Earthquakes and Structures, *Vol. 18, No. 4 (2020) 481-492*
DOI: https://doi.org/10.12989/eas.2020.18.4.481

Bending analysis of softcore and hardcore functionally graded sandwich beams

Lazreg Hadji*[1,2] and Abdelkader Safa[3]

[1]*Department of Mechanical Engineering, University of Tiaret, BP 78 Zaaroura, Tiaret,14000, Algeria*
[2]*Laboratory of Geomatics and Sustainable Development, University of Tiaret, Algeria*
[3]*Department of Civil Engineering, Ahmed Zabana University Centre, Relizane (48000), Algeria*

(*Received November 24, 2019, Revised January 25, 2020, Accepted February 4, 2020*)

Abstract. A New hyperbolic shear deformation theory is developed for the bending analysis of softcore and hardcore functionally graded sandwich beams. This theory satisfies the equilibrium conditions at the top and bottom faces of the sandwich beam and does not require the shear correction factor. The governing equations are derived from the principle of virtual work. Sandwich beams have functionally graded skins and two types of homogenous core (softcore and hardcore). The material properties of functionally graded skins are graded through the thickness according to the power-law distribution. The Navier solution is used to obtain the closed form solutions for simply supported FGM sandwich beams. The accuracy and effectiveness of proposed theory are verified by comparison with previous research. A detailed numerical study is carried out to examine the influence of the deflections, stresses, and sandwich beam type on the bending responses of functionally graded sandwich beams.

Keywords: FG sandwich beams; bending; stress; hardcore; softcore; navier solution

1. Introduction

In recent years, there is a rapid increase in the use of functionally graded (FG) sandwich structures in aerospace, marine and civil engineering due to high strength-to-weight ratio. Functionally graded materials (FGMs) are new inhomogeneous materials which have been widely used in many engineering applicants such as nuclear reactors and high-speed spacecraft industries (Yamanouchi *et al.* (1990)). The mechanical properties of FGMs vary smoothly and continuously from one surface to the other. Typically these materials are made from a mixture of ceramic and metal or from a combination of different materials. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity. The ductile metal constituent on the other hand, prevents fracture caused by stresses due to the high temperature gradient in a very short period of time. Furthermore a mixture of ceramic and metal with a continuously varying volume fraction can be easily manufactured (Fukui 1991, Koizumi 1997). The analysis of these materials has been considered by many researchers.

The FGM sandwich can alleviate the large interfacial shear stress concentration because of the gradual variation of material properties at the facesheet-core interface. The effects of the FGM core were studied by Venkataraman and Sankar (2001) and Anderson (2003) on the shear stresses at the facesheet-core of the FGM sandwich beam. Pan and Han (2005) analyzed the static response of the multilayered rectangular plate made of the functionally graded, anisotropic, and linear magneto-electro-elastic materials. Bennai *et al.* (2015) used a new higher-order shear and normal deformation theory for functionally graded sandwich beams. Chaabane *et al.* (2019) developed an analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation. Alimirzaei *et al.* (2019) investigated the nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions. Berghouti *et al.* (2019) analyze the vibration of nonlocal porous nanobeams made of functionally graded material. Bourada *et al.* (2019) investigated the dynamic of porous functionally graded beam using a sinusoidal shear deformation theory. Batou *et al.* (2019) studied the wave dispersion properties in imperfect sigmoid plates using various HSDTs. Tlidji *et al.* (2019) analyze the vibration of different material distributions of functionally graded microbeam. Salah *et al.* (2019) investigated the thermal buckling properties of ceramic-metal FGM sandwich plates using 2D integral plate model. Boussoula *et al.* (2020) used a simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates. Adda Bedia *et al.* (2019) used a new Hyperbolic Two-Unknown Beam Model for bending and buckling analysis of a nonlocal strain gradient nanobeams. Karami *et al.* (2019a) studied the wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation. Karami *et al.* (2019b) investigated the resonance behavior of functionally graded polymer composite nanoplates reinforced with grapheme nanoplatelets. Karami *et al.* (2019c) used the Galerkin's approach for buckling analysis of functionally graded

*Corresponding author, Ph. D.
E-mail: had_laz@yahoo.fr

http://www.techno-press.org/?journal=eas&subpage=7
ISSN: 2092-7614 (Print), 2092-7622 (Online)

anisotropic nanoplates/different boundary conditions. Karami *et al*. (2019d) analyze the exact wave propagation of triclinic material using three dimensional bi-Helmholtz gradient plate model. Karami *et al*. (2019e) studied the pre stressed functionally graded anisotropic nanoshell in magnetic field. Meksi *et al*. (2019) developed an analytical solution for bending, buckling and vibration responses of FGM sandwich plates. Hellal *et al*. (2019) analyze the dynamic and stability of functionally graded material sandwich plates in hygro-thermal environment using a simple higher shear deformation theory.

Since the shear deformation effects are more pronounced in these structures, the first-order shear deformation theory and higher-order shear deformation theories should be used. By using these theories, although many papers have been devoted to study static, vibration and buckling analysis of FG structures such as shells (Viola *et al*. 2014, Fazzolari 2014), beams (Simsek 2009, Thai and Vo 2012, Mirza 2018, S.R Mahmoud 2017, Akbaş 2014), and plates (Draiche *et al*. 2019, Medani *et al*. 2019, Abualnour *et al*. 2019, Draoui *et al*. 2019, Semmah *et al*. 2019, Hussain *et al*. 2019, Belbachir *et al*. 2019, Mahmoud *et al*. 2019, Sahla *et al*. 2019). In addition, in recent years, many researchers have dealt the effect of stretching the thickness on FGM structures (Addou *et al*. 2019, Boutaleb *et al*. 2019, Khiloun *et al*. 2019, Zarga *et al*. 2019, Boulefrakh *et al*. 2019, Boukhlif *et al*. 2019, Mahmoudi *et al*. 2019, Zaoui *et al*. 2019).

The FGM sandwich construction exists in two types: the FGM facesheet-homogeneous core and the homogeneous facesheet-FGM core. For the case of the homogeneous core, the softcore is commonly employed because of the light weight and high bending stiffness in the structural design. The homogeneous hardcore is also employed in other fields, such as control or thermal environments.

Nowadays, functionally graded sandwich beams are widely used in many industries including nuclear engineering. Therefore, accurate structural analysis of FG sandwich beams is required to predict their correct bending behavior.

As far as we know, there has been little investigation for the bending analysis of softcore and hardcore functionally graded sandwich beams using hyperbolic shear deformation beam theory. In this paper, a new hyperbolic shear deformation theory is developed for the bending analysis of softcore and hardcore functionally graded beams. The most interesting feature of this theory is that it does not require the shear correction factor and satisfies equilibrium conditions at the top and bottom faces of the sandwich beam. The governing equations are derived from the principle of virtual work. Sandwich beams have functionally graded skins and two types of homogenous core (softcore and hardcore). The material properties of functionally graded skins are graded through the thickness according to the power-law distribution. The Navier solution is used to obtain the closed form solutions for simply supported FGM sandwich beams. The non-dimensional numerical values of displacements and stresses are obtained for uniformly distributed loads and various values of the power law index and skin-core-skin thickness ratios. Effects of softcore and hardcore on the non-dimensional displacements and stresses are carefully discussed. The effects of various variables, such as gradient index, span-to-depth ratio and sandwich beam type are all discussed.

2. New hyperbolic shear deformation theory for FGM sandwich beams

2.1 Geometrical configuration

Consider a FG sandwich beam composed of three elastic layers. Top and bottom layers of beam (layers 1 and 3) are made up of FG material, whereas middle layer (layer 2) is made up of isotropic material as shown in Fig. 1.

The beam is of length L in x-direction, and total thickness h in z-direction. Thickness coordinates for the top surface is h_1, the bottom surface is h_4 and for layer interfaces are h_2 and h_3. The beam is subjected to transverse load $q_{(x)}$ on the top surface. For brevity, the ratio of the thickness of each layer from bottom to top is denoted by the combination of three numbers, i.e., “1-0-1”, “2-1-2” and so on. Through-the-thickness variation in elastic properties (E, ν, G) for two types of beams, are expressed by the rule of mixture as

Type A (Hardcore):

$$E(z) = E_m + (E_c - E_m)V^{(n)} \tag{1}$$

Type B (Softcore):

$$E(z) = E_c + (E_m - E_c)V^{(n)} \tag{2}$$

where $V^{(n)}$ n(1,2,3) represents function of volume fraction for nth layer; E_m and Ec are the Young’s moduli of metal and ceramic, respectively. Functions of volume fraction for three elastic layers are assumed to obey a power law as follows:

Layer 1:

$$V^{(1)} = \left(\frac{z-h_1}{h_2-h_1}\right)^k \tag{3a}$$

Layer 2:

$$V^{(2)} = 1 \quad \text{for} \quad z \in [h_2, h_3] \tag{3b}$$

Layer 3:

$$V^{(3)} = \left(\frac{z-h_4}{h_3-h_4}\right)^k \quad \text{for} \quad z \in [h_3, h_4] \tag{3c}$$

where k is the power-law index:

2.2 Basic assumptions

The assumptions of the present theory are as follows:

• The displacements are small in comparison with the plate thickness. Therefore, the strains involved are infinitesimal.

• The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x and time t only.

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{4}$$

The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

• The axial displacement u in x-direction and v in the y-direction, consists of extension, bending, and shear components.

$$u = u_0 + u_b + u_s \tag{5}$$

• The bending component u_b is assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for ub can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \tag{6}$$

•The shear component u_s give rise, in conjunction with w_s, to the hyperbolic variation of shear strain γ_{xz} and hence to shear stress τ_{xz} through the thickness of the beam in such a way that shear stress τ_{xz} is zero at the top and bottom faces of the beam. Consequently, the expression for u_s can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \tag{7}$$

where

$$f(z) = z - \frac{z\cosh\left(\frac{1}{2}\pi\right) - \frac{h\sinh\left(\frac{\pi z}{h}\right)}{\pi}}{\cosh\left(\frac{1}{2}\pi\right) - 1} \tag{8}$$

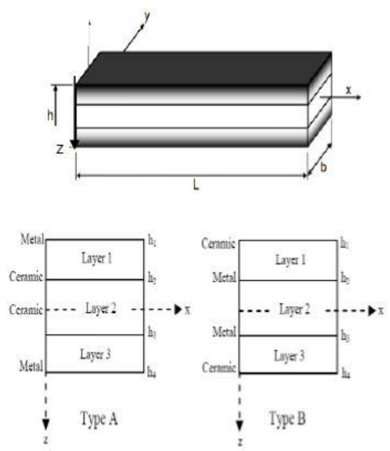

Fig. 1 Geometry, coordinate system and material gradation of functionally graded sandwich beams

2.3 Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (4) - (8) as

$$\begin{aligned} u(x,z,t) &= u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ w(x,z,t) &= w_b(x,t) + w_s(x,t) \end{aligned} \tag{9}$$

The strains associated with the displacements in Eq. (9) are

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z\,k_x^b + f(z)\,k_x^s \\ \gamma_{yz} &= g(z)\,\gamma_{yz}^s \\ \varepsilon_z &= 0 \end{aligned} \tag{10}$$

Where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \\ &\gamma_{xz}^s = \frac{\partial w_s}{\partial x},\ g(z) = 1 - f'(z), \end{aligned} \tag{11}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x^{(n)} = Q_{11}(z)\varepsilon_x \ \text{ and } \tau_{xz}^{(n)} = Q_{55}(z)\gamma_{xz} \tag{12}$$

Where

$$Q_{11}(z) = E^{(n)}(z) \ \text{ and } \ Q_{55}(z) = \frac{E^{(n)}(z)}{2(1+\nu)} \tag{13}$$

2.4 Governing equations

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The principle of virtual work in the present case yields

$$\int_{-h/2}^{h/2}\int_0^L \left[\sigma_x^{(n)}\delta\varepsilon_x + \tau_{xz}^{(n)}\delta\gamma_{xz}\right]dx\,dz - \int_0^L q(x)\delta w\,dx = 0 \tag{14}$$

where q is the applied transverse load.

Substituting Eqs. (10) and (12) into Eq. (14) and integrating through the thickness of the beam, Eq. (14) can be rewritten as

$$\begin{aligned} &\int_0^L \left(N_x\frac{d\delta u_0}{dx} - M_x^b\frac{d^2\delta w_b}{dx^2} - M_x^s\frac{d^2\delta w_s}{dx^2} + Q_{xz}\frac{d\delta w_s}{dx}\right)dx \\ &-\int_0^L q(\delta w_b + \delta w_s)dx = 0 \end{aligned} \tag{15}$$

Where N_x, M_x^b, M_x^s and Q_{xz} are the stress resultants defined as

$$(N_x, M_x^b, M_x^s) = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f(z))\, \sigma_x dz \tag{16a}$$

and

$$Q_{xz} = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} g\, \tau_{xz} dz \tag{16b}$$

The governing equations of equilibrium can be derived from eq. (15) by integrating the displacement gradients by parts and setting the coefficients where δu_0, δw_b, δw_s, zero. Thus, one can obtain the equilibrium equations associated with the present hyperbolic shear deformation beam theory

$$\begin{aligned} \delta u_0 &: \frac{dN_x}{dx} = 0 \\ \delta w_b &: \frac{d^2 M_x^b}{dx^2} + q = 0 \\ \delta w_s &: \frac{d^2 M_x^s}{dx^2} + \frac{dQ_{xz}}{dx} + q = 0 \end{aligned} \tag{17}$$

Eq. (17) can be expressed in terms of displacements (U_0, w_b, w_s) by using Eqs. (12) and (16) as follows

$$\begin{aligned} & A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_b}{\partial x^3} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} = 0 \\ & B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} + q = 0 \\ & B_{11}^s \frac{\partial^3 u_0}{\partial x^3} - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} + A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + q = 0 \end{aligned} \tag{18}$$

Where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz \tag{19a}$$

and

$$A_{55}^s = \sum_{n=1}^{3} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55} \left[g(z)\right]^2 dz \tag{19b}$$

2.5 Analytical solution

Closed-form solutions for simply-supported FG sandwich beam subjected to uniform transverse load $q(x)$ are obtained using Navier's solution technique. The variables u_0, w_b, w_s can be written by assuming the following variations:

$$\begin{aligned} u_0 &= \sum_{m=1}^{\infty} U_m \cos(\lambda x) \\ w_b &= \sum_{m=1}^{\infty} W_{bm} \sin(\lambda x) \\ w_s &= \sum_{m=1}^{\infty} W_{sm} \sin(\lambda x) \end{aligned} \tag{20}$$

Where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, and $\lambda = m\pi / L$. The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \tag{21}$$

where Q_m is the load amplitude calculated from

$$Q_m = \frac{2}{L} \int_0^L q(x) \sin(\lambda x) dx \tag{22}$$

The coefficients Q_m are given below for some typical loads

$$m = 1 \text{ and } Q_1 = q_0 \quad \text{Sinusoidal load} \tag{23a}$$

$$Q_m = \frac{4q_0}{m\pi} \quad \text{Uniform load} \tag{23b}$$

Substituting the expansions of u_0, w_b, w_s and q from Eqs. (20) and (21) into the equations of motion Eq. (18), the analytical solutions can be obtained from the following equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ Q_m \end{Bmatrix} \tag{24}$$

where

$$\begin{aligned} & a_{11} = A_{11}\lambda^2,\ a_{12} = -B_{11}\lambda^3,\ a_{13} = -B_{11}^s\lambda^3, \\ & a_{22} = D_{11}\lambda^4,\ a_{23} = D_{11}^s\lambda^4,\ a_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2 \end{aligned} \tag{25}$$

3. Numerical results and discussions

3.1 Numerical results

The accuracy of the present formulation based on the hyperbolic shear deformation beam theory is proved by applying it for the static analysis of FG sandwich beams. The following non-dimensional forms available in the

Table 1 Dimensionless center deflections $\bar{w}$ at ((x=L/2, z=0) of the different functionally graded sandwich beams subjected to uniformly distributed load (L/h=5)

		Type A: Hardcore				Type B: Softcore			
k	Theory	1-0-1	2-1-2	1-1-1	1-2-1	1-0-1	2-1-2	1-1-1	1-2-1
Ceramic	Present	3.1652	3.1652	3.1652	3.1652	17.182	17.182	17.182	17.182
	PSDBT*	3.1654	3.1654	3.1654	3.1654	17.183	17.183	17.183	17.183
	TSDBT*	3.1649	3.1649	3.1649	3.1649	17.180	17.180	17.180	17.180
	HSDBT*	3.1633	3.1633	3.1633	3.1633	17.172	17.172	17.172	17.172
	FSDBT*	3.1657	3.1657	3.1657	3.1657	17.185	17.185	17.185	17.185
	CBT*	2.8783	2.8783	2.8783	2.8783	15.625	15.625	15.625	15.625
1	Present	7.8375	6.9340	6.2697	5.4114	4.229	4.6438	5.0464	5.7692
	PSDBT*	7.8352	6.9328	6.2693	5.4122	4.2415	4.6654	5.0726	5.7924
	TSDBT*	7.8317	6.9304	6.2682	5.4125	4.2552	4.6907	5.1025	5.8163
	HSDBT*	7.8355	6.9330	6.2694	5.4122	4.3113	4.6631	5.0697	5.7900
	FSDBT*	7.8197	6.9159	6.2470	5.3807	4.0202	4.3301	4.6495	5.2774
	CBT*	7.4152	6.5604	5.9181	5.0798	3.6157	3.8610	4.1245	4.6605
2	Present	11.347	9.6815	8.3908	6.7572	3.6101	3.9302	4.2684	4.9127
	PSDBT*	11.341	9.6779	8.3893	6.7579	3.6193	3.9504	4.2967	4.9438
	TSDBT*	11.333	9.6730	8.3869	6.7582	3.6297	3.9748	4.3307	4.9787
	HSDBT*	11.342	9.6784	8.3895	6.7578	3.7194	3.9482	4.2936	4.9405
	FSDBT*	11.354	9.6842	8.3831	6.7345	3.4629	3.6822	3.9278	4.4418
	CBT*	10.829	9.2602	8.0074	6.4056	3.1340	3.2955	3.4899	3.9167
5	Present	15.193	13.212	11.231	8.5138	3.2904	3.5123	3.7801	4.3409
	PSDBT*	15.179	13.203	11.227	8.5136	3.2953	3.5279	3.8058	4.3764
	TSDBT*	15.161	13.192	11.221	8.5131	3.3005	3.5469	3.8379	4.4188
	HSDBT*	15.181	13.204	11.228	8.5137	3.4121	3.5262	3.8030	4.3725
	FSDBT*	15.227	13.251	11.249	8.5036	3.1980	3.3333	3.5111	3.9220
	CBT*	14.480	12.726	10.811	8.1409	2.9208	3.0043	3.1354	3.4651
10	Present	16.308	14.636	12.572	9.4060	3.2154	3.3928	3.6259	4.1442
	PSDBT*	16.290	14.623	12.566	9.4050	3.2182	3.4053	3.6488	4.1799
	TSDBT*	16.267	14.608	12.557	9.4034	3.2207	3.4205	3.6776	4.2233
	HSDBT*	16.293	14.625	12.566	9.4051	3.3370	3.4039	3.6464	4.1759
	FSDBT*	16.313	14.693	12.605	9.4038	3.1452	3.2446	3.3924	3.7581
	CBT*	15.386	14.104	12.132	9.0232	2.8865	2.9366	3.0395	3.3266
Metal	Present	17.182	17.182	17.182	17.182	3.1652	3.1652	3.1652	3.1652
	PSDBT*	17.183	17.183	17.183	17.183	3.1654	3.1654	3.1654	3.1654
	TSDBT*	17.180	17.180	17.180	17.180	3.1649	3.1649	3.1649	3.1649
	HSDBT*	17.172	17.172	17.172	17.172	3.1633	3.1633	3.1633	3.1633
	FSDBT*	17.185	17.185	17.185	17.185	3.1657	3.1657	3.1657	3.1657
	CBT*	15.625	15.625	15.625	15.625	2.8783	2.8783	2.8783	2.8783

*Results from Ref. (Sayyad and Ghugal 2019)

PSDBT: parabolic shear deformation beam theory; TSDBT: trigonometric shear deformation beam theory; HSDBT: hyperbolic shear deformation beam theory; FSDBT: first-order shear deformation beam theory; CBT: classical beam theory.

literature are used for the purpose of presenting the numerical results

$$\bar{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right),\ \bar{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2},\frac{h}{2}\right),$$
$$\bar{\tau}_{xz} = \frac{h}{q_0 L}\tau_{xz}(0,0) \tag{26}$$

The material properties used in the present study are: Ceramic (Alumina, Al_2O_3): E_c=380 GPa; v=0.3. Metal (Aluminium, Al): E_m=70 GPa;v=0.3.

For the numerical study, hardcore (Type A) and softcore (Type B) FG sandwich beams.

a- 1–0-1: This scheme is made of two layers. Take h_1=-h/2, h_2=0, h_3=0 and h_4=h/2.

b- 2–1-2: This scheme is made of three layers. Take h_1=-h/2, h_2=-h/10, h_3=h/10 and h_4=h/2.

c- 1–1-1: This scheme is made of three layers. Take h_1=-h/2, h_2=-h/6, h_3=h/6 and h_4=h/2.

d- 1–2-1: This scheme is made of three layers. Take h_1=-h/2, h_2=-h/4, h_3=h/4 and h_4=h/2.

3.2 Discussion on numerical results

In order to prove the validity of the present hyperbolic shear deformation theory, the non-dimensional numerical results according to the present formulation using different shape functions in terms of thickness coordinate are presented in Tables 1 through 3 and plotted in Fig. 2 through 5. Two types of FG sandwich beams (hardcore and softcore) are considered in the present study. Non-dimensional forms for presenting numerical results are given in Eq. (26). From a numerical and graphical results,

Table 2 Dimensionless axial stresses $\bar{\sigma}_x$ at (x=L/2 z=-h/2) of the different functionally graded sandwich beams subjected to uniformly distributed load (L/h=5)

		Type A: Hardcore				Type B: Softcore			
k	Theory	1-0-1	2-1-2	1-1-1	1-2-1	1-0-1	2-1-2	1-1-1	1-2-1
Ceramic	Present	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990
	PSDBT*	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028
	TSDBT*	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061
	HSDBT*	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010
	FSDBT*	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501
	CBT*	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501
1	Present	1.7956	1.5889	1.4339	1.2320	4.8047	5.1455	5.5036	6.2169
	PSDBT*	1.7969	1.5901	1.4352	1.2332	4.8130	5.1563	5.5156	6.2288
	TSDBT*	1.7980	1.5911	1.4361	1.2342	4.8215	5.1677	5.5282	6.2406
	HSDBT*	1.7968	1.5900	1.4351	1.2331	4.7748	5.1552	5.5144	6.2277
	FSDBT*	1.7797	1.5745	1.4204	1.2192	4.7109	5.0305	5.3738	6.0721
	CBT*	1.7797	1.5745	1.4204	1.2192	4.7109	5.0305	5.3738	6.0721
2	Present	2.6183	2.2389	1.9372	1.5517	4.1600	4.3923	4.6625	5.2388
	PSDBT*	2.6198	2.2403	1.9385	1.5531	4.1669	4.4021	4.6743	5.2513
	TSDBT*	2.6209	2.2413	1.9396	1.5542	4.1739	4.4128	4.6871	5.2644
	HSDBT*	2.6197	2.2402	1.9384	1.5530	4.1514	4.4012	4.6731	5.2501
	FSDBT*	2.5991	2.2225	1.9218	1.5374	4.0832	4.2937	4.5469	5.1031
	CBT*	2.5991	2.2225	1.9218	1.5374	4.0832	4.2937	4.5469	5.1031
5	Present	3.4991	3.0721	2.6113	1.9694	3.8678	3.9977	4.1868	4.6412
	PSDBT*	3.5007	3.0734	2.6127	1.9709	3.8732	4.0062	4.1978	4.6543
	TSDBT*	3.5017	3.0743	2.6138	1.9721	3.8785	4.0153	4.2101	4.6686
	HSDBT*	3.5006	3.0733	2.6126	1.9708	3.8781	4.0054	4.1967	4.6530
	FSDBT*	3.4753	3.0545	2.5949	1.9539	3.8055	3.9143	4.0851	4.5147
	CBT*	3.4753	3.0545	2.5949	1.9539	3.8055	3.9143	4.0851	4.5147
10	Present	3.7222	3.4035	2.9284	2.1815	3.8168	3.9024	4.0549	4.4558
	PSDBT*	3.7241	3.4047	2.9298	2.1830	3.8215	3.9100	4.0652	4.4689
	TSDBT*	3.7253	3.4055	2.9307	2.1842	3.8258	3.9181	4.0768	4.4835
	HSDBT*	3.7240	3.4047	2.9297	2.1829	3.8337	3.9093	4.0642	4.4676
	FSDBT*	3.6929	3.3853	2.9118	2.1656	3.7608	3.8261	3.9601	4.3343
	CBT*	3.6929	3.3853	2.9118	2.1656	3.7608	3.8261	3.9601	4.3343
Metal	Present	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990	3.7990
	PSDBT*	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028	3.8028
	TSDBT*	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061	3.8061
	HSDBT*	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010	3.8010
	FSDBT*	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501
	CBT*	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501	3.7501

*Results from Ref. (Sayyad and Ghugal 2019)

the following observations are made.

1- Table 1 show the comparison of non-dimensional transverse deflections for FG sandwich beams subjected to uniform load. It is clear that that the non-dimensional transverse deflections increases with an increase in power law index k for homogenous hardcore (Type A), whereas the opposite nature of variation in the non-dimensional transverse deflections is observed for homogenous softcore (Type B). This leads to the important observation that if core is made of soft material, non-dimensional transverse deflection is less for FG sandwich structures. It is also pointed out that the increase in power law index increases flexibility of the type A sandwich structures, whereas it increases the stiffness of type B sandwich structures.

(Type A: at k=0, beam is fully ceramic, whereas at k=∞, beam is fully metallic. This means, increase in power law index increases the flexibility of type A sandwich beams. Type B: at k=0, beams is fully metallic, whereas at k=∞, beam is fully ceramic. This means an increase in power law index increases the stiffness of type B sandwich beams).

2- It can be observed that the values of the transverse deflections obtained using various shear deformation beam theories (i.e., PSDBT, TSDBT, HSDBT, FSDBT) are in good agreement with the those given by the present theory for all power law index k and for the both models (Type A and B). Due to ignoring the shear deformation effect, FSDBT and CBT underestimates deflection of moderately deep beams(L/h=5). It is also pointed out that for a hardcore FG sandwich beams, transverse deflection is minimum for 1–2–1 and maximum for 1–0–1, whereas for softcore FG sandwich beams, transverse deflection is minimum for 1–0

Table 3 Dimensionless transverse shear stresses $\bar{\tau}_{xz}$ at (x=0 z=0) of the different functionally graded sandwich beams subjected to uniformly distributed load (L/h=5)

		Type A: Hardcore				Type B: Softcore			
k	Theory	1-0-1	2-1-2	1-1-1	1-2-1	1-0-1	2-1-2	1-1-1	1-2-1
Ceramic	Present	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146
	PSDBT*	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305
	TSDBT*	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524
	HSDBT*	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246
	FSDBT*	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922
1	Present	1.0164	0.8963	0.8415	0.7933	0.2858	0.3665	0.4342	0.5279
	PSDBT*	1.0279	0.9075	0.8540	0.8081	0.2984	0.3859	0.4578	0.5531
	TSDBT*	1.0472	0.9261	0.8740	0.8306	0.3152	0.4116	0.4888	0.5853
	HSDBT*	1.0259	0.9060	0.8521	0.8061	0.3413	0.3835	0.4549	0.5501
	FSDBT*	0.8313	0.7306	0.6760	0.6183	0.1531	0.1776	0.1988	0.2336
2	Present	1.2521	1.0073	0.9102	0.8311	0.2201	0.2946	0.3638	0.4714
	PSDBT*	1.2584	1.0140	0.9197	0.8446	0.2294	0.3112	0.3862	0.4987
	TSDBT*	1.2726	1.0278	0.9367	0.8663	0.2420	0.3335	0.4162	0.5341
	HSDBT*	1.2570	1.0128	0.9180	0.8425	0.2823	0.3093	0.3835	0.4954
	FSDBT*	1.0791	0.8713	0.7722	0.6760	0.1245	0.1464	0.1658	0.1988
5	Present	1.7644	1.1806	1.0031	0.8763	0.1697	0.2338	0.2984	0.4109
	PSDBT*	1.7628	1.1784	1.0067	0.8873	0.1757	0.2464	0.3176	0.4383
	TSDBT*	1.7686	1.1822	1.0172	0.9066	0.1838	0.2635	0.3437	0.4747
	HSDBT*	1.7615	1.1780	1.0057	0.8855	0.2324	0.2449	0.3153	0.4352
	FSDBT*	1.5373	1.0791	0.9002	0.7457	0.1049	0.1245	0.1422	0.1730
10	Present	2.2973	1.3068	1.0608	0.9007	0.1509	0.2093	0.2703	0.3819
	PSDBT*	2.3005	1.2991	1.0604	0.9099	0.1554	0.2198	0.2874	0.4084
	TSDBT*	2.3128	1.2967	1.0663	0.9270	0.1615	0.2341	0.3106	0.4439
	HSDBT*	2.2994	1.2989	1.0600	0.9083	0.2116	0.2186	0.2854	0.4053
	FSDBT*	1.9050	1.2103	0.9736	0.7823	0.0979	0.1166	0.1336	0.1634
Metal	Present	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146	0.7146
	PSDBT*	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305	0.7305
	TSDBT*	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524	0.7524
	HSDBT*	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246	0.7246
	FSDBT*	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922	0.4922

*Results from Ref. (Sayyad and Ghugal 2019)

1 and maximum for 1–2–1.

3- Table 2 shows the non-dimensional inplane normal stresses when FG sandwich beams is subjected to uniformly distributed load. It is important to note that in Table 2, the non-dimensional inplane normal stresses at the top surface of the beams is mentioned, those are not maximum in-plane stresses. Examination of the results presented in these table reveals that all the models predict more or less the same values of stresses. It is pointed out that non-dimensional normal stresses are same for fully ceramic and metal beams. This leads to the important conclusion that for single-layer FG beams, non-dimensional normal stresses are independent of Young's modulus of the material. It is also pointed out that the FSDBT and CBT predict same values of non-dimensional in-plane normal stresses for all power law indices.

4- Examination of Fig. 2 reveals that when the top surface is ceramic (rigid) and core material is metal (softcore), variations of the in-plane stresses within each face sheet obey a distinct trend, i.e. the in-plane stresses are reduced from the top surface to towards the mid-plane. Whereas examination of Fig. 3 reveals that when top surface is made up of metal (soft material) and core is of ceramic (hardcore), the stresses have encountered both reduction and growth trends within each individual skin, for the higher values of the volume fraction indices. The in-plane normal stresses are tensile in nature at the top surface and compressive in nature at the bottom surface of the beams.

5- Note that the in-plane normal stresses in beams corresponding to k=0 and k=∞ yield the maximum in-plane normal stresses at the top surfaces, i.e., z=-h/2. This is because, at these values of power law index, materials of beam become fully homogenous and isotropic (either ceramic or metal). According to Table 2, considering in-plane stresses at the top or bottom surfaces of the beams, it is also observed that among four lamination schemes for hardcore FG sandwich beams, maximum inplane normal stresses at the top surface are observed for 1–0–1 and minimum for 1–2–1, whereas for softcore beams, in-plane normal stresses at the top surface are minimum for 1–0–1 and maximum for 1–2–1. However, the maximum values of in-plane stresses through-the-thickness of FG sandwich

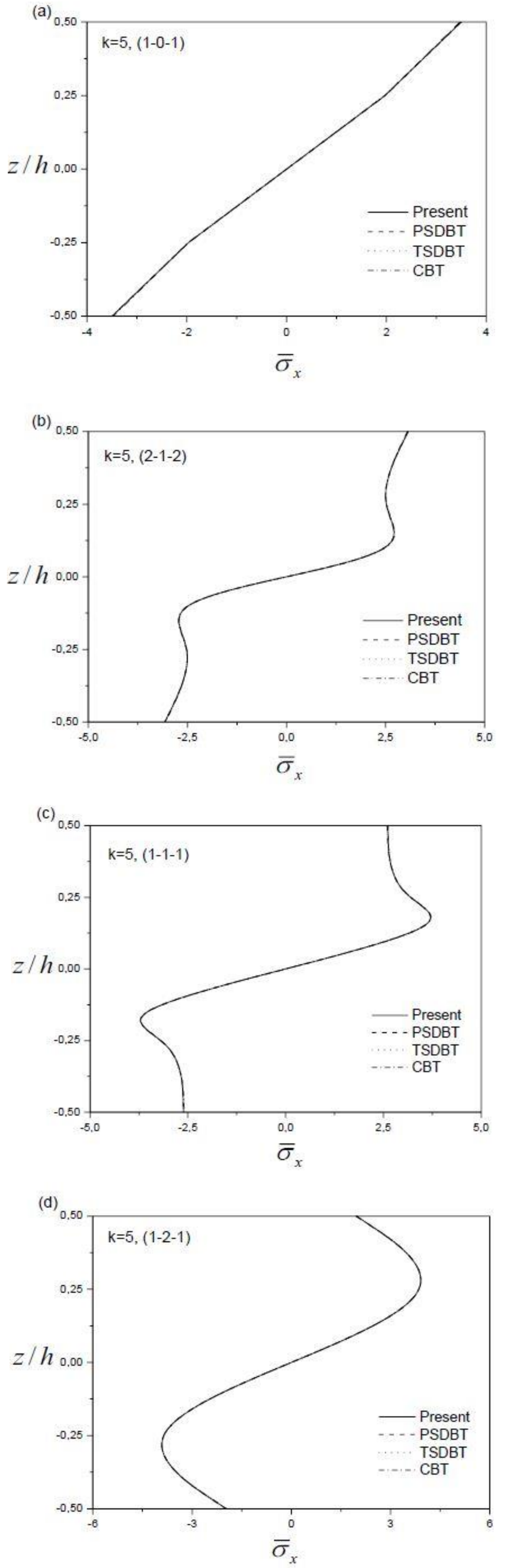

Fig. 2 The variation of the axial stress $\overline{\sigma}_x$ in different types of functionally graded sandwich beams (Type A: hardcore).

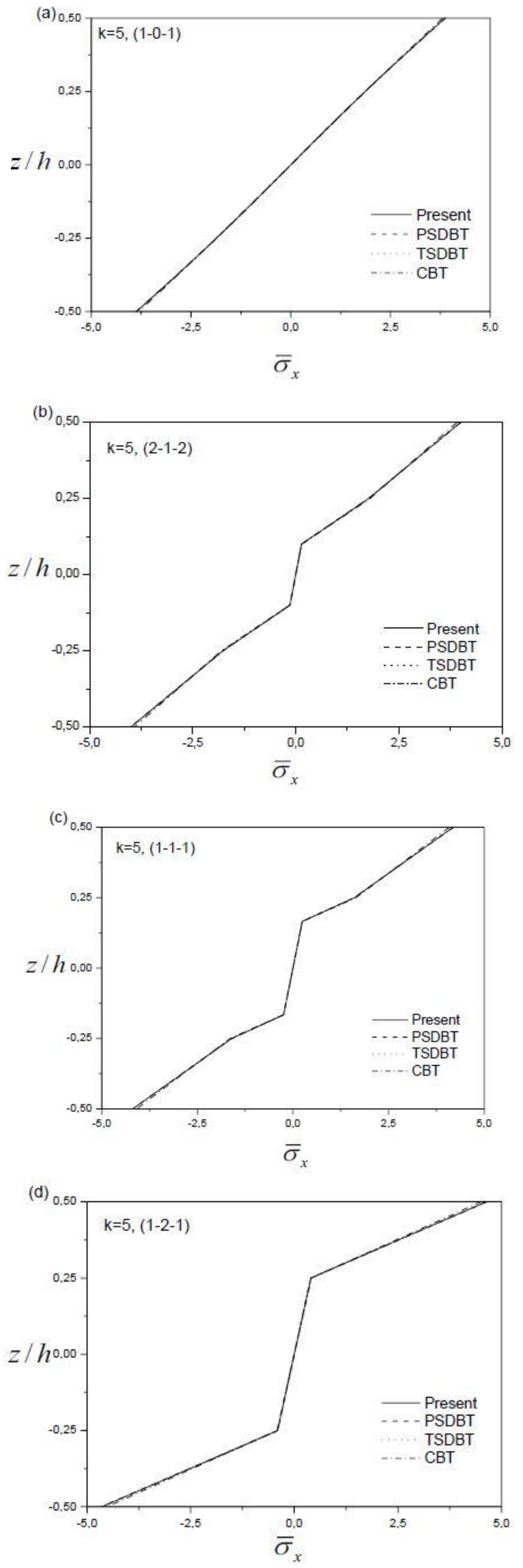

Fig. 3 The variation of the axial stress $\overline{\sigma}_x$ in different types of functionally graded sandwich beams (Type B: softcore)

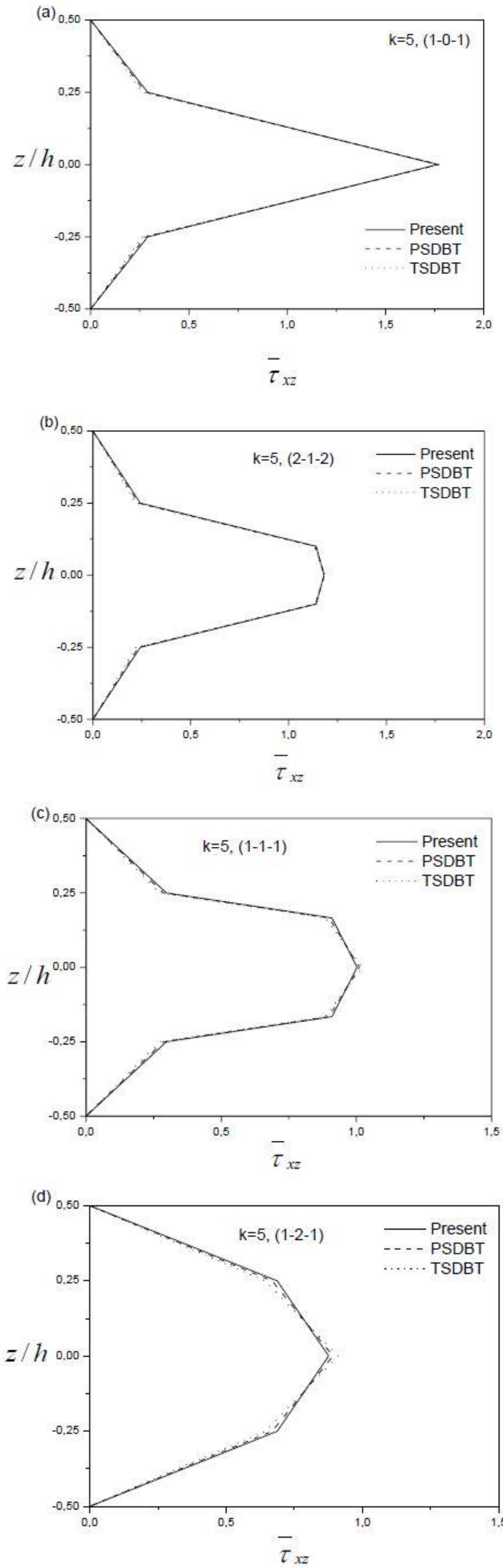

Fig. 4 The variation of the transverse shear stress $\bar{\tau}_{xz}$ in different types of functionally graded sandwich beams (Type A: hardcore)

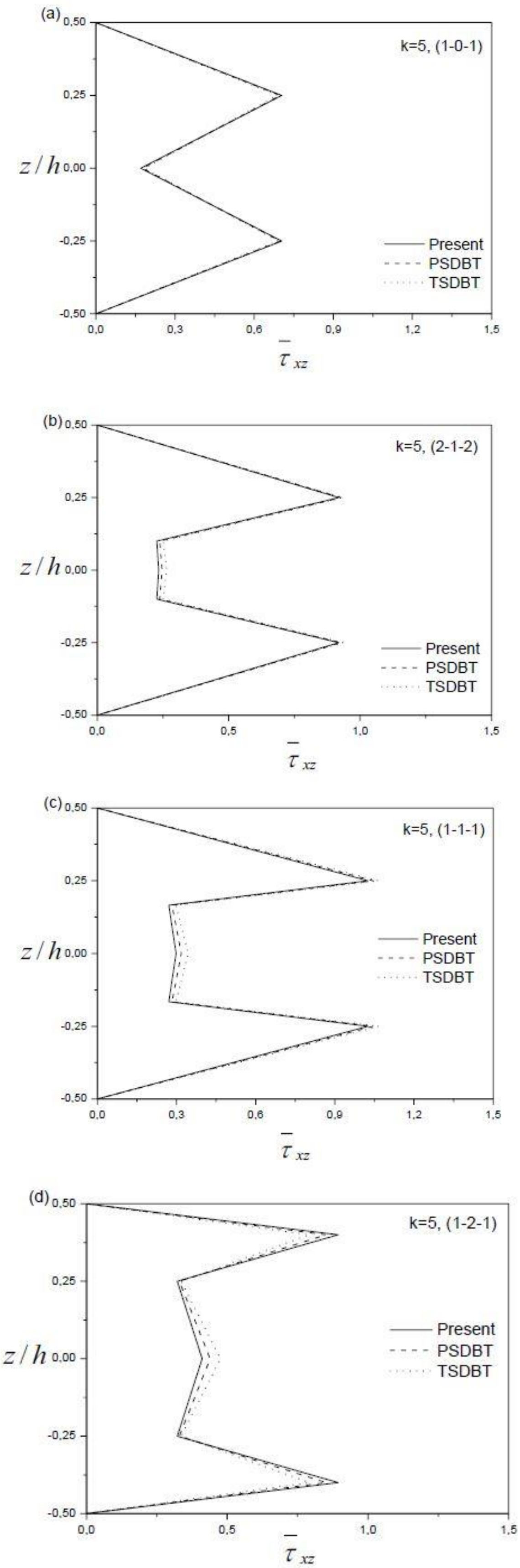

Fig. 5 The variation of the transverse shear stress $\bar{\tau}_{xz}$ in different types of functionally graded sandwich beams (Type B: softcore)

beams are observed in the middle layer, i.e. core. Figure 2 show that in-plane normal stresses are maximum at top or bottom surfaces (z=±h/2) for 1-0-1 and maximum at layer interface for other lamination schemes such as 2–1–2 at ±h/2, 1–1–1 at ±h/6 and 1–2–1 at ±h/4. This trend is observed for both hardcore and softcore beams.

5- Table 3 show the comparison of non-dimensional transverse shear stresses obtained using the present theory. Values are presented at mid-plane, i.e., z=0. It is observed that transverse shear stresses predicted by TSDBT are on higher side compared to other higher order models. FSDBT does not satisfy traction free conditions at top and bottom surfaces. Through-the-thickness distributions of transverse shear stresses for FG sandwich beams are shown in Figures 4 through 5.

6- Through the thickness distributions of transverse shear stresses reveal that for softcore FG sandwich beams, transverse shear stresses are maximum in skins and minimum in core, whereas for hardcore FG sandwich beams and plates, transverse shear stresses are minimum in skins and maximum in core.

4. Conclusions

A New hyperbolic shear deformation theory is developed for the bending analysis of FG sandwich beams. The theory takes account of transverse shear effects and hyperbolic distribution of the transverse shear strains through the thickness of the FG sandwich beam. Hence it is unnecessary to use shear correction factors. The power-law FGM sandwich beams with the FGM facesheet and the Hardcore and the sandwich beams with the FGM facesheet and softcore are considered. Obtained results were presented in figures and tables and compared with references and these demonstrate the accuracy of present theory. It can be said that the present hyperbolic beam theory is much simpler, straightforward and can be easily applied for wide range of problems for static and analyses of FGM sandwich beams and the same is recommended for analyses of FGM sandwich plates.

References

Addou, F.Y., Meradjah, M., Bousahla, A.A., Benachour, A., Bourada, F., Tounsi, A. and Mahmoud, S.R. (2019), “Influences of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation using quasi 3D HSDT”, *Comput. Concrete*, **24**(4), 347-367. https://doi.org/10.12989/cac.2019.24.4.347.

Abualnour, M., Chikh, A., Hebali, H., Kaci, A., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2019), “Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory”, *Comput. Concrete*, **24**(6). 489-498.

Adda Bedia, W., Houari, M.S.A., Bessaim, A., Bousahla, A.A., Tounsi, A., Saeed, T. and Alhodaly, M.S. (2019), “A New Hyperbolic Two-Unknown Beam Model for Bending and Buckling Analysis of a Nonlocal Strain Gradient Nanobeams”, *J. Nano Res.*, **57**, 175-191. https://doi.org/10.4028/www.scientific.net/JNanoR.57.175.

Akbaş, Ş.D. (2014), “Large post-buckling behavior of Timoshenko beams under axial compression loads”, *Struct. Eng. Mech. Int. J.*, **51**(6), 955-971. https://doi.org/10.12989/sem.2014.51.6.955.

Alimirzaei, S., Mohammadimehr, M. and Tounsi, A. (2019), “Nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions”, Struct. Eng. Mech., **71**(5), 485-502. https://doi.org/10.12989/sem.2019.71.5.485.

Anderson, T.A. (2003), “A 3-D elasticity solution for a sandwich composite with functionally graded core subjected to transverse loading by a rigid sphere”, *Compos. Struct.*, **60**(3), 265-274. https://doi.org/10.1016/S0263-8223(03)00013-8.

Batou, B., Nebab, M., Bennai, R., Ait Atmane, H., Tounsi, A. and Bouremana, M. (2019), “Wave dispersion properties in imperfect sigmoid plates using various HSDTs”, *Steel Compos. Struct.*, **33**(5), 699-716. https://doi.org/10.12989/scs.2019.33.5.699.

Belbachir, N., Draiche, K., Bousahla, A.A., Bourada, M., Tounsi, A. and Mahmoud, S.R. (2019), “Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings”, *Steel Compos. Struct.*, **33**(1), 913-924. https://doi.org/10.12989/scs.2019.33.1.081.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), “An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities”, *Earthq. Struct.*, **13**(3), 255-265. https://doi.org/10.12989/eas.2017.13.3.255.

Benferhat, R., Daouadji, T.H., Mansour, M.S. and Hadji, L. (2016c), “Effect of porosity on the bending and free vibration response of functionally graded plates resting on Winkler-Pasternak foundations”, *Eartq. Struct.*, **10**(5), 1429-1449. https://doi.org/10.12989/eas.2016.10.6.1429.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), “A new higher-order shear and normal deformation theory for functionally graded sandwich beams”, *Steel Compos. Struct.*, **19**(3), 521-546. https://doi.org/10.12989/scs.2015.19.3.521.

Berghouti, H., Adda Bedia, E.A., Benkhedda, A. and Tounsi, A. (2019), “Vibration analysis of nonlocal porous nanobeams made of functionally graded material”, *Advan. Nano Res.*, **7**(5), 351-364. https://doi.org/10.12989/anr.2019.7.5.351.

Boukhlif, Z., Bouremana, M., Bourada, F., Bousahla, A.A., Bourada, M., Tounsi, A. and Al-Osta, M.A. (2019), “A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation”, *Steel Compos. Struct.*, **31**(5), 503-516. https://doi.org/10.12989/scs.2019.31.5.503.

Boulefrakh, L., Hebali, H., Chikh, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2019), “The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate”, *Geomech. Eng.*, **18**(2), 161-178. https://doi.org/10.12989/gae.2019.18.2.161.

Bourada, F., Bousahla, A.A., Bourada, M., Azzaz, A., Zinata, A., and Tounsi, A. (2019), “Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory”, *Wind Struct.*, **28**(1), 19-30. https://doi.org/10.12989/was.2019.28.1.019.

Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A. A. and Tounsi, A. (2020), “A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates”, *Smart Struct. Syst.*, (Accepted). https://doi.org/10.12989/sss.2020.25.2.197.

Boutaleb, S., Benrahou, K.H., Bakora, A., Algarni, A., Bousahla, A.A., Tounsi, A., Tounsi, A. and Mahmoud, S.R. (2019), “Dynamic Analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT”, *Advan. Nano Res.*, **7**(3), 189-206. https://doi.org/10.12989/anr.2019.7.3.191.

Chaabane, L.A., Bourada, F., Sekkal, M., Zerouati, S., Zaoui, F.Z., Tounsi, A., Derras, A., Bousahla, A.A. and Tounsi, A. (2019), "Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation", *Struct. Eng. Mech.*, **71**(2), 185-196. https://doi.org/10.12989/sem.2019.71.2.185.

Draiche, K., Bousahla, A.A., Tounsi, A., Alwabli, A. S., Tounsi, A. and Mahmoud, S.R. (2019), "Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory", *Comput. Concrete*, **24**(4), 369-378. https://doi.org/10.12989/cac.2019.24.4.369.

Draoui, A., Zidour, M., Tounsi, A. and Adim, B. (2019), "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)", *J. Nano Res.*, **57**, 117-135. https://doi.org/10.4028/www.scientific.net/JNanoR.57.117.

Fazzolari, F.A. (2014), "A refined dynamic stiffness element for free vibration analysis of cross-ply laminated composite cylindrical and spherical shallow shells", *Compos. Part B*: *Eng*., **62**, 143-158. https://doi.org/10.1016/j.compositesb.2014.02.021.

Fukui, Y. (1991), "Fundamental investigation of functionally gradient material manufacturing system using centrifugal force", *Jpn. Soc. Mech. Eng. Int. J*., **34**(1), 144-148. https://doi.org/10.1299/jsmec1988.34.144.

Hellal, H., Bourada, M., Hebali, H., Bourda, F., Tounsi, A., Bousahla, A.A. and Mahmour, S.R. (2019), "Dynamic and stability analysis of functionally graded material sandwich plates in hygro-thermal environment using a simple higher shear deformation theory", *J. Sandwich Struct. Mat*., (Accepted), https://doi.org/10.1177/1099636219845841.

Hussain, M., Naeem, M.N., Tounsi, A. and Taj, M. (2019), "Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity", *Advan. Nano Res.*, **7**(6), 431-442. https://doi.org/10.12989/anr.2019.7.6.431.

Karami, B., Janghorban, M. and Tounsi, A. (2019a), "Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation", *Struct. Eng. Mech.*, **7**(1), 55-66. https://doi.org/10.12989/sem.2019.70.1.055.

Karami, B., Shahsavari, D., Janghorban, M. and Tounsi, A. (2019b), "Resonance behavior of functionally graded polymer composite nanoplates reinforced with grapheme nanoplatelets", *Int. J. Mecha. Sci.*, **156**, 94-105. https://doi.org/10.1016/j.ijmecsci.2019.03.036.

Karami, B., Janghorban, M. and Tounsi, A. (2019c), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, **35**(4), 1297-1316. https://doi.org/10.1007/s00366-018-0664-9.

Karami, B., Janghorban, M. and Tounsi, A. (2019d), "On exact wave propagation analysis of triclinic material using three dimensional bi-Helmholtz gradient plate model", *Struct. Eng. Mech.*, **69**(5), 487-497. https://doi.org/10.12989/sem.2019.69.5.487.

Karami, B., Janghorban, M. and Tounsi, A. (2019e), "On pre stressed functionally graded anisotropic nanoshell in magnetic field", *J. Brazil. Soc. Mech. Sci. Eng.*, **41**(11), 495. https://doi.org/10.1007/s40430-019-1996-0.

Khiloun, M., Bousahla, A.A., Kaci, A., Bessaim, A., Tounsi, A. and Mahmoud, S.R. (2019), "Analytical modeling of bending and vibration of thick advanced composite plates using a four-variable quasi 3D HSDT", *Eng. Comput.*, **1-15**, https://doi.org/10.1007/s00366-019-00732-1.

Koizumi, M. (1997), "FGM activities in Japan", *Compos. Part B: Eng*., **28**(1-2), 1-4. https://doi.org/10.1016/S1359-8368(96)00016-9.

Mahmoud, S.R. and Tounsi, A. (2019), "On the stability of isotropic and composite thick plates", *Steel Compos. Struct.*, **33**(4), 551-568. https://doi.org/10.12989/scs.2019.33.4.551.

Mahmoudi, A., Benyoucef, S., Tounsi, A., Benacour, A., Adda Bedia, E.A. and Mahmoud, S.R. (2019), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *J. Sandwich Struct. Mat.*, **21**(6), 1906-1926. https://doi.org/10.1177%2F1099636217727577.

Medani, M., Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate", *Steel Compos. Struct.*, **32**(5), 595-610. https://doi.org/10.12989/scs.2019.32.5.595.

Meksi, R., Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2019), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater*., **21**(2), 727-757. https://doi.org/10.1177%2F1099636217698443.

Mirza, S.B., Yaqoob, Y. and Hasan Mohammad, K. (2018),"Analysis of Laminated and FGM Beams using Various Theories", *IOP Confer. Ser. Mat. Sci. Eng.*, **404**(1), 012030.

Pan, E. and Han, F. (2005), "Exact solution for functionally graded and layered magneto-electro-elastic plates", *Int. J. Eng. Sci.*, **43**(3-4), 321-339. https://doi.org/10.1016/j.ijengsci.2004.09.006.

Reddy, J.N. (2004), "Mechanics of laminated composite plates and shells, theory and analysis", CRC Press. Ohio. U.S.A.

Sahla, M., Saidi, H., Draiche, K., Bousahla, A.A., Bourada, F. and Tounsi, A. (2019), "Free vibration analysis of angle-ply laminated composite and soft core sandwich plates", *Steel Compos. Struct*., **33**(5), 663-679. https://doi.org/10.12989/scs.2019.33.5.663.

Salah, F., Boucham, B., Bourada, F., Benzair, A., Bousahla, A.A. and Tounsi, A. (2019), "Investigation of thermal buckling properties of ceramic-metal FGM sandwich plates using 2D integral plate model", *Steel Compos. Struct.*, **33**(6), 805-822. https://doi.org/10.12989/scs.2019.33.6.805.

Semmah, A., Heireche, H., Bousahla, A.A. and Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Advan. Nano Res.*, **7**(2), 89-98. https://doi.org/10.12989/anr.2019.7.2.089.

Sayyad, A.S. and Ghugal, Y.M. (2019), "A unified five-degree-of freedom theory for the bending analysis of softcore and hardcore functionally graded sandwich beams and plates", *J. Sandwich Struct. Mat*., **2019**, https://doi.org/10.1177%2F1099636219840980.

Simsek, M. (2009), "Static analysis of a functionally graded beam under a uniformly distributed load by Ritz method", *Int. J. Eng. Appl. Sci.*, **1**(3),1-11.

Thai, H.T. and Vo, T.P. (2012), "Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories", *Int. J. Mech. Sci*., **62**(1), 57-66. https://doi.org/10.1016/j.ijmecsci.2012.05.014.

Tlidji, Y., Zidour, M., Draiche, K., Safa, A., Bourada, M., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2019), "Vibration analysis of different material distributions of functionally graded microbeam", *Struct. Eng. Mech*., **69**(6), 637-649. https://doi.org/10.12989/sem.2019.69.6.637.

Venkataraman, S. and Sankar, B.V. (2001), "Analysis of sandwich beams with functionally graded core", *Proceedings of the 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Struct. Dyn., Mat. Conference, Seattle, AIAA*, **1281**, 16-19.

Viola, E., Rossetti, L., Fantuzzi, N. and Tornabene, F. (2014), "Static analysis of functionally graded conical shells and panels using the generalized unconstrained third order theory coupled with the stress recovery", *Compos. Struct.*, **112**, 44-65. https://doi.org/10.1016/j.compstruct.2014.01.039.

Yamanouchi, M., Koizumi, M. and Shiota, I. (1990), "Functionally gradient materials", *Proceedings of the 1st International Symposium on Functionally Gradient Materials*.

Zaoui, F.Z., Ouinas, D. and Tounsi, A. (2019), "New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations", *Compos. Part B*, **159**, 231-247. https://doi.org/10.1016/j.compositesb.2018.09.051.

Zarga, D., Tounsi, A., Bousahla, A.A., Bourada, F. and Mahmoud, S.R. (2019), "Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory", *Steel Compos. Struct*., **32**(3), 389-410. https://doi.org/10.12989/scs.2019.32.3.389.

CC