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Expected damage for SDOF systems in soft soil sites: an energy-based approach

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Abstract. The seismic response of structures to strong ground motions is a complex problem that has been studied for decades. However, most of current seismic regulations do not assess the potential level of damage that a structure may undergo during a strong earthquake. This will happen in spite that the design objectives for any structural system are formulated in terms of acceptable levels of damage. In this article, we analyze the expected damage in single-degree-of-freedom systems subjected to long-duration ground motions generated in soft soil sites, such as those located in the lakebed of Mexico City. An energy-based methodology is formulated, under the consideration of input energy as the basis for the evaluation process, to estimate expected damage. The results of the proposed methodology are validated with damage curves established directly with nonlinear dynamic analyses.

Keywords: seismic energy; expected damage; energy-based approach

1. Introduction

The accepted worldwide approach for seismic design states that structural damage should be minimized for frequent low-intensity events, and that collapse should be avoided for rare high-intensity ones. To achieve this, seismic regulations usually establish, for the design ground motion(s), acceptance criteria in terms of strength and stiffness, in order to satisfy the design objective(s). Whereas few seismic regulations establish criteria for multiple performance levels, most formulate criteria for collapse prevention with no explicit consideration to continuous operation and loss control.

As it has been evidenced after important earthquakes worldwide (Mexico 1985, Northridge 1994, Kobe 1995, Taiwan 1999, Chile 2010, Japan 2011, Ecuador 2016, Mexico 2017, among others) economic losses due to seismic damage are excessive for a wide part of society, including the insurance, real estate and industrial sectors. Although the design for life safety is considered, production losses, downtime and indirect costs (repair, insurance), among others, are not. Because the improvement of current design criteria requires the explicit quantification of structural damage, there is a need to formulate practical and simple methodologies that can be implemented within a practical setting.

After extensive research, it has been understood that the effects of cumulative damage are relevant in sites capable of generating long-duration ground motions. Among many others, Akiyama (2003), Bommer *et al*. (2004), Cornell (1997), Hancock and Bommer (2007), Iervolino *et al*. (2006), Oyarzo-Vera and Chouw (2008), Raghunandan and Liel (2013), Chandramohan *et al*. (2016); found that cumulative damage has a strong relationship with ground motion duration, even if peak demands (usually expressed in terms of displacement and acceleration) are not affected by this parameter. Particularly, short-duration ground motions tend to have a relative low destructive potential, even when exhibiting large ground acceleration (Eads *et al*. 2013). Likewise, it has been found that conventional single-parameter measures, usually used to characterize the severity of ground motions, do not describe their destructive potential (Takizawa and Jennings 1980; Shinozuka *et al*. 2000, Villaverde 2007). Despite this, current seismic regulations do not consider duration as a relevant design parameter, and very few take it into account, indirectly, during the definition of accelerograms to carry out performance-based evaluations (ASCE 2010, PEER 2010, FEMA 2012).

Cumulative damage can be evaluated by using the hysteretic plastic energy dissipated by the structural system during the entire duration of the seismic record. Akiyama and Takahashi (1992), Fajfar (1992), Chai and Fajfar (2000) studied cumulative damage in structures using an input energy approach. Likewise, Malhotra (2002), Kunnath and Chai (2004), Chai (2005), Teran-Gilmore and Jirsa (2005), Arroyo and Ordaz (2006), Kalkan and Kunnath (2007), Choi and Kim (2009), Leelataviwat *et al*. (2009), Benavent-Climent (2011), Mollaioli *et al*. (2011), Mollaioli and Bosi (2012), and Donaire-Ávila *et al*. (2017), among others, extended this concept by incorporating the effect of low-cycle fatigue into duration-dependent inelastic design spectra. Hancock and Bommer (2005); Kashani *et al*. (2017); studied the influence of strong motion duration and number of plastic cycles on the behavior of structural

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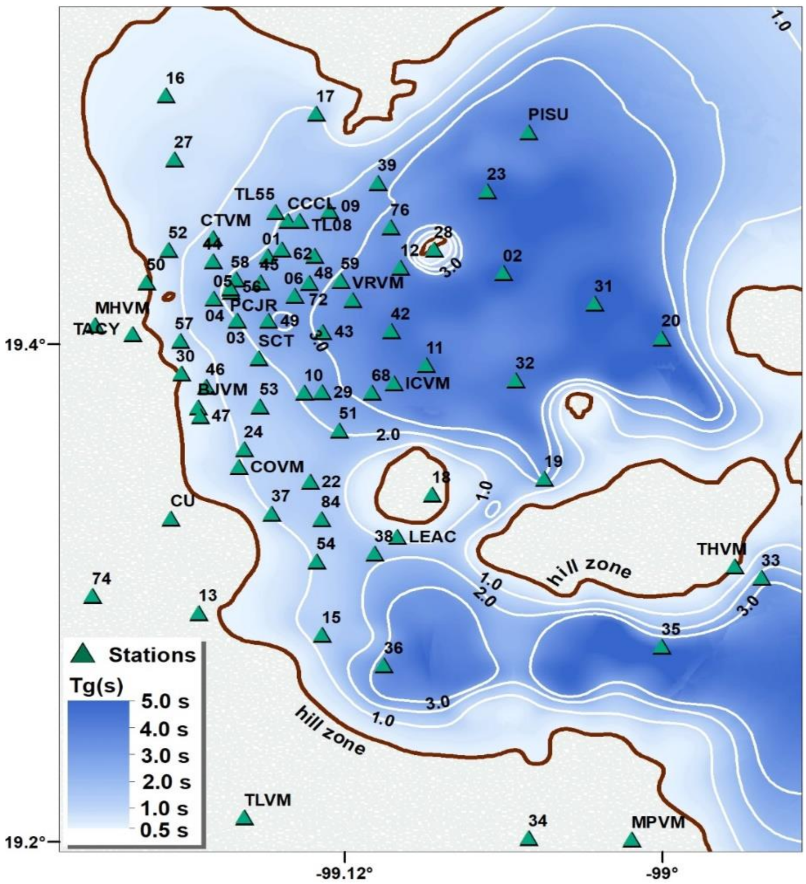


Fig. 1 Accelerometric stations under consideration and dominant soil periods for Mexico City lakebed-zone

systems; and found that it is relevant for the seismic design of structures located at soft soil sites. An alternative to consider the duration of the event is an energy-based approach (Mollaioli *et al*. 2011, Goertz *et al*. 2018, Quinde 2019).

The influence of ground motion duration on cumulative structural damage is particularly important in very soft soils, similar to those located in the lakebed of Mexico City. Structural damage and collapse of structures located there are significantly influenced by the frequency and energy contents of the ground motions (Reinoso and Ordaz 1999; Arroyo and Ordaz 2007; Bojorquez *et al*. 2009). This has been corroborated after the occurrence of the September 19, 1985 and 2017 earthquakes (Ordaz *et al*. 1988; Singh *et al*. 2018; Quinde 2019).

In this paper, damage curves are studied to analyze structural damage in single-degree-of-freedom (SDOF) systems subjected to long duration narrow-banded motions. Based on this, an energy-based methodology to estimate expected damage in SDOF systems is developed and discussed. Elastic input energy is considered, within this setting, the basis for the evaluation process. Finally, the results obtained with the proposed methodology are compared with damage curves established directly with nonlinear dynamic analyses.

2. Accelerometric stations and ground vibration periods

Fig. 1 shows the location, within Mexico City, of the 78 accelerometric stations under consideration in this study. Also shown are the ground periods (T_g). Note that the soil properties vary significantly with short distances within the lakebed zone. One technique to measure the soil dominant period was presented by Reinoso and Ordaz (1999). For each instrumented site (Fig. 1) Fourier spectral ratios are computed with respect to the average Fourier spectra at hill zone sites, and the dominant soil period is that corresponding to the highest amplification. These soil periods are consistent with the those computed from geotechnical information and ambient noise vibration.

Because of the wide range of soil periods, Reinoso (2002), classified sites at Mexico City into six groups: Groups 1 and 2 correspond to sites having T_g smaller than 1s; Groups 3, 4 and 5 cover period ranges of 1.0-1.8, 1.8-2.5 and 2.5-3.8s, respectively. Group 6 covers sites having T_g longer than 3.8s (T_g can reach values as high as 5.5s). Groups 1 and 2 were not considered in the studies reported in this article.

3. Consideration of cumulative damage

Current seismic design regulations usually assess structural damage through maximum displacement demands (Chai 2005). The disadvantage of using a spectral acceleration, and its corresponding displacement, as the main design parameter is that ground motions having

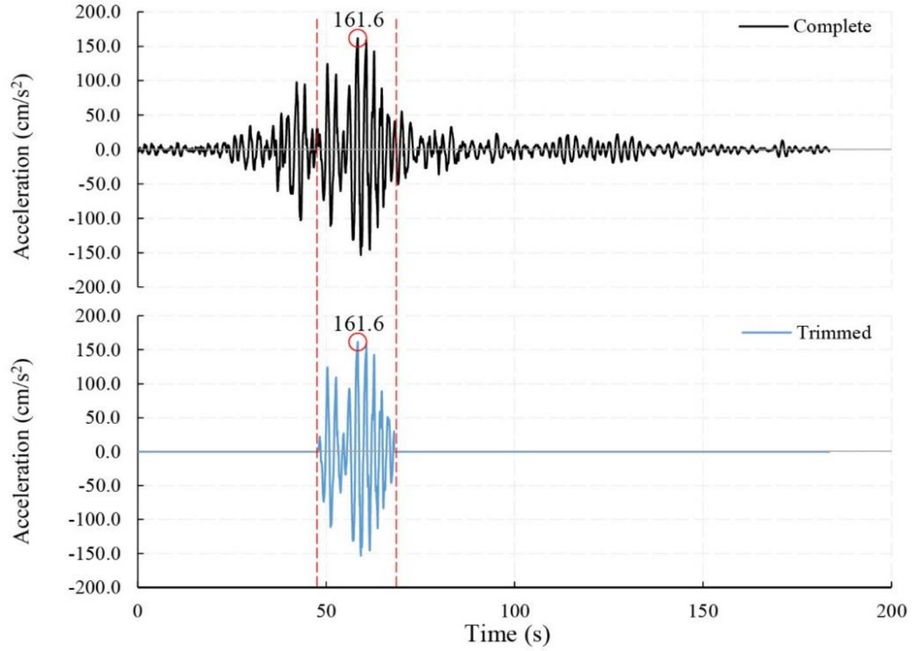


Fig. 2 Accelerometric record for the September 19, 1985 earthquake, for SCT site (T_g=1.9s) in Mexico City lakebed. Complete record (upper location) and trimmed record (bottom)

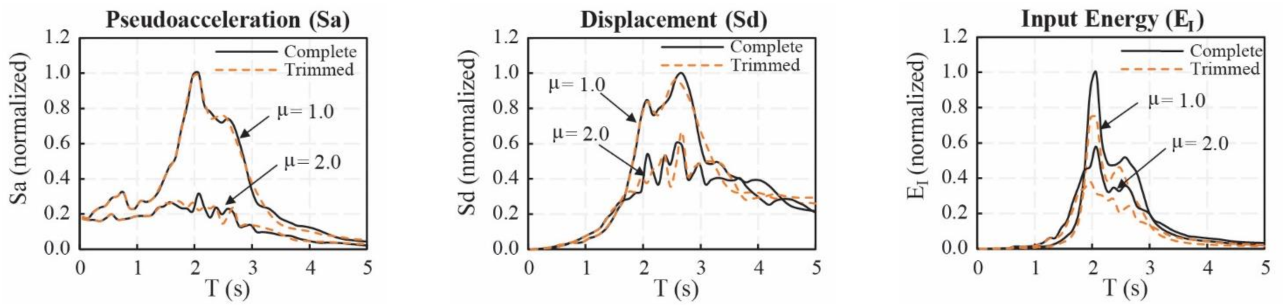


Fig. 3 Pseudoacceleration response spectra (Sa), spectral displacement (Sd) and input energy (E_I). The spectra have been computed with the complete and trimmed records of Fig. 2, for elastic (μ=1.0) and nonlinear (μ=2.0) responses

similar peak spectral responses may exhibit significantly different damage potential. This is particularly true for long-duration ground motions (Hancock and Bommer 2005, Hancock and Bommer 2006). Excessive deterioration due to the accumulation of severe plastic deformations may cause the structure to fail at deformation levels significantly smaller than those corresponding to monotonic loading. This phenomenon is termed *low-cycle fatigue*.

Fig. 2 shows the complete and trimmed records of the motion recorded in the EW-direction at the SCT site (T_g=1.9s) during the September 19, 1985 earthquake. The trimmed record, delimited in the figure by the broken lines, corresponds to the 20 seconds of the most intense phase of the motion.

Fig. 3 shows pseudoacceleration (Sa), displacement (Sd) and input energy (E_I) spectra for 5% of critical damping. Elastic spectra and spectra for a ductility μ=2.0 are shown for both the complete and trimmed records. All spectra are normalized by their peak value. Note that while the Sa and Sd demands are similar for both records, the input energy, E_I, exhibits noticeable differences, particularly for systems having a period close to that of the ground motion. Fig. 4 shows the hysteretic behavior of SDOF systems subjected to both, the complete and trimmed, records. Two hysteretic behaviors are considered: a) elasto-perfectly plastic (EPP); and, b) stiffness degrading (DMP). Although the maximum displacement demand is similar for both records, the number of cycles is larger (for both hysteretic behaviors) when the complete record is used.

The results summarized in Figs. 2 to 4 show that, in some cases, a peak demand can lead to an underestimation of damage. When analyzing a structure located in sites capable of generating long-duration ground motions, it is better to incorporate into the design process, parameters (such as duration and energy demands), that can consider explicitly the effects of cumulative damage (Fajfar 1992, Bernal 1993, Teran-Gilmore 1996, Chai 2005, Arroyo and Ordaz 2006, Bojorquez *et al*. 2009).

4. Expected damage curves for soft soils

The procedures used to calculate fragility curves that allow for the estimation of structural damage is discussed in

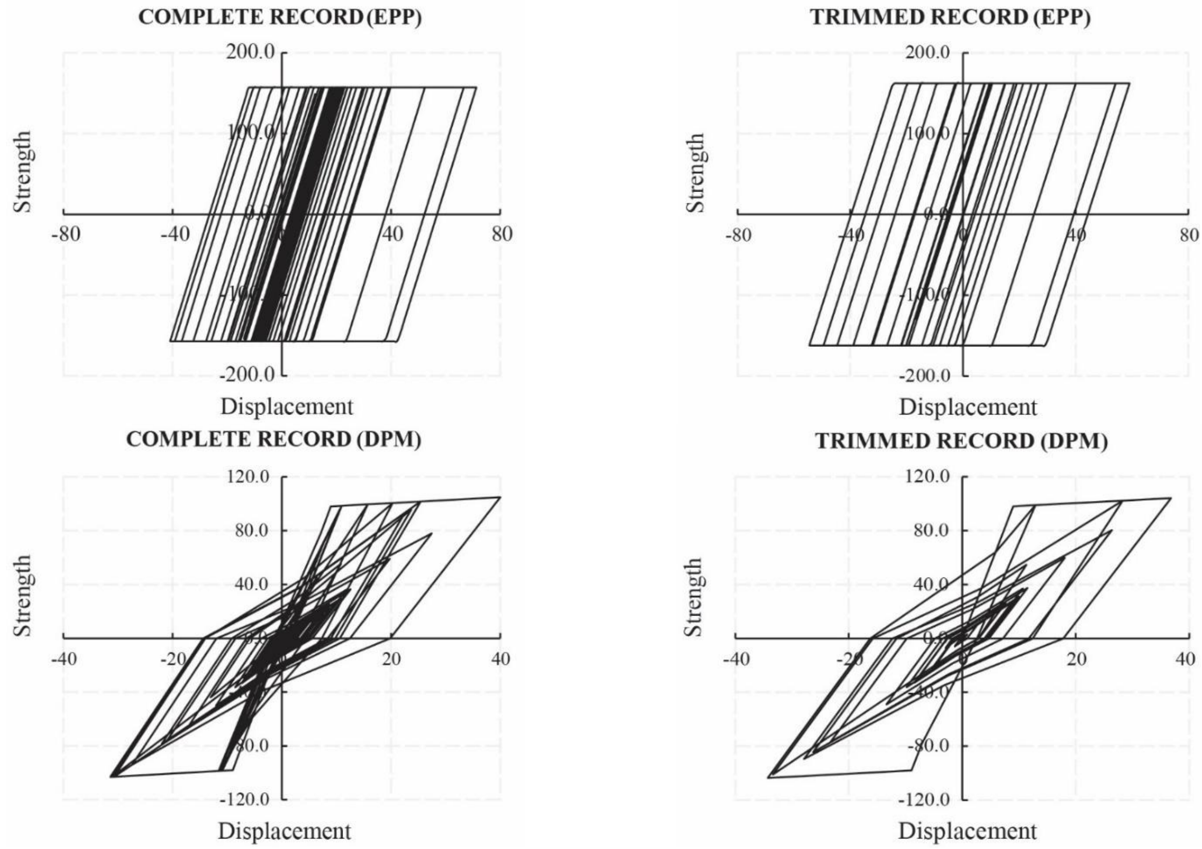


Fig. 4 Cyclic behavior for the complete and trimmed records of Fig. 2. The first row corresponds to elasto-perfectly-plastic behavior, the second one to stiffness degrading behavior

the following section. The single-degree-of-freedom (SDOF) systems and nonlinear dynamic analysis methodologies used to evaluate their structural performance for different intensities are described. Likewise, the damage models and intensity measures used in the computation of the expected damage curves are presented.

4.1 Single-Degree-Of-Freedom systems

A damage analysis was performed for SDOF systems having elasto-perfectly-plastic behavior, 5% of critical damping, and 70 different structural periods, ranging from 0.05 to 5.0s. The lateral strength of the systems was established, according to the design requirements of the Mexico City Building Code, for specific values of the seismic behavior factor (Q). In terms of defining the design spectra, Q can be considered the maximum ductility demand, in such a manner that Q=1 implies elastic behavior.

4.2 Nonlinear dynamic analysis

Several approaches can be used to obtain relevant data to establish a fragility function. A widely used one is the incremental dynamic analysis (IDA), in which a ground motion set is repeatedly and linearly scaled to establish the dependence, up to the collapse, of a relevant seismic demand with respect to an intensity measure (IM) of the motion (Vamvatsikos and Cornell 2002, FEMA 2009). A second approach is the multiple stripe analysis (MSA), where the computation is made for specific sets of the IM, each of which has a unique set of ground motions (Jalayer, 2003).

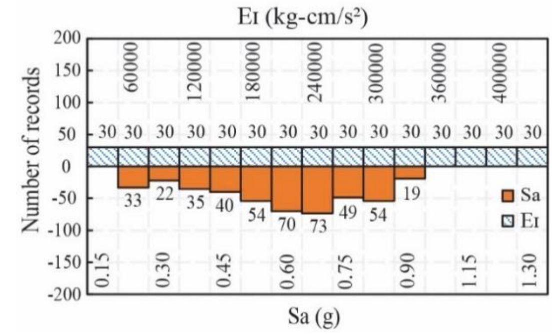
4.2.1 Incremental dynamic analysis

An IDA aims at estimating the structural performance of a system by linearly scaling each seismic record under the consideration of multiple intensity levels, until the structure reaches a state of collapse. The structural performance is studied, in terms of a previously defined IM, for each scaled record (Villaverde 2007). In this way, the seismic demands and overall performance of the system can be assessed simultaneously.

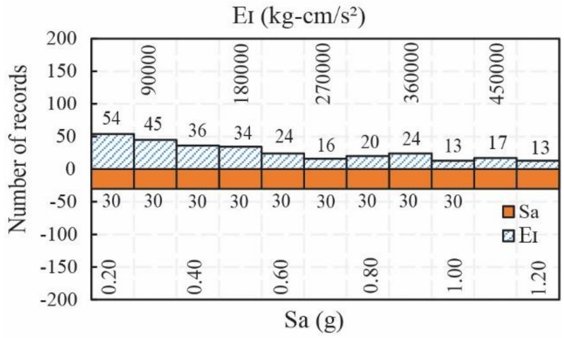
Vamvatsikos and Cornell (2002, 2004) proposed a detailed algorithm to perform efficiently an IDA. Likewise, they explain how to use its results to assess the structural performance and reliability of a system. For soft soils, similar to those located in the lakebed of Mexico City, the choice of a scaling methodology is essential for a pertinent assessment of structural performance, since the characteristics of the ground motions generated there strongly depend upon their intensity. Under these circumstances, the scaling method used should be seismological-based (Quiroz-Ramírez *et al*. 2014; Quinde 2019).

4.2.2 Multiple stripe analysis

The MSA provides statistical information on the



(a) Using input energy as target IM



(b) Using pseudoacceleration as target IM

Fig. 5 Histograms of the frequency of records for each intensity measure

response and performance of the structure for a wide range of ground motion intensities. Unlike the IDA, an MSA is constructed from a set of intensity strips (Jalayer 2003), and is used when the properties of the ground motions significantly change with their intensity (Bradley 2010, Sarieddine and Lin 2013, Baker 2015). Within this context, the choice of seismic records is not arbitrary and has a noticeable influence on the quantification of structural performance. The MSA provides an improved picture of how the general trend and dispersion of the response evolves with a gradually increasing ground motion intensity.

4.3 Intensity measures

A fundamental step in damage analysis is to express damage in probability terms as a function of a properly selected intensity measure (IM). Any ground motion parameter can be used as IM; among others: peak ground acceleration (PGA), pseudoacceleration (Sa), displacement (Sd), Input energy (E_I), and duration. Also, it is possible to use a combination of parameters to improve the description of the ground motion characteristics at different demand levels. For instance, Ibarra *et al*. (2005), proposed a methodology to evaluate the capacity and collapse of a structure, by considering a measure of relative intensity $S_a(T_1)/g/\gamma$ (where T_1 is the fundamental period of the structure; g, the acceleration of gravity; $\gamma = V_y/W$; V_y, the base shear of the structure at first yield; and W, the weight of the structure). Likewise, Baker and Cornell (2005), proposed the use of a vector-valued IM defined in terms of a spectral acceleration and the parameter ε (that represents a measure of the difference, for a structural period of interest, between the spectral acceleration of a record and the average of a ground motion prediction equation).

In this article, expected damage was evaluated under the consideration of two IMs. Particularly, Sa and E_I were considered, at different times, the target IM. A histogram offers an interesting perspective to analyze a damage distribution with respect to the IM. Fig. 5 shows histograms for both IMs, with the aim of showing the changes in the intensity distribution. The intensities were simulated for the SCT station. Fig. 5(a) shows a histogram established by considering E_I the target IM (mesh rectangles), using 30 seismic records for each intensity level. The corresponding histogram in terms of Sa, using the same 30 seismic records, is shown in the same figure (filled rectangles). Likewise, Fig. 5(b) shows a similar concept but considering Sa the target IM.

As can be seen in Fig. 5(a), while the bar sizes in terms of Sa tend to be taller for intensities between 0.50 and 0.75 g, for extreme intensities (large and small values of Sa), the number of occurrences decreases. As shown in Fig. 5(b), the distribution of occurrences in terms of E_I shows a different tendency when Sa is considered the target IM. Particularly, a larger number of occurrences can be observed for small values of E_I. This change in the histogram behavior leads to the expected damage curves being different and dependent on the selected IM.

4.4 Damage models

Even though several researchers have used $NE_{H\mu}$ to develop recommendations and formulations for earthquake-resistant design (Scribner and Wight 1980, Darwin and Nmai 1986, Krawinkler and Nassar 1992) it has been repeatedly observed that are several issues may affect the structural damage estimation. As mentioned by Terán and Jirsa (2005), one issue to consider during design is that the plastic energy dissipated up to the failure of an element may change significantly with the amplitude of the plastic cycles. In this article, damage indexes that aim at assessing global damage through $NE_{H\mu}$ without going into details regarding how this energy is dissipated, are used. This allows for a fairly simple yet reasonable estimation of structural damage, in a setting in which a normalized index, ranging from 0 to 1, allows for a direct comparison of significantly different structural responses.

Two damage indices were used. The Park and Ang damage index (Park and Ang 1985) has been widely used due to its large experimental support. It has been formulated in terms of a linear combination of the maximum displacement and the plastic hysteretic energy demand

$$ID_{PA} = \frac{\mu_{\max}}{\mu_u} + \beta \frac{NE_{H\mu}}{\mu_u} \tag{1}$$

where $\mu_{\max}$ is the maximum ductility demand, defined as the ratio of the maximum and yielding displacements; β, a parameter that characterizes the stability of the hysteretic cycle (0.15 is considered for ductile behavior); and μu, the ultimate ductility the system can undergo when subjected to monotonic deformation. $NE_{H\mu}$ is the normalized hysteretic

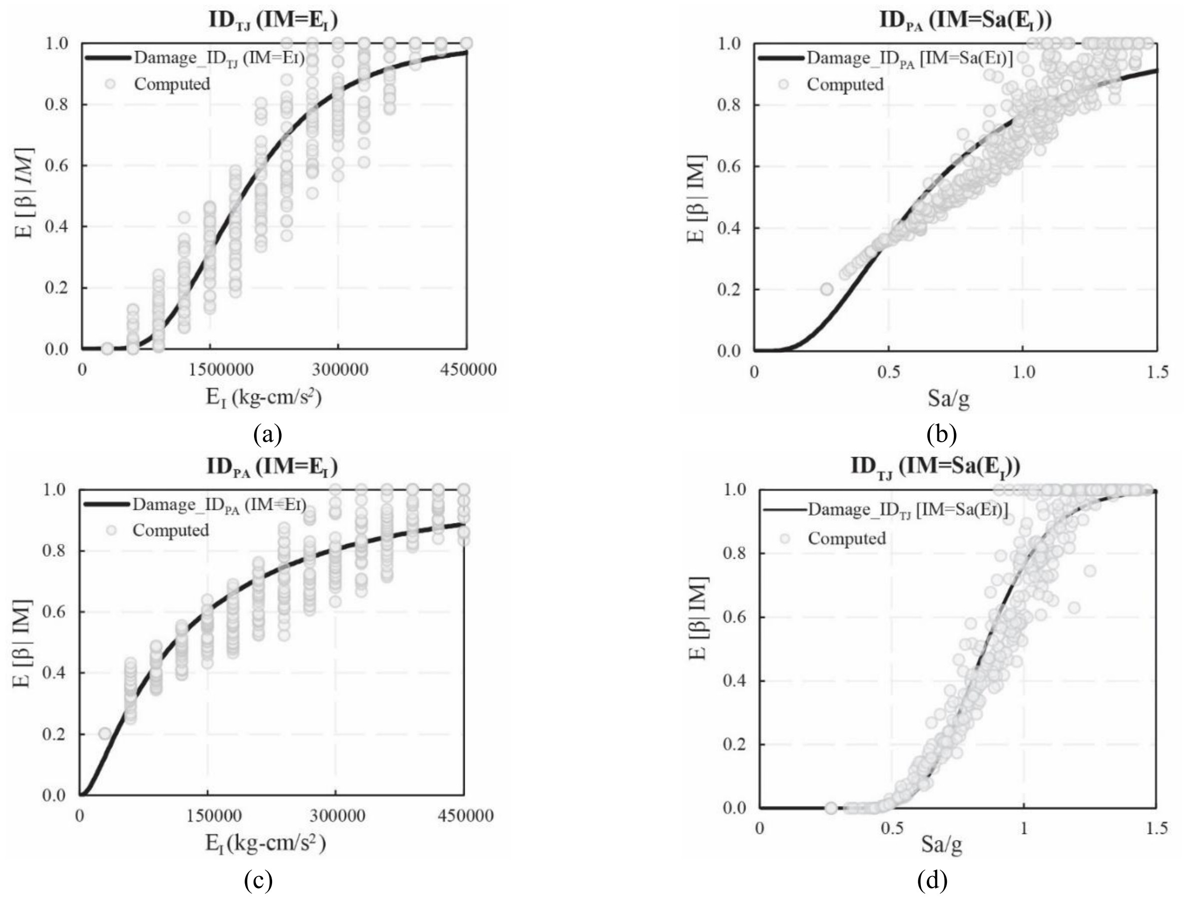


Fig. 6 Damage curves for SCT site (T_g=1.9s). (a) and (c) for the Expected damage using E_I as objective IM, using (a) the damage models of Terán and Jirsa (ID_{TJ}) and (b) Park and Ang (ID_{PA}); expected damage using Sa as intensity measure, computed for the same damage models used in figures (a) and (c)

energy

$$NE_{H\mu} = \frac{E_{H\mu}}{F_y x_y} \quad (2)$$

where $E_{H\mu}$ is the hysteretic plastic energy dissipated during the ground motion; and F_y and x_y, the strength and displacement at first yield, respectively. For an elasto-perfectly-plastic system, $NE_{H\mu}$ is equal to the sum of all plastic displacements developed by the system during the ground motion, normalized by x_y.

Terán-Gilmore and Jirsa (2005) established a simple damage model from the linear cumulative damage theory. For this model, the $NE_{H\mu}$ that the system can accommodate up to failure is given by

$$NE_{H\mu u} = \frac{1.5}{(2-b)}(\mu_u - 1) \quad (3)$$

where b is a parameter that characterizes the stability of the hysteretic cycle. b of 1.5 corresponds to ductile members, and thus, has a close correspondence to β of 0.15 for the Park and Ang index. The demand/capacity ratio in terms of normalized hysteretic energy can be used to establish an energy-based damage index as follows

$$ID_{TJ} = \frac{NE_{H\mu}}{NE_{H\mu u}} \quad (4)$$

Fig. 6 shows damage curves for the SCT site (T_g=1.9s), established with MSAs of a SDOF system with T=2.0s and designed for Q=3.0. 40 records were considered for each one of 15 intensity stripes. First, E_I was used as the target IM. While the black continuous line plots a beta function adjusted to the mean damage, a circle corresponds to the damage estimated with a nonlinear dynamic analysis for one ground motion. Fig. 6(a) and 6(c) were established, for the damage indexes of Teran and Jirsa (ID_{TJ}) and Park and Ang (ID_{PA}), respectively, by using E_I as the IM. The IM under consideration in Fig. 6(b) and 6(d) is the *Sa* associated to the records used to establish Fig. 6(a) and 6(c).

Fig. 7 shows damage curves similar to those of Fig. 6, but now *Sa* is considered the target intensity measure. While Fig. 7(a) and 7(c) are formulated in terms of *Sa*, Fig. 7(b) and 7(d) consider E_I. Fig. 8 compares all damage curves plotted in Figs. 6 and 7. Note that the IM in each curve has been normalized by its respective peak value, so as to allow a direct comparison. Fig. 8(a) shows that if ID_{TJ} is considered to assess damage, the use of E_I as *IM* results in larger damage up to values of $E[\beta|IM]$=0.8. Although significant differences can be observed when E_I and Sa are used as *IM* for $E[\beta|IM]$<0.6; for the assessment of collapse, $E[\beta|IM]$>0.8, both E_I and *Sa* yield similar results (collapse is associated to a similar value in terms of the normalized *IM*). As shown in Fig. 8(b), the selection of E_I or *Sa* as *IM*

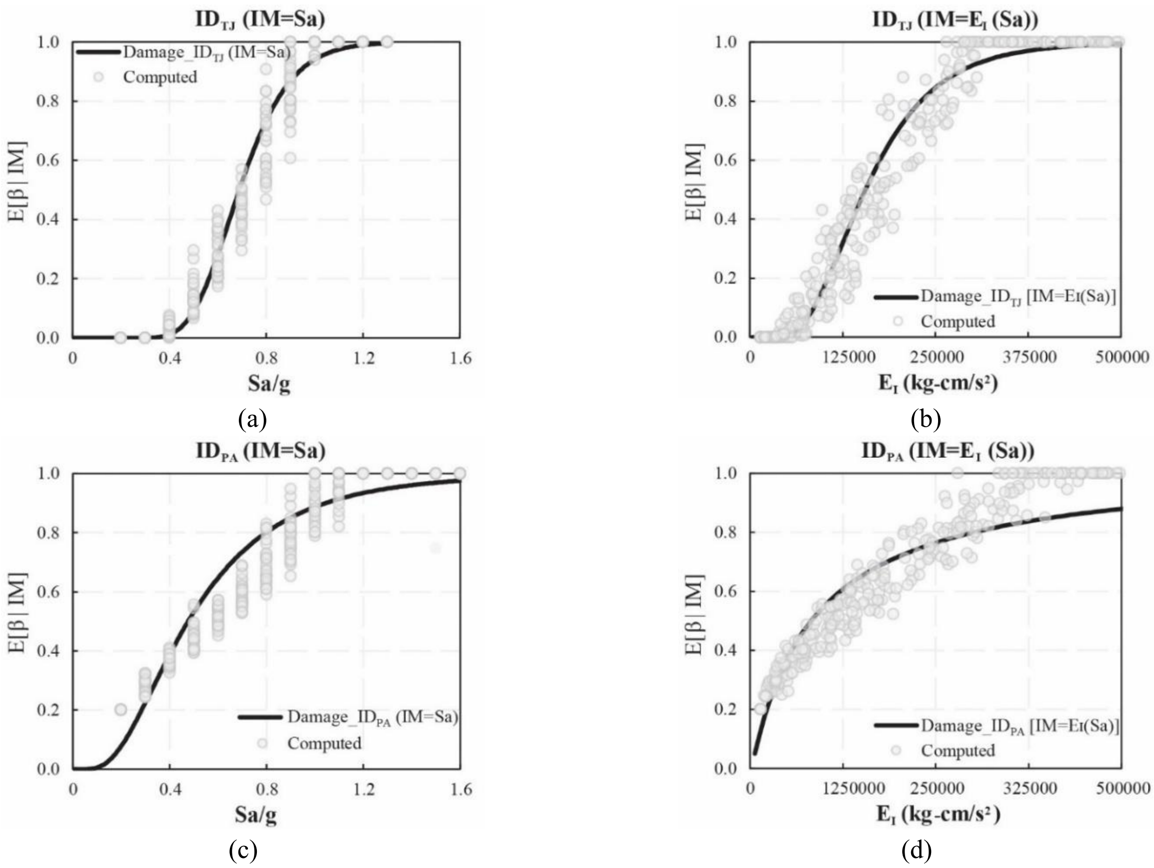


Fig. 7 Damage curves for SCT site (T_g=1.9s). (a) and (c) for the expected damage using Sa as objective IM, using (a) the damage models of Terán and Jirsa (ID_{TJ}) and (b) Park and Ang (ID_{PA}); expected damage using E_I as intensity measure, computed for the same damage models used in figures (a) and (c)

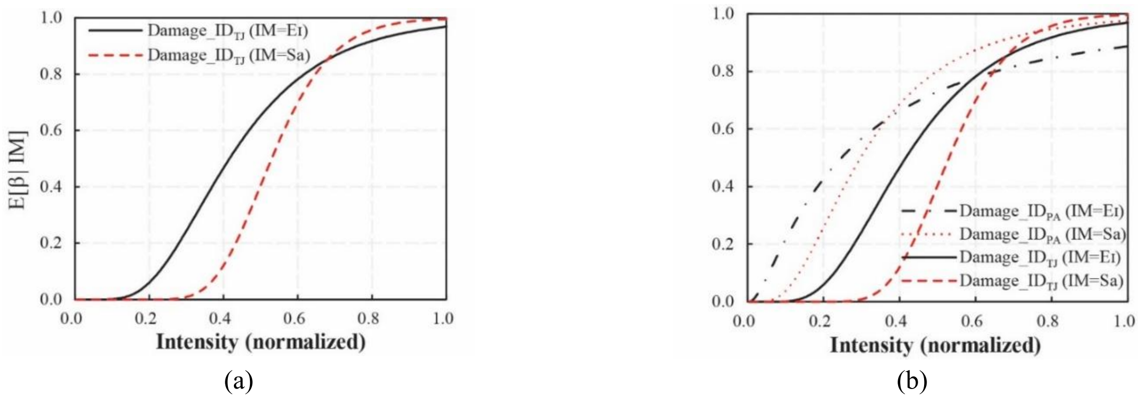


Fig. 8 Comparison of damage curves (normalized) considering different intensity measures (Sa and E_I)

have a similar effect on damage estimation with both damage indexes, in that the use of E_I results in larger damage for small and moderate intensities, and slightly smaller damage for large intensities. This figure also shows significant differences between the damage estimates established with ID_{PA} and ID_{TJ} for normalized intensities ranging from 0.1 to 0.6. For normalized intensities ranging from 0.1 to 0.6. For normalized intensities larger than 0.6, the differences become much smaller.

To select a well-suited damage index for the estimation of structural damage for structures located in soft soils, the formulation of the indexes models was analyzed. The ID_{PA} was calibrated from experimental tests that considered generic loading protocols in which most of the plastic energy is dissipated in cycles of large amplitude. ID_{TJ} was developed from three sets of seismic records which include long-duration ground motions recorded in the lakebed zone of Mexico City, for which a large percentage of plastic energy is dissipated in low amplitude cycles. Even though both damages indexes consider the use of seismic energy and ID_{PA} is widely used for damage assessment, the index ID_{TJ} is agreeable with Mexico City seismotectonic environment and local soil effects. Because of this, from here on, only the results related to ID_{TJ} will be presented.

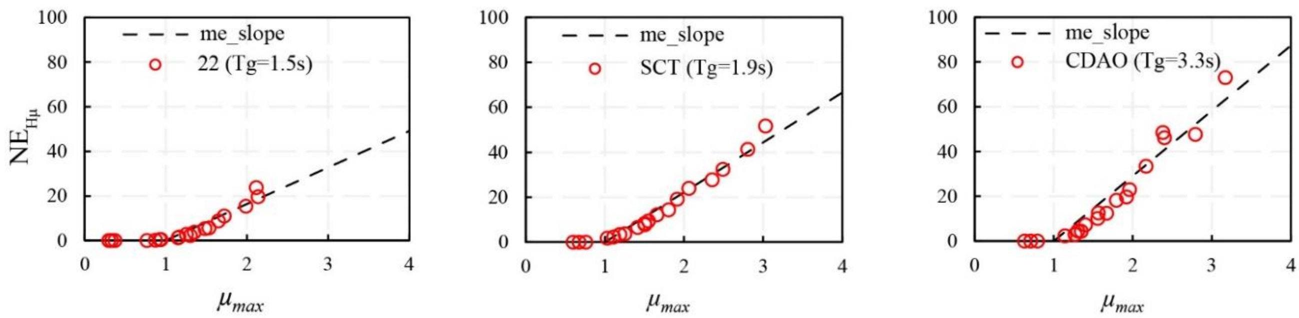


Fig. 9 μ_{max}-$NE_{H\mu}$ relationship for an elasto-perfectly-plastic model for three different sites. The lateral strength was computed for Q=3.0

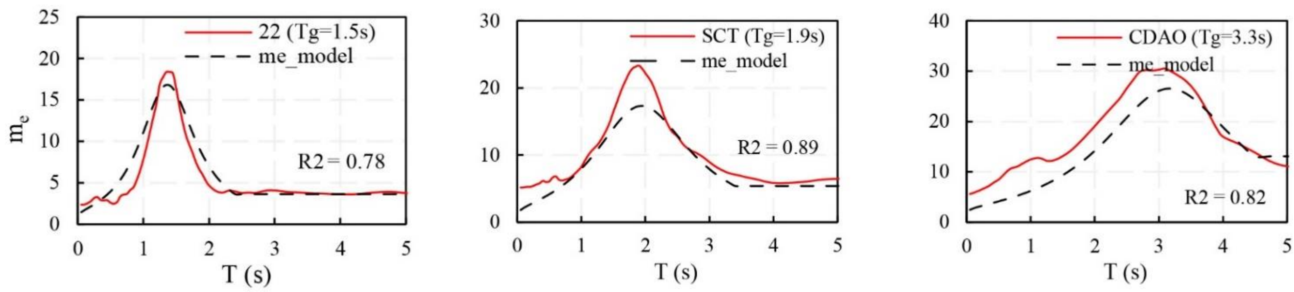


Fig. 10 Comparisons of the m_e model (dashed line) and the calculated values (continuous line) for three sites in the Mexico City, 22 (T_g=1.5s), SCT (T_g=1.9s) and CDAO (T_g=3.3s), for a lateral strength computed with Q=3.0

5. Relationship between plastic energy and maximum displacement demands

An important aspect of the methodology proposed herein, is the relationship that exists between the plastic energy, in terms of $NE_{H\mu}$, and the maximum ductility (μ_{max}) demands. As shown in Fig. 9, a straight-line can be used to numerically characterize this relationship for μ_{max}≥1; m_e denotes the slope of this straight-line. The figure shows m_e for sites 22 (T_g=1.5s), SCT (T_g=1.9) and CDAO (T_g=3.3s) for a SDOF system with T=2.0 and a lateral strength corresponding to Q=3.0. As shown, the computed responses (circles) reasonably fit a straight line (dashed line) for $\mu_{max} \geq 1$.

Note that plastic energy is not dissipated until yielding occurs ($\mu_{max} = 1$). The value of m_e depends on the period and lateral strength of the system, and the dominant period of motion (T_g)

$$m_e = \frac{\alpha\left(\frac{T}{T_g}\right)^{\beta}}{\gamma + \left(\frac{T}{T_g} - 1\right)^2} \tag{5}$$

The values of γ, α and β are obtained by regression analysis under the consideration of all the sites indicated in Fig. 1. Fig. 10 compares results obtained from the m_e model under consideration herein (dashed line) with computed values (continuous line) for sites 22 (T_g=1.5s), SCT (T_g=1.9s) and CDAO (T_g=3.3s). Likewise, the values of the coefficient R^2 are included to show the model fit.

As can be seen in Fig. 10, the m_e model reasonably fits the computed results. High values of R^2 are associated to the fit. An in-depth discussion on the m_e model and the results corresponding to all sites under consideration in Fig. 1 can be found in (Quinde 2019).

Table 1 Values for parameters of input energy model

	For soil periods (1<Tg<5.5)				
	μ=1.5	μ=2.0	μ=2.0	μ=2.0	μ=2.0
σ	0.04	0.07	0.10	0.11	0.16
α	1.28	1.17	1.12	1.08	1.11
β	0.96	0.96	0.89	0.81	0.65
γ	0.90	0.82	0.73	0.68	0.66

6. Energy-based methodology to estimate expected damage in single-degree-of-freedom systems

The proposed methodology is based on an energy approach that accounts for the energy demands for the complete (not trimmed) ground motion. The strong motion duration and the frequency and energy contents of the seismic excitation are considered indirectly through the energy spectra used as input data.

The traditional earthquake-resistant design seeks to avoid collapse and does not consider the explicit estimation of the expected level of structural damage. Safer and more transparent structural designs can be achieved by using a performance-based approach that explicitly accounts for probable damage. To make this possible, it is necessary to formulate simple methodologies for the evaluation of structural damage, capable of considering the impact of cumulative and residual deformations in the reliability and life-cycle cost of the structural system. The methodology discussed next addresses this issue through an energy

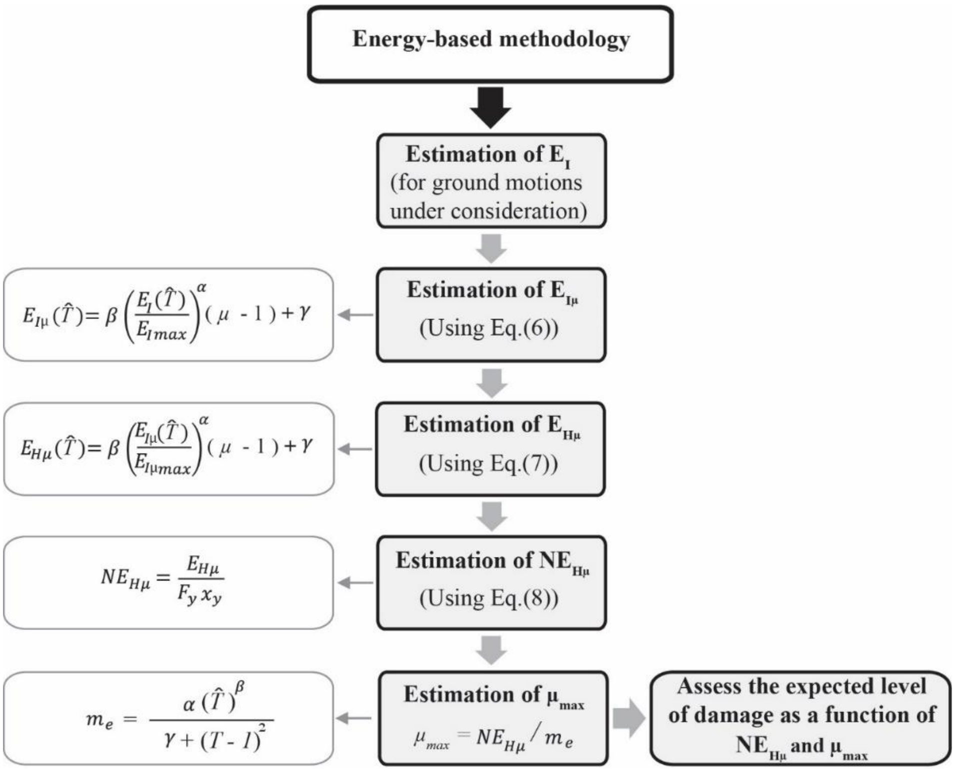


Fig. 11 Step-diagram for the energy-based methodology proposed in this article to evaluate accumulated and residual damage

approach that considers the elastic input energy (E_I) demand as the basis for damage evaluation.

First, the structural system needs to be designed according to code (herein, the Mexico City Building Code is used). This implies establishing the periods of the structural system (T) and of the ground at the site (T_g), and the lateral strength of the system according to a design spectrum corresponding to the value of maximum ductility selected for design (μ_d).

Once the structural properties of the system are known, a set of ground motions, capable of representing the seismicity at the construction site, should be used. If a database of recorded accelerograms is not available, synthetic ground motions are required. The aim is that a broad range of intensities, related to different levels of structural damage, should be covered. For each ground motion, damage assessment is carried out in six steps:

1) An elastic input energy is established for the ground motion under consideration.

2) An inelastic input energy demand is estimated, as discussed in Quinde *et al*. (2016), for $\mu = \mu_d$

$$E_{I\mu}(\hat{T}) = \beta\left(\frac{E_I(\hat{T})}{E_{I_{max}}}\right)^{\alpha}(\mu - 1) + \gamma \tag{6}$$

where $\hat{T} = T/T_g$, and the values of the parameters γ, α and β are taken from Table 1. An in-depth discussion on how these values were obtained can be found in Quinde *et al*. (2016).

3) The corresponding plastic hysteretic energy demand ($E_{H\mu}$) is estimated, as discussed in Quinde *et al*. (2016), for $\mu = \mu_d$

Table 2 Values for parameters of plastic hysteretic energy model

	For soil periods (1<*Tg*<5.5)				
	μ=1.5	μ=2.0	μ=2.0	μ=2.0	μ=2.0
σ	0.09	0.06	0.07	0.05	0.05
α	0.20	0.18	0.14	0.17	0.18
β	0.87	0.59	0.34	0.25	0.11
γ	0.00	0.00	0.00	0.00	0.00

$$E_{H\mu}(\hat{T}) = \beta\left(\frac{E_{I\mu}(\hat{T})}{E_{I\mu_{max}}}\right)^{\alpha}(\mu - 1) + \gamma \tag{7}$$

The values of the parameters γ, α and β are taken from Table 2. An in-depth discussion on how these values were obtained can be found in Quinde *et al*. (2016).

4) The normalized hysteretic energy ($NE_{H\mu}$) is estimated as

$$NE_{H\mu} = \frac{E_{H\mu}}{F_y x_y} \tag{8}$$

where F_y and x_y are the strength and displacement at first yield, respectively.

The actual maximum ductility demand is established $\mu_{max} = NE_{H\mu}/m_e$, whelore m_e is given by Ec. (5). For instance, the values of parameters α, β and γ for sites with dominant soil period between 1.8s<T_g<2.8s are

$$\alpha = (6.270 - 1.837T_g)(0.5Q) \tag{9}$$

$$\beta = 0.125\ (0.5Q) \tag{10}$$

$$\gamma = (0.405 - 0.128T_g)(0.5\ Q) \tag{11}$$

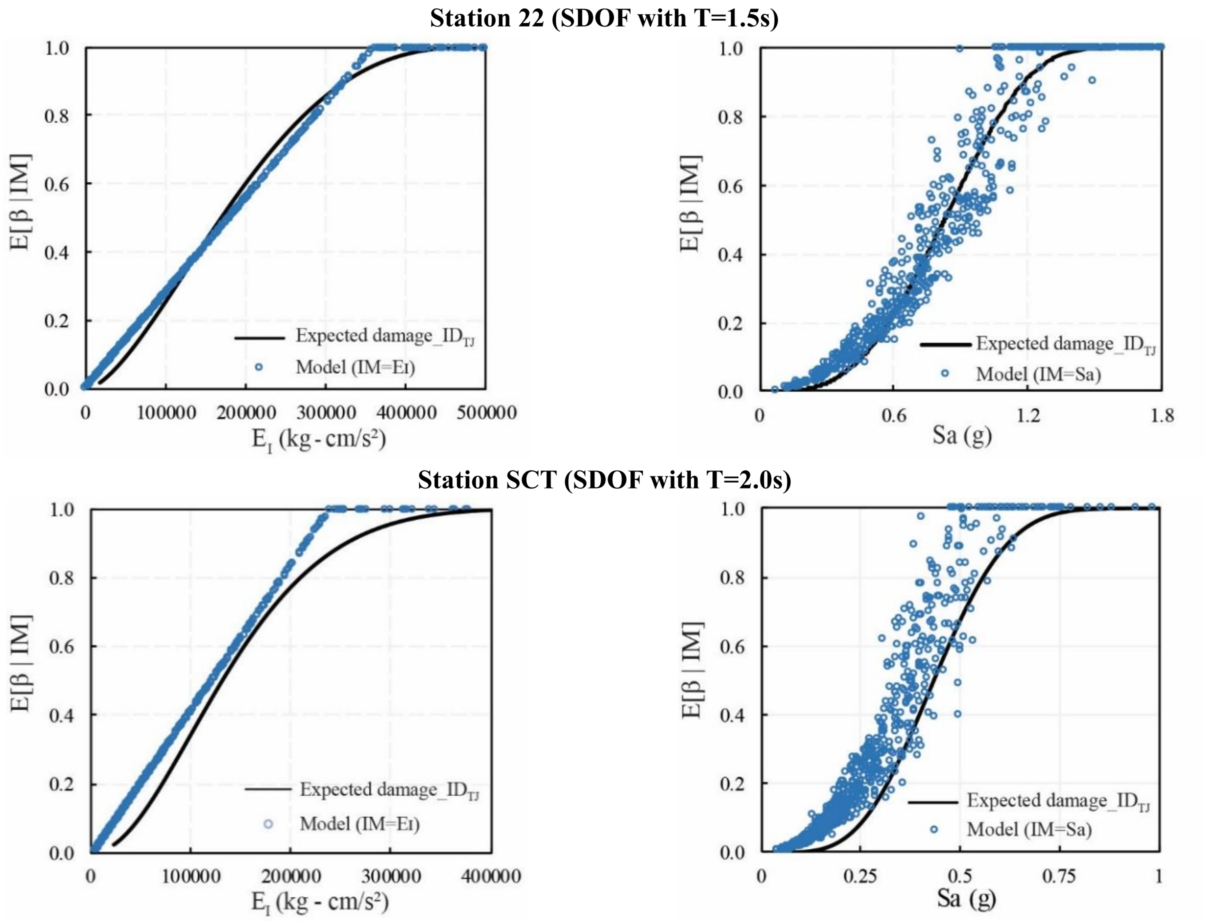


Fig. 12 Comparison of the expected damage computed with the energy-based proposed methodology (circles) and traditional non-linear methodologies (continuous line). For sites 22 (T_g=1.5s) and SCT (T_g=1.9s) for SDOF systems stated in each figure and a Q=3.0. The Terán and Jirsa damage index was used

An in-depth discussion on Eqs. (9)-(11) can be found in Quinde (2019). Note that μ_{max} is not necessarily equal to μ_d because the Sa design spectrum does not have a perfect correspondence to the actual spectrum for the ground motion under consideration.

5) Assess the expected level of damage as a function of $NE_{H\mu}$ and $\mu_{\max}$ using and adequate damage index (i.e., Park and Ang 1985 or Terán-Gilmore and Jirsa 2005).

Although it may be necessary to generate a synthetic strong ground motion database, the proposed methodology does not require hundreds of nonlinear analyzes to establish expected damage curves. Fig. 11 shows a step-diagram for the proposed energy-based methodology. All calibrations under consideration in this article consider elasto-perfectly-plastic behavior.

7. Validation of energy-based methodology

The methodology was used to establish damage curves for SDOF systems having elasto-perfectly-plastic behavior and located at different sites. Figs. 12 and 13 compare damage curves established with the proposed methodology (circles) and expected damage curves (continuous line) established from hundreds of nonlinear analyses and β functions. ID_{TJ} is used to evaluate damage since it provides, for soft soils, a more consistent assessment of structural collapse. The sites under consideration are 22 (T_g=1.5s), SCT (T_g=1.9s), CDAO (T_g=3.3s) and 11 (T_g=4.5s). The periods of the SDOF systems are indicated in each figure. Q=3.0 was used to compute the lateral strength of the systems. Figs. 12 and 13 were established under the consideration of E_I as the target IM. Note that the methodology introduced herein yields damage curves that fit reasonably well the actual expected damage curves.

Table 3 Errors between expected damage curves using the proposed methodology and traditional non-linear methodologies

Group	22	SCT	CDAO	11
σ_{ln}(0.1-1.0) =	0.36	0.17	0.27	0.2
$\sigma_{ln_{max}}$(0.1-1.0) =	1.04	0.56	0.73	0.63
σ_{ln}(0.2-0.8) =	0.18	0.09	0.24	0.14
$\sigma_{ln_{max}}$(0.2-0.8) =	0.47	0.38	0.56	0.41
σ(0.1-1.0) =	0.3	0.13	0.24	0.17
σ_{max}(0.1-1.0) =	1.83	0.74	1.08	0.92
σ(0.2-0.8) =	0.23	0.06	0.23	0.08
σ_{max}(0.2-0.8) =	0.6	0.24	0.62	0.19

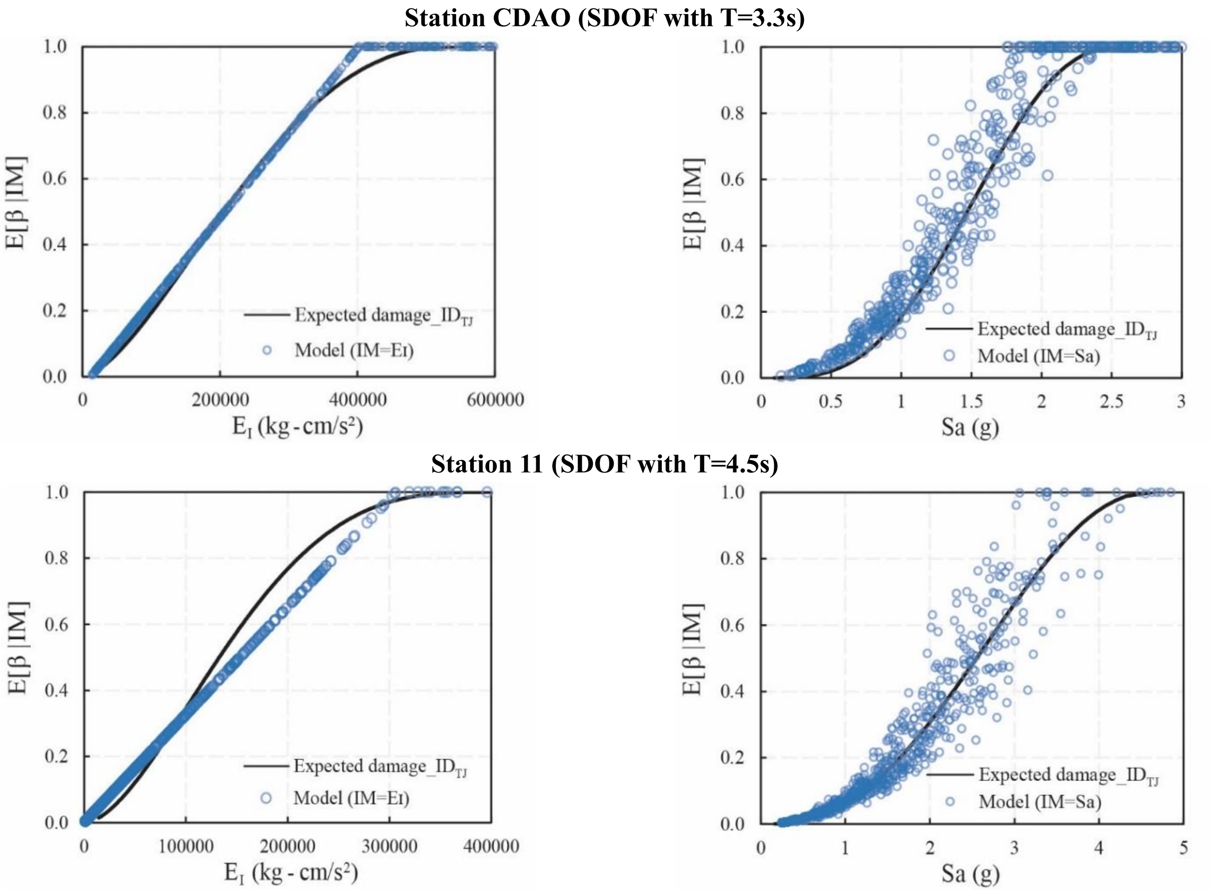


Fig. 13 Comparison of the expected damage computed with the energy-based proposed methodology (circles) and traditional non-linear methodologies (continuous line). For sites CDAO (T_g=3.3s) and 11 (T_g=4.5s) for SDOF systems stated in each figure and a Q=3.0. The Terán and Jirsa damage index was used

This conclusion is also valid for the case in which Sa is considered the IM.

Errors associated with the use of the methodology are summarized in Table 3. These errors correspond to the average logarithmic error associated with damage levels larger than 10%, σ_{ln}(0.1-1.0), the maximum logarithmic error for damage levels greater than 0.1, $\sigma_{ln_{\max}}$(0.1-1.0), the average logarithmic error in damage levels ranging from 0.2 to 0.8, σ_{ln}(0.2-0.8), and arithmetic errors for the same intervals.

The assessment of damage that results from the proposed methodology fits reasonably well the results derived from a traditional methodology. The largest error obtained is 0.39 for damage levels greater than 0.1. However, in the range of damage of most interest (0.2 to 0.8), logarithmic errors less than 0.25, on average, are estimated. Fig. 14 shows estimated and actual damage curves along with the standard deviation.

Although the differences remain similar for groups of stations 3, 4 and 5, with reasonably low values; for group 6 the deviations increase. For sites with very deep clay layers (T_g>3.8s), the soil behavior is complex and requires a particular interpretation, even higher vibration modes of the soil influence the response. However, the results obtained reasonably fit the damage estimated by dynamic nonlinear methodologies.

8. Conclusions

In this paper, a methodology to estimate expected damage in SDOF systems was presented. The strong motion duration and the frequency and energy contents of the seismic excitation are considered indirectly through the use of elastic input energy spectra as the basis of the evaluation process.

This energy-based methodology was used to estimate cumulative damage for SDOF systems located in the Mexico City lakebed, where soil periods can even exceed T_g=5s. The estimated damage fits reasonably the expected damage curves established directly with nonlinear dynamic analyses. The proposed methodology could be used to estimate the structural damage in the initial stages of earthquake-resistance design.

The traditional earthquake-resistant design seeks to avoid collapse and does not consider the explicit estimation of the expected level of structural damage. At each stage of the proposed methodology, relevant information (maximum displacement, ductility demand, dissipated plastic energy, and residual displacement) can be obtained, so that structural analysis can be made for different performance levels. This model is simple enough to apply to common structural systems.

Although the basis for the evaluation process in the

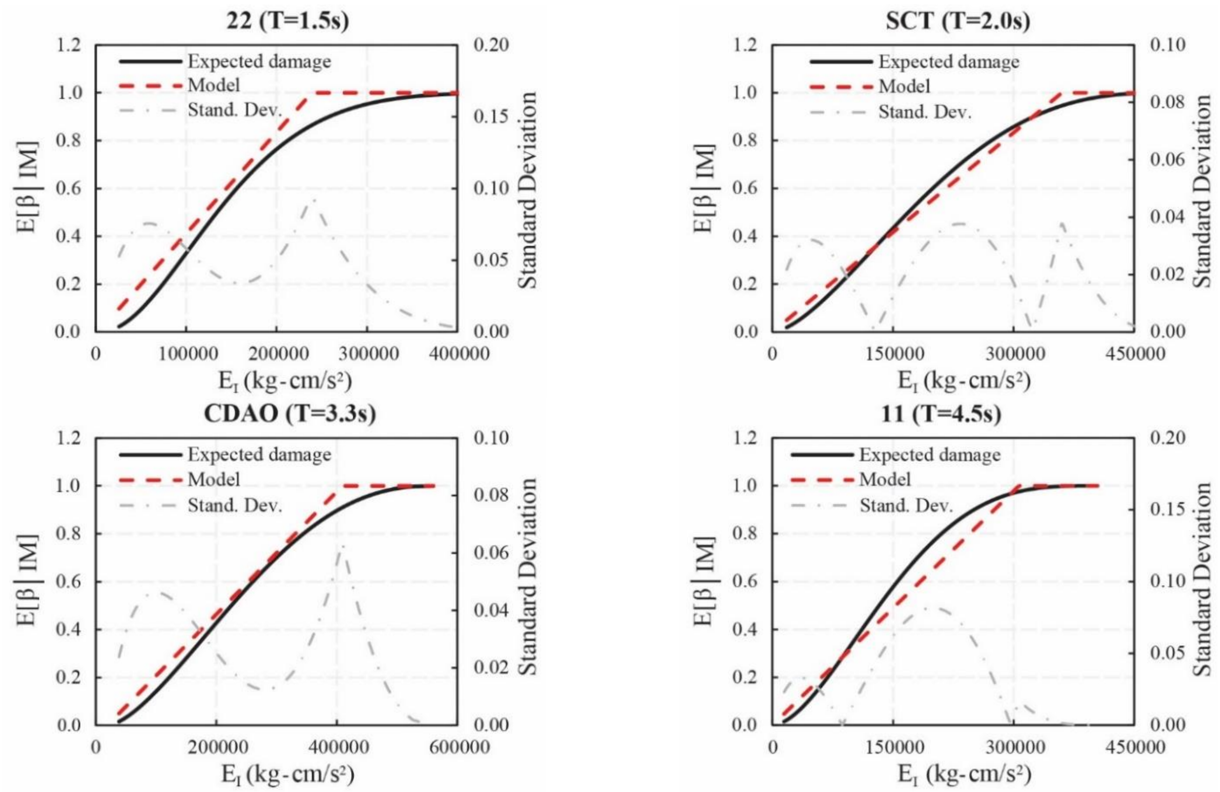


Fig. 14 Expected damage curves for different sites using both methodologies with their respective standard deviations

proposed methodology is the input energy, the results of the expected damage, using the pseudoacceleration as an intensity measure, fit reasonably well the actual expected damage curve. Although significant differences were observed when input energy and pseudoacceleration were used as intensity measure, for collapse values (expected damage larger than 0.8), the two intensity measures show similar results.

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