SK29101, S4Z2, and SDF5A, respectively, according to the Rules of Nomenclature for Transmission Poles and Towers (CEPP 2013). Unless labelled ground wires, the transmission lines shown in Fig. 5 are conductors in each prototype. Table 1 presents the properties of the transmission lines (ground wires and conductors) included in each cluster model. Fig. 7 compares the percentages of the masses from the pylon trunk, pylon crossarms and tributary transmission lines in each single-pylon model.

Prototype A is a 1000 kV electricity transmission system in Shandong Province, China. The entire system extends about 1048.5 km. Three spans of transmission lines and four pylons were included in the cluster model as Fig. 5(a) shows. The three spans of transmission lines considered in the model have the same span of 600 m. As Fig. 6(a) depicts, the pylon of interest has three crossarms at the elevations of 75 m, 94.5 m, and 113.8 m, respectively.

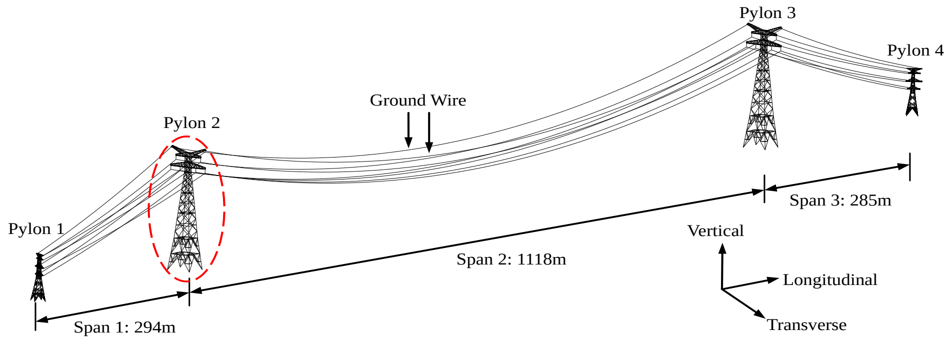
Prototype B is a 220 kV electricity transmission system in Jiangsu Province, China. The entire system extends about 11.7 km. Three spans of transmission lines with the same span of 400m and four pylons were included in the cluster model as Fig. 5(b) shows. Pylon 2 from Prototype B was selected since it has the highest number of crossarms among the three example pylons as compared in Fig. 6. Note that a



(a) Isolated from Prototype A

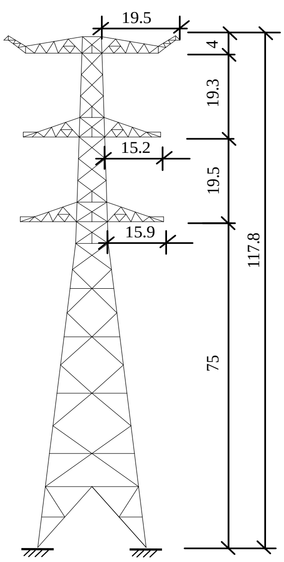


(b) Isolated from Prototype B

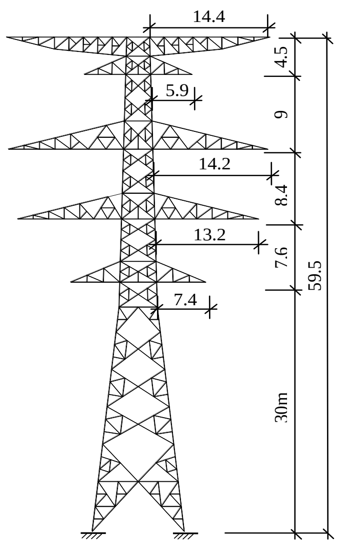


(c) Isolated from Prototype C

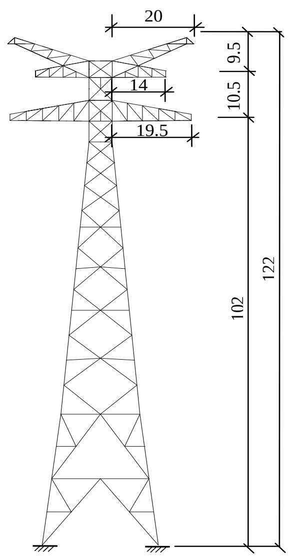
Fig. 5 Illustration of the cluster models



(a) From Prototype A



(b) From Prototype B



(c) From Prototype C

Fig. 6 Elevations of the example pylons (unit: m)

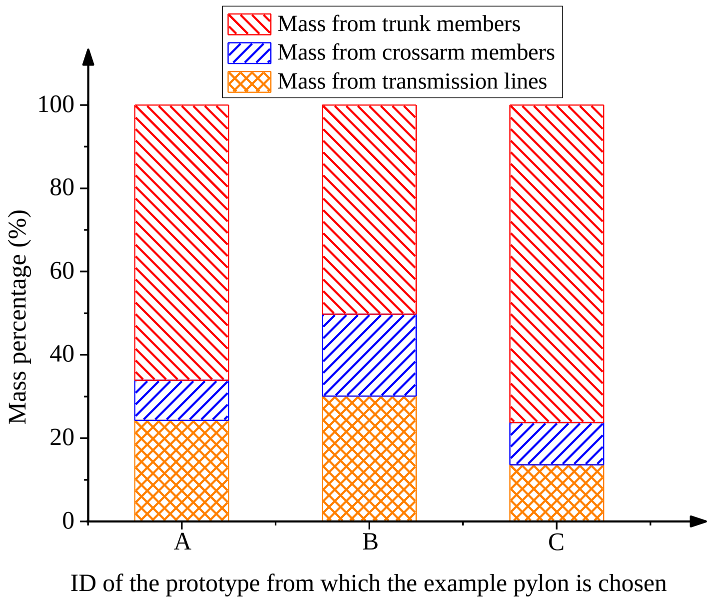


Fig. 7 Masses from different members in each single-pylon model

Table 1 Summary of properties of transmission lines

Prototype	Category of transmission lines	Designation	Modulus of elasticity (MPa)	Cross-section area (mm^2)	Mass per unit length (kg/km)
A	Conductor	JL/GIA-630/45	63000	674	2079.2
	Ground wire	JLB20A-185	147220	182.8	1221.5
B	Conductor	LGJ-300/40	73000	339	1133
	Ground wire	JLB40-150	103600	148	696
C	Conductor	LHBGJ-400/95	78000	501.02	1856.7
	Ground wire	OPGW-180	170100	175.2	1286

higher number of crossarms tends to amplify the overturning action and the flexural deformation of the pylon when it vibrates laterally.

Prototype C is a 220 kV electricity transmission system in Shandong Province, China. The entire system extends about 58.4 km. Three spans of transmission lines and four pylons were included in the cluster model as Fig. 5(c) shows. The three spans of transmission lines are 294 m, 1118 m, and 285 m, respectively. Pylon 2 from this system was chosen because it supports extremely long-span (exceeding 1000 m) transmission lines from one side and it is also the highest among the three example pylons considered. Notably, the interior span of the transmission lines in the cluster model shown in Fig. 5(c) crosses the Yellow River which is the 2_{nd} longest river in China and also the 5_{th} longest river in the world. Incidentally, the cluster model selected from Prototype C has been analyzed for other research issues by the authors. Further information about the model and the prototype can be found elsewhere (Tian *et al*. 2017, Tian *et al*. 2018).

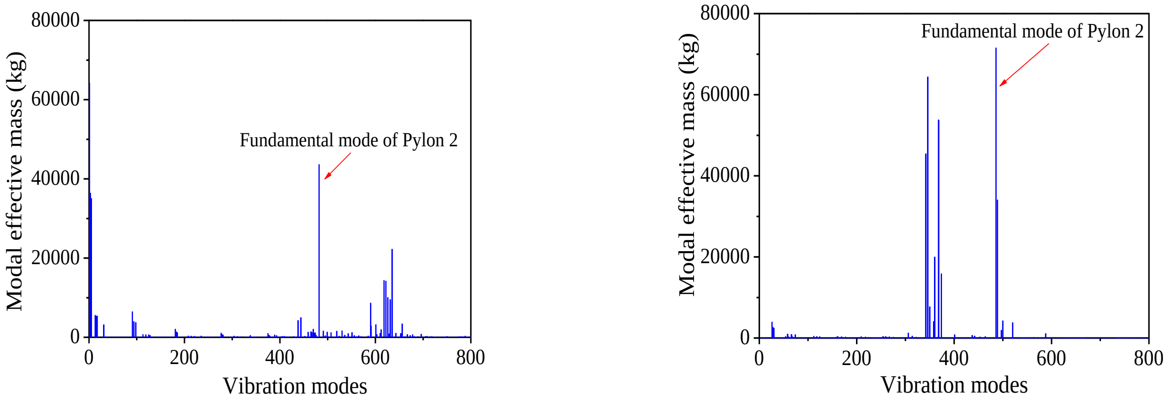
Abaqus (Version 6.12), which is a commercially available software package, was used to develop the cluster model for each example pylon. In each cluster model, beam elements (B31) and truss elements (T3D2) were adopted for the pylon members and transmission lines, respectively. Up to 120 elements were used for the transmission line in a single span in these cluster models. Isotropic elastic material and tension-only elastic material properties were assigned to the pylon members and transmission lines, respectively. Masses were introduced to each cluster model through assigning mass densities to the materials for the pylon members and the transmission lines.

SAP 2000 (Version 14), which is also a commercial software package, was used to develop the single-pylon models. Elastic frame elements were used for the pylon members. The forces associated with a certain acceleration influence vector were assigned to the corresponding DOFs in each single-pylon model. The displacements due to the application of $\mathbf{f}$ were used to construct the corresponding shape vector ϕ, followed by calculation of the approximate fundamental periods.

Table 2 summarizes the fundamental periods predicted from the eigenvalue analyses of the cluster models and these from the simplified method based on the single-pylon models. The predictions based on $\mathbf{A}_h$ were also included in Table 2 for comparison. Note that the periods associated with the longitudinal and transverse directions were compared separately. As shown, when the simplified method is implemented, the predictions associated with $\mathbf{A}_r$ govern in all cases (i.e., between $\mathbf{A}_v$ and $\mathbf{A}_r$, adoption of $\mathbf{A}_r$ produces larger fundamental periods in the simplified method). Along the transverse direction, the errors of the

Table 2 Comparison of analysis results

Prototype ID	Pylon ID	Vibration direction of interest	Period from eigenvalue analysis of the cluster model (sec)	Simplified method		
				Acceleration influence vector	Prediction (sec)	Error* (%)
A	2	Transverse	0.8627	$\mathbf{A_h}$	0.7598	11.93
				$\mathbf{A_v}$	0.2568	70.23
				$\mathbf{A_r}$	0.8619	0.08
		Longitudinal	0.8362	$\mathbf{A_h}$	0.7831	6.35
				$\mathbf{A_v}$	0.2568	69.29
				$\mathbf{A_r}$	0.8101	3.12
B	2	Transverse	0.7167	$\mathbf{A_h}$	0.6063	15.4
				$\mathbf{A_v}$	0.5731	20.04
				$\mathbf{A_r}$	0.7121	0.64
		Longitudinal	0.6535	$\mathbf{A_h}$	0.6034	7.67
				$\mathbf{A_v}$	0.5731	12.3
				$\mathbf{A_r}$	0.6323	3.24
C	2	Transverse	0.9714	$\mathbf{A_h}$	1.3582	39.82
				$\mathbf{A_v}$	0.3327	65.75
				$\mathbf{A_r}$	0.9537	1.82
		Longitudinal	0.9201	$\mathbf{A_h}$	1.3479	46.49
				$\mathbf{A_v}$	0.3327	63.84
				$\mathbf{A_r}$	0.9137	0.7



(a) Transverse direction (b) Longitudinal direction

Fig. 8 Modal effective mass in each vibration mode of the cluster model of Prototype A

governing predictions from the simplified method against the results from the rigorous eigenvalue analyses of the cluster models are 0.08%, 0.64% and 1.82% for the example pylons from Prototypes A to C, respectively. Along the longitudinal direction, the observed errors are 3.12%, 3.24% and 0.7% for the example pylons from Prototypes A to C, respectively. The observed errors are all within the acceptable range, suggesting the adequacy of the simplified method.

As briefly mentioned in the introduction section, eigenvalue analyses of the cluster models are onerous. Taking the cluster model from Prototype A as an example, a total of 482 and 486 eigenvalues have to be extracted in the analyses in the two orthogonal vertical planes to achieve the fundamental periods of Pylon 2 along the transverse and longitudinal directions, respectively. To identify the eigenvalues for the vibration modes of Pylon 2, a database of the modal effective masses shown in Fig. 8 were generated. Note that each peak value in the graph suggests a vibration mode involving participation of a significant portion of the masses from the system (possibly participation of a pylon but not necessarily the one of interest). Based on a visual inspection of each mode associated with the peak modal effective mass, the mode numbers and eigenvalues associated with the fundamental vibration modes of Pylon 2 were identified. Compared with the eigenvalue analyses of the cluster model, the simplified method apparently requires much reduced modelling efforts and computational costs.

In the simplified method, $\mathbf{A_v}$ is introduced for the case in which vertical vibration of a pylon turns to be its fundamental vibration mode in a vertical plane of interest.

Among the three example pylons considered in this research, none exhibits such a fundamental vibration mode. Nevertheless, it is recommended to consider $\mathbf{A_v}$ when implementing the simplified method since such a case theoretically can occur in a pylon with extremely flexible crossarms. Moreover, $\mathbf{A_h}$ was introduced for comparison purpose since the force vector associated with $\mathbf{A_h}$ is occasionally used in practice for approximating the fundamental periods of simple systems such as shear buildings (Clough and Penzien 1993). Although Rayleigh's quotient usually provides excellent estimates of the fundamental periods of conventional civil structures even with a mediocre force vector (Chopra 2011), the errors in the fundamental periods of the pylons predicted using $\mathbf{A_h}$ vary from 11.93% to 39.82% and from 6.35% to 46.49% along the transverse and longitudinal directions, respectively. This observation suggests that the force vectors acceptable for conventional civil structures should be used with caution for pylons since the two systems have significantly different mass and stiffness distributions.

4. Conclusions

This research focused on development and validation of a simplified method for determination of the fundamental periods of pylons in electricity transmission systems. The theoretical basis underlying the simplified method is Rayleigh's quotient. Compared with the rigorous but excessively onerous approach adopted in the current practice, the simplified method decreases the needed modelling effort and computational cost. Moreover, the simplified method can be conveniently implemented via a conventional computer program that includes a simple elastic structural analysis module. Based on the analyses conducted in this investigation, the following conclusions may be drawn:

- Through analyses of three example pylons selected from the representative overhead electricity transmission systems having different design parameters, the simplified method was found to provide the fundamental periods of the pylons that are similar to the predictions from the rigorous eigenvalue analyses of the cluster models. Therefore, the simplified method can be used as a convenient surrogate for the tedious eigenvalue analyses.
- The stiffness and mass distributions in pylons are significantly different from these in conventional civil structures. As such, the force vectors that can be used in Rayleigh's quotient to reasonably approximate the fundamental periods of conventional civil structures may be inadequate for pylons.

Acknowledgements

This research was financially supported by the National Natural Science Foundation of China (under Awards No. 51778347, No. 51778348, No. 51578325 and No. 51578324), and the Young Scholars Program of Shandong University (under Award No. 2017WLJH33).

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