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Effect of two-temperature on the energy ratio at the boundary surface of inviscid fluid and piezothermoelastic medium

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Abstract. The phenomenon of reflection and transmission of plane waves at an interface between fluid half space and orthotropic piezothermoelastic solid half-space with two-temperature has been investigated. Energy ratios of various reflected and transmitted waves are computed with the use of amplitude ratios. The law of conservation of energy across the interface has been justified. It is found that the energy ratios are the functions of angle of incidence, frequency of independent wave and depend on the different piezothermoelastic material. A piezothermoelastic material has been considered which is in welded contact with water. Variations of energy ratios corresponding to the reflected waves and transmitted waves are computed and shown graphically for the two different models. A particular reduced case of interest is also discussed.

Keywords: reflection; piezothermoelastic; orthotropic; transmission; amplitude ratios

1. Introduction

Chen and Gurtin (1968), Chen and Williams (1968), Chen *et al*. (1969) introduced the two-temperature theory of thermoelasticity in the heat conduction equation which depends on two distinct temperatures, the conductive temperature φ and the thermodynamic temperature T. The existence of conductive and thermodynamic temperatures is due to the thermal and mechanical processes, respectively occurring between the particles and the layers of the elastic material. For static problems, the difference between these two temperatures is proportional to the heat supply. However, in the absence of any heat supply, the two temperatures are identical (Chen and Gurtin 1968, Chen and Williams 1968). In contrast, for dynamic problems, particularly problems involving the phenomenon of wave propagation, φ and T both temperatures are generally different irrespective of the presence of heat supply. Awad (2011) described the spatial decay estimates in non-classical linear thermoelastic semi-cylindrical bounded domains. Miranville and Quintanilla (2016) investigated the spatial behavior in two-temperature generalized thermoelastic theories.

Kumar et al. (2018) studied the propagation of plane waves in an anisotropic thermoelastic medium with void and two-temperature in the context of three phase lag theory of thermoelasticity. Ezzat et al. (2018) studied the two-temperature theory in Green–Naghdi thermoelasticity with fractional phase-lag heat transfer. Deswal et al. (2019) investigated the reflection of plane waves from the free surface of a homogeneous, anisotropic, fiber-reinforced thermoelastic rotating medium with the consideration of effects of two-temperature and dual-phase-lag parameters. Lotfy (2019) discussed the interaction between thermal-elastic-plasma waves through photothermal process under the effects of magnetic field and two-temperature parameter.

Piezoelectricity as one of the branches of crystal physics is now the base of the modern engineering practice in various technologies like frequency control, signal processing, sound and ultrasound microphones and speakers, ultrasonic imaging, hydrophones, actuators and motors based on the converse effect, detection of pressure variations in the form of sound etc. By continuously monitoring deformation, the sensors can record operational loads, compute fatigue and estimate remaining component life. The theory of thermopiezoelectric material was first proposed by Mindlin (1974) and derived governing equations of a thermopiezoelectric plate. The physical laws for the thermopiezoelectric material have been explored by Nowacki (1978, 1979). Chandrasekharaiah (1984) used generalized Mindlin's theory of thermopiezoelectricity to account for the finite speed of propagation of thermal disturbances. Majhi (1995) studied the transient thermal response of the semi-infinite piezoelectric rod subjected to the heat source. Sharma (2010a, 2010b) investigated the propagation of inhomogeneous waves in anisotropic piezo-thermoelastic media and discussed the piezoelectric effect on the velocities of waves in an anisotropic piezo-poroelastic medium. Vashishth and Sukhija (2014, 2015) studied the inhomogeneous waves at the boundary of an anisotropic piezo-thermoelastic medium and, reflection and transmission of plane waves from fluid piezothermoelastic solid interface. Marin and Nicaise (2016) studied the existence and stability results for thermoelastic dipolar bodies with double porosity. Vashishth and Sukhija (2017) also discussed the inhomogeneous waves in porous piezo-thermoelastic solids Marin and Craciun (2017) derived the

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uniqueness results for a boundary value problem in dipolar thermoelastic model. Marin (2017) also studied the effect of microtemperatures for micropolar thermoelastic bodies. Kumar and Kaur (2017) investigated the reflection of plane waves at micropolar piezothermoelastic solids. Kumar and Sharma (2017) investigated the reflection and transmission of plane waves at an elastic half space and piezothermoelastic solid half space with fractional order derivative. Marin and Ochsner (2017) studied an initial boundary value problem for modeling a piezoelectric dipolar body. Sharma (2018) investigated the phenomenon of reflection–refraction of attenuated waves at the interface between a thermo-poroelastic medium and a thermoelastic medium. Wang (2018) studied transient responses of laminated anisotropic piezo-thermoelastic plates and cylindrical shells with interfacial diffusion and sliding in cylindrical bending. Sangwan et al. (2018) discussed the reflection and transmission of plane waves at an interface between elastic and micropolar piezoelectric solid half-spaces. Kumar and Sharma (2019) studied the response of fractional order derivative on the energy ratios at the boundary surface of fluid-piezothermoelastic medium.

Youssef and Bassiouny (2008) proposed the generalised two temperature theory of thermoelasticity to solve the boundary value problems of one dimensional piezothermoelastic half-space with heating its boundary with different types of heating. Ezzat et al. (2010) formulated two temperature theory of thermoelasticity for piezoelectric/piezomagnetic materials. Bassiouny and Sabry (2013) investigated the propagation of thermal wave through a semi-infinite slab subjected to thermal loading of fractional order of exponential type applied for finite period of time.

In this paper, the problem of reflection and transmission of plane waves from a fluid-orthotropic piezothermoelastic solid interface with two-temperature is discussed. Energy ratios corresponding to the reflected and transmitted waves are computed with the aid of amplitude ratios. Further, effects of angle of incidence and two-temperature parameter on the energy ratios are observed analytically and shown graphically. A particular case of interest is also discussed.

2. Basic equations

Following Kumar *et al*. (2018) and Vashishth and Sukhija (2015), the basic equations for a homogeneous, anisotropic, thermally conducting, piezoelectric elastic medium in the absence of body forces and free charge density are as follows.

Constitutive equations

$$\sigma_{ij} = c_{ijkl}\,\varepsilon_{kl} - e_{ijk}E_k - \alpha_{ij}T, \tag{1}$$

$$D_i = \xi_{ij}E_j + e_{ijk}\varepsilon_{jk} + \tau_i T, \tag{2}$$

$$E_i = -\phi_{,i}, \quad (i, j, k, l = 1, 2, 3). \tag{3}$$

Equations of motion:

$$\sigma_{ij,j} - \rho\,\ddot{\mathrm{u}}_i = 0. \tag{4}$$

Equation of heat conduction

$$K_{ij}\,\varphi_{,ij} - \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C_e T + \alpha_{ij}\mathrm{u}_{i,j}T_0 - \tau_i\phi_{,i}T_0) = 0, \tag{5}$$

such that $\varphi - T = a_{ij}\varphi_{,ij}$.

Gauss equation:

$$D_{i,i} = 0, \quad (i, j = 1, 2, 3). \tag{6}$$

Following Achenbach (1973), the constitutive relations for the inviscid fluid half space are

$$\sigma_{ij}^f = \lambda_f\,\mathrm{u}_{k,k}^f\,\delta_{ij}, \ (i, j, k, l = 1, 2, 3), \tag{7}$$

and the equations of motion are

$$\sigma_{ij,j}^f - \rho_f\,\ddot{\mathrm{u}}_i^f = 0, \ (i, j = 1, 2, 3), \tag{8}$$

where C_{ijkl} are elastic parameters, α_{ij} is tensor of thermal moduli respectively. ρ, C_e are, respectively , the density and specific heat at constant strain, T_0 is the reference temperature, T is the absolute temperature and φ is the conductive temperature of the medium, a_{ij} are the two-temperature parameters, τ_o is the thermal relaxation time, which will ensure that the heat conduction equation will predict finite speeds of heat propagation of matter from one medium to other. u_i, u_i^f are the components of displacement vector in the solid and fluid half spaces, $\sigma_{ij}(=\sigma_{ji})$, $\sigma_{ij}^f(=\sigma_{ji}^f)$ are the components of the stress tensor in the solid and fluid half spaces, $\varepsilon_{ij} = \frac{1}{2}(\mathrm{u}_{i,j} + \mathrm{u}_{j,i})(= \varepsilon_{ji})$ are the components of the strain tensor, $K_{ij}(=K_{ji})$ are the components of thermal conductivity, E_i is the electric field intensity, D_i is the electric displacement, ϕ is the electric potential, τ_i are the pyroelectric constants, e_{ijk}, ξ_{ij} are tensors of piezothermal moduli, ρ_f and λ_f are the density and the bulk modulus of the fluid, respectively. The piezothermal coefficients c_{ijk}, K_{ij} and C_e are positive. The symbol comma "," followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot "." denotes the derivative with respect to time.

3. Formulation of the problem

We consider an orthotropic piezothermoelastic half space (OPTHS) and the inviscid fluid half space (FHS) as shown in Fig 1. PTHS occupies the region $x_3 \geq 0$, and the FHS occupies region $x_3 \leq 0$.

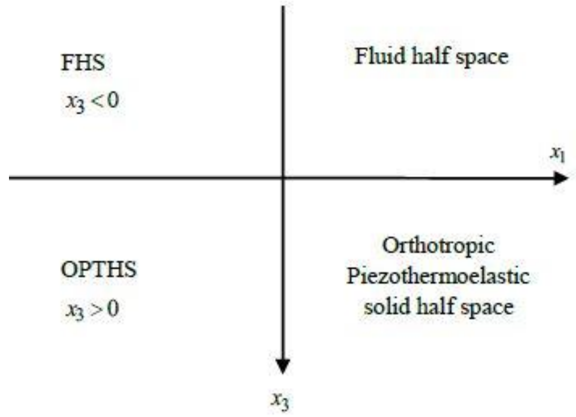


Fig. 1 Geometrical representation of FHS and OPTHS

Following Tzou and Bao (1995), the constitutive relations in an orthotropic piezothermoelastic medium in x_1-x_3 plane are

$$\sigma_{11}=c_{11}\,\varepsilon_1+c_{13}\,\varepsilon_3-e_{31}E_3-\alpha_{11}T, \tag{9a}$$

$$\sigma_{33}=c_{13}\,\varepsilon_1+c_{33}\,\varepsilon_3-e_{33}E_3-\alpha_{33}T, \tag{9b}$$

$$\sigma_{13}=2c_{55}\,\varepsilon_5-e_{15}E_1, \tag{9c}$$

$$D_1=\xi_{11}E_1+2e_{15}\varepsilon_5, \tag{9d}$$

$$D_3=\xi_{33}E_3+e_{31}\varepsilon_1+e_{33}\varepsilon_3+\tau_3T, \tag{9e}$$

$$E_1=-\phi_{,1}, \tag{9f}$$

$$E_3=-\phi_{,3}, \tag{9g}$$

and, constitutive relations for fluid half space

$$\sigma_{11}^f=\lambda_f\,(\mathrm{u}_{1,1}^f+\mathrm{u}_{3,3}^f), \tag{10a}$$

$$\sigma_{33}^f=\lambda_f\,(\mathrm{u}_{1,1}^f+\mathrm{u}_{3,3}^f). \tag{10b}$$

Substituting the constitutive relations (9a) - (9g) into the field Eqs. (4) - (6) without body forces, heat sources, yield

$$c_{11}\,\mathrm{u}_{1,11}+c_{13}\,\mathrm{u}_{3,13}+e_{31}\phi_{,31}+c_{55}(\mathrm{u}_{1,33}+\mathrm{u}_{3,13})-\alpha_{11}(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,1}+e_{15}\phi_{,13}-\rho\,\ddot{\mathrm{u}}_1=0, \tag{11a}$$

$$c_{55}(\mathrm{u}_{1,31}+\mathrm{u}_{3,11})+c_{13}\,\mathrm{u}_{1,13}+c_{33}\,\mathrm{u}_{3,33}-\alpha_{33}(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,3}+e_{15}\phi_{,11}+e_{33}\phi_{,33}-\rho\,\ddot{\mathrm{u}}_3=0, \tag{11b}$$

$$(K_{11}\varphi_{,11}+K_{33}\varphi_{,33})-\left(1+\tau_0\frac{\partial}{\partial t}\right)T_0(\alpha_{11}\,\dot{\mathrm{u}}_{1,1}+\alpha_{33}\,\dot{\mathrm{u}}_{3,3}-\tau_3\dot{\phi}_{,3}+r(\dot{\varphi}-a_{11}\dot{\varphi}_{,11}-a_{33}\dot{\varphi}_{,33}))=0, \tag{11c}$$

$$-\xi_{11}\phi_{,11}+e_{15}(\mathrm{u}_{1,31}+\mathrm{u}_{3,11})-\xi_{33}\phi_{,33}+e_{31}\,\mathrm{u}_{1,31}+e_{33}\,\mathrm{u}_{3,33}+\tau_3(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,3}=0. \tag{11d}$$

Using constitutive Eqs. (10a, 10b) in Eq. (8), the field equations for fluid half space can be written as

$$\left.\begin{aligned}\lambda_f\left(\mathrm{u}_{1,11}^f+\mathrm{u}_{3,31}^f\right)-\rho_f\ddot{\mathrm{u}}_1^f&=0,\\ \lambda_f\left(\mathrm{u}_{1,13}^f+\mathrm{u}_{3,33}^f\right)-\rho_f\ddot{\mathrm{u}}_3^f&=0.\end{aligned}\right\} \tag{12}$$

We introduce the following dimensionless quantities

$$\begin{gathered}\left(x_1',x_3',\mathrm{u}_1',\mathrm{u}_3'\right)=\frac{\omega_1}{c_1}\left(x_1,x_3,\mathrm{u}_1,\mathrm{u}_3\right),\\ (\mathrm{u}_1^f{}',\mathrm{u}_3^f{}')=\frac{\omega_1}{c_1}(\mathrm{u}_1^f,\mathrm{u}_3^f),\left(t',\tau_0'\right)=\omega_1\left(t,\tau_0\right),\\ \left(T',\varphi'\right)=\frac{\alpha_{11}}{\rho c_1^2}\left(T,\varphi\right),\phi'=\frac{\omega_1 e_{31}\phi}{c_1\alpha_{11}T_0},\left(\sigma_{ij}',\sigma_{ij}^f{}'\right)\\ =\frac{1}{\alpha_{11}T_0}\left(\sigma_{ij},\sigma_{ij}^f\right),(P',P^f{}')=\frac{1}{\alpha_{11}T_0c_1}(P,P^f),\\ (a_{11}',a_{33}')=\frac{\omega_1^2}{c_1^2}(a_{11},a_{33}),\end{gathered} \tag{13}$$

where $c_1=\sqrt{\dfrac{c_{11}}{\rho}}$, $\omega_1=\dfrac{\rho C_e c_1^2}{K_{11}}$.

Introducing the dimensionless quantities (13), in the system of Eqs. (11a) - (12), with the removal of prime ('), reduces to the following form

$$c_{11}\,\mathrm{u}_{1,11}+(c_{13}+c_{55})\,\mathrm{u}_{3,13}+c_{55}\,\mathrm{u}_{1,33}+\frac{\alpha_{11}T_0}{e_{31}}(e_{31}+e_{15})\phi_{,13}-\rho c_1^2(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,1}-\rho c_1^2\,\ddot{\mathrm{u}}_1=0, \tag{14a}$$

$$(c_{55}+c_{13})\,\mathrm{u}_{1,31}+c_{55}\,\mathrm{u}_{3,11}+c_{33}\,\mathrm{u}_{3,33}-\rho c_1^2\frac{\alpha_{33}}{\alpha_{11}}(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,3}+\frac{\alpha_{11}T_0}{e_{31}}(e_{15}\phi_{,11}+e_{33}\phi_{,33})-\rho c_1^2\,\ddot{\mathrm{u}}_3=0, \tag{14b}$$

$$\frac{\rho\omega_1}{\alpha_{11}}(K_{11}\varphi_{,11}+K_{33}\varphi_{,33})-\left(1+\tau_0\frac{\partial}{\partial t}\right)T_0$$
$$(\alpha_{11}\dot{\mathrm{u}}_{1,1}+\alpha_{33}\dot{\mathrm{u}}_{3,3}-\tau_3\dot{\phi}_{,3}+r(\dot{\varphi}-a_{11}\dot{\varphi}_{,11}$$
$$-a_{33}\dot{\varphi}_{,33}))=0, \tag{14c}$$

$$-\frac{\alpha_{11}T_0}{e_{31}}(\xi_{11}\phi_{,11}+\xi_{33}\phi_{,33})+e_{15}(\mathrm{u}_{1,31}+\mathrm{u}_{3,11})+$$
$$e_{31}\mathrm{u}_{1,31}+e_{33}\mathrm{u}_{3,33}+\tau_3\frac{\rho c_1^2}{\alpha_{11}}(\varphi-a_{11}\varphi_{,11}-a_{33}\varphi_{,33})_{,3}=0, \tag{14d}$$

$$\lambda_f\left(\mathrm{u}^f_{1,11}+\mathrm{u}^f_{3,31}\right)-c_1^2\rho_f\ddot{\mathrm{u}}^f_1=0, \tag{15a}$$

$$\lambda_f\left(\mathrm{u}^f_{1,13}+\mathrm{u}^f_{3,33}\right)-c_1^2\rho_f\ddot{\mathrm{u}}^f_3=0. \tag{15b}$$

For the solution of the plane harmonic waves propagating in x_1-x_3 plane, the displacement components, electric potential and temperature are written as follows

$$\left(\mathrm{u}_1,\mathrm{u}_3,\phi,\varphi\right)=\left(U,A,B,C\right)\exp\left[i\omega\left(-\frac{x_1}{c}-qx_3+t\right)\right], \tag{16}$$

where ω is the circular frequency, c is the apparent phase velocity and q is the unknown slowness parameter. U,A,B and C are the unknown amplitude vectors with respect to the waves and that are independent of time t and spatial coordinates. The system of Eqs. (14a) - (14d), with the aid of Eq. (16), yield another system of equations

$$-\omega^2\left\{\left(\frac{c_{11}}{c^2}+c_{55}q^2-\rho c_1^2\right)U+(c_{13}+c_{55})\frac{q}{c}A\right.$$
$$\left.+\frac{\alpha_{11}T_0}{e_{31}}(e_{31}+e_{15})\frac{q}{c}B\right\}+\frac{i\omega}{c}\rho c_1^2 \tag{17a}$$
$$\left(1+a_{11}\frac{\omega^2}{c^2}+a_{33}\omega^2q^2\right)C=0,$$

$$-\omega^2\left\{(c_{55}+c_{13})\frac{q}{c}U+\left(\frac{c_{55}}{c^2}+c_{33}q^2-\rho c_1^2\right)A+\right.$$
$$\left.\frac{\alpha_{11}T_0}{e_{31}}\left(\frac{e_{15}}{c^2}+e_{33}q^2\right)B\right\}+i\omega q\rho c_1^2\frac{\alpha_{33}}{\alpha_{11}} \tag{17b}$$
$$\left(1+a_{11}\frac{\omega^2}{c^2}+a_{33}\omega^2q^2\right)C=0,$$

$$i\omega\left\{\frac{t_{11}\alpha_{11}}{c}U+\alpha_{33}t_{11}qA-q\frac{t_{11}\alpha_{11}\tau_3T_0}{e_{31}}B\right\}-\frac{\rho}{\alpha_{11}}$$
$$\left\{\omega^2\omega_1\left(\frac{K_{11}}{c^2}+K_{33}q^2\right)+t_{11}rc_1^2\left(1+a_{11}\frac{\omega^2}{c^2}+a_{33}\omega^2q^2\right)\right\} \tag{17c}$$
$$C=0,$$

$$-\omega^2\left\{(e_{15}+e_{31})\frac{q}{c}U+\left(\frac{e_{15}}{c^2}+e_{33}q^2\right)A-\frac{\alpha_{11}T_0}{e_{31}}\right.$$
$$\left.\left(\frac{\xi_{11}}{c^2}+\xi_{33}q^2\right)B\right\}-i\omega q\rho c_1^2\frac{\tau_3}{\alpha_{11}} \tag{17d}$$
$$\left(1+a_{11}\frac{\omega^2}{c^2}+a_{33}\omega^2q^2\right)C=0,$$

where $t_{11}=T_0(i\omega-\omega^2\tau_0)$.

The above system of the equations can be written in the matrix form as VS=o, where

$$\boldsymbol{V}=\begin{bmatrix} c_{55}q^2+x_{11} & x_{12}q & x_{13}q & x_{15}q^2+x_{14} \\ x_{12}q & c_{33}q^2+x_{17} & x_{18}+x_{19}q^2 & x_{21}q^3+x_{20}q \\ x_{22} & x_{23}q & x_{24}q & x_{30}q^2+x_{29} \\ x_{31}q & e_{33}q^2+x_{32} & x_{34}q^2+x_{33} & x_{36}q^3+x_{35}q \end{bmatrix}, \tag{18}$$

And $\boldsymbol{S}$ = [U, A, B, C]^{tr}. Here, superscript symbol "*tr*" represents transpose of the matrix. The symbols used in matrix $\boldsymbol{V}$ are mentioned in the Appendix A. This system of the equations has a non- trivial solution if the determinant of the matrix $\boldsymbol{V}$ vanishes i.e.

$$\det\boldsymbol{V}=0. \tag{19}$$

Eq. (19) yields a characteristic equation in q^2 On Solving this characteristic equation, we obtain q^1, q^2, q^3, q^4 correspond to the roots of the Eq. (18) whose imaginary parts are positive, and q_5,q_6,q_7,q_8 denote the roots whose imaginary parts are negative. The eigen values are arranged in descending order such that q^1, q^2 and q^3 corresponds to the propagating quasi longitudinal (qP) wave mode, quasi transverse (qS) wave mode, quasi thermal (qT) wave mode and q^4 corresponds to the electric potential component wave mode (eP) of wave propagation, respectively. For each $q_i(i=1,2,...,8)$, the corresponding eigen vectors U_i, A_i, B_i and C_i can be written as

$$W_i=\frac{cof\left(V_{42}\right)_{q_i}}{cof\left(V_{41}\right)_{q_i}},\ \Phi_i=\frac{cof\left(V_{43}\right)_{q_i}}{cof\left(V_{41}\right)_{q_i}},\ \Theta_i=\frac{cof\left(V_{44}\right)_{q_i}}{cof\left(V_{41}\right)_{q_i}}, \tag{20}$$

where

$$W_i = \frac{A_i}{U_i},\ \Phi_i = \frac{B_i}{U_i},\ \Theta_i = \frac{C_i}{U_i}, \tag{21}$$

and $cof(V_{ij})_{qi}$ denotes the cofactor of V_{ij} to the eigen value q_i. The amplitudes (U_i, A_i, B_i and C_i) of the plane harmonic waves decrease as these waves propagates in a piezothermoelastic medium. The amplitudes of the plane harmonic waves propagating in a piezothermoelastic medium also depend on the frequency. The formal solution for the mechanical displacement, the electric potential, temperature, the stress components and electric displacement becomes

$$(\mathrm{u}_1, \mathrm{u}_3, \phi, \varphi) = \sum_i (1, W_i, \Phi_i, \Theta_i) U_i \exp\left\{ i\omega\left(-\frac{x_1}{c} - q_i x_3 + t \right)\right\},$$
$$(\sigma_{33}, \sigma_{31}, D_3) = i\omega \sum_i (D_{1i}, D_{2i}, D_{6i}) U_i \exp\left\{ i\omega\left(-\frac{x_1}{c} - q_i x_3 + t \right)\right\}. \tag{22}$$

The solution of wave in the fluid medium can be expressed as

$$(\mathrm{u}_1^f, \mathrm{u}_3^f) = (U^f, A^f) \exp\left(i\omega\left(-\frac{x_1}{c} \mp q_f x_3 + t \right)\right), \tag{23}$$

where $q_f = \frac{1}{c}\sqrt{\frac{c^2}{c_f^2} - 1}$ is the unknown slowness parameter, U^f and A^f are the associated amplitudes, and c_f is the longitudinal wave velocity in the fluid medium. The formal solution for the displacements in fluid medium become

$$(\mathrm{u}_1^f, \mathrm{u}_3^f) = \sum_{p=1,2} (1, W_p^f) U_p^f \exp\left(i\omega\left(-\frac{x_1}{c} + (-1)^p q_f x_3 + t \right)\right), \tag{24}$$

where $p = 1$ corresponds to the incident wave, $p = 2$ corresponds to the reflected wave, and $W_1^f = q_f c,\ W_2^f = -q_f c,\ c = \frac{c_f}{\sin\theta}$.

The normal stress becomes

$$\sigma_{33}^f = i\omega \sum_{p=1,2} \rho_f c U_p^f \exp\left(i\omega\left(-\frac{x_1}{c} + (-1)^p q_f x_3 + t \right)\right). \tag{25}$$

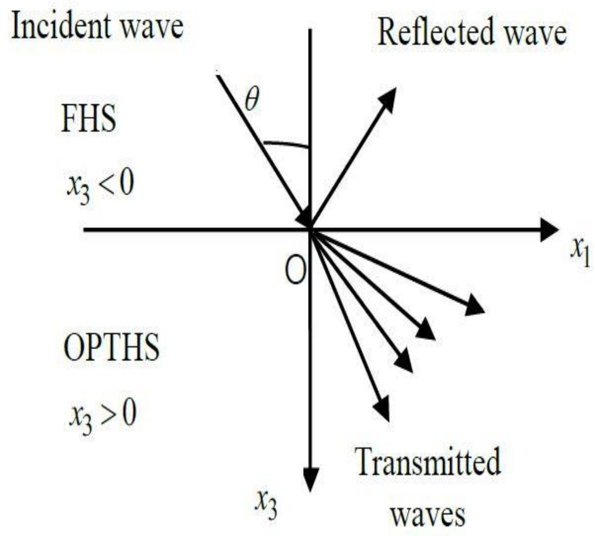


Fig. 2 Reflection and transmission of plane wave

4. Reflection and transmission coefficients

4.1 Amplitude ratios

We consider an OPTHS which is in contact with FHS at $x_3 = 0$. One reflected longitudinal wave in FHS and four transmitted waves in OPTHS are observed when a plane longitudinal wave is incident at the interface by an angle θ to the normal. These transmitted waves are quasi longitudinal (qP) mode, quasi transverse (qS) mode, quasi thermal (qT) mode and electric potential (eP) mode.

The boundary conditions at the interface $x_3 = 0$ are as follows:

Continuity of normal stress

$$\sigma_{33} = \sigma_{33}^f, \tag{26a}$$

(i) Vanishing of the tangential stress

$$\sigma_{13} = 0, \tag{26b}$$

(iii) Continuity of normal velocity component

$$\dot{\mathrm{u}}_3 = \dot{\mathrm{u}}_3^f, \tag{26c}$$

(iv) Vanishing of electric displacement

$$D_3 = 0, \tag{26d}$$

(v) Isothermal boundary

$$T = 0. \tag{26e}$$

Using Eqs. (22), (24) and (25) and, the constitutive Eqs. (1) – (3), the boundary conditions (26a) - (26e) result in a non-homogeneous system

$$\boldsymbol{AX} = \boldsymbol{B}, \tag{27}$$

where

$$\boldsymbol{A}=\begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & -\rho_f c \\ D_{21} & D_{22} & D_{23} & D_{24} & 0 \\ D_{31} & D_{32} & D_{33} & D_{34} & q_f c \\ D_{41} & D_{42} & D_{43} & D_{44} & 0 \\ D_{51} & D_{52} & D_{53} & D_{54} & 0 \end{bmatrix},$$

$$\boldsymbol{X}=\left[X_1, X_2, X_3, X_4, X_5^f\right]^{tr},$$

$$\boldsymbol{B}=\left[\rho_f c, 0, q_f c, 0, 0\right]^{tr}.$$

The elements of 5×5 matrix $\boldsymbol{A}$ and notations used in $\boldsymbol{X}$ are given in the Appendix B. After solving the system (27), the transmitted and the reflected amplitude ratios are obtained.

4.2 Energy ratios

Energy ratios are used to describe system energy output related to the system energy input. The average energy flux of the incident, reflected and transmitted waves helps in knowing the distribution of energy between reflected and transmitted waves across a surface element of unit area. Following Vashishth and Sukhija (2015), the normal acoustic flux P in a piezothermoelastic solid is

$$P=-\mathrm{Re}\left(\sigma_{31}\bar{\dot{u}}_1+\sigma_{33}\bar{\dot{u}}_3-\phi\bar{\dot{D}}_3+K_{33}\bar{T}_{,3}\frac{T}{T_0}\right). \quad (28)$$

The time average of P over a period denoted by $<P>$ represents the average energy transmission per unit surface area per unit time. The average energy flux of the

(a) Incident wave is

$$\left\langle P_I\right\rangle=\frac{1}{2}\omega^2 q_f\rho_f c^2\left|U_1^f\right|^2, \quad (29)$$

(b) Reflected wave is

$$\left\langle P_R\right\rangle=-\frac{1}{2}\omega^2 q_f\rho_f c^2\left|U_2^f\right|^2, \quad (30)$$

(c) Transmitted waves are

$$\left\langle P_s\right\rangle=-\frac{1}{2}\omega^2\,\mathrm{Re}\Big(D_{2s}+D_{1s}\bar{W}_s+\bar{D}_{4s}\Phi_s+ \frac{i}{\omega}\frac{K_{33}}{T_0}\bar{D}_{6s}\left(t_{14}+t_{15}q_s^2\right)\Theta_s\Big)\left|U_s\right|^2, s=1,2,3,4. \quad (31)$$

The energy ratios of the reflected and transmitted waves are defined as

$$E_R=\frac{\left\langle P_R\right\rangle}{\left\langle P_I\right\rangle},\ E_s=\frac{\left\langle P_s\right\rangle}{\left\langle P_I\right\rangle},\ s=1,2,3,4. \quad (32)$$

Due to interaction between different fields and displacements corresponding to transmitted waves, the interaction energy ratios are described as $E_{st}=\dfrac{\left\langle P_{st}\right\rangle}{\left\langle P_I\right\rangle}$,

where

$$\left\langle P_{st}\right\rangle=-\frac{1}{2}\omega^2\,\mathrm{Re}\Big(D_{2s}U_s\bar{U}_t+D_{1s}\bar{W}_t U_s\bar{U}_t+ \bar{D}_{4s}\Phi_t U_t\bar{U}_s+\frac{i}{\omega}\frac{K_{33}}{T_0}\bar{D}_{6s}\left(t_{14}+t_{15}q_s^2\right)\Theta_t\bar{U}_s U_t\Big), \quad (33)$$

$$s=1,2,3,4.$$

The resultant interaction energy between the transmitted waves is $E_{\mathrm{int}}=\sum_{\substack{s,t=1\\ s\neq t}}^{4}E_{st}$.

Since

$$\sum_{s=1}^{4}E_s+E_{\mathrm{int}}+E_R=1, \quad (34)$$

therefore, the law of conservation is verified.

Particular case

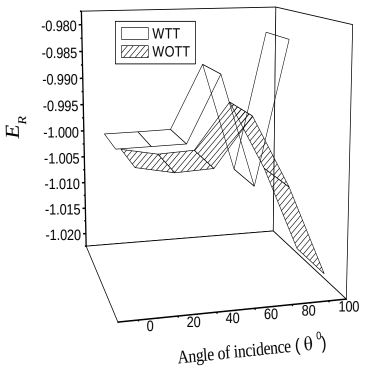
If we put $a_{11}=a_{33}=0$, in system of equations (11) then the matrix $\boldsymbol{V}$ reduces to

$$\boldsymbol{V}=\begin{bmatrix} c_{55}q^2+x_{11} & x_{12}q & x_{13}q & 0 \\ x_{12}q & c_{33}q^2+x_{17} & x_{18}+x_{19}q^2 & 0 \\ x_{22} & x_{23}q & x_{24}q & x_{26}q^2+x_{25} \\ x_{31}q & e_{33}q^2+x_{32} & x_{34}q^2+x_{33} & 0 \end{bmatrix},$$

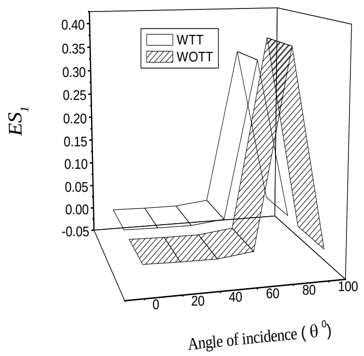
such that for the non-trivial solution, $|\boldsymbol{V}|=0$, yielding a characteristic equation $m_{11}q^8+m_{12}q^6+m_{13}q^4+m_{14}q^2+m_{15}=0$, where the notations $m_{1i},(i=1,2,3,4,5)$ are mentioned in the Appendix A. Solving the characteristic equation we obtain the unknown amplitude of the respective waves and hence, energy ratios at the interface of elastic and orthotropic piezothermoelastic half spaces can be obtained verifying the law of conservation of energy.

5. Numerical results and discussion

For the purpose of numerical calculation, we consider the homogeneous orthotropic piezothermoelastic media. The amplitude ratios and energy ratios for the reflected and transmitted waves and the interaction energy ratios are computed with the help of the software Matlab 9.0 and Origin 6.1 and, graphs of energy ratios are shown depicting the effect of two temperature. Further, the law of conservation of energy is verified. Following Vashishth and Sukhija (2015), the numerical values of cadmium selenide (CdSe) have been taken. Elastic constants (in units of GPa)

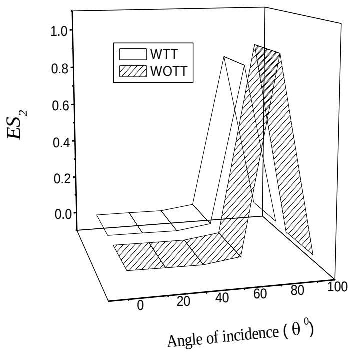


(a) Reflected wave

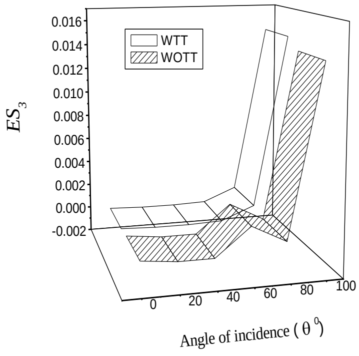


(b) Transmitted qP wave

Fig. 3 Variation of energy ratio with respect to θ



(a) Transmitted qS wave



(b) Transmitted qT wave

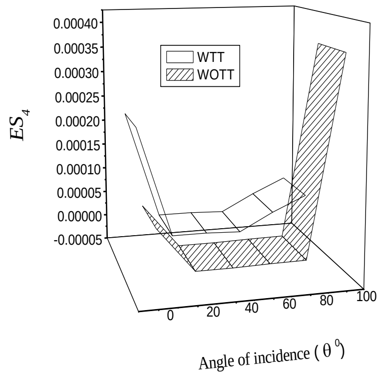
Fig. 4 Variation of energy ratio with respect to θ

are $c_{11} = 74.1$, $c_{13} = 39.3$, $c_{33} = 83.6$, $c_{55} = 15.1$. Thermoelastic coupling constants ($10^6\, PaK^{-1}$) are given by $\alpha_{11} = 0.621$, $\alpha_{33} = 0.551$. Electric permittivity constants ($10^{-11}\, Fm^{-1}$) are given by $\xi_{11} = 8.26$, $\xi_{33} = 9.03$. Thermal conductivity constants ($Wm^{-1}K^{-1}$) are given by $K_{11} = 9$, $K_{33} = 9$. Piezoelectric constants ($10^{-3}Cm^{-2}$) are given by $e_{15} = 3$, $e_{31} = 35$, $e_{33} = 34$. Pyroelectric constant is $\tau_3 = -2.6\times10^{-6}\, Cm^{-2}K^{-1}$. Numerical values for the remaining constants are $C_e = 260\, JKg^{-1}K^{-1}$, $c_f = 1500\, ms^{-1}$, $a_{11} = 10^{-20}$, $a_{33} = 2\times10^{-22}$, $\tau_0 = 2\times10^{-5}\, s$, $\omega = 2\pi\times10^3\,\mathrm{Hz}$, $\rho = 5504\, Kgm^{-3}$, $\rho_f = 1500\, Kgm^{-3}$, $T_0 = 298K$.

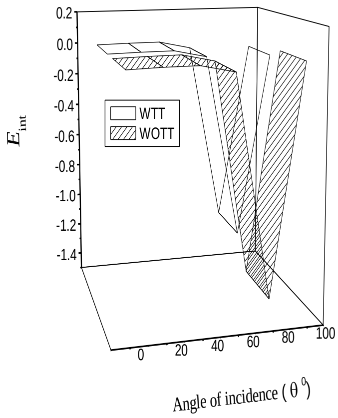
In all the graphs, notations ▭, WTT, ▨, WOTTdenote the energy ratio curves corresponding to orthotropic piezothermoelastic solid with two-temperature and orthotropic piezothermoelastic solid, respectively.

It is noticed from Fig. 3(a) that the energy ratio (E_R) of the reflected P wave attains constant magnitude values for $0 \le \theta \le 40^0$. For WTT, it increases when $40^0 \le \theta \le 60^0$ and $80^0 \le \theta$ while decreases for $60^0 \le \theta \le 80^0$. In case of WOTT model, it increases when angle of incidence varies from 40^0 to 60^0 and for $60^0 \le \theta$, it shows decreasing behaviour. It implies that when the medium is considered with two temperature effect, energy ratio of reflected wave gains energy. Fig. 3(b) illustrates that the behaviour of energy ratio (ES_1) of the transmitted wave (qP) is similar for both models with different magnitude values. At $\theta = 80^0$, its magnitude value for WOTT is greater than for WTT. As θ increases above 80^0, it shows monotonically decreasing behaviour.

Fig. 4(a) shows that the energy ratio (ES_2) of the transmitted wave (qS) varies as angle of incidence increases. For both mediums, it shows similar behaviour with variations in magnitude values with respect to θ. At $\theta = 80^0$, in the absence of two temperature effect the numerical value of ES_2 is greater than for WTT. In the beginning, for $\theta \le 60^0$, the plot shows constant behaviour of ES_2 then its value increases when θ varies from 60^0 to 80^0 . As θ increases above 80^0 , it shows monotonically decreasing behaviour. Fig. 4(b) depicts that for WTT, the energy ratio (ES_3) of the transmitted wave (qT) shows similar behaviour with variations in magnitude values for $0 \le \theta \le 40^0$. For WTT, it increases



(a)Transmitted energy ratio of eP wave mode



(b) Interaction energy

Fig. 5 Variation of energy ratio and energy with respect to θ

monotonically as angle of incidence increases above 40^0.

In case of WOTT, it shows variations in behaviour when θ varies from 40^0 to 60^0. As θ increases above 80^0, it shows monotonically increasing behaviour.

Fig. 5(a) illustrates that for both models the energy ratio (ES_4) of the transmitted wave (eP) initially decreases for lower values of θ and further, for $20^0 \le \theta \le 60^0$, the plot is stationary in case of both with slight difference in magnitude values. However, when θ varies from 60^0 to 80^0, ES_4 is still showing stationary mode in the absence of two temperature effect. Then, it shows monotonically behaviour for $60^0 \le \theta$ in case of WTT medium and for $80^0 \le \theta$, when WOTT medium is considered. It is noticed that $80^0 \le \theta$, energy ratio of the transmitted wave (eP) gains higher energy in case of WOTT in comparison to WTT. Fig. 5(b) clearly shows that the interaction energy E_{int} shows the similar behaviour with different numerical values for both WTT and WOTT mediums as of ES_1 and ES_2 illustrated by figures 3(b) and 4(a). As angle of incidence varies from 0 to 80^0, it shows monotonically decreasing behaviour but increases as θ increases above 80^0.

6. Conclusions

The mathematical study is to discuss the reflection and transmission phenomenon of elastic waves at an interface of fluid half space – orthotropic piezothermoelastic half space. This phenomenon is studied with the consideration of two temperature theory. The energy ratios of various reflected and transmitted waves are obtained by using the amplitude ratios and these energy ratios are discussed analytically and represented graphically to show the effect of two-temperature. The following conclusions are made from the above study.

- Amplitude ratios and energy ratios are affected by the frequency, angle of incidence, two- temperature parameter and piezothermoelastic properties of the material. The nature of this dependence is different for reflected and transmitted waves.
- Piezothermoelastic and two-temperature parameter have a significant influence on the energy ratios.
- Principle of conservation of energy has been justified.
- It is found that sum of these energy ratios is approximately unity at each angle of incidence. This shows that there is no dissipation of energy during reflection and transmission phenomenon.
- Energy ratio of the transmitted qP wave possesses the maximum value for WOTT model. As angle of incidence increases, the energy ratio of the transmitted waves increases in comparison to the reflected wave and interaction energy ratio.
- Numerical results show that the reflection and transmission coefficient along with energy ratios of various reflected and transmitted waves are affected significantly by the two-temperature parameter.

References

Achenbach, J.D. (1973), “Wave propagation in elastic solids”, North Holland Publ., Amsterdam.

Awad, E.S. (2011), “A note on the spatial decay estimates in non-classical linear thermoelastic semi-cylindrical bounded domains”, *J. Therm. Stresses*, **34**(2), 147-160. https://doi.org/10.1080/01495739.2010.511942.

Bassiouny, E. and Sabry, R. (2013), “Fractional order two temperature thermo-elastic behaviour of piezoelectric materials”, *J. Appl. Math. Phys.*, **1**(5), 110-120. http://dx.doi.org/10.4236/jamp.2013.15017.

Bassiouny, E. and Youssef, H.M. (2008), “Two-temperature generalised thermopiezoelasticity of finite rod subjected to different types of thermal loadings”, *J. Therm. Stresses*, **31**(1), 233-245. https://doi.org/10.1080/01495730701737902.

Chandrasekharaiah, D.S. (1984), “A generalised linear thermoelasticity theory of piezoelectric media”, *Acta Mech.*, **71**, 293-349. http://10.1007/bf01173936.

Chen, P.J. and Gurtin, M.E. (1968), “On a theory of heat conduction involving two temperatures”, *Z. Angew. Math. Phys.*, **19**(4), 614-627. https://doi.org/10.1007/BF01594969.

Chen, P.J. and Williams, W.O. (1968), “A note on non-simple heat conduction”, *Z. Angew. Math. Phys.*, **19**(6), 969-970. https://doi.org/10.1007/BF01602278.

Chen, P.J., Gurtin, M.E. and Williams, W.O. (1969), "On the thermodynamics of non-simple elastic materials with two temperatures", *Z. Angew. Math. Phys*., **20**(1), 107-112. https://doi.org/10.1007/BF01591120.

Deswal, S., Sheokand, S.K. and Kalkal, K.K. (2019), "Reflection at the free surface of fiber-reinforced thermoelastic rotating medium with two-temperature and phase-lag", *Appl. Math. Model*., **65**, 106-119. https://doi.org/10.1016/j.apm.2018.08.004.

Ezzat, M.A., El-Karamany, A.S. and Awad, E.S. (2010), "On the coupled theory of thermo-piezoelectric/ piezomagnetic materials with two-temperature", *Can. J. Phys*., **88**(5), 307-315. https://doi.org/10.1139/P10-015.

Ezzat, M.A., El-Karamany, A.S and El-Bary, A.A. (2018), "Two-temperature theory in Green–Naghdi thermoelasticity with fractional phase-lag heat transfer", *Microsyst. Technol*., **24**(2), 951-961. https://doi.org/10.1007/s00542-017-3425-6.

Kumar, R. and Kaur, M. (2017), "Reflection and transmission of plane waves at micropolar piezothermoelastic solids", *J. Solid Mech*., **9**(3), 508-526.

Kumar, R. and Sharma, P. (2017), "Effect of fractional order on energy ratios at the boundary surface of elastic-piezothermoelastic media", *Coupled Syst. Mech*., **6**(2), 157-174. https://doi.org/10.12989/csm.2017.6.2.157.

Kumar, R. and Sharma, P. (2019), "Response of fractional order on energy ratios at the boundary surface of fluid-piezothermoelastic media", *Appl. Math. Comp*., **358**(1), 194-203. https://doi.org/10.1016/j.amc.2019.04.020.

Kumar, R., Vashishth, A.K. and Ghangas, S. (2018), "Waves in anisotropic thermoelastic medium with phase lag, two-temperature and void", *Mater. Phys. Mech*., **35**, 26-138.

Lotfy, Kh. (2019), "A novel model for photothermal excitation of variable thermal conductivity semiconductor elastic medium subjected to mechanical ramp type with two-temperature theory and magnetic field", *Sci. Rep*., **9**(1), 3319. https://doi.org/10.1038/s41598-019-39955-z.

Majhi, M.C. (1995), "Discontinuities in generalized thermo elastic wave propagation in a semi- infinite piezoelectric rod", *J. Tech. Phys*., **36**(3), 269-278. http://pascal-francis.inist.fr/vibad/index.php?action=getRecordDetail&idt=2941280.

Marin, M. (2017), "Effect of microtemperatures for micropolar thermoelastic bodies", *Struct. Eng. Mech*., **61**(3), 381-387. https://doi.org/10.12989/sem.2017.61.3.381.

Marin, M. and Craciun, E.M. (2017), "Uniqueness results for a boundary value problem in dipolar thermoelasticity to model composite materials", *Comput. Part B: Eng*., **126**, 27-37. https://doi.org/10.1016/j.compositesb.2017.05.063.

Marin, M. and Nicaise, S. (2016), "Existence and stability results for thermoelastic dipolar bodies with double porosity", *Continuum Mech. Thermodyn*., **28**(6), 1645-1657. https://doi.org/10.1007/s00161-016-0503-4.

Marin, M. and Ochsner, A. (2017), "An initial boundary value problem for modelling a piezoelectric dipolar body", *Continuum Mech. Thermodyn*., **32**(2), 267-278. doi: 10.1007/s00161-017-0599-1.

Mindlin, R. D. (1974), "Equation of high frequency of thermopiezoelectric crystals plates", Int. J. Solids Struct., 625-637. https://doi.org/10.1007/s00161-017-0599-1.

Miranville, A. and Quintanilla, R. (2017), "On the spatial behavior in two-temperature generalized thermoelastic theories", *Z. Angew. Math. Phys*., **68**(5). https://doi.org/10.1007/s00033-017-0857-x.

Nowacki, W. (1979), "Foundation of Linear Piezoelectricity", Interactions in Elastic Solids, Springer, Wein, Chapter 1.

Nowacki, W. (1978), "Some general theorems of thermo-piezoelectricity", *J. Therm. Stresses*, **1**(2), 171-182. https://doi.org/10.1080/01495737808926940.

Sangwan, A., Singh, B. and Singh, J. (2018), "Reflection and transmission of plane waves at an interface between elastic and micropolar piezoelectric solid half-spaces", *Tech. Mech*., **38**(3), 267-285. https://doi.org/10.24352/UB.OVGU-2018-034.

Sharma, M.D. (2010a), "Propagation of inhomogeneous waves in anisotropic piezo-thermoelastic media", *Acta Mech*., **215**(1-4), 307-318. https://doi.org/10.1007/s00707-010-0336-3.

Sharma, M.D. (2010b), "Piezoelectric effect on the velocities of waves in an anisotropic piezo-poroelastic medium", *Proc. R. Soc. A*, **466**(2119), 1977-1992. https://doi.org/10.1098/rspa.2009.0534.

Sharma, M.D. (2018), "Reflection–refraction of attenuated waves at the interface between a thermo-poroelastic medium and a thermoelastic medium", *Waves Random Complex Medium*, **28**(3), 570-587. https://doi.org/10.1080/17455030.2017.1370154.

Tzou, H.S. and Bao, Y. (1995), "A theory on anisotropic piezothermoelastic shell laminates with sensor/actuator applications", *J. Sound Vib*., **184**(3), 453-473. https://doi.org/10.1006/jsvi.1995.0328.

Vashishth, A.K. and Sukhija, H. (2014), "Inhomogeneous waves at the boundary of an anisotropic piezo-thermoelastic medium", *Acta Mech*., **225**(12), 3325-3338. https://doi.org/10.1007/s00707-014-1139-8.

Vashishth, A.K. and Sukhija, H. (2015), "Reflection and transmission of plane waves from fluid piezothermoelastic solid interface", *Appl. Math. Mech. Engl. Ed*., **36**(1), 11-36. https://doi.org/10.1007/s10483-015-1892-9.

Vashishth, A.K. and Sukhija, H. (2017), "Inhomogeneous waves in porous piezo-thermoelastic solids", *Acta Mech*., **228**(5), 1891-1907. https://doi.org/10.1007/s00707-017-1805-8.

Wang, X. (2018), "Transient responses of laminated anisotropic piezothermoelastic plates and cylindrical shells with interfacial diffusion and sliding in cylindrical bending", *Math. Mech. Solution*, 1-26. https://doi.org/10.1177%2F1081286518769480.

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Appendix A

$$t_{11}=T_0(i\omega-\omega^2\tau_0),\ t_{12}=\frac{\alpha_{11}T_0}{e_{31}},\ t_{13}=-\frac{i\rho c_1^2}{\omega c},$$

$$t_{14}=1+\omega^2\frac{a_{11}}{c^2},\ t_{15}=\omega^2 a_{33},\ t_{16}=-\frac{i\rho c_1^2\alpha_{33}}{\omega\alpha_{11}},$$

$$t_{17}=-\frac{\rho}{\alpha_{11}},\ t_{18}=t_{17}\omega_1\omega^2,\ t_{19}=-t_{11}c_1^2 r,\text{where}$$

$$r=\frac{\rho C_e}{T_0},\ t_{20}=\frac{i\rho c_1^2\tau_3}{\omega\alpha_{11}}.$$

$$x_{11}=\frac{c_{11}}{c^2}-\rho c_1^2,\ x_{12}=\frac{(c_{13}+c_{55})}{c}=x_{16},$$

$$x_{13}=\frac{(e_{31}+e_{15})}{c}t_{12},\ x_{14}=t_{13}t_{14},\ x_{15}=t_{13}t_{15},$$

$$x_{17}=\frac{c_{55}}{c^2}-\rho c_1^2, x_{18}=\frac{t_{12}e_{15}}{c^2},\ x_{19}=t_{12}e_{33},$$

$$x_{20}=t_{14}t_{16},\ x_{21}=t_{15}t_{16},\ x_{22}=\frac{i\omega\alpha_{11}t_{11}}{c},$$

$$x_{23}=i\omega\alpha_{33}t_{11},\ x_{24}=-i\omega\tau_3 t_{12}t_{11},$$

$$x_{25}=\frac{t_{18}K_{11}}{c^2},\ x_{26}=t_{18}K_{33},\ x_{27}=t_{14}t_{19},$$

$$x_{28}=t_{15}t_{19},\ x_{29}=x_{25}+x_{27},\ x_{30}=x_{26}+x_{28},$$

$$x_{31}=\frac{(e_{31}+e_{15})}{c}, x_{32}=\frac{e_{15}}{c^2},\ x_{33}=\frac{-t_{12}\xi_{11}}{c^2},$$

$$x_{34}=-t_{12}\xi_{33},\ x_{35}=t_{14}t_{20},\ x_{36}=t_{15}t_{20}.$$

$$b_{11}=x_{17}x_{33}-x_{18}x_{32}, b_{12}=c_{33}x_{33}+x_{17}x_{34},$$

$$b_{13}=-(x_{19}x_{32}+x_{18}e_{33}), b_{14}=b_{12}+b_{13},$$

$$b_{15}=c_{33}x_{34}-x_{19}e_{33}, b_{16}=x_{12}x_{33}-x_{18}x_{31},$$

$$b_{17}=x_{12}x_{34}-x_{19}x_{31},\ b_{18}=x_{12}x_{32}-x_{17}x_{31},$$

$$b_{19}=x_{12}e_{33}-c_{33}x_{31}, b_{20}=c_{55}b_{11}+x_{11}b_{14}-x_{12}b_{16}$$
$$+x_{13}b_{18}, b_{21}=c_{55}b_{14}+x_{11}b_{15}-x_{12}b_{17}+x_{13}b_{19}.$$

$$m_{11}=c_{55}x_{26}b_{15},\ m_{12}=b_{15}x_{25}c_{55}+x_{26}b_{21}, m_{13}=b_{21}x_{25}$$
$$+b_{20}x_{26},\ m_{14}=b_{20}x_{25}+x_{26}x_{11}b_{11},\ m_{15}=x_{11}x_{25}b_{11}.$$

Appendix B

$$D_{1i}=-\frac{c_{13}}{c}-c_{33}q_iW_i-\frac{\alpha_{11}T_0e_{33}}{e_{31}}q_i\Phi_i-\frac{\alpha_{33}\rho c_1^2}{i\omega\alpha_{11}}\Big(1+$$
$$\omega^2\frac{a_{11}}{c^2}+\omega^2a_{33}q_i^2\Big)\Theta_i, D_{2i}=-c_{55}\left(\frac{W_i}{c}+q_i\right)+\frac{e_{15}c_1\alpha_{11}T_0}{ce_{31}\omega_1}\Phi_i,$$

$$D_{3i}=W_i,\ D_{5i}=\left(1+\omega^2\frac{a_{11}}{c^2}+\omega^2a_{33}q_i^2\right)\Theta_i,$$

$$D_{4i}=-\frac{e_{31}}{c}-e_{33}q_iW_i+\frac{\xi_{33}T_0\alpha_{11}}{e_{31}}q_i\Phi_i+\frac{\tau_3\rho c_1^2}{i\omega\alpha_{11}}$$
$$\left(1+\omega^2\frac{a_{11}}{c^2}+\omega^2a_{33}q_i^2\right)\Theta_i,$$

$$D_{6i}=\left(1+\omega^2\frac{a_{11}}{c^2}+\omega^2a_{33}q_i^2\right)q_i\Theta_i,$$

$$X_i=\frac{U_i}{U_1^f},(i=1,2,3,4),\ X_5^f=\frac{U_2^f}{U_1^f}.$$