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Seismic reliability analysis of structures based on cumulative damage failure mechanism

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Abstract. Non-stationary random seismic response and reliability of multi-degree of freedom hysteretic structure system are studied based on the cumulative damage failure mechanism. First, dynamic Eqs. of multi-degree of freedom hysteretic structure system under earthquake action are established. Secondly, the random seismic response of a multi-degree freedom hysteretic structure system is investigated by the combination of virtual excitation and precise integration. Finally, according to the damage state level of structural, the different damage state probability of high-rise frame structure is calculated based on the boundary value of the cumulative damage index in the seismic intensity earthquake area. The results show that under the same earthquake intensity and the same floor quality and stiffness, the lower the floor is, the greater the damage probability of the building structure is; if the structural floor stiffness changes abruptly, the weak layer will be formed, and the cumulative damage probability will be the largest, and the reliability index will be relatively small. Meanwhile, with the increase of fortification intensity, the reliability of three-level structure fortification is also significantly reduced. This method can solve the problem of non-stationary random seismic response and reliability of high-rise buildings, and it has high efficiency and practicability. It is instructive for structural performance design and estimating the age of the structure.

Keywords: cumulative damage failure mechanism; hysteretic structure system; pseudo excitation method; precise integration method; seismic reliability analysis

1. Introduction

Studies have shown that there are two types of structural dynamic damage forms: the first transcendence damage and cumulative damage of structures (Liu *et al*. 1993, Zhu 2005). Cumulative damage refers to the maximum amplitude of the dynamic response (displacement, velocity, acceleration, etc.) of the structure. Although it does not reach the damage limit, under the long-term dynamic random load, the final structural performance (strength, stiffness, energy consumption, etc.) produces irreversible cumulative damage, causing structural collapse and destruction (Ni 1999). Therefore, under the reciprocating action of the engineering structure, the main damage is caused by the fatigue damage of the structure, and the fatigue damage is the result of the cumulative damage of the structure (Huang *et al*. 2017). Based on the above reasons for structural failure, the criterion of fatigue cumulative damage must be determined in reliability analysis. The early theory of fatigue cumulative damage is Miner's linear cumulative damage criterion, which can be obtained the fatigue reliability of structures based on the probabilistic model of cumulative damage or fatigue life. According to Miner's linear cumulative damage theory, Miles developed the fatigue reliability problem and proposed an Eq. to calculate the cumulative damage expectation for narrow-band processes. Moreover, Powell also studied the application of the extreme probability density function of narrow-band processes to calculate fatigue reliability, and the results are safe. In addition, for the fatigue reliability of linear systems with stationary broadband random disturbances, Shinozuka *et al*. deduced the fatigue failure probability ranges of two failure thresholds and extended the conclusions to time-varying boundary problems (Shen *et al*. 1997). Traditionally, the linear correlation coefficient was frequently used to describe the dependence among random variables because of its simplicity and convenience of implementation. However, it is difficult to use the linear correlation coefficient if the joint probability distribution of random variables is not a Gaussian distribution. Therefore, the fatigue reliability of the nonlinear structural system with non-stationary broadband random disturbances is complex and difficult to calculate.

In view of this, many scholars have introduced stochastic analysis methods into engineering structure analysis and dynamic reliability theory (Li *et al*. 2017, Osama *et al*. 2018, Parham *et al*. 2018). Li *et al*. (1993) proposed four basic Eqs. for reliability analysis of seismic structures, his research work is groundbreaking in structural dynamic reliability analysis. Lin *et al*. (2014) proposed a virtual excitation method, which has efficient and accurate solutions to both stationary and non-stationary random vibration problems so that the deterministic dynamic analysis method can be used to solve the random vibration. Moreover, Du *et al*. (2006) also used the virtual excitation to systematically study the stationary seismic response and dynamic reliability of the structure. Li and Chen (2009)

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developed a class of probability density evolution methods and evaluated the random seismic response and seismic reliability of the structure. Based on the complex modal analysis theory, the stochastic seismic response of the multi-degree-of-freedom hysteretic system is investigated, and the dynamic reliability analysis is carried out based on the fatigue failure criterion (ductility and energy dissipation) (Guo and Wang 1999). By discretizing the probability of peak acceleration corresponding to each earthquake intensity and combining the dynamic analysis results of random vibration with the static reliability analysis, a formula for calculating the structural reliability based on the peak acceleration distribution of ground motion corresponding to probabilistic seismic intensity is proposed (Yang *et al*. 2011). Based on the first transcendence mechanism, Sun Wei *et al*. (2011) investigated the reliability of the high-rise isolated structures under the non-stationary random earthquake excitation under the 8-degree and 9-degree rare earthquakes. However, the above dynamic reliability analysis is based on the limited assumptions of damage criteria, structural characteristics (stiffness degradation, etc.), and load sequence properties. Many approximate methods are proposed for structural damage reliability analysis. But for the multi-degree of the freedom structure system, the analysis of structural damage reliability is very difficult because of the correlation between the responses (William *et al*. 1997). At the same time, how to use an accurate and efficient algorithm to solve the non-stationary random seismic response and reliability of engineering structure is very necessary for the reliability analysis of engineering structure, and it is also worth further study (Li *et al*. 2019, Mariano and Lorenzo 2019).

For this reason, based on the structural dynamic reliability analysis of the cumulative damage failure mechanism, the Eqs. of motion of the hysteretic system of the stiffness-degenerate restoring force model reflecting the cumulative damage of the structure is established. Furthermore, the virtual excitation and the fine integration are combined to calculate the random response of the multi-degree-of-freedom hysteretic structure system, which makes the calculation significantly simplified and greatly improves the calculation efficiency. Finally, based on the structural cumulative damage (two parameters of displacement and energy consumption) failure criterion, the dynamic reliability analysis of the nonlinear hysteretic system was carried out. The research results provide a reference for fatigue life prediction and seismic reliability design of engineering structures.

2. Random seismic response of multi-degree-of-freedom hysteretic structural systems

2.1 Eq. of motion of multi-degree-of-freedom hysteretic structural systems under earthquake

The multi-degree-of-freedom hysteretic system considering the stiffness degradation resilience model can be expressed as

$$\begin{aligned}&[M]\ddot{X}(t)+[\mathrm{C}]\dot{\mathrm{X}}(t)+[k]X(t)+[K_z]Z(t)\\&=-[M][I]\ddot{X}_g(t)\end{aligned} \tag{1}$$

Where, $[M]$, $[K]$, $[C]$ are the mass, stiffness, and damping matrix of the system, respectively, $X(t)$, $\dot{X}$, $\ddot{X}(t)$ represent the displacement response, velocity, and acceleration response of each floor. Considering the stiffness degradation of reinforced concrete structures, the Bouc-Wen model is used to simulate，$[K_z]$ is a hysteretic stiffness matrix, $Z(t)$ is the hysteretic displacement vector of the structure. $\ddot{X}_g(t)$ is the ground motion input acceleration. $[I]$ is the unit column vector.

The expression for the Bouc-Wen model considering structural stiffness degradation is

$$\dot{z}_i=(1/\eta)[A\dot{x}_i-v(\beta\left|\dot{x}_i\right|\left|z_i\right|^{\mathrm{n-1}}z_i+\gamma\dot{x}_i\left|z_i\right|^n)] \tag{2}$$

Where, z_i, $\dot{z}_i$ indicates the hysteresis displacement of each floor and its derivative, $\dot{x}_i$ is the speed of each floor. A, v, η are hysteretic displacement degradation characteristic parameters, n is the control parameter of the hysteretic displacement skeleton curve, β, γ are the control parameters of the hysteresis curve area.

In this paper, the modal analysis is used to solve the stochastic response of the hysteretic structural system. The Eq. (1) is obtained by matrix transformation to deduce the reduced-order state equation, and the general solution method of the state vector matrix is further calculated (Guo and Wang 1999.).

$$\{\dot{U}\}=[H]\{U\}+\{E\}\ddot{X}_g(t) \tag{3}$$

Where, The {U}, [H] and {E}vectors are

$$\{\mathrm{U}\}=\begin{Bmatrix}\{x_t\}\\\{\dot{x}_t\}\\\{z_t\}\end{Bmatrix}\qquad\{E\}=\begin{Bmatrix}\{0\}\\-\{\delta x\}\\\{0\}\end{Bmatrix}$$
$$[H]=\begin{bmatrix}[0]&[I]&[0]\\-[M]^{-1}[K]&-[M]^{-1}[C]&-[M]^{-1}[K_z]\\ [0]&[C_e]&[K_e]\end{bmatrix} \tag{4}$$

Where , among them

$$[M]=\begin{bmatrix}m_1&0&\cdots&0\\m_2&m_2&\cdots&0\\\vdots&\vdots&\ddots&0\\m_j&m_j&\cdots&m_j\end{bmatrix}\qquad[c]=\begin{bmatrix}c_1&-c_2&\cdots&0\\0&c_2&\cdots&0\\\vdots&\vdots&\ddots&-c_j\\0&0&\cdots&c_j\end{bmatrix}$$

$$[\delta x]=\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}\qquad [K]=\begin{bmatrix}\alpha_1 k_1 & -\alpha_2 k_2 & \cdots & 0\\ 0 & \alpha_2 k_2 & \cdots & 0\\ \vdots & \vdots & \ddots & -\alpha_j k_j\\ 0 & 0 & \cdots & \alpha_j k_j\end{bmatrix}$$

$$K_z=\begin{bmatrix}(1-\alpha_1)k_1 & -(1-\alpha_1)k_1 & \cdots & 0\\ 0 & (1-\alpha_2)k_2 & \cdots & 0\\ \vdots & \vdots & \ddots & -(1-\alpha_j)k_j\\ 0 & 0 & \cdots & (1-\alpha_j)k_j\end{bmatrix}$$

Where, $m_1 \sim m_j$, $c_1 \sim c_j$, $k_1 \sim k_j$, $a_1 \sim a$ are the mass, damping, stiffness, and second stiffness factor of each layer. $[C_e]$ and $[K_e]$ are the equivalent damping and stiffness matrices of the structural hysteresis model.

2.2 Structural random seismic response calculation based on virtual excitation method

The non-stationary random ground motion model $f(t)$ can be expressed as the product of the stationary stochastic process $r(t)$ and the modulation function $g(t)$, and the Eq. can be expressed as

$$f(t)=g(t)r(t) \tag{5}$$

Construct the virtual stimulus $f(t)$ with the product of the unit harmonic excitation $e^{i\omega t}$ and the constant $\sqrt{S_{rr}(\omega)}$:

$$f(t)=g(t)\tilde{r}(t)=g(t)\sqrt{S_{rr}(w)}e^{iwt} \tag{6}$$

Then the response of the structural system at the time t under the virtual stimulus is

$$\tilde{y}(\omega,t)=\sqrt{S_{rr}(\omega)}I(\omega,t) \tag{7}$$

Where, $S_{rr}(\omega)$ is the self-spectral density function of seismic excitation , $I(\omega,t)$ is the response of the deterministic stimulus to the initial stationary system at the time t. Then the self-spectral density function of the structural response $y(\omega,t)$ is expressed as

$$S_{yy}(\omega,t)=\tilde{y}^*(\omega,t)\tilde{y}(\omega,t)=\left|\tilde{y}(\omega,t)\right|^2 \tag{8}$$

Then, the variance of the structural response $y(\omega\ t)$ is expressed as

$$\sigma_y^2(t)=\int_{-\infty}^{+\infty}S_{yy}(\omega,t)d\omega \tag{9}$$

According to the basic principle of the virtual excitation (Lin *et al*. 2004, Du *et al*. 2006), the variance of the velocity response and the hysteretic displacement response can be further determined

$$\sigma_{\dot{x}}^2(t)=\int_{-\infty}^{+\infty}S_{\dot{x}\dot{x}}(\omega,t)d\omega \tag{10}$$

$$\sigma_z^2(t)=\int_{-\infty}^{+\infty}S_{zz}(\omega,t)d\omega \tag{11}$$

Then the covariance of the velocity response and the hysteretic displacement response is

$$\mathrm{cov}(\dot{X},Z)=\rho_{\dot{X}Z}\sigma_{\dot{X}}(t)\sigma_Z(t) \tag{12}$$

Where, p_{xz} represents the correlation between velocity and hysteresis displacement, $\sigma_{\dot{x}}(t)$ and $\sigma_z(t)$ are the standard deviations of velocity and hysteresis displacement, respectively.

The combination of virtual excitation and precise integration is used to solve the seismic response problem of structures under non-stationary random excitation (Lin *et al*. 2004, Zhong, 1994). The response quantity of the structural dynamic equation of Eq. (1) is solved by the precise time-history integral method. The dynamic Eq. (1) of the structure is obtained by matrix transformation, and the state Eq. (3) of reduced-order is obtained. In the reasonable range of integral step size, stability or rigidity problem will not be occurred by the precision integral method. Therefore, the precision integral method is a high-precision numerical integral method.

3. Seismic reliability analysis of structures based on cumulative damage threshold

According to the structural damage index and the failure criterion, it is more reasonable to use the displacement and energy two-parameter indicators to determine the structural damage for the structure that is the cumulative damage of fatigue (Ni 1999, Yang *et al*. 2010). Therefore, this paper considers the Park damage index based on the maximum displacement response and cumulative plastic energy between the two layers and calculates the probability of structural damage state corresponding to the damage index limit value, thus completing the structural seismic performance evaluation.

3.1 Statistic and the probability distribution of maximum displacement and plastic cumulative energy reaction parameters between layers

From Eq. (9), the statistics of the peak displacement of the interlayer displacement can be obtained

$$\mu_Y=[\sqrt{2l(f_0T_d)}+\frac{0.5772}{\sqrt{2l(f_0T_d)}}]\sigma_Y \tag{13}$$

$$\sigma_Y=\frac{\pi\sigma_Y}{\sqrt{12l(f_0T_d)}}\qquad f_0=\frac{\sigma_{\dot{y}}}{\pi\sigma_y} \tag{14}$$

Where, μ_y and σ_Y are the mean and standard deviation of the eak displacement response of the interlayer, respectively. T_d indicates the duration of the earthquake action, f_0 is the zero-crossing rate of the inter-layer displacement response.

Define the cumulative hysteresis energy ε_i of the structure i -th layer in the T_d period:

$$\varepsilon_i = (1-\alpha_i)k_i\int_0^{T_d} z_i(t)\dot{u}_i(t)dt \tag{15}$$

Where, $Z_i(t)$ is the hysteresis displacement of the first layer of the structure, $u_i(t)$ and $\dot{u}_i(t)$ are the displacement and velocity of the first layer of the structure.

When the structural response is a stationary stochastic process, the expected and variance values of the cumulative hysteretic energy dissipation of the structure are

$$E(\varepsilon_{m,i}) = (1-\alpha_i)k_iT_dE\left(z_i(t)\dot{u}_i(t)\right) \tag{16}$$

$$\sigma_{\varepsilon_i}^2 = 2(1-\alpha_i)^2k_i^2\int_0^{t_d}(T_d-\tau)\left[\rho_{z_i}(\tau)\rho_{\dot{u}_i}(\tau)+\rho_{z_i\dot{u}_i}(\tau)\rho_{\dot{u}_iz_i}(\tau)\right]d\tau \tag{17}$$

Where, $\rho_{z_i}(\tau), \rho_{\dot{u}_i}(\tau), \rho_{z_i\dot{u}_i}(\tau), \rho_{\dot{u}_iz_i}(\tau)$ refer to the density distribution of structural parameters (such as retardation displacement, interlayer velocity, etc.) at a certain time.

For deterministic structures, it can be considered that the randomness of seismic input has a much greater influence on the seismic reliability of the structure than the random parameters of the structure, so only u_{m} (inter-layer maximum displacement) and ε (structure cumulative energy loss) are considered in the Park damage index. The two variables are random variables and were assumed to be fully correlated. For the convenience of calculation, μ_y (yield displacement) and Q structural strength) can be considered as the determined quantity, and the damage index D_i is a linear combination of random variables. The damage index D obeys a lognormal distribution, and its mean and variance are

$$\mu_{Di} = \left[(\frac{\mu_{u_m}}{u_y})^2 + 3.21(\frac{\mu_\varepsilon}{Qu_y})^{1.47}\right]^{1.471/2} \tag{18}$$

$$\sigma_{Di}^2 = (\frac{\partial D}{\partial u_m})^2\sigma_{u_m}^2 + (\frac{\partial D}{\partial_\varepsilon})^2\sigma_\varepsilon^2 + 2(\frac{\partial D}{\partial u_m})(\frac{\partial D}{\partial_\varepsilon})\sigma_{u_m}\sigma_\varepsilon \tag{19}$$

Where, u_y is yield displacement, Q indicates structural strength. The probability density function based on the Park damage indicator is

$$f_{Di}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\ln D}x}\exp\left[-\frac{1}{2}(\frac{\ln x-\mu_{\ln D_i}}{\sigma_{\ln D_i}})\right] \tag{20}$$

3.2 Seismic reliability analysis of structures based on Park damage criterion

For earthquake-resistant building structures, the damage state can be classified into five types of damage levels according to the damage index. According to the damage index range of each damage level, the probability of the cumulative damage state of the structure under earthquake action is calculated, and the seismic reliability of the structure is analyzed.

According to the reliability theory, the structural function of each layer of the building structure are

$$Z_i = R_i - S_i = g(u_{m,i}, u_{y,i}, \varepsilon_i, Q_i) \tag{21}$$

The probability of failure (destruction) of each i -layer of the building structure is

$$P_{f,i} = P_i(Z_i < 0) \tag{22}$$

The two-parameter Park damage criterion using displacement and cumulative plastic energy, the failure index of structure (or interlayer structure) is

$$D = \frac{u_m}{u_u} + \beta\frac{\varepsilon_m}{F_yu_u} \tag{23}$$

Where, u_u indicates the deformation between the limits; u_m indicates the maximum interlayer deformation; F_y indicates yield strength; ε_m indicates the maximum cumulative hysteresis energy between layers; β is an energy consumption factor (its value is in the range of 0 to 0.85, and the mean value is 0.15).

This model was the seismic damage model of the combination of maximum deformation and accumulated hysteretic energy proposed by park and a.h.s.ang, which was based on a large number of reinforced concrete beam-column test components. The model could reflect the seismic damage of the interlayer structure (each storey), and reflect the stiffness degradation of each storey. Therefore, in this study, it was considered that the seismic damage model of the interlayer structure (each storey) could better reflect the seismic damage mechanism of the structure (Lu *et al.*, 2001).

Through the previous section, the statistical characteristics of the displacement and cumulative plastic energy and the limit value D of various damages were calculated. Given the magnitude of the seismic intensity I_j, the probability of Class n damage occurring in the i-story of a building structure in the I_d seismic intensity zone during the design base period

$$P_{f,i,n} = \sum_j P_{i,j,n}(D_n^b < D < D_n^t)P(I_j \mid I_d) \tag{24}$$

Where, D_n^b and D_n^t indicates the upper and lower limits of the structural damage index when n-level failure occurs.

It is worth noting that due to the large difference in power spectral density between ground motion samples with different seismic intensities, and seismic intensity is discretized, and the seismic intensity probability distribution of major earthquake regions in China is counted (Yang *et al.* 2010). The power spectral density function of the relatively uniform seismic intensity over the 50-year design period was obtained.

Under the action of seismic intensity I_j, the reliability of the *i*-storey of a building structure in the I_d seismic intensity zone within the design reference period is

$$P_{si} = 1 - P_{f,i,n}(I_j \mid I_d) \quad (25)$$

Furthermore, the probability that the overall structure of the house will undergo *n*-level damage during the design reference period is

$$\boldsymbol{P_f = P_{f,n}(D_n \mid I_d)} \quad (26)$$

Under the influence of seismic intensity I_j, the building structure in the I_d earthquake fortification area is based on the reliability of the three-level fortification target

$$\boldsymbol{P_s = 1 - P_f(I_j \mid I_d)} \quad (27)$$

4. Results and discusstion

The above method is applied to the random seismic response and dynamic reliability analysis of high-rise structures. Taking a 12-storey frame structure as the research object, its damping ratio is 0.05, the infilled wall of the frame structure is made of hollow concrete bricks, the structural characteristic parameters (floor mass and rigidity) are shown in Table 1, the height of each floor is 3.0 m, and the site soil category is class II. The random seismic response of the structure under different earthquake intensity is analyzed, and its dynamic reliability is compared.

The Bouc-Wen model of stiffness degradation is adopted in this structural system.The parameters of hysteretic model for each floor are taken as follows according to reference (Baber and Wen 2001): n=1, A=1,β=0.8,γ=0. 2, υ=1, η=1, α_1～α_j=0. 2. In this study, the stiffness degradation of the hysteretic model is only considered in the calculation of this structural system.

For the class II soil site, the power spectrum model of stationary random ground motion was used in this study (Du *et al*., 1998). This model overcomes the shortcoming of unbounded ground velocity and displacement caused by the Tajimi-Kanai (Tajimi-Kanai spectrum) model (Ou *et al*. 1994), and can determine the parameters in the model by using the basic parameters based on the current seismic design，and the model is expressed as follows

$$S(\omega) = \frac{1}{1+(D\omega)^2} \cdot \frac{\omega^4}{(\omega^2+\omega_0^2)^2} \times \frac{1+4\xi_g^2(\omega/\omega_g)^2}{\left(1-(\omega/\omega_g)^2\right)^2+4\xi_g^2(\omega/\omega_g)^2} S_0 \quad (28)$$

Where, ω_g and ζ_g are the preeminent circular frequency and damping ratio of the site soil, respectively. S_0 is the self-spectral density of bedrock acceleration. *D* and ω_0 are the spectral parameters related to the earthquake magnitude.

Generally, the statistical mean value should be taken: *D*= (*1/28*) *π*, ω_0=*1.83 rad/s*. Fig. 1 shows the acceleration power spectral density function under different field conditions. According to the suggestion of reference (Du *et al*. 1998, Ou *et al*. 1991), the predominant circl e frequency and damping ratio of site soil correspondin g to the parameters of the ground motion power spectr um model for class II site soil are as follows: ζ_g=0.72, ω_g=20.94 *rad/ s*.

Table 1 Characteristic parameters of each layer of the 12-storey frame structure

Floor number	1	2	3	4	5～10	11	12
Mass /10^3 kg	2800	2800	2800	2850	2800	2750	2750
Stiffness /10^3 kN/m	7000	7000	7000	6000	6800	6000	6800

Table 2 Range of damage state index

Damage grade	Grade 1 Basic intact	Grade 2 Minor damage	Grade 3 Medium damage	Grade 4 Serious damage	Grade5 Collapse
Damage index range	0.0-3.3	3.3-3.6	3.6-6.0	6.0-11.1	> 11.1

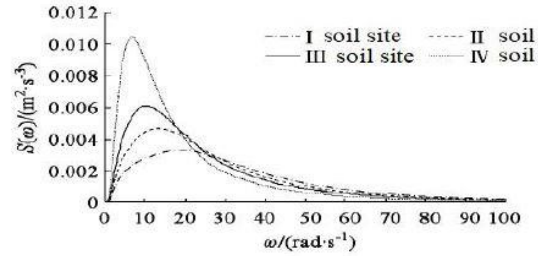

Fig. 1 Acceleration power spectrum density function

Due to the structure of this study adopts a concrete porous brick masonry wall, there was a lack of a large number of actual engineering seismic damage data to t est and calibrate the "damage index." According to the test and analysis results of the concrete porous brick m asonry wall, the average damage index was 15.56, whi ch was less than the average damage index of clay sol id brick (Lin *et al*. 2012). In addition, the standard of defining the average damage index of solid clay brick was more conservative based on the results of the liter ature research (Jiang *et al*. 1985). Therefore, in this st udy, according to the principle of defining the average value of the above two wall damage state indicators (Yang *et al*. 2006, Yang *et al*. 2010), the damage inde xes of frame structures (concrete multi-empty brick fill ed walls) with different earthquake disaster levels were finally taken, as shown in Table 2.

In this study, the calculation of the random seismic dynamic response of structure was mainly obtained by solving the dynamic differential equation with Simulink of Matlab toolbox. In the Simulink interface, various state modules were inserted to establish the model of state vector conforming to the dynamic equation. The output results were further studied and analyzed by giving the experimental factors and the initial state values. Further, based on the numerical output results of simulation in Simulink, the JC algorithm was used to solve the reliability index and the probability of different damage levels

Table 3 Failure probability of each floor structure of the 12-storey building with earthquake intensityVIII (%)

Floor	Damage grade				
	Grade 1 Basic intact	Grade 2 Minor damage	Grade 3 Medium damage	Grade 4 Serious damage	Grade5 Collapse
1	9.834	28.135	48.468	12.523	2.931
2	10.563	33.478	44.384	11.263	2.103
3	13.456	43.562	41.929	5.936	1.365
4	9.986	48.726	48.564	12.362	3.213
5	18.536	45.637	30.213	4.562	1.236
6	22.361	46.526	28.654	3.563	0.981
7	29.562	49.286	27.561	2.564	0.765
8	36.420	29.836	23.53	2.130	0.564
9	46.578	28.356	20.125	1.960	0.465
10	52.456	34.542	18.965	1.362	0.223
11	15.362	49.562	46.532	11.231	2.962
12	72.315	45.630	5.236	0.837	0.116

Table 4 Failure probability of the first floor of 12-storey building among different protected areas (%)

Earthquake intensity	Damage grade				
	Grade 1 Basic intact	Grade 2 Minor damage	Grade 3 Medium damage	Grade 4 Serious damage	Grade5 Collapse
6	51.665	19.653	9.653	1.765	0.278
7	16.360	23.231	12.327	2.563	1.362
8	9.568	26.657	16.653	7.634	2.578

Table 5 Reliability of each floor under 8-degree rare earthquake

Floor number	1	2	3	4	5	6	7	8	9	10	11	12
Reliability	0.83	0.75	0.71	0.45	0.76	0.83	0.86	0.48	0.91	0.95	0.48	0.96

Table 6 Reliability of each floor under 9-degree rare earthquake

Floor number	1	2	3	4	5	6	7	8	9	10	11	12
Reliability	0.65	0.38	0.36	0.15	0.4	0.45	0.51	0.21	0.55	0.61	0.21	0.68

Table 7 Reliability based on three-level fortification object

Seismic fortification intensity	No damage under small earthquake	Repairable under moderate earthquake	No collapsing under strong earthquake
8	0.812	0.765	0.912
9	0.560	0.486	0.663

occurring on each floor of the building structure with different seismic intensity in the design reference period was calculated.

The specific calculation process is as follows: the response statistics of random variables are calculated by Section 3.1 formula. Based on the cumulative damage criterion, the probability of different failure stages occurring in the design reference period is calculated by the Eq. (24) and Eq. (26). By calculating, we can get the probability of different failure levels of each floor in a 7-degree area of 12-storey building structure in the design reference period, as shown in Table 3. The probability of different damage levels occurring on the first floor of 12-storey building structures with different seismic intensity (6, 7, 8 degrees) in the design reference period is shown in Table 4.

For a 12-storey frame structure without special isolation design, the dynamic reliability analysis of the structure with the cumulative damage mechanism is carried out. From Table 3, it can be seen that under earthquake action, with the increase of structural floors, the probability of damage of grade 1 (basically intact) increases, while the probability of damage of grade 3 (moderate damage), grade 4 (serious damage) and grade 5 (collapse damage) decreases. The results show that the lower the floor is, the greater the probability of medium damage, serious damage, and collapse damage is when the floor stiffness and mass are the same for the same building structure, which indicates that the first floor of the structure has formed a weak layer. At the same time, under the same conditions, the damage probability of damage grade above 2 (slight damage) on the fourth and eleventh floors of the building structure is significantly higher than that of other floors. The main reason is that the stiffness of the two floors is obviously reduced, and it is also a weak floor.

From Table 4, with the increase of the probability of large intensity earthquakes occurring in the earthquake area, the probability of damage to building structures increases, especially in the light, medium, serious, and collapse damage grades.

According to China's code for seismic design of buildings (GB-50011, 2010), the limit value B of inter-storey displacement of structures is taken as 1/120 of the limit value of the elastic-plastic displacement angle. From Eq. (25), the dynamic reliability of each storey of a 12-storey structure under rare earthquakes of 8 degrees and 9 degrees can be obtained, as shown in Table 5 and Table 6. Eq. (27) is used to calculate the dynamic reliability of structures based on three-level defensive targets, as shown in Table 7.

From Table 5 and Table 6, under the rare earthquake of 8 degrees and 9 degrees, the reliability index of some floors of the structure is lower, and the reliability index of the fourth and eleventh floors of the structure are the smallest. The main reason is that the stiffness of the fourth and eleventh floors of the structure has a sudden change, which shows that the two stories are weak stories, so the cumulative damage probability is the largest. Therefore, the seismic design of the structure should be strengthened in the design process. From Table 6, it can also be seen that under the rare 9-degree earthquake, most of the floor displacements have exceeded the limit, the reliability index of the whole structure is very small, and the whole structure has basically failed. The above analysis shows that the lower the floor is, the greater the probability of structural failure is under the same magnitude earthquake, and the lower the reliability of the floor is, the worse the reliability of the structure is. At the same time, with the increase of the probability of large intensity earthquakes occurring in seismic areas, the probability of cumulative damage and damage of structures increase.

From Table 7, under the same fortification intensity, the reliability index of the waterproofing level for two levels of a small earthquake and large earthquake is larger, while the reliability of repairable earthquake is smaller. The main reason is that the displacement response and accumulated energy of each floor of the structure exceed the set limit, which leads to the low-reliability index of the structure system. However, when the seismic fortification intensity of the frame structure is 9-degrees, the reliability of the three-level fortification level has been significantly reduced. At this time, the structure must be strengthened. Otherwise, it cannot meet the requirements of 9-degree seismic fortification.

5. Conclusions

For the hysteretic multi-degree-of-freedom structure system, the random response of the structure is calculated by combining the virtual excitation method with the precise integration method. Based on the damage level of the structure, the limit values of the cumulative damage index (the maximum displacement response between layers and the cumulative plastic energy) of each damage level are used to analyze the seismic reliability of the structure. The results show that the lower the floor is, the greater the probability of cumulative damage will be. If the floor stiffness changes abruptly, the weaker floor will be formed, and the probability of damage will be the largest, and the reliability index of the floor will be smaller. With the increase of earthquake intensity, the cumulative damage probability of the structure is increased. At the same time, with the increase of seismic fortification intensity, the reliability of three-level structure fortification has been significantly reduced, so the structure must be strengthened. Otherwise, it cannot meet the requirements of seismic fortification. The combination of virtual excitation method and precise integration method can solve the problem of non-stationary random seismic response and reliability of high-rise buildings. The method is simple, efficient, and widely applicable.

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