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A displacement-based seismic design procedure for buildings with fluid viscous dampers

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Abstract. This paper presents a displacement-based seismic design procedure for new structures with fluid viscous dampers and/or for existing structures, where these devices are required as a retrofit measure and damage control. To consider the non-proportional damping produced by these devices in a conventional modal spectral analysis, the effect of the fluid viscous dampers is approximated as the sum of a proportional damping matrix and a complementary matrix which is representative of non-proportional damping matrix. To illustrate the application of this procedure and evaluate the performance of structures designed with the procedure proposed, five regular plane frames: 8, 12, 17, 20 and 25-storey, and an 8-storey building are designed. The seismic demands used for design and validation were the records obtained at the SCT site during the 1985 Michoacan earthquake, and that of the 2017 Morelos - Puebla earthquake obtained at the Culhuacan site, both stations located on soft soil sites. To validate the procedure proposed, the performances and damage distributions used as design targets were compared with the corresponding results from the nonlinear step-by-step analyses of the designed structures subjected to the same seismic demands.

Keywords: displacement-based seismic design; damage control; linear fluid viscous dampers; non-proportional damping approximation; modal spectral analysis

1. Introduction

The main objective of seismic design is to guarantee adequate behaviour of a structure by accomplishing a design performance objective (PO), when subjected to earthquake scenarios which may occur during its service life. For this purpose, the building code design has relied primarily on force-based design procedures, in which the structures are analysed for a set of seismic design forces defined via static analysis or modal spectral analysis, from which the structural components are designed. At the final stage or the process, the interstorey drifts are checked that they do not exceed the thresholds associated with the limit states that comprise the PO, if the threshold is exceeded the structural members shall be modified to satisfy such restriction.

However, it has been recognized that the aforementioned force-based design approach may not guarantee adequate control of seismic performance under design conditions. For this reason, performance-based design procedures have been developed, particularly simplified procedures based on displacements as these parameters are acknowledged can be well correlated with structural performance e.g., Panagiotakos and Fardis (1999), Priestley *et al.* (2007), Ayala *et al.* (2012), Lopez *et al.* (2015), Vamvatsikos and Aschheim (2016), Katsanos and Vamvatsikos (2017), Lenza *et al.* (2017), among others. Displacement-based procedures consist on defining first a design displacement configuration of the structure where the design interstorey threshold is not exceeded, from which the design forces are derived, the structural analysis, and design of elements are carried out.

In the application of these procedures there are many situations where it is quite challenging to design structures that satisfy the PO considered due to architectural and geometric restrictions to the structural elements (e.g., beams). For this reason, it is necessary to consider other design alternatives to satisfy the PO, such as the use of passive energy dissipation devices e.g., the fluid viscous dampers (FVDs), which dissipate part of the input earthquake energy, and as a consequence reduce the displacements of the structure as well as the corresponding earthquake induced damage. An essential part in the seismic design of structures with FVDs is their size, location, and distribution particularly in the vertical direction.

According to Hwang (2013) and Lin and Wu (2013) there are two main approaches to distribute such devices. The first approach is to define the location and the optimal distribution of the devices via genetic algorithms e.g., the procedure proposed by Takewaki (2009), which consist in obtaining the minimum transfer functions of interstorey drifts and/or top floor absolute accelerations. On the other hand, Lopez-Garcia (2001) and Lopez-Garcia and Soong

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(2002) propose a simplified sequential search algorithm, which consists in assign the FVDs with the same damping coefficient (damper's size) are placed at the storey with the maximum interstorey velocity until the velocity at that storey no longer occurs at the storey. Then dampers are assigned where the interstorey velocity becomes the largest. The procedure is repeated until the necessary total damping coefficient required is added to the structure. Even though, optimal design procedures are efficient for obtaining the number and distribution of FVDs (Huang 2018) with respect another such as stochastic procedures (e.g., Lavan and Levy 2006, 2009), they are neither simple nor practical (Lopez-Garcia and Soong 2002).

Due to the above the second approach arises, the simplified procedures in which modal spectral analysis can be used. The problem of this analysis is that, in general, the damping matrix of structures with passive energy dissipation devices is non-proportional leading to non-classical eigenvalues and vectors inconsistent with modal spectral analysis where classical damping model is employed. To overcome this limitation, in Constantinou and Symans (1992) proposed an approximate simplified procedure, in which the behaviour of the FVDs is considered as supplemental modal viscous damping. Also, the approximate amount of damping associated with the FVDs is easily determined in terms of the dynamic properties of the structure such as the mode shapes and periods.

From the original propose Constantinou and Symans (1992) several procedures have been developed where the main objective is not to determine the amount of damping necessary to comply with the PO, but to determine the placement defining the vertical arrangement and size of the devices in proportion to the distribution of a considered Engineering Demand Parameter (EDP). Pekcan *et al.* (1999) propose a distribution of the damping coefficient in proportion to the storey shear force of a structure. Along the same lines, Hwang *et al.* (2013) propose a damper distribution based on storey shear strain energy. In Landi *et al.* (2015) the influence of some EDPs (e.g., interstorey drifts, storey stiffness, among others) in the design of FVDs is evaluated, concluding that the structural response does not present significant variations between the parameters considered.

Usually, the design approach towards structures with FVDs is to concentrate all energy dissipation in the devices and restrict the behaviour of structural elements to the elastic range. However, in cases where a damping amount is required such that it is not practical or reasonable i.e., with values greater than 15% (Hwang and Huang 2003) it is necessary to accept damage to the structure. Based on this idea, in Liang *et al.* (2012) presents a direct displacement-based procedure (e.g., Priestley *et al*. 2007), in which a structure equipped with FVDs can exhibit inelastic behaviour. The supplementary damping provided by the devices can be obtained with the procedure proposed Constantinou and Symans (1992) or Ramirez *et al.* (2000), which considers that the behaviour of the FVDs can be approximated by supplementary proportional damping with the objective to apply modal spectral analysis. However, this assumption is not always the most appropriate, since significant errors can occur in the determination of structural performance (Veletsos and Ventura 1986).

Due to the above, this paper presents a displacement-based seismic design procedure based on the formulation of Ayala *et al.* (2012), for new structures and/or for retrofit of existing structures with FVDs considering explicitly inelastic behaviour. This procedure can be applied in structures with irregularities in plan and/or elevation where these passive dissipation devices are required as a retrofit measure. The goal of the procedure is that the designed structure complies with the interstorey drift threshold associated with the considered ultimate limit state (ULS).

The supplemental damping required by the structure under design conditions considering a predetermined damage distribution e.g., strong column-weak beam, is estimated by means of a simplified criterion based on the characterization of the viscous damping matrix as the sum of a proportional damping matrix and a complementary non-proportional damping matrix. As the design application is focused on interstorey drift control, the vertical arrangement of the devices is defined in proportion to the modal interstorey drift i.e., the storey damping coefficients are estimated using the relative weights of the modal interstorey drifts along the height of the structure.

To illustrate the application of this procedure and evaluate the performance of structures designed with the procedure proposed, five regular plane frames: 8, 12, 17, 20, 25-storey, and an 8-storey building are designed. The procedure proposed can be used design spectrums as indicated in the building codes; however, with the objective to validate this procedure, the seismic demand used for the design of the frames was spectrum corresponding to a particular record. The seismic demands used for design and validation were the records obtained at the SCT site during the 1985 Michoacan earthquake, and that of the 2017 Morelos - Puebla earthquake obtained at the Culhuacan site, both stations located on soft soil sites. The validity of the procedure proposed is assessed by comparing the target interstorey drifts, base shears, and corresponding design damage distribution with those obtained via non-linear dynamic step-by-step analysis of the structures designed. Finally, some conclusions about the design procedure, and the results obtained are presented stressing the most relevant advantages of the displacement-based design of structures with FVDs and damage control.

2. Fundamentals of the procedure proposed

2.1 Reference SDOF system

The procedure proposed is based on the assumption that the performance of a non-linear multi-degree of freedom (MDOF) structure with or without FVDs may be approximated via a reference bilinear single-degree of freedom (SDOF) system with supplemental viscous damping, normally associated with the fundamental mode of the structure in both its elastic and inelastic stages of behaviour (Ayala *et al.* 2012, Lopez *et al.* 2015). Because of the above, the main tool to characterize the behaviour of a

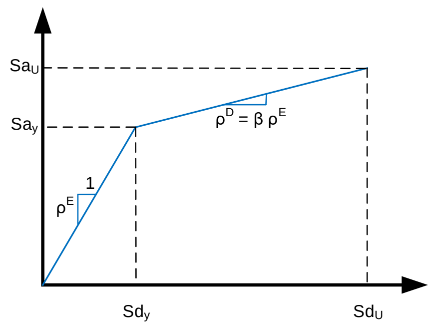


Fig. 1 Behaviour curve of the reference SDOF system for performance objective

MDOF structure is the plot of spectral displacement (Sd), vs. spectral pseudo-acceleration (Sa), i.e., the so-called behaviour curve of the reference SDOF oscillator Lopez *et al.* (2015). The design approach in the procedure proposed is the definition of a design behaviour curve that provides the stiffness, strength required by the structure, along with a particular viscous damping ratio to satisfy a given PO. For the case of a PO comprised of an ULS a design bilinear behaviour curve is built (Fig. 1). The characteristic points that define this curve are: origin $(0,0)$, yield (Sd_y, Sa_y), and ultimate (Sd_u, Sa_u) Lopez *et al.* (2015).

The first branch of this curve defines the elastic stiffness of the structure. Its slope (ρ^E), is limited in such a way that the interstorey drift threshold for the PO is not exceeded for the corresponding demand level. The slope of the second branch (ρ^D), represents the stiffness of a design damage state corresponding to the ULS, obtained using an accepted damage distribution under these design conditions e.g., using the strong column-weak beam principle. The slope of this inelastic branch, is described in terms of the post-yield to elastic stiffness ratio (α). The yield and ultimate characteristic points are defined so that the interstorey drift of the PO considered is not exceeded for the corresponding demand level. With the objective to satisfy the target PO of the structure with FVDs, the design spectrum is modified to consider the added damping due at the effect of these energy dissipation devices.

2.2 Simplified models

The design procedure proposed uses two simplified linear models to characterize the behaviour of the structure in both the elastic stage and the inelastic stage (Ayala *et al.* 2012). The simplified elastic model is defined by the elastic properties of the structural elements, such as the nominal moments of inertia of the sections for the steel elements, or the cracked inertias for the concrete elements. The simplified model that represents the non-linear behaviour herein referred as damaged model, which is a replica of the elastic model, in which by means of the strong column-weak beam principle plastic hinges are introduced at the ends of the elements that are more assumed to experiment damage in accordance with to the design performance level considered. The dynamic properties are obtained from modal spectral analysis of both models and the design demands are obtained from the superposition of the results of modal spectral analysis of both models, as shall be described in full detail in Section 3.

2.3 Definition of the seismic design demands

Since the procedure proposed uses an inelastic SDOF system as a reference to the behaviour of an inelastic MDOF system, the seismic demands used during the design are obtained from inelastic design spectra, of the constant ductility type, as in current building codes. However, in the procedure proposed the ductility (μ), and the post-yield to elastic stiffness ratio (α), of the design spectrum to be used in a particular design application should be that corresponding to the required ductility demand, as shall be explained in detail in the following section. Furthermore, the design modal damping ratio is derived from the added FVDs.

2.4 Added damping consideration

As mentioned above, most seismic analysis procedures for structures with FVDs consider the effect of these devices in the response of the structure by assuming a proportional damping matrix, approximation of the actual non-proportional damping matrix. This assumption, however, is not strictly valid, as the response of the structure may show significant errors when compared against that obtained using a dynamic step-by-step analysis of the structure with non-proportional damping matrix report in Veletsos and Ventura (1986). To minimize the errors involved in the procedure proposed by Constantinou and Symans (1992) this paper proposes the following:

The equation that describes the dynamic equilibrium of a MDOF structure can be written as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{\iota\}\ddot{u}_g(t) \tag{1}$$

where

$[M]$	=	mass matrix
$[C]$	=	damping matrix
$[K]$	=	stiffness matrix
$\{\ddot{u}\}, \{\dot{u}\}$ y $\{u\}$	=	acceleration, velocity, and displacement vectors
$\{\iota\}$	=	influence vector
$\ddot{u}_g$	=	ground acceleration

For a structure equipped with FVDs, the damping matrix can be represented as:

$$[C] = [C_0] + [C_d] \tag{2}$$

where

$[C_0]$	=	inherent damping matrix (assumed as proportional)
$[C_d]$	=	damping matrix associated with the FVDs (non-proportional)

Since the damping matrix associated with the FVDs is non-proportional, in this work it is characterized as the sum of a proportional damping matrix $([C_{dp}])$, and a complementary, which is representative of non-proportional damping matrix $([C_{dnp}])$:

$$[C_d] = [C_{dp}] + [C_{dnp}] \tag{3}$$

The first order approximation of the proportional matrix also referred as Rayleigh damping matrix (mass-proportional damping and stiffness-proportional damping) may be written as

$$[C_{dp}] = a_{0d}[M] + a_{1d}[K] \tag{4}$$

where

$a_{0d} + a_{1d} =$ constants with units s^{-1} and s

To uncouple the equilibrium equations given by Eq. (1) the following change of coordinates is required $\{u\} = [\Phi]\{x\}, \{\dot{u}\} = [\Phi]\{\dot{x}\}\ and\ \{\ddot{u}(t)\} = [\Phi]\{\ddot{x}\}$, to give

$$[M][\Phi]\{\ddot{x}\} + [C_0 + C_{dp} + C_{dnp}][\Phi]\{\dot{x}\} + [K][\Phi]\{x\} = -[M]\{\iota\}\ddot{u}_g(t) \tag{5}$$

Where

$[\Phi]$ = modal matrix

Pre-multiplying each term in Eq. (5) by $[\Phi]^T$ gives:

$$[\Phi]^T[M][\Phi]\{\ddot{x}\} + [\Phi]^T[C_0 + C_{dp} + C_{dnp}][\Phi]\{\dot{x}\} + [\Phi]^T\ [K][\Phi]\{x\} = -[\Phi]^T[M]\{\iota\}\ddot{u}_g(t) \tag{6}$$

or

$$[M^*]\{\ddot{x}\} + [C_0{}^*]\{\dot{x}\} + [C_{dp}{}^*]\{\dot{x}\} + [C_{dnp}{}^*]\{\dot{x}\} + [K^*]\{x\} = -[\Phi]^T[M]\{\iota\}\ddot{u}_g(t) \tag{7}$$

Where $[M^*] = [\Phi]^T[M][\Phi], [C_0{}^*] = [\Phi]^T[C_0][\Phi]$, $[C_{dp}{}^*] = [\Phi]^T[C_{dp}][\Phi]$, and $[K^*] = [\Phi]^T[K][\Phi]$ are diagonal matrices, but the term $[C_0{}^*] = [\Phi]^T[C_{dnp}][\Phi]$ is not a diagonal matrix, which invalidates the classic modal analysis. However, with the purpose of applying this analysis in the same way as stipulated in the building codes, and to approximate the damping provided by the FVDs, this term is assumed as diagonal.

Dividing Eq. (7) by $[M^*]$, and assuming that the addition of FVDs in the structure does not significantly modify its modal characteristics, a set of uncoupled dynamic equilibrium equations expressed in terms of modal coordinates $x_i(t)$, is obtained:

$$\ddot{x}_i(t) + 2(\xi_{0i} + \xi_{dpi} + \xi_{dnpi})\omega_i\dot{x}_i(t) + \omega_i^2 x_i(t) = -PF\ddot{u}_g(t) \tag{8}$$

where

ξ_{0i} = inherent damping ratio of the structure which is often assumed to be 5% in most buildings codes

ξ_{dpi} = proportional viscous damping ratio corresponding to the added devices for mode i

ξ_{dnpi} = damping ratio of mode i corresponding to the complementary damping matrix

ω_i = frequency for mode i

The ξ_{dnpi} can write a

$$\xi_{dnpi} = \frac{\{\phi\}_i^T[C_{dnp}]\{\phi\}_i}{2\omega_i} \tag{9}$$

Therefore, the effective damping ratio of the structure corresponding with the mode i is obtained by the Eq. (10)

$$\xi_{effi} = \xi_{0i} + \xi_{dpi} + \xi_{dnpi} \tag{10}$$

The proportional viscous damping ratio corresponding to the added devices (ξ_{dpi}) can be calculated using the energy based approximation proposed by Constantinou and Symans (1992), considering the damaged model, Eq. (11)

$$\xi_{dpi} = \frac{T_i \sum_{j=1}^{nd} C_j f_j^2\ (\phi_{rj,i}^D)^2}{4\ \pi \sum_{q=1}^{n} m_q (\phi_{q,i}^D)^2} \tag{11}$$

where

T_i = period of the vibration mode i

m_q = mass of the storey q

f_j = amplification factor relating to the geometrical arrangement of the damper j e.g., $cos(\theta)$ for diagonal-brace (Hwang 2013)

$\phi_{rj,}^D$ = difference between the modal ordinates connected by the damper j of the vibration mode i corresponding to the damaged model

$\phi_{q,i}^D$ = horizontal modal displacements of the storey q of the mode i corresponding to the damaged model

nd = number of devices

n = number of storeys

C_j = damping coefficient of the device j

2.5 Higher modes contribution

The most commonly accepted procedure to consider the contribution of higher modes to the seismic performance of structures equipped with FVDs is that proposed by Ramirez *et al.* (2000). This procedure involves a modal combination of two components; the first corresponding to the performance of the structure with an effective damping ratio associated to the fundamental mode, which is defined by Eq. (10) with $i = 1$ i.e., ξ_{eff1}. The second component is the performance of the structure with damping ratio (ξ_R) associated to the so-called residual mode (ϕ_R), this mode approximately considers the contribution of the higher modes and is defined by Eqs. (12) - (13):

$$\{\phi_R\} = \frac{1}{FR_R}\{\iota\} - \frac{FR_1}{FR_R}\{\phi_1\} \tag{12}$$

with

$$FR_R = 1 - FR_1 \tag{13}$$

Where

$\{\phi_R\}$ = residual mode vector

FR_R = participation factor corresponding to the residual mode

The corresponding damping ratio ξ_R is defined as the sum of the inherent damping ratio of the structure corresponding to residual mode (ξ_{0R}), and the damping ratio provided by the devices which associated to the residual mode (ξ_{dR}), Eq. (14)

$$\xi_R = \xi_{0R} + \xi_{dR} \tag{14}$$

To apply ξ_R in the procedure proposed, ξ_{dR} is divided in two parts the first is associated to the proportional damping (ξ_{dpR}), and the second associated to the complementary damping (ξ_{dnpR}) provided by the FVDs, Eq. (15)

$$\xi_R = \xi_{0R} + \xi_{dpR} + \xi_{dnpR} \tag{15}$$

According to Ramirez *et al.* (2000) the effective period associated to the residual mode (T_R) may be defined through the period of the second mode (T_2); or by a percentage of the period of the fundamental mode (T_1); as the 40% recommended in ASCE 7-10 (2010), Eq. (16)

$$T_R = 0.40\ T_1 \tag{16}$$

With the components of the performance associated to the fundamental and the residual mode, the modal combination is carried out e.g., using the square root of the sum of the squares (SRSS).

2.6 Arrangement of the FVDs

The distribution of the devices significantly influences its effectiveness on the control of structural performance. For this reason, several investigations have focused on developing procedures to distribute "strategically" the devices, usually based on an EDP. The approach employed in most of these procedures is to estimate the required storey damping, thus, defining the damping coefficient for each level. These coefficients are estimated using the formulation proposed by Constantinou and Symans (1992), Eq. (11), which is modified according to the considered EDP e.g., shear strain energy Hwang *et al.* (2013), shear force Pekcan *et al.* (1999), interstorey drifts, stiffness Landi *et al.* (2015), among others.

Since the procedure presented in this paper uses the interstorey drift as the EDP, the vertical distribution of the FVDs is defined for such parameter. Accordingly, the storey damping coefficients is considered proportional to modal interstorey drift of the damaged model (section 2.2) hence, in a similar manner as carried out by Hwang *et al.* (2013) and Landi *et al.* (2015). The damping coefficient of storey j necessary to satisfy the PO can be defined as

$$C_j = p\ \lambda_j \tag{17}$$

where

p = proportionality constant

λ_j = modal drift of the storey j corresponding to the damaged model

The total damping coefficient of the structure is equal to the sum of the damping coefficient of each storey:

$$\sum_{j=1}^{nd} C_j = p \sum_{j=1}^{nd} \lambda_j \tag{18}$$

Substituting Eq. (18) into Eq. (17), one can obtain the relationship between the damping coefficient at each storey and the total damping coefficient of the structure.

$$C_j = \frac{\lambda_j}{\sum_{j=1}^{nd} \lambda_j} \sum_{j=1}^{nd} C_j \tag{19}$$

Substituting Eq. (19) into Eq. (11), and considering a damage structure, we obtain:

$$\xi_{dpi} = \frac{T_i \sum_{j=1}^{nd} \lambda_j f_j^2 \,(\phi_{rj,i}^D)^2 \sum_{j=1}^{nd} C_j}{4\pi \sum_{q=1}^{n} m_q (\phi_{q,i}^D)^2 \sum_{j=1}^{n} \lambda_j} \tag{20}$$

Solving the total amount of the damping coefficients of Eq. (20), and substituting in Eq. (19) the damping coefficient to each storey j:

$$C_k = \frac{4\pi\ \lambda_j\ \xi_{dpi} \sum_{q=1}^{n} m_q (\phi_{q,i}^D)^2}{T_i \sum_{j=1}^{nd} \lambda_j f_j^2\,(\phi_{rj,i}^D)^2} \tag{21}$$

Eq. (21) provides a reasonable estimate of storey damping for the displacement control of structures under seismic loading. However, to assure that this approach leads to a realistic estimation of storey damping forces, the following criteria regarding the physical location of the FVDs in the structure is recommended:

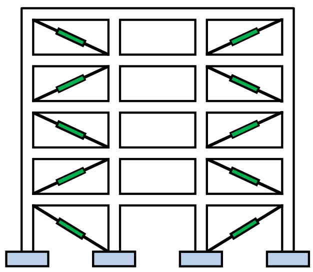
• Place symmetrically with respect to the geometry of the structure.

• Place minimum two devices for each level of the structure.

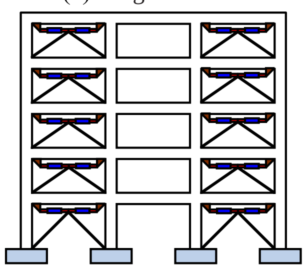
Place the damper between each level the according with the existent configurations e.g., diagonal-brace, k-brace, among others (see Fig. 2).

3. Procedure proposed

In accordance with the aforementioned concepts, the application of the proposed displacement-based design procedure for structures with FVDs intended to satisfy a given PO, can be summarized in the following steps:



(a) Diagonal-brace



(b) K-brace

Fig. 2 Arrangement of the FVDs

1) Structural pre-dimensioning according to engineering judgement or designer experience. The purpose of this step is to define a realistic stiffness distribution of the structural elements throughout the height of the structures.
2) Modal analysis of the elastic structural model defined in the previous step. From this analysis the modal participation factor (PF^E), the fundamental period of the structure (T_E), and the fundamental modal shape (ϕ_1^E), are obtained. The spectral yield displacement of the reference SDOF (Sd_y), is calculated using the Eq. (22)

$$Sd_y = \frac{\Delta_y \ H_z}{PF_1^E\left(\phi_{z,1}^E - \phi_{z-1,1}^E\right)} \tag{22}$$

Where
Δ_y = yield interstorey drift
H_z = height of the critical storey z (where maximum drift occurs)
PF_1^E = modal participation factor of the fundamental mode (elastic structure)
$\phi_{z,1}^E$ = modal shape ordinate of the critical interstorey z (elastic structure)

The yield interstorey drift for a reinforced concrete structure may be calculated using the Eq. (23) Priestley *et al.* (2007)

$$\Delta_y = \frac{0.5 \ \varepsilon_y \ L_1}{h_{v1}} \tag{23}$$

where
ε_y = yield strain of the reinforcing steel
L_1 = beam length
h_{v1} = beam depth
3) Definition of a design damage distribution e.g., strong-column weak-beam criterion for the PO in accordance with the characteristics of the structure, the design demands using a strategy and considering the contribution of the FVDs added to the structure. From such distribution the damage model is defined; in which plastic hinges expected to develop are characterized by hinges or reduced rotational springs whose stiffness is equal to realistic values of post-yield stiffness of structural elements.
4) Modal analysis of the damaged model from which the fundamental modal shape (ϕ_1^D), participation factor (PF^D), and period, (T_D), are obtained. Subsequently the target spectral displacement of the reference SDOF (Sd_U), is obtained using the Eq. (24)

$$Sd_U = \frac{\Delta_U \ H_z}{PF_1^D\left(\phi_{z,1}^D - \phi_{z-1,1}^D\right)} \tag{24}$$

where
Δ_U = ultimate interstorey drift
$\phi_{z,1}^D$ = modal shape coordinate of the critical storey z corresponding to the damaged model
5) Calculation of the target yield and ultimate spectral displacements of the reference SDOF system Sd_U, and Sd_y, respectively corresponding to the fundamental mode using the results of modal analysis. The design ductility (μ), and post-yielding to initial stiffness ratio (α), is obtained using Eqs. (25) - (26):

$$\mu = \frac{Sd_U}{Sd_y} \tag{25}$$

$$\alpha = \left(\frac{T_E}{T_D}\right)^2 \tag{26}$$

6) Modification of the effective viscous damping ratio from the inelastic displacement spectrum for the given μ, and α, until the spectral displacement is equal to the target spectral displacement (Sd_U), of the structure (ξ_{Design}), see Fig. 3(a). At this step, existing structures can be retrofitted adding the necessary damping in order to satisfy the PO for a determined state of damage.
7) Determination of the yield strength (Sa_y), in the inelastic strength spectrum, corresponding to the values of μ, and α, previously calculated, see Fig. 3(b).
8) Calculation of the ultimate strength (Sa_U), of the reference system using the Eq. (27)

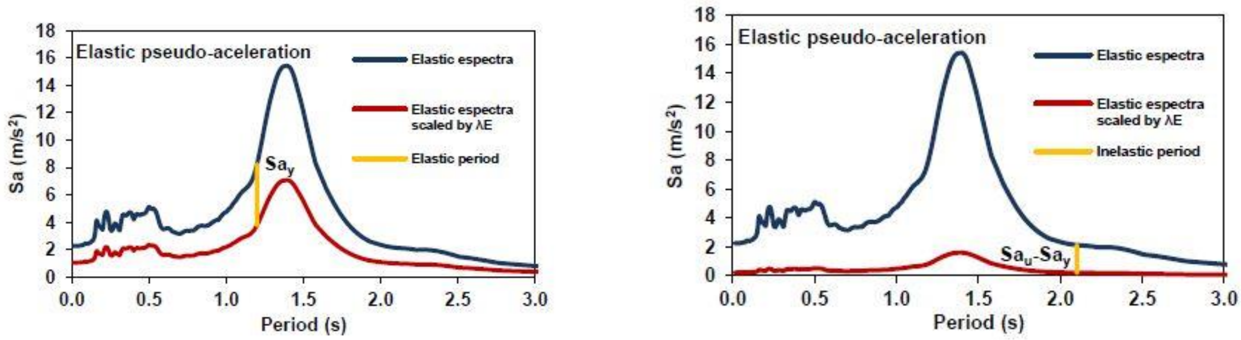
$$Sa_U = Sa_y[1 + (\mu - 1)] \tag{27}$$

9) Definition of the behaviour curve of the reference SDOF system with the characteristic points (Sd_y, Sa_y), and (Sd_U, Sa_U), see Fig. 1.
10) Calculation of the damping coefficients of the FVDs, using Eq. (21) for the fundamental mode i.e., $\xi_{eff1} = \xi_{design}$, where the proportional part (ξ_{dp1}), may be defined using the Eq. (20), and the complementary part (ξ_{dnp1}), as defined by Eq. (9) with $i = 1$. The damping coefficient for each storey is calculated in proportion to the relative modal displacement normalized with the storey height of the building corresponding to the damaged model.
11) Calculation of the damping corresponding to the residual mode (ξ_R), using the Eq. (15) with the damping coefficients of the devices determined in the previous step, according to the section 2.5.
12) Determination of the design forces of the elements using the results of three different analyses

- Gravity load analysis of the undamaged structure considering the dead and live loads according to the use of the structure as well as the building code considered.
- Modal spectral analysis of the undamaged structure, using the elastic design spectrum scaled by the ratio of the strength per unit mass at the yield point of the behaviour curve, and the elastic pseudo-acceleration for the initial period (λ_E), see Fig. 4(a).
- Modal spectral analysis of the damaged structure, using the elastic spectrum scaled by the ratio of the difference of the ultimate and yield strengths per unit mass as well as the pseudo-acceleration for the period of the damaged structure (λ_D), see Fig. 4(b).

Each modal spectral analysis considers the total effective damping ratio for the fundamental mode (ξ_{eff1}), as well as the damping corresponding to the residual mode (ξ_R). The design forces are obtained by adding the forces due to gravity loads, and the forces of the modal spectral analyses for each damping ratio of the undamaged and damaged structure.

13) Modal combination of the response obtained from the modal spectral analyses carried out in the previous step of the structure with ξ_{eff1} and ξ_R, according to the sections



(a) Undamaged model (b) Damaged model

Fig. 4 Strength spectra used for the modal spectral analyses

Table 1 Geometry of the element sections of the example models

Description	Levels	Beams(m)	Levels	Columns(m)
8-storey	1 – 4	0.60 *x* 0.30	1 – 4	0.60 *x* 0.60
	5 – 8	0.50 *x* 0.25	5 – 8	0.50 *x* 0.50
3D 8-storey	1 – 8	0.60 *x* 0.30	1 – 8	0.70 *x* 0.70
12-storey	1 – 4	0.55 *x* 0.30	1 – 4	0.70 *x* 0.70
	5 – 8	0.50 *x* 0.25	5 – 8	0.60 *x* 0.60
	9 – 12	0.45 *x* 0.25	9 – 12	0.50 *x* 0.50
17-storey	1 – 5	0.65 *x* 0.65	1 – 5	1.20 *x* 1.20
	6 – 9	0.50 *x* 0.50	6 – 9	1.00 *x* 1.00
	10 – 13	0.45 *x* 0.45	10 – 13	0.80 *x* 0.80
	14 – 17	0.40 *x* 0.40	14 – 17	0.70 *x* 0.70
20-storey	1 – 4	0.50 *x* 0.50	1 – 5	1.20 *x* 1.20
	5 – 8	0.45 *x* 0.45	6 – 8	1.00 *x* 1.00
	9 – 12	0.40 *x* 0.40	9 – 12	0.80 *x* 0.80
	13 – 16	0.35 *x* 0.35	13 – 20	0.70 *x* 0.70
	17 – 20	0.30 *x* 0.30		
25-storey	1 – 5	0.50 *x* 0.50	1 – 5	1.20 *x* 1.20
	6 – 10	0.45 *x* 0.45	6 – 10	1.10 *x* 1.10
	11 – 15	0.40 *x* 0.40	11 – 15	1.00 *x* 1.00
	16 – 20	0.35 *x* 0.35	16 – 20	0.90 *x* 0.90
	21 – 25	0.30 *x* 0.30	21 – 25	0.80 *x* 0.80

2.4 - 2.5.

14) Design of structural elements with the forces obtained from the sum of the three analyses of the simplified models using the applicable design rules. The design process must be carried out in such a way that the design criteria of the code do not alter significantly the expected performance.

4. Application examples

4.1 Characteristics of frame examples

Fig. 5 show the frames considered in this study, their roof mass was 98 kN·s^2/m and 108 kN s^2/m for the other storeys. For the building depicted in Fig. 6 the roof translational mass considered was 611 kN·s^2·m, and 771 kN·s^2·m in the others levels; 83529 kN·m·s^2 and 105323 kN·m·s^2 as rotational mass respectively, and the mass eccentricity is 10% for all levels. The nominal properties of the materials used in the design were concrete compressive strength f'c=2.45x10^4 kN·m^2, concrete modulus of elasticity E_c=21.70x10^6 kN·m^2, reinforced concrete weight density γ=23.53 kN/m^3, steel reinforcement yield stress f_y=4.50x10^5 kN/m^2, and steel modulus of elasticity E_s=2.00x10^8 kN/m^2. Based on the results of the preliminary design of the frames, the sections of the structural elements for the frames were defined in the Table 1; gravitational loads were assigned according to what is recommended in NTCS-17.

4.2 Arrangement of the FVDs and damage states

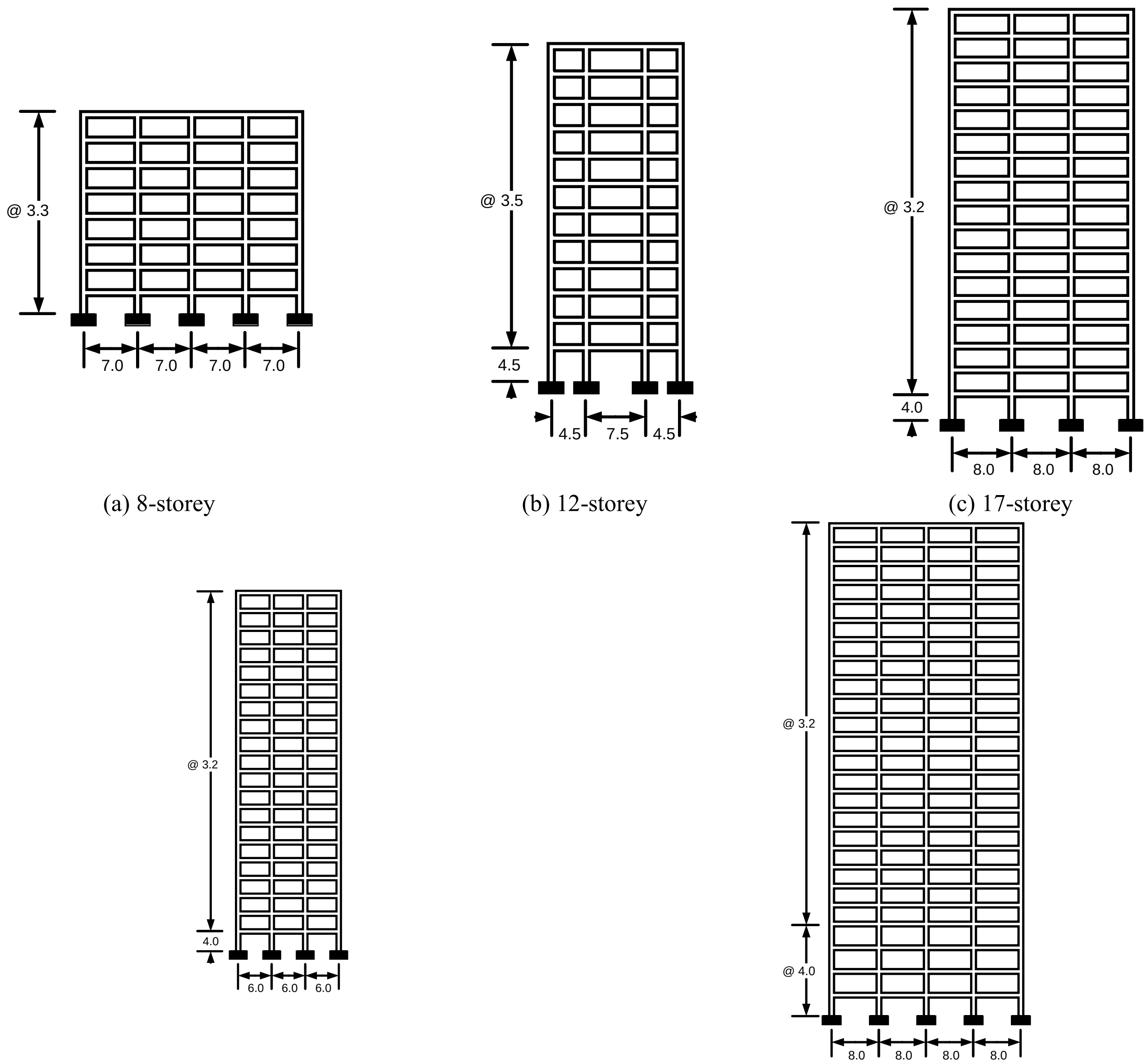
Diagonal FVDs were placed in all storeys in one or several spans in a symmetric arrangement as recommended in section 2.6 as shown in Fig. 10. The size of the FVDs i.e., the damping coefficients were calculated according to step 11 of procedure proposed. For each model, a damage distribution consistent with the seismic design philosophy of strong column-weak beam is proposed (Fig. 10).

4.3 Design seismic demands

For the purpose of validating the effectiveness of the

Table 2 Properties of the behaviour curve for application examples (units: m, s)

Model		T_E	T_D	α	μ	Sd_u	Sa_y	Sa_U	λ_E	λ_D	ξ_{Design}
8-storey		1.3	4.7	0.08	2	0.20	3.29	3.54	0.24	0.75	0.13
3D 8-storey	direction X	1.5	2.9	0.26	2	0.22	3.40	3.5	0.28	0.10	0.17
	direction Y	1.2	2.1	0.35	2	0.26	6.50	6.7	0.66	0.11	0.10
12-storey		2.0	8.1	0.06	2	0.31	1.49	1.59	0.15	0.58	0.13
17-storey		2.1	6.1	0.12	2	0.37	1.67	1.86	0.19	0.59	0.13
20-storey		2.3	6.1	0.14	2	0.41	1.54	1.78	0.08	0.77	0.11
25-storey		2.6	7.4	0.12	2	0.59	2.47	2.77	0.35	1.21	0.10



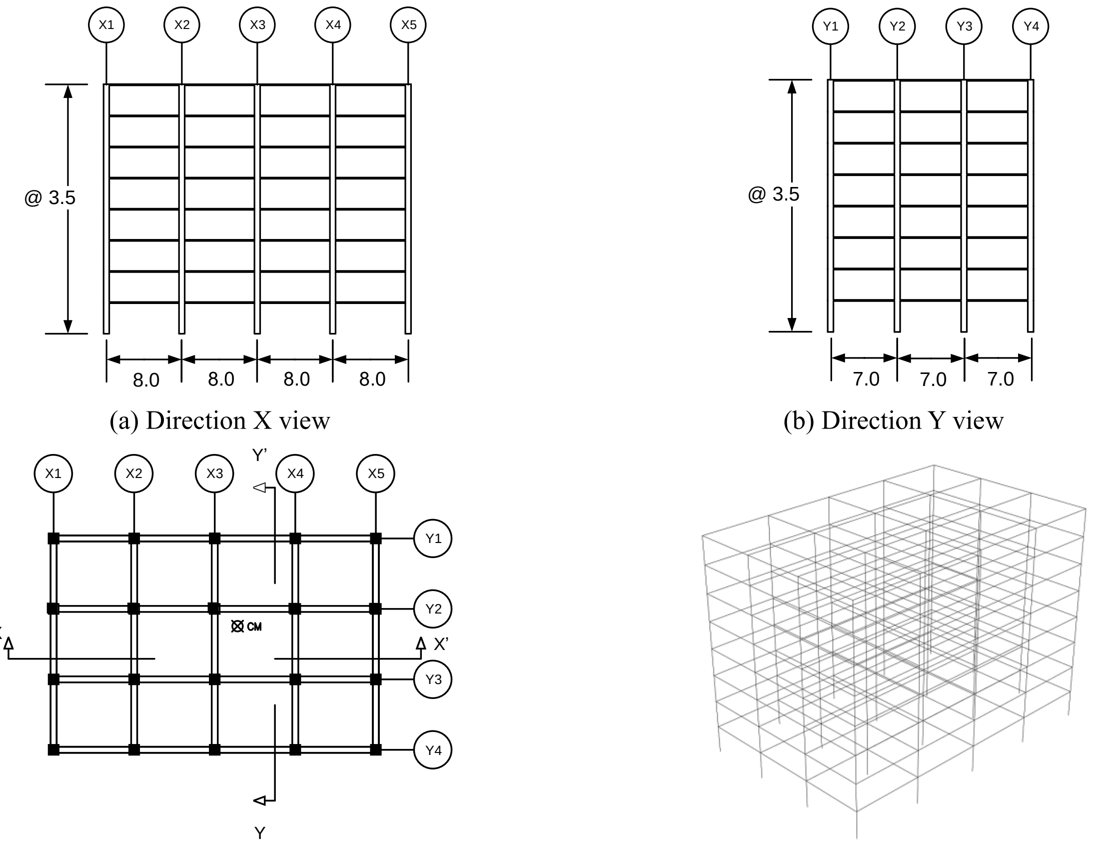
(a) 8-storey (b) 12-storey (c) 17-storey

(d) 20-storey (e) 25-storey

Dimensions in m

Fig. 5 Geometry of the example frames

procedure, the response spectra corresponding to the two records considered were used as seismic demands, which were obtained from the Valley of Mexico. For the frames the accelerogram of the EW component of the September 19, 1985 Michoacan Earthquake recorded at the SCT station was considered as the seismic demand. For the 8-storey and building, the 2017 Morelos - Puebla earthquake in Mexico recorded at the Culhuacan site was used.



(a) Direction X view (b) Direction Y view

(c) Plan view (d) 3D view

Fig. 6 Geometry of the 8-storey building

4.4 Design of the case studies using the procedure proposed

For each frame the elastic model of the bare structures was built considering the preliminary structural elements and the modal analysis was carried out. From these results, the elastic fundamental periods, and modal shapes were attained subsequently, the damage distribution was proposed. The dynamic properties in the inelastic stage, fundamental period, and shapes were obtained from modal analyses of the damaged model. With the objective to consider the contribution of the higher modes in the application examples, the procedure of Ramirez *et al.* (2000) was applied; the results obtained showed that the structural response does not present significant variations when only the participation of the fundamental mode is considered. Thus the contribution of the higher modes of the structure to its total response may be ignored. The properties of the behaviour curve for the structures complying with the PO of the collapse prevention limit state (CPLS) are shown in the Table 2

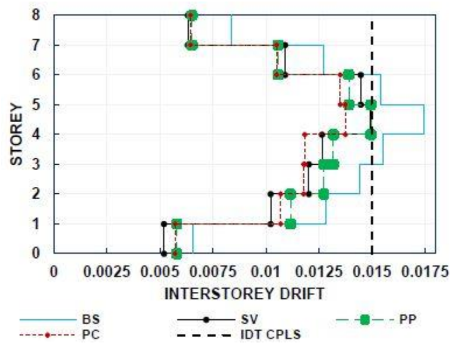
4.5 Validation of the procedure proposed

In order to validate the results obtained from the displacement-design procedure proposed in this paper 8, 12, 17, 20, and 25-storey frames; and 8-storey framed building all with FVDs were designed. For the purpose of assessing the seismic performance of the designed structure, the maximum drifts obtained from the non-linear step-by-step analysis of the structure were compared with the maximum drift considered as design target. The non-linear step-by-step analysis was carried out with the following considerations:

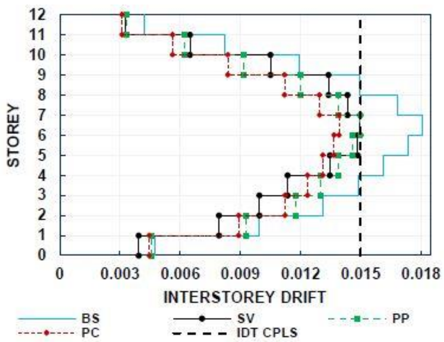
• Elasto-plastic bilinear stable hysteretic behaviour for all beams and columns.
• Non-proportional damping matrix due to the incorporation of the FVDs.
• P -Δ effects no considered.
• Yield moments for beam and columns are those obtained from the design method without any standardization.
• The interstorey drift considered as design target (Δ_U) was 0.015, as specified by NTCS-17 to satisfy the CPLS.

In order to validate the procedure proposed, the interstorey drift obtained from the dynamic step-by-step analysis was compared for the followings cases:

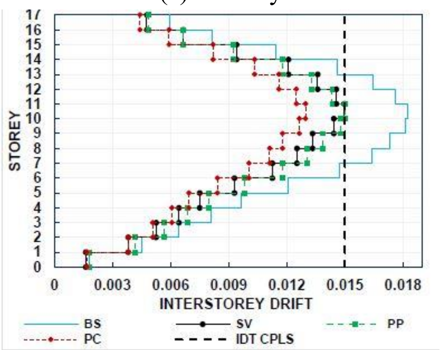
• Bare structure, (BS).
• Structure with supplemental viscous damping (damping that "approximates" the behaviour of the devices, (SV).
• Structure with FVDs designed as a function of the interstorey drift using the procedure proposed, (PP).
• Structure with FVDs designed as a function of the interstorey drift using the procedure proposed by Constantinou and Symans (1992), which is the procedure indicated technical documents such as ASCE 7-10 (2010), (PC).



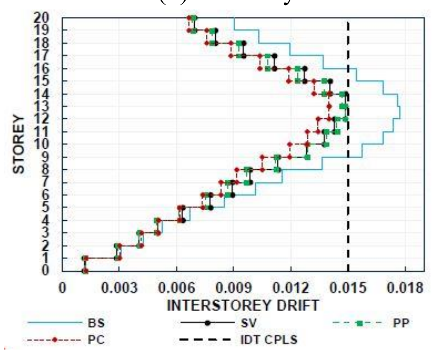
(a) 8-storey



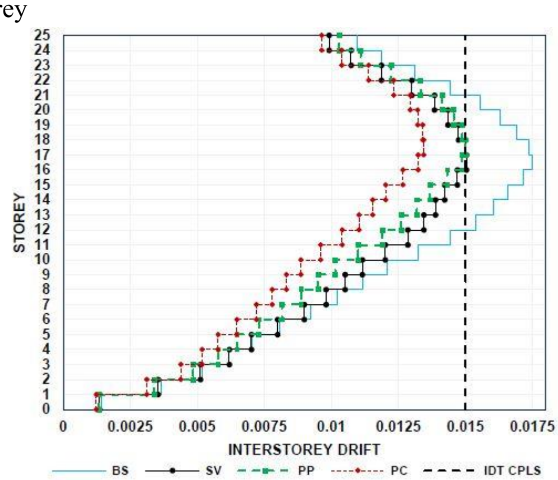
(b) 12-storey



(c) 17-storey

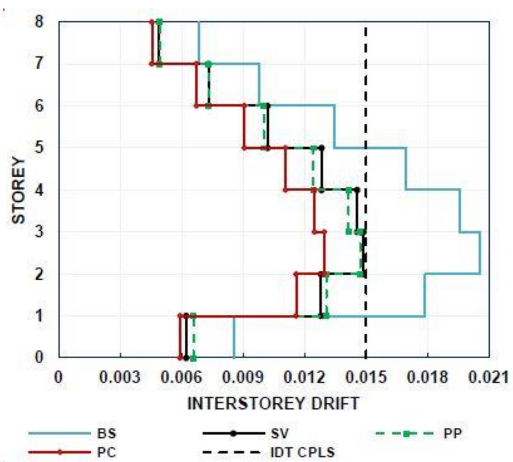


(d) 20-storey

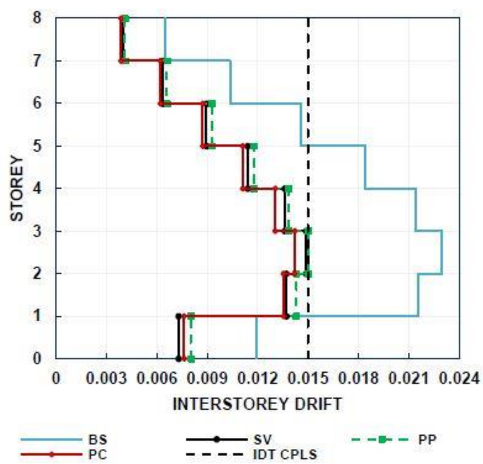


(e) 25-storey

Fig. 7 Interstorey drifts of the example frames

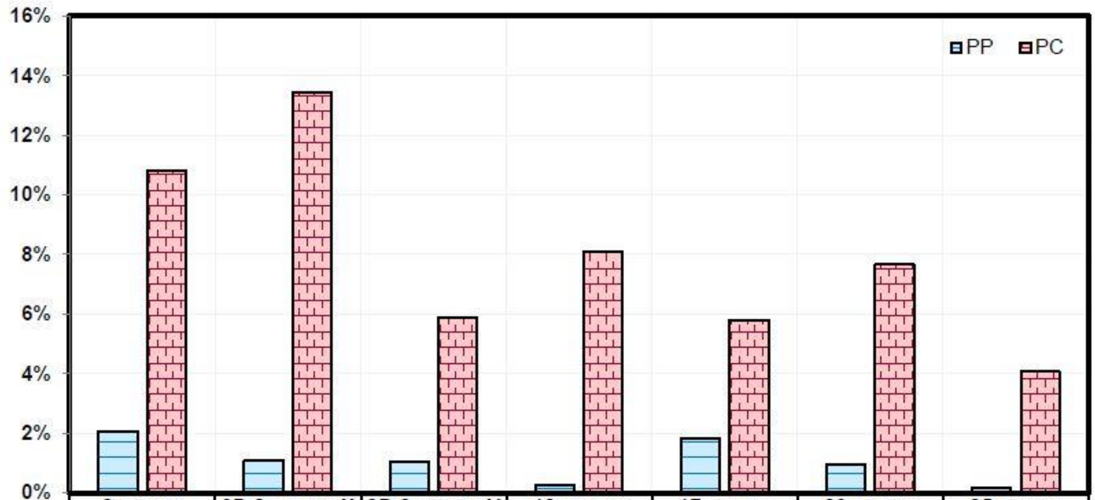


(a) Direction X



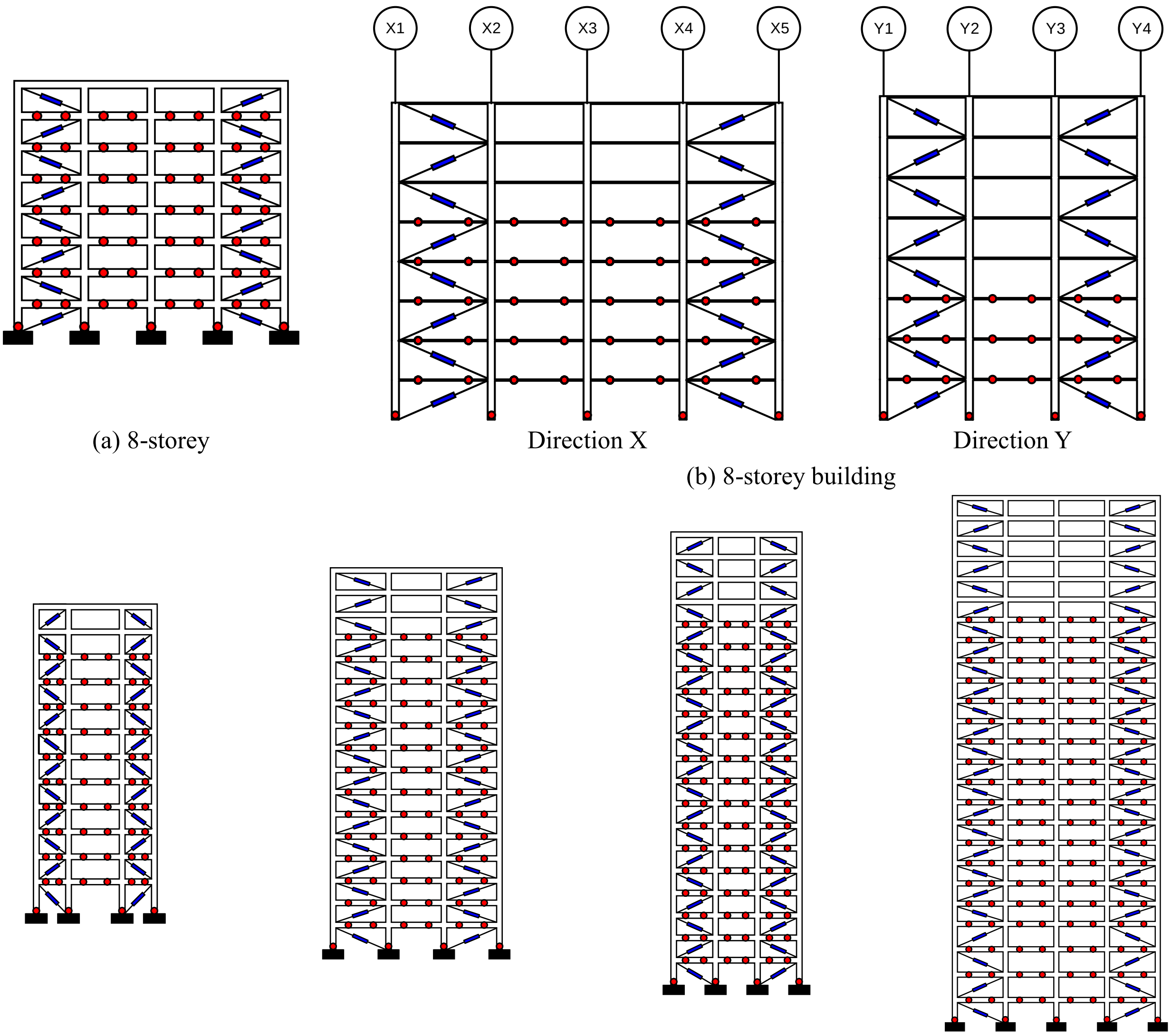
(b) Direction Y

Fig. 8 Interstorey drifts of the 8-storey building



	8-storey	3D 8-storey X	3D 8-storey Y	12-storey	17-storey	20-storey	25-storey
PP	2.0%	1.1%	1.0%	0.3%	1.8%	0.9%	0.2%
PC	10.8%	13.4%	5.9%	8.1%	5.8%	7.7%	4.1%

Fig. 9 Relative errors of interstorey drift



(a) 8-storey

Direction X

Direction Y

(b) 8-storey building

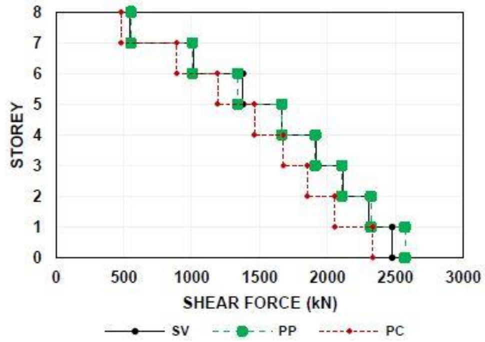
(c) 12-storey

(d) 17-storey

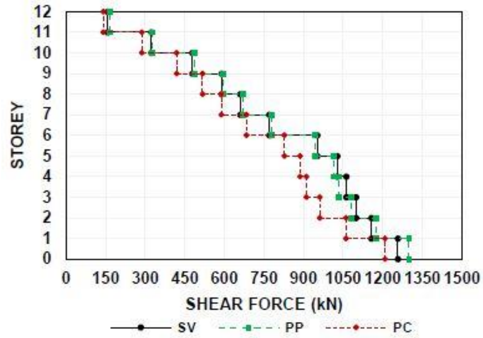
(e) 20-storey

(f) 25-storey

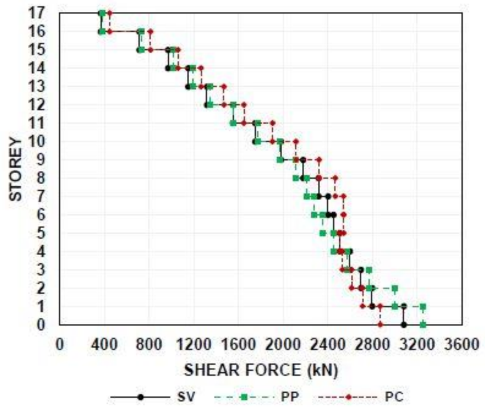
Fig. 10 Damage distribution and arrangement of FVDs of the application examples



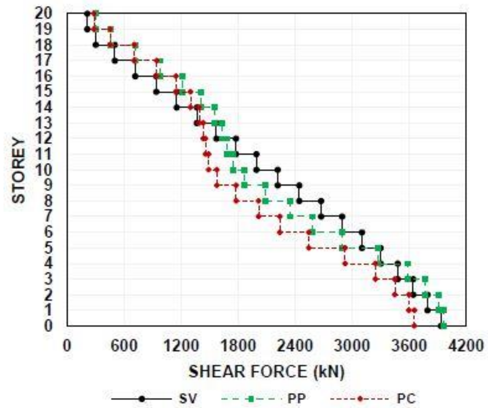
(a) 8-storey frame



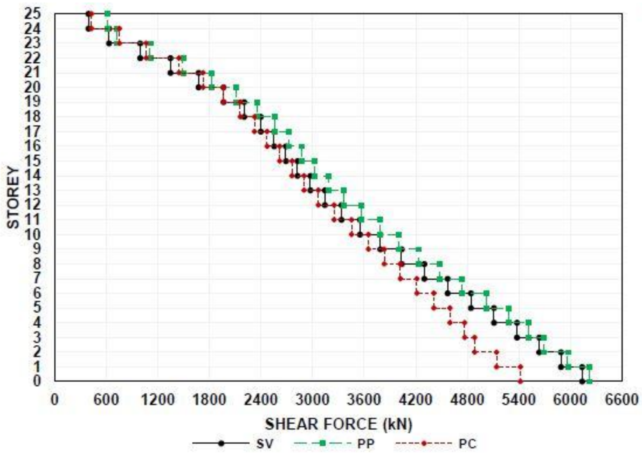
(b) 12-storey frame



(c) 17-storey frame

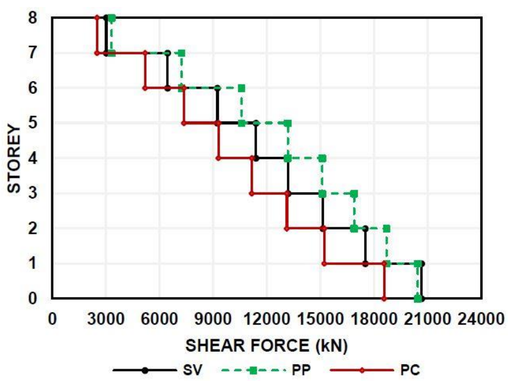


(d) 20-storey frame

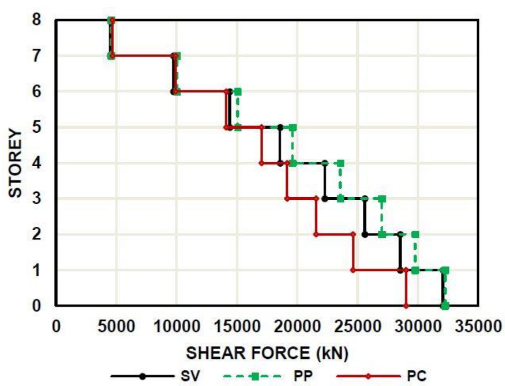


(e) 25-storey frame

Fig. 11 Interstorey shear force of the frame examples



(a) Direction X



(b) Direction Y

Fig. 12 Interstorey shear force for 8-storey building

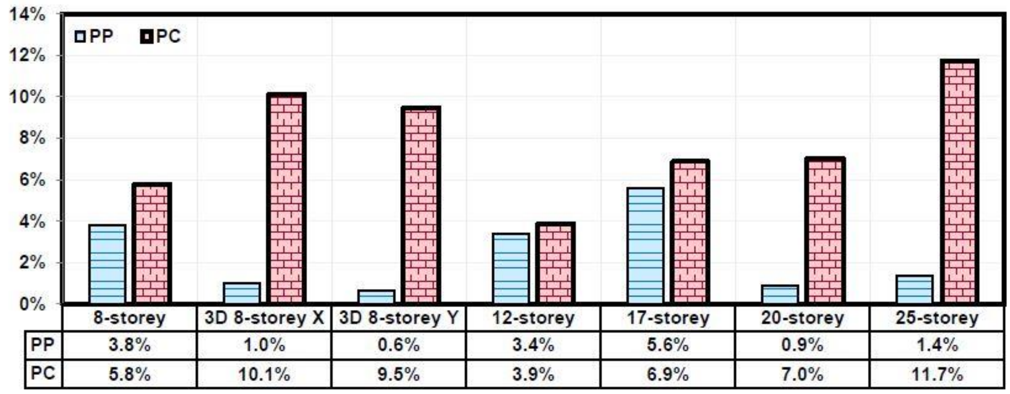


Fig. 13 Relative errors of base shear

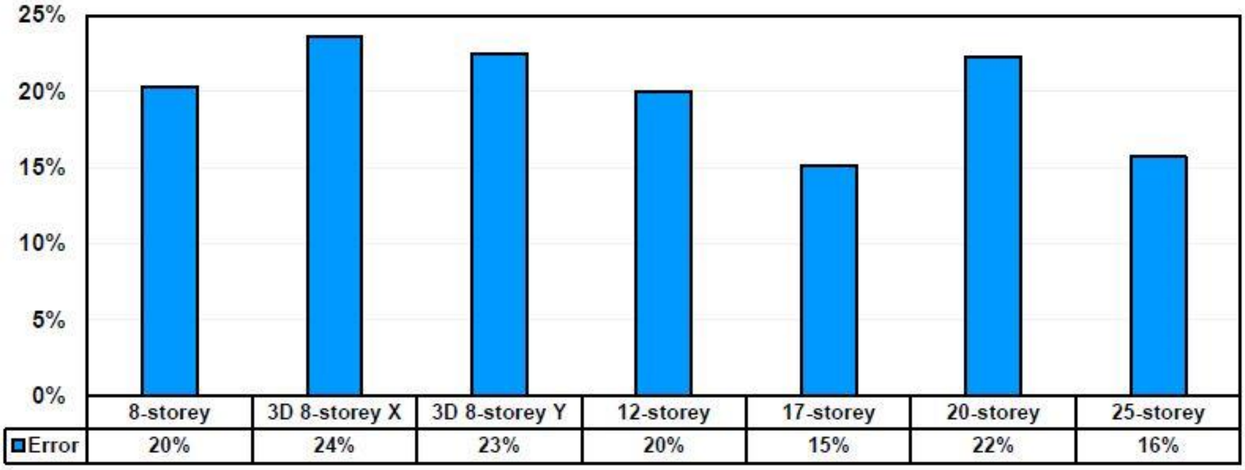


Fig. 14 Relative errors of total damping coefficient

4.6 Evaluation of results

The results of the interstorey drifts of the applications examples are shown in Figs. 7 and 8, the variations of this parameter obtained with the procedure proposed in the storey where the maximum value occurs are not significant when considering the effect of the dampers as proportional damping matrix.

The Fig. 9 shows the error obtained from the interstorey drifts of the models studied in the storey where the maximum drift occurs. The error was determined for the procedure proposed, and the procedure proposed by Constantinou and Symans (1992) with respect to the CPLS. The maximum error that presents in the procedure proposed was 2%, in the case of the procedure proposed by Constantinou and Symans (1992); the maximum error was 13%. In addition, the interstorey shears forces were determined for the studied models (Figs. 11 and 12) where, as in the interstorey drifts, the errors for the procedure proposed were lower compared to the procedure proposed by Constantinou and Symans (1992), see Fig. 13. The damage distributions obtained are approximately equal to those proposed as targets (see Fig. 10). In general, the proposed damaged distributions are maintained.

As can be seen in Figs 7 and 8 and Figs. 11 and 12, both the maximum interstorey drifts and the basal shears do not show significant differences between of the designs obtained with the procedure proposed and the proposal of Constantinou and Symans (1992). However, the difference is more noticeable in the sum of the damping coefficients of the devices in the structure, where there were differences that vary from 15% to 24% for the same supplementary damping amount $\left(\xi_{Design}\right)$ to satisfy the CPLS (see Fig. 14).

5. Conclusions

This paper presented the formulation, and application of a displacement-based seismic design procedure for new structures and/or for existing structures equipped with FVDs, with irregularities in plan and/or elevation where these passive dissipation devices are required as a retrofit measure. To validate the design procedure proposed, the performance of the case studies considered designed with the procedure proposed was assessed by means of non-linear step-by-step dynamic analyses employing ground motions consistent with the seismic demands used for their design. From the analysis of such results, the following conclusions were derived:

• The maximum interstorey drifts of the structures produced by the design demand applying the approximation of the effect of the FVDs as proportional damping proposed in this paper, are approximates to those obtained from the non-linear step-by-step analyses of the structures considering a non-proportional damping matrix. However, to guarantee this conclusion it is necessary to carry out additional evaluation/design examples, considering structures subjected to different seismic demands.
• The distributions of damage proposed in the design processes, which are in itself a design target were approximately reproduced in the results of the non-linear step-by-step analysis of the structures with a non-proportional damping matrix.
• The most notable difference between the results of the procedure proposed, and the procedure recommended in the technical documents such as ASCE 7-10 (2010) is in the sum of the damping coefficients of the devices in the structure, where the total damping coefficient obtained by the procedure proposed is lower compared to the proposal of Constantinou and Symans (1992), which varied from 15% to 24%.
• The comparison of the effort involved in the application of this procedure using computational tools available in most design offices such as: SAP2000, CSI (2006), among others and the quality of results obtained with those of other design procedures place it as an excellent design tool for the design of structures with viscous damping, since requires only two modal spectral analyses for its application.

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